Physics Cases For Muon Colliders

Joint Presentation by

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Muon Colliders Physics Workshop
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The Tasks

Precision Tests of New Physics found at the LHC and Finding New Physics not accessible at the LHC

We do not (yet) know if the landscape of new physics is relatively accessible and not too hidden or if it requires finding special hidden valleys or scaling extreme (energy/luminosity) heights.

There is growing ”suspicion” that the latter picture is the most relevant.
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<td>200 GeV</td>
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<td>ATLAS-CONF-2010-021</td>
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*Only a selection of the available results shown*
Of course, there is one more possibility, that is at the same time very problematical for the LHC and a special opportunity for high energy lepton colliders:

**Strong $W^+W^-$ Interactions**

A bit amorphous, maybe some ”resonance” peaks, but otherwise a desert. Will there be funding to explore this landscape?
The Timeline

- By the end of the current LHC run, we will have seen new physics or know:

  • it is hiding in special valleys, hidden by reason of weak signals or large backgrounds (for which large $L$ might help); or

  • it is at $>1$ TeV (in many instances, such as new $Z'$ or $W'$, even higher) mass scales (for which higher LHC $\sqrt{s}$ might be needed).

If we find nothing, or just a SM-like Higgs boson, by the end of 2012, and there is a 2 year LHC shutdown, then it will be 2014 before there will be a clear guideline for the muC.
The New Physics List:

From the HEPAP report, "The Quantum Universe"

1. What is the origin of mass for fundamental particles?
2. Are there undiscovered principles of nature: new symmetries, new physical laws, ...?
3. Can we understand Dark Matter? Can we make it at colliders?
4. What is Dark Energy?
5. Are there extra spatial dimensions?
6. Do the forces unify at high energy scales?
7. Why so many kinds of particles? Is there a 4th family?
8. What are neutrino masses and mixings telling us?
9. How did the universe begin?
10. Why is the universe dominated by matter?

I will focus on 1, 2, 5 and 7.
I will focus on issues associated with high $\sqrt{s}$. If there is a light SM Higgs or light MSSM SM-like Higgs boson, the LHC will see it. If it is very SM-like and heavy supersymmetry is seen, we will presume that the $H, A$ exist but are heavy. The LHC has a problem if the $H, A$ are significantly more massive than 500 GeV. At $\sqrt{s} = 14$ GeV we had the plots below:

The status of $t\bar{t}h, h \rightarrow b\bar{b}$ is still under discussion.
The $H$ and $A$

- Increasingly degenerate as $m_A \sim m_H \to \text{large}$.

Color code: $\tan \beta = 2$ black; $\tan \beta = 5$ red; $\tan \beta = 8$ blue; $\tan \beta = 10$ green; $\tan \beta = 12$ magenta; $\tan \beta = 20$ cyan.
- Therefore, increasingly hard to observe separated peaks.

Separating $A$ from $H$. Beamstrahlung=0.01%, bremsstrahlung included. $L = .01$ fb$^{-1}$ at any given $\sqrt{s}$. Ok for $m_A = 400$ GeV; Impossible for $m_A = 900$ GeV.
Beamstrahlung = 0.1% ⇒ makes $m_A = 400$ GeV separation somewhat marginal at $\tan\beta = 5$, but still ok for $\tan\beta \in [8, 12]$.

Even at high mass, there is info in total width, $t\bar{t}$ vs. $b\bar{b}$ relative rates, total event rate, .... Tao will discuss the strategy we have developed.

- Bremstrahlung tail discovery is possible for $m_H, m_A \lesssim 0.96\sqrt{s}$ provided $R \lesssim 0.1\%$ and $b\bar{b}$ mass resolution is of order $\pm 5$ GeV.
• Pair production at high $\sqrt{s}$ is a good discovery option. Below is an illustration for $m_A \sim m_H \sim 1$ TeV. Need $\sqrt{s} \gtrsim 2.4 m_A$ and $L = 100 - 1000$ fb$^{-1}$ (detailed study needed).

![Graph showing $e^+e^-$ or $\mu^+\mu^-$ Collisions with various decay channels and their corresponding cross-sections.]  

SUGRA: $\tan\beta = 2; \mu < 0$

$m_0 = 2m_{1/2} = 0.5$ TeV

• Both of the above options would be good in the case of a general two-Higgs-doublet model.
A Fourth Generation?

• Precision electroweak, Yukawa perturbativity, .... require $m_{t'} , m_{b'} \lesssim 550$ GeV.

• LHC will soon either exclude or detect the 4th generation quarks.

• If a 4th generation exists then threshold scans of $b'b'$ and $t't'$ production will give the best mass determinations. Especially important might be the precise determination of $m_{t'} - m_{b'}$ which will give a crucial contribution to $\Delta T$ that might allow a heavier SM-like Higgs boson (as predicted in the MSSM context for a 4th generation).

• Meanwhile, if we see a light SM-like Higgs boson at the LHC with expected rates in the $gg \rightarrow h \rightarrow WW$ and $gg \rightarrow h \rightarrow \gamma \gamma$ final states, we will exclude a 4th generation based on non-decoupling loop effects. We will also exclude a sequential $W'$ with "SM-like" couplings to the light Higgs.

Defining ratios relative to SM expectations, $R_{WW}$ and $R_{\gamma \gamma}$, a 4th generation and/or sequential $W'$ will result in $R$ values substantially $> 1$. These increases derive from the loop triangle diagrams.

- The $gg \rightarrow h$ coupling counts heavy colored fermions in the loop.
- The $\gamma\gamma \rightarrow h$ coupling counts heavy charged fermions with negative sign and charged $W$ and $W'$ with larger coefficient and positive sign.

The solid black curve shows $R_{WW}$ in the presence of a 4th generation. The (a) long dash – short dash red (b) dotted blue, (c) long dash magenta curves show $R_{\gamma\gamma}$ for the cases: (a) 4th generation only, (b) sequential $W'$ only, (c) 4th generation plus sequential $W'$. All curves are for a Higgs boson with SM-like couplings and SM final decay states.
• Implications for MSSM Higgs physics are dramatic.
  1. In the context of any 2HDM(II), including the MSSM Higgs sector, strict Yukawa perturbativity requires $1/2 \lesssim \tan \beta \lesssim 2$.
  2. $m_h \sim 400-500$ GeV is predicted and a relatively high $\sqrt{s}$ muC would be appropriate for detailed studies.

![Graph](image)

$$MSSM, \ n_G=4, \ n_W=1$$

$m_h$ vs. $m_A$ for $\tan \beta = 1.5, 2$ and 3.

3. The $A$ could be the lightest MSSM Higgs and it will have unexpectedly large $BR(A \rightarrow \gamma\gamma)$ because of 4th generation loop contributions.
If a 4th generation is present, a 0.5 − 1.5 TeV lepton collider will be an ideal machine for detailed studies of the 4th generation quarks and leptons and/or the "light" $h$ of the MSSM.
No Higgs or Higgs-like states: the Strongly-Interacting Electroweak Scenario

Much of the following material is based on papers by the Muon-Quartet (Barger, Berger, Gunion, Han).

• If no Higgs boson exists with $m_H < 600$ GeV, then, naively, partial wave unitarity of $W_L W_L \rightarrow W_L W_L$ will be violated at large $s_{WW}$. The $W_L W_L \rightarrow W_L W_L$ scattering amplitude behaves as

$$A \sim \begin{cases} 
\frac{m_H^2}{v^2} & \text{if light Higgs}, \\
\frac{s_{WW}}{v^2} & \text{if no light Higgs}.
\end{cases} \quad (1)$$

Understanding the manner in which unitarity violation is avoided at high energies will be crucial.

• $W_L W_L \rightarrow W_L W_L$ scattering will be probed via
• Energy reach is a critical matter here with subprocess energies $\sqrt{s_{WW}} \gtrsim 1.5$ TeV is needed to probe strong $WW$ scattering. Since $E_\mu \sim (3-5)E_W$, this condition implies

$$\sqrt{s_{\mu\mu}} \sim (3-5)\sqrt{s_{WW}} \gtrsim 4 \text{ TeV}.$$ (2)

• The ultimate goal is to determine all the different weak isospin amplitudes, in terms of which the physical scattering amplitudes can be written as

$$\mathcal{M}(W_L^+W_L^- \to Z_LZ_L) = \frac{1}{3}[T(0) - T(2)]$$

$$\mathcal{M}(Z_LZ_L \to W_L^+W_L^-) = \frac{1}{3}[T(0) - T(2)]$$

$$\mathcal{M}(W_L^+W_L^- \to W_L^+W_L^-) = \frac{1}{6}[2T(0) + 3T(1) + T(2)]$$

$$\mathcal{M}(Z_LZ_L \to Z_LZ_L) = \frac{1}{3}[T(0) + 2T(2)]$$

$$\mathcal{M}(W_L^\pm Z_L \to W_L^\pm Z_L) = \frac{1}{2}[T(1) + T(2)]$$

$$\mathcal{M}(W_L^\pm W_L^\pm \to W_L^\pm W_L^\pm) = T(2).$$
In principle, a muC can access all but the last channel provided we can clearly separate \( W \)'s from \( Z \)'s. Given that there are only 3 independent \( T \)'s we then have 2 cross checks. The sizes of the signals in the different processes thus depend on the resonant or non-resonant interactions in the different isospin channels. Expectations in different types of models appear in the Table.

Sizes of SEWS signals in \( W_L^+W_L^- \rightarrow \) various vector boson final states: \( L \) (large), \( M \) (medium), \( S \) (small). Scalar=\( T(0) \); Vector=\( T(1) \).

<table>
<thead>
<tr>
<th>final state</th>
<th>resonant scalar ((H^0))</th>
<th>resonant vector ((\rho_{TC}))</th>
<th>non-resonant ((LET))</th>
</tr>
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<tbody>
<tr>
<td>( W_L^+W_L^- )</td>
<td>( L )</td>
<td>( L )</td>
<td>( S )</td>
</tr>
<tr>
<td>( Z_LZ_L )</td>
<td>( L )</td>
<td>( S )</td>
<td>( M )</td>
</tr>
<tr>
<td>( W_L^\pm Z )</td>
<td>( S )</td>
<td>( L )</td>
<td>( S )</td>
</tr>
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</table>

- A first (over?) estimate of SEWS effects is provided by using the Standard Model with a heavy Higgs as a prototype of the strong scattering sector.
- The SM with a light Higgs is an appropriate definition of the electroweak background since only transversely polarized \( W \)'s contribute to vector boson scattering when the Higgs has a small mass.
For a 1 TeV SM Higgs boson, the signal is thus defined as

\[ \Delta \sigma = \sigma(m_{h_{SM}} = 1 \text{ TeV}) - \sigma(m_{h_{SM}} = 10 \text{ GeV}). \]  

(3)

Results for \( \Delta \sigma \) are shown below for \( \sqrt{s} = 1.5 \text{ TeV} \) and 4 TeV. The strong scattering signal is relatively small at energies of order 1 TeV, but grows substantially as multi-TeV energies are reached. Thus, the highest energies in \( \sqrt{s} \) that can be reached at a muon collider could be critically important.

Strong electroweak scattering signals in \( W^+W^- \rightarrow W^+W^- \) and \( W^+W^- \rightarrow ZZ \) at future lepton colliders.

<table>
<thead>
<tr>
<th>( \sqrt{s} )</th>
<th>( \Delta \sigma(W^+W^-) )</th>
<th>( \Delta \sigma(ZZ) )</th>
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</thead>
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<tr>
<td>1.5 TeV</td>
<td>8 fb</td>
<td>6 fb</td>
</tr>
<tr>
<td>4 TeV</td>
<td>80 fb</td>
<td>50 fb</td>
</tr>
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With 1000 fb\(^{-1} \) per year the muC will allow comprehensive studies to be made of any SEWS signals.

- Many other models for the strongly interacting gauge sector have been constructed in addition to the SM, including:
  1. a ("Scalar") model in which there is a scalar Higgs resonance with \( M_S = 1 \text{ TeV} \) but non-SM width of \( \Gamma_S = 350 \text{ GeV} \);
2. a (“Vector”) model in which there is no scalar resonance, but rather a vector resonance with $M_V = 1 \text{ TeV}$ and $\Gamma_V = 35 \text{ GeV};$

3. a model, denoted by “LET” or “$m_{h_{SM}} = \infty$”, in which the SM Higgs is taken to have infinite mass and the partial waves simply follow the behavior predicted by the low-energy theorems;

4. a model (denoted by “LET-K”) in which the LET behavior is unitarized via $K$-matrix techniques.

- Total numbers of $W^+W^-, ZZ \to 4$-jet signal $S$ and background $B$ events calculated for a 4 TeV muC collider with integrated luminosity 200 fb$^{-1}$. Events are summed over the mass range $0.5 < M_{WW} < 1.5 \text{ TeV}$ except for the $W^+W^-$ channel with a narrow vector resonance for which $0.9 < M_{WW} < 1.1 \text{ TeV}$. The statistical significance $S/\sqrt{B}$ is also given. The hadronic branching fractions of $WW$ decays and the $W^\pm/Z$ identification/misidentification are included.

<table>
<thead>
<tr>
<th>channels</th>
<th>$m_{h_{SM}}$</th>
<th>$M_S$</th>
<th>$M_V$</th>
<th>$m_{h_{SM}}$</th>
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<td>$S(\mu^+\mu^- \to \bar{\nu}\nu W^+W^-)$</td>
<td>1900</td>
<td>1400</td>
<td>370</td>
<td>230</td>
</tr>
<tr>
<td>$B$(backgrounds)</td>
<td>1100</td>
<td>1100</td>
<td>110</td>
<td>1100</td>
</tr>
<tr>
<td>$S/\sqrt{B}$</td>
<td>57</td>
<td>42</td>
<td>35</td>
<td>6.9</td>
</tr>
<tr>
<td>$S(\mu^+\mu^- \to \bar{\nu}\nu ZZ)$</td>
<td>970</td>
<td>700</td>
<td>220</td>
<td>350</td>
</tr>
<tr>
<td>$B$(backgrounds)</td>
<td>160</td>
<td>160</td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>$S/\sqrt{B}$</td>
<td>77</td>
<td>55</td>
<td>17</td>
<td>28</td>
</tr>
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1. A broad Higgs-like scalar will enhance both $W^+W^-$ and $ZZ$ channels with $\sigma(W^+W^-) > \sigma(ZZ)$;

2. a $\rho$-like vector resonance will manifest itself through $W^+W^-$ but not $ZZ$;

3. while the $m_{h_{SM}} = \infty$ (LET) amplitude will enhance $ZZ$ more than $W^+W^-$.

4. The $m_{h_{SM}} = \infty$ signal for $W^+W^-$ is visible, although still far from robust; the ratio $S/B$ can be enhanced by making a higher mass cut (e.g. $M_{WW} > 0.7$ TeV), but the significance $S/\sqrt{B}$ is not improved.
Histograms for the signals and backgrounds in strong vector boson scattering in the (a) $W^+W^-$ and (b) $ZZ$ final states. The background is given by the strictly electroweak $m_{h_{SM}} = 0$ limit of the Standard Model. The three signals shown are (I) a vector resonance with $M_V = 1$ TeV, $\Gamma_V = 35$ GeV, (II) the SM Higgs with $m_{h_{SM}} = 1$ TeV, and (III) the SM with $m_{h_{SM}} = \infty$ (LET model). In the figure the shorthand notation $h$ is used for $h_{SM}$. 
The complementarity of the $W^+W^-$ and $ZZ$ final state modes is clear from the figure.

- We note that event numbers are such that not only could a substantial overall signal be observed, but also at high $L$ the shape of the excess in the distribution in vector boson pair mass could be measured over a broad interval in the 1 TeV range.
  - For instance, from the top plot in the figure, in the case of $m_{h_{SM}} = \infty$, a 100 GeV interval from 1.4 TeV to 1.5 TeV would contain $L \times 100 \text{ GeV} \times (4 \times 10^{-3} \text{ fb/ GeV}) = 400$ signal events for $L = 1000 \text{ fb}^{-1}$, thereby allowing a 5% measurement of the $m_{W^+W^-}$ signal distribution in this bin.
  - The level of accuracy in this one bin alone would distinguish this model from the Vector or $m_{h_{SM}} = 1$ TeV models.
  - The difference between the three different distributions plotted could be tracked in both channels.
  - The ability to measure the distributions with reasonable precision would allow detailed insight into the dynamics of the strongly interacting electroweak sector when the collider achieves energies substantially above 1 TeV.
Events vs. $M_{VV}$ for two SEWS models (including the combined backgrounds) and for the combined backgrounds alone in the ZZ final states. Signals shown are: (i) the SM Higgs with $m_{h_{SM}} = 1$ TeV, $\Gamma_H = 0.5$ TeV; (ii) the Scalar model with $M_S = 1$ TeV, $\Gamma_S = 0.35$ TeV. Results are for $L = 1000$ fb$^{-1}$ and $\sqrt{s} = 4$ TeV. Sample error bars are shown at $m_{VV} = 1.02, 1.42, 1.82, 2.22$ and $2.62$ TeV for the illustrated 80 GeV bins. Results are for $L = 1000$ fb$^{-1}$ and $\sqrt{s} = 4$ TeV.
• Even an amorphous LET type of SEWS scenario gives very decent signatures if $\sqrt{s}$ is in the $\gtrsim 3$ TeV range.

• Angular distributions of the jets in the $WW \rightarrow 4$jet final state will provide a powerful discrimination of SEWS from the light Higgs theory, as illustrated in the figure below. Here $\theta^*$ is the angle of the $q - \bar{q}$ from $W$-decays in the two $W$-rest frames, relative to the $W$-boost direction in the $WW$ c.m., averaged over all configurations.
• Thus, if some signals for a strongly interacting sector emerge at the LHC, a $\sqrt{s} = 3 - 4$ TeV $\mu C$ collider will be essential.
• The $W_L^+ W_L^- \rightarrow \bar{t}t$ channel is another valuable domain for SEWS studies, since $W_L^+ W_L^- \rightarrow \bar{t}t$ also violates unitarity at high energies. The figure below illustrates expected cross sections.

Cross section vs. $\sqrt{s}$ for $\mu^+ \mu^- \rightarrow \nu \bar{\nu} tt, \mu^+ \mu^- t \bar{t}$ for Higgs masses $m_H = 0.1$ TeV, 1 TeV, and $\infty$. 
A note on Higgsless Models (Terning, Csaki, et al.)

In these models, $W_LW_L$ unitarity is cured by $W'$ and $Z'$ extra-dimensional recurrences.

But, a successful cure requires relatively low $Z'$, $W'$ masses. As limits on $M_{Z'}$ start to exceed 1 TeV the Higgsless solution becomes very problematical.
First, let me paraphrase Langacker’s Physics Report. \( Z' \) includes new resonances associated with a gauge symmetry, KK excitations, etc.

- A new \( U(1)' \) gauge symmetry is one of the best motivated extensions of the standard model.

- For example, \( U(1)' \)s occur frequently in superstring constructions.

- If there is supersymmetry at the TeV scale, then both the electroweak and \( Z' \) scales are usually set by the scale of soft supersymmetry, so it is natural to expect \( M_{Z'} \) in the TeV range.

- TeV-scale \( U(1)' \)s (or Kaluza-Klein excitations of the photon and \( Z \)) frequently occur in models of dynamical symmetry breaking, Little Higgs models, and models with TeV\(^{-1}\)-scale extra dimensions.
• Other constructions, such as non-supersymmetric grand unified theories larger than $SU(5)$, also lead to extra $U(1)'$'s, but in these cases there is no particular reason to expect breaking at the TeV scale (and breaking below the GUT may lead to rapid proton decay).

The observation of a $Z'$ would have consequences far beyond just the existence of a new gauge boson.

• Anomaly cancellation would imply the existence of new fermions.
  These could just be right-handed neutrinos, but usually there are additional particles with exotic electroweak quantum numbers.

• There must also be at least one new SM scalar whose VEV breaks the $U(1)'$ symmetry.
  This scalar could be a singlet under the SM gauge groups, but needn’t be.
This scalar could mix with the Higgs doublet(s) and significantly alter the collider phenomenology.

**Corollary:** a) we better hope we have seen some regular Higgs boson responsible for the $Z$ mass and b) we must be prepared to search for the Higgs boson(s) associated with generating mass for the $Z'$.  

- The $Z'$ couplings could be family nonuniversal, allowing new tree-level contributions, e.g., to $t$, $b$, and $\tau$ decays.  

- In the supersymmetric case the $U(1)'$ could solve the $\mu$ problem by replacing $\mu$ by a dynamical variable linked to the $U(1)'$ breaking, and the allowed MSSM parameter range would be extended.  

- The singlets and exotics would be parts of chiral supermultiplets, and there would be extended neutralino sectors associated with the new
singlino and gaugino, modifying the collider physics and cold dark matter possibilities.

- The $U(1)'$ symmetry would also constrain the possibilities for neutrino mass and might be related to proton stability and $R$-parity conservation.

- A $Z'$ might also couple to a hidden sector and could play a role in supersymmetry breaking or mediation.

- Finally, a dynamical $\mu$ would allow a strong first order electroweak phase transition and new sources of CP violation in the Higgs sector, making electroweak baryogenesis more likely than in the SM or the MSSM, with the ingredients observable in the laboratory.

There are large classes of $Z'$ models, distinguished by the chiral charges of the quarks, leptons, and Higgs fields, as well as the Higgs and exotic spectrum, gauge coupling, $Z'$ mass, and possible mass and kinetic mixing.
In string constructions, for example, $U(1)'$'s that do not descend through $SO(10)$ or left-right symmetry can have seemingly random charges.

There is no simple classification or parametrization that takes into account all of the possibilities.

One (model independent) approach, valid for family universality, is to take a conventional value for the new gauge coupling, and regard the charges of the left-handed quarks ($Q_L$), leptons ($L_L$), and antiparticles $u^c_L, d^c_L,$ and $e^+_L$, as well as $M_{Z'},$ $\Gamma_{Z'}$ and the mixing angle $\theta$ as free parameters relevant to experimental searches.

However, 8 parameters are too many for most purposes, so one must resort to specific models or lower-dimensional parametrizations to illustrate the possibilities.

Within the context of supersymmetry, the observation of a $Z'$ could completely alter the paradigm of having just the MSSM at the TeV scale, with a desert up to a scale of grand unification or heavy Majorana neutrino
masses, and would suggest a whole range of new laboratory and cosmological consequences.

In the nonsupersymmetric case, a $Z'$ might be one of the first experimental manifestations of a new TeV scale sector of physics.

In short, if there is a $Z'$ or similar at the TeV scale we better be sure that any future collider can study it in detail.

How to find and study a $Z'$.

- The LHC will detect a $Z'$ if $M_{Z'} < 4 - 6$ TeV.

  This implies that any $Z'$ accessible to a muC will have been discovered.

- Off-resonance production of a $Z'$ resonance ($\sqrt{s} > M_{Z'}$) via the bremsstrahlung/beamstrahlung luminosity tail is significant. But, it seems likely that we would run the muC on or close to the resonance peak.
An energy scan through $Z'$ mass would certainly give large signals and various associated measurements would allow us to fully characterize its properties, just as we did for the $Z$.

Advantages and disadvantages of a muC for $Z'$ study.

- The disadvantage of low polarization for the muon beams at a muC could inhibit our ability to separate left and right handed couplings to the muon and to the quarks and leptons and other objects to which it decays.

  Study is needed (Raja).

- Scan energy resolution could be useful.

1. Consider a RS-style recurrence of the $Z$. Such a $Z'$ will interfere at amplitude level with the normal $Z$. All kinds of interesting patterns can emerge depending upon the model. A plot from Rizzo (hep-ph/0001140) is below.
Cross sections for $\mu^+\mu^- \rightarrow e^+e^-, b\bar{b}, c\bar{c}$ as functions of energy in both the "conventional" scenario and an Arkani-Hamed/Schmaltz (AS) scenario for which the quarks have opposite sign couplings. The red curve applies for the $\mu^+\mu^-$ final state in either model whereas the green (blue) and cyan (magenta) curves label the $b\bar{b}$ and $c\bar{c}$ final states for the "conventional" (AS) scenario.

To discriminate between curves for the two models would require excellent energy resolution as one scans in $\sqrt{s}$. 
A measurement of $A_{FB}$ would allow more straightforward discrimination, but to what extent this would be possible at a muC is uncertain. So, this is an example where an $e^+e^-$ collider might be superior.
2. A Randall-Sundrum type scenario where there are excitations of both the gauge bosons and the graviton. Typically, a muon collider would see both at the same time. A plot from hep-ph/0006041 (Rizzo + Hewett) is below:

![Graph showing excitations](image)

The $Z$ excitations are shown by the black curve. The different colored curves correspond to gravition excitations for different $k/\mathit{M}_{\text{Planck}}$ values.

To measure the small $k/\mathit{M}_{\text{Planck}}$ excitation curves would require excellent energy resolution.
One can also learn something from angular distributions.

Angular distributions for KK graviton recurrences (ignore hadron collider error bars) vs. other spins: dashes=spin 0; dots=various spin 1 cases.
Note: The above plots are for values of a certain parameter in the model $\nu > -0.5$.

For more negative values of $\nu$, couplings of both the gauge excitations and the graviton excitations to normal fermions becomes very small.

Couplings of gauge fields to KK excitations are not exponentially suppressed, but only very small!

Limits on the masses of the gauge and graviton KK excitations will go away, but so will the signals!

This situation can arise in other $Z'$ models as well.

- Another possibility is the following: in arXiv:1002.1754 (Wise+Perez) they construct a model in which the $Z'$ couples only to $U(1)_L$ ($L =$ gauged lepton number).

In this model, hadron colliders will not detect the $Z'$ but a lepton collider would.

Such a model would be truly perverse!