Theory Overview of Higgs
Subtitle: Why B Factories Must Search for $\Upsilon \to \gamma + Higgs$.

Jack Gunion
U.C. Davis

Higgs and Exotics Workshop, SLAC, October 29, 2007

Motivations and Theoretical Background

1. **Models with extra Higgs fields abound.**

   - **String theory reductions that yield the SM quarks and gauge bosons almost always have one or more Higgs fields in addition to the one doublet required for generating mass.**
     Especially common are Higgs fields that are singlets under $SU(3) \times SU(2) \times U(1)$.
     - **There is a kind of bound that says the natural number of 'axionic'-like singlet fields, N, is of order $N \sim M^2_{\rm Planck}/M^2_{\rm String}$, which for many cases (basically all models so far other than the $G2$ based models) is a fairly large number.**
     - **The idea is that each axionic like field corresponds to a wrapping around a cycle of area $M^2_{\rm String}$ (in field moduli space) and that there are roughly $M^2_{\rm Planck}/M^2_{\rm String}$ such cycles when the the overall size available for cycles is of order $M_{\rm Planck}$.**
     - **If N is larger than this bound then $M^2_{\rm Planck}$ itself receives large corrections from axionic loops that 'restores' the bound.**

– Often, the mass of the Higgs bosons contained within these fields is small because the mass is zero in some symmetry limit of the theory and the symmetry is softly broken.

- Supersymmetric models without singlets have many problems.
  All the problems are readily cured if one adds one or more SM gauge group singlet fields.
  As an example, the considerable advantages of the NMSSM vs. the MSSM will be discussed later.
- In the absence of low-energy supersymmetry, multiple Higgs fields can delay the naturalness / hierarchy problem.
  This is not so well known and so I will explain briefly.
  – The dominant quadratic divergence arises from a virtual top quark loop,

$$\delta m^2_{h_{\rm SM}} = -\frac{3}{4\pi^2}\frac{m_t^2}{v^2}\Lambda_t^2 \,, \tag{1}$$

where Λ_t is the high energy cutoff and $v = 176~\mathrm{GeV}$.
This creates the hierarchy/fine-tuning issue in that the SM Higgs mass is very sensitive to the cutoff Λ_t. A formal definition of fine tuning

with respect to Λ_t is (for numerics, we take $m_t \sim v \sim 174~\mathrm{GeV}$)

$$F_t(m_{h_{\rm SM}}) = \left| \frac{\partial \delta m^2_{h_{\rm SM}}}{\partial \Lambda_t^2} \frac{\Lambda_t^2}{m_h^2} \right| = \frac{3}{4\pi^2} \frac{\Lambda_t^2}{m^2_{h_{\rm SM}}} \equiv K \frac{\Lambda_t^2}{m^2_{h_{\rm SM}}} \,. \qquad (2)$$

Too large a value of F_t at a given Λ_t implies that you must look for new physics at or below the scale

$$\Lambda_t \lesssim \frac{2\pi v}{\sqrt{3} m_t} m_{h_{\rm SM}} F_t^{1/2} \sim 400~\text{GeV} \left(\frac{m_{h_{\rm SM}}}{115~\text{GeV}} \right) F_t^{1/2} , \qquad (3)$$

$F_t > 10$ is deemed problematical, implying (for the precision electroweak preferred SM $m_{h_{\rm SM}} \sim 100~\mathrm{GeV}$ mass) new physics somewhat below $1~\mathrm{TeV}$, in principle well within LHC reach.

– Adding Higgs fields can raise Λ_t, thereby postponing the need for truly new physics.

  One possible scenario is the addition of many singlets. (J. R. Espinosa and J. F. Gunion, Phys. Rev. Lett. 82, 1084 (1999) [arXiv:hep-ph/9807275].)

  We imagine that the singlets mix with the $h_{\rm SM}$ so that the resulting eigenstates, h_i share all the WW, ZZ, $f\overline{f}$ couplings according to their overlap fraction f_i: $h_{\rm SM} = f_i h_i + \ldots$, where $\sum_i f_i^2 = 1$ is required.

Precision electroweak constraints require $m_{EW} \sim 70 - 80$ GeV (central) or $m_{EW} < 170$ GeV (95% cL), where

$$\log[m_{EW}] = \sum_i f_i^2 \log m_i\,, \quad \text{or, equivalently} \quad m_{EW} = \prod_i m_i^{f_i^2}\,. \tag{4}$$

An appropriate m_{EW} can be maintained in many ways. For example, we can have a mixture of small and large m_i with roughly equal f_i.

LEP bounds do not necessarily apply even for small m_i — overlapping signals and Higgs to Higgs-pair decays (see later discussion) can obscure the standard signals.

Meanwhile, each h_i has its top quark loop mass correction scaled by f_i^2 and thus

$$F_t^i = f_i^2 F_t(m_i) = K f_i^2 \frac{\Lambda_t^2}{m_i^2} \tag{5}$$

i.e. significantly reduced.

Thus, multiple mixed Higgs allow a much larger Λ_t for a given maximum acceptable common F_t^i.

Also, large Λ_t implies significant corrections to low-E phenomenology from Λ_t-scale physics is less likely.

A model with one doublet plus 4 singlets can allow $\Lambda_t \sim 5$ TeV before the hierarchy problem becomes significant.

– At the LHC, such Higgs would be very hard to make and detect. The ILC would find the "Higgs continuum" with enough luminosity. Prior to that, a B factory might find the lightest of the h_i and a_j if sufficiently luminosity is accumulated to be sensitive to small f_i.

2. In the simplest cases, the Higgs fields are doublets or singlets (so as to maintain $\rho \sim 1$ as a prediction rather than input to the theory).

   But, one cannot exclude triplets and so forth.

   If a singlet or triplet is to couple to b's it must mix with a doublet Higgs.

   - In the triplet case, this requires a non-zero vev for the neutral field of the triplet which means $\rho \sim 1$ is no longer a prediction of the theory: ρ is renormalized and $\rho \simeq 1$ must be input.

   In any case, such mixing may or may not be present — often a symmetry can be imposed that prevents such mixing, but such symmetries are typically broken at some level.

3. Multi-Higgs models will usually have tri-linear Higgs self couplings among the different Higgs bosons. If phase space allowed, Higgs to Higgs-pair decays can easily dominate the decay of any Higgs with $mass < 2m_W$.

If we ignore phase space suppression, assume SM-like $h_2 b\bar{b}$ and $h_2 WW$ couplings, and write $g_{h_2 h_1 h_1} = c\frac{g m_{h_2}^2}{2 m_W}$, then one finds

$$\Gamma(h_2 \to h_1 h_1) = c^2 \frac{g^2 m_{h_2}^3}{128\pi m_W^2} \sim 0.17 c^2 \text{ GeV} \left(\frac{m_{h_2}}{100 \text{ GeV}}\right)^3 \quad vs. \tag{6}$$

$$\Gamma(h_2 \to b\bar{b}) \sim 0.003 \text{ GeV} \left(\frac{m_{h_2}}{100 \text{ GeV}}\right) \quad and \tag{7}$$

$$\Gamma(h_2 \to ZZ) = \frac{1}{2}\Gamma(h_2 \to WW) = \frac{g^2 m_{h_2}^3}{128\pi m_W^2} . \tag{8}$$

For $m_{h_2} = 100$ GeV:

$c \sim 0.13$ **implies** $\Gamma(h_2 \to h_1 h_1) = \Gamma(h_2 \to b\bar{b})$**;**

$c \sim 0.42$ **implies** $\Gamma(h_2 \to h_1 h_1) = 10 \times \Gamma(h_2 \to b\bar{b})$**.**

Largish c's are common in models. Thus, Higgs pair modes are likely to dominate until we pass above the WW threshold. (J. R. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D 39, 844 (1989), followed by J. F. Gunion, H. E. Haber and T. Moroi, Snowmass 1996, [arXiv:hep-ph/9610337], and B. A. Dobrescu and K. T. Matchev, JHEP 0009, 031 (2000) [arXiv:hep-ph/0008192] and B. A. Dobrescu, G. Landsberg and K. T. Matchev, Phys. Rev. D 63, 075003 (2001) [arXiv:hep-ph/0005308]. Now many more are rediscovering.)

Why is this important for this talk? A particularly attractive setup is to have a roughly $100~\mathrm{GeV}$ h_2 with SM-strength couplings that decays to two light Higgs bosons with $m_{h_1} < 2m_b$.

- **Perfect for precision electroweak.**
- **A choice for c of modest size gives $B(h_2 \to b\bar{b}) \sim 0.1$, which fits perfectly the 2.3σ peak at $M_{b\bar{b}} \sim 100~\mathrm{GeV}$ in $Z + b\bar{b}$ events at LEP.**

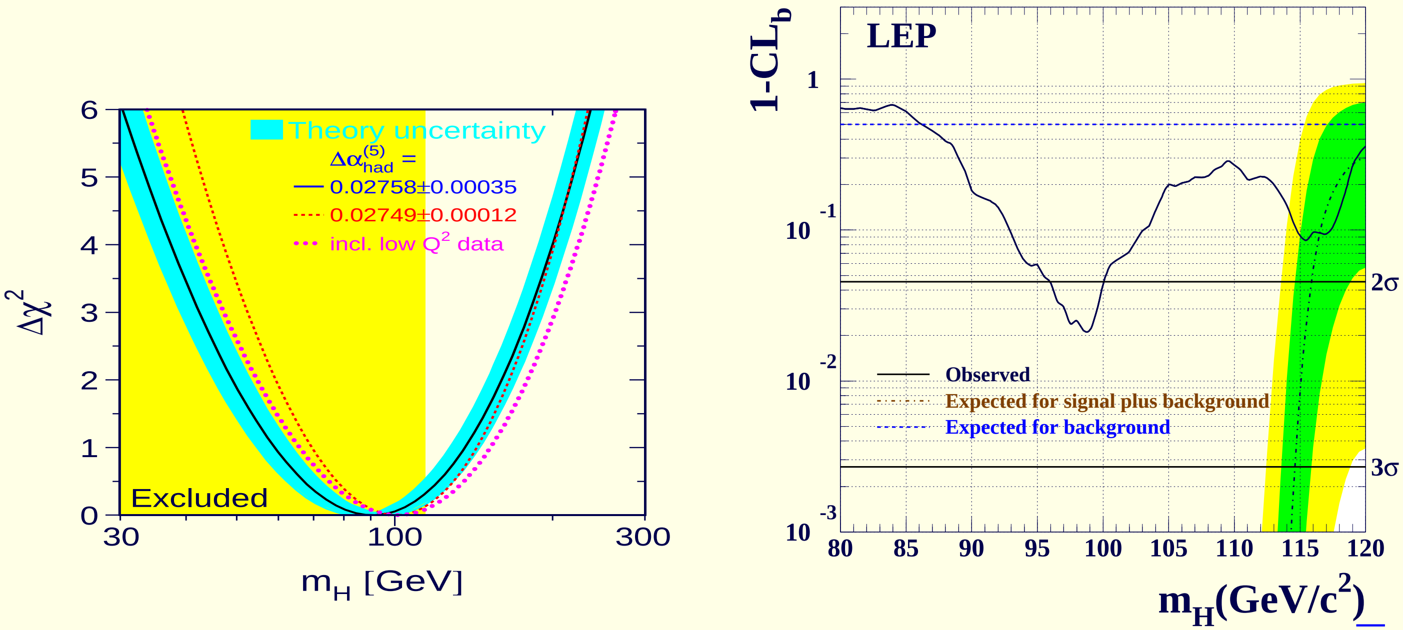

Figure 1: Preferred SM Higgs mass and $1 - CL_b$ for the $Zb\bar{b}$ final state.

- **The main $h_2 \to h_1 h_1$ decay channel yields 4τ or $4j$ final states, for which LEP limits only require $m_{h_2} > 87~\mathrm{GeV}$ and $82~\mathrm{GeV}$, respectively.**
- **If the h_1 is pure singlet, this does not work since LEP limits on the $h_2 \to h_1 h_1 \to invisible$ decay give $m_{h_2} > 114~\mathrm{GeV}$.**

An h_1 with some, but not necessarily large, non-singlet component will have fermionic couplings, including $b\bar{b}$ and $\tau^+\tau^-$.

- **If $m_{h_1} > 2m_b$, then LEP limits on $Z+4b$ enter and give $m_{h_2} > 110\,\mathrm{GeV}$, and we lose the explanation of the 2.3σ peak and have somewhat less perfect agreement with precision electroweak constraints.**

Net result: Look for h_1 with $m_{h_1} < 2m_b$ with coupling to $b\bar{b}$ and decays to $\tau^+\tau^-$ or $2j$ or in an extreme to $2e, 2\mu$. $\Rightarrow$ a B factory search.

Some non-SUSY sample models

1. **The two doublet model in special limits.** For this discussion,

   - Φ_1 and Φ_2 are the two doublet Higgs fields — the most general Higgs potential is

$$\begin{aligned} V &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + h.c.] \\ &+\frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+ \left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + [\lambda_6(\Phi_1^\dagger\Phi_1 + \lambda_7(\Phi_2^\dagger\Phi_2)]\Phi_1^\dagger\Phi_2 + h.c.\right\} \end{aligned} \tag{9}$$

   - H^0 and h^0 are the heavy and light CP-even Higgs bosons, respectively;
   - A^0 is the CP-odd Higgs boson

   It is conventional to think of the usual decoupling limit of $m_{A^0} \to \infty$ with h^0 becoming very SM-like. Choosing m_{h^0} small enough (but $> 114~\mathrm{GeV}$) gives good agreement with precision electroweak data.

   However, there are some alternative possibilities.

(a) (P. H. Chankowski, T. Farris, B. Grzadkowski, J. F. Gunion, J. Kalinowski and M. Krawczyk, Phys. Lett. B 496, 195 (2000) [arXiv:hep-ph/0009271]. See also J. F. Gunion and H. E. Haber, Phys. Rev. D 67, 075019 (2003) [arXiv:hep-ph/0207010].)

$$V_{quartic}(\Phi_1,\Phi_2) = \frac{1}{2}\lambda_1 \left|\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2\right|^2 + \frac{1}{2}\lambda_5 \left|\Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1\right|^2 , \quad (10)$$

with $\lambda_5 > \lambda_1$. This gives

$$m_{H^\pm}^2 = m_{H^0}^2 = \lambda_5 v^2 > m_{h^0}^2 = \lambda_1 v^2 > m_{A^0}^2 \quad (11)$$

with $> \rightarrow \gg$ being the case of interest. The h^0 is exactly SM-like, but m_{A^0} is arbitrarily small and $m_{H^\pm} \sim m_{H^0}$ is arbitrarily large.
$h^0 \to A^0 A^0$ decays are characterized by $c = 1$, so $m_{h^0} \sim 100~\mathrm{GeV}$ would have escaped LEP for $m_{A^0} < 2m_b$.

(b) (J. F. Gunion and H. E. Haber, Phys. Rev. D 67, 075019 (2003) [arXiv:hep-ph/0207010].)
There is another potential form with $\lambda_3 > -2\lambda_5 > 0$ for which

$$m_{H^0}^2 = (\lambda_3 + \lambda_5)v^2 > m_{h^0}^2 = m_{A^0}^2 = m_{H^\pm}^2 = -\lambda_5 v^2 , \quad (12)$$

where the H^0 is the SM-like Higgs and all the other Higgs bosons are lighter.

$B(H^0 \to H^+H^-, h^0h^0, A^0A^0)$ would all be substantial provided that $\lambda_3 \gg -2\lambda_5$ so that $m_{H^0} \gg m_{h^0} = m_{H^\pm} = m_{A^0}$.

If $m_{h^0} = m_{H^\pm} = m_{A^0} < 2m_b$, an H^0 with $m_{H^0} \sim 100~\mathrm{GeV}$ would have escaped LEP.

The small λ_5 limit of the potential is particularly simple since in this limit $\lambda_1 \simeq \lambda_2 \simeq \lambda_3$:

$$V_{quartic}(\Phi_1, \Phi_2) = \frac{1}{2}\lambda_3 \left| \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 \right|^2 . \quad (13)$$

(c) In another approach (R. Barbieri and L. J. Hall, arXiv:hep-ph/0510243), the m_{EW} game was pursued in the two doublet model.

The hierarchy problem is reduced as described earlier, with H^0 having the bulk of the WW, ZZ coupling.

In the model $H^0 \to h^0h^0$ decays are important, but m_{h^0} is too large for $\Upsilon \to \gamma h^0$.

2. **A single doublet plus one singlet.** For this particular discussion we employ the following notation:

- a for a (possibly light) CP-odd Higgs boson;
- h for a (possibly light) CP-even Higgs boson;

- s for a (possibly light) singlet Higgs boson;
- S for a singlet Higgs field, whether complex or real.
- H for a doublet Higgs field.

For a (non-supersymmetric) model, the general Higgs potential for the usual doublet field and the singlet field (assumed real) is:

$$\begin{aligned} V &= \frac{1}{2}\lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2 + \frac{1}{2}m_S^2 S^2 + \frac{1}{2}k(H^\dagger H)S^2 + \frac{1}{4!}hS^4 \\ &\quad +\alpha S + \delta(H^\dagger H)S + \kappa S^3\,, \end{aligned} \tag{14}$$

where the terms of the last line would be absent if we imposed a Z_2 symmetry of V under $S \to -S$. (A more restricted form is considered in V. Barger, P. Langacker, M. McCaskey, M. J. Ramsey-Musolf and G. Shaughnessy, arXiv:0706.4311 [hep-ph].)

There are now several possibilities:

(a) Z_2 is imposed *and* $m_S^2 > 0$ (so that $\langle S\rangle = 0$).
In this case, there is no SH mixing and S reduces to its quantum dof the s Higgs boson field where s is absolutely stable and a (viable as one can show) dark matter candidate. (see e.g. the NMSM of H. Davoudiasl, R. Kitano, T. Li and H. Murayama, Phys. Lett. B 609, 117 (2005) [arXiv:hep-ph/0405097].)

The most important coupling to focus on is then that from the k term. Writing $S = s$ and $H^0 = \frac{v}{\sqrt{2}} + \frac{1}{\sqrt{2}}(h + ia)$ (this a is absorbed by EWSB) we have an interaction

$$\frac{1}{2} k v h s^2 \,, \tag{15}$$

implying that $h \to ss$ decays are possible (and invisible).

Of course, additional new physics could make the s unstable: (S. Chang, P. J. Fox and N. Weiner, Phys. Rev. Lett. 98, 111802 (2007) [arXiv:hep-ph/0608310]. Similar ideas in M. J. Strassler, arXiv:hep-ph/0607160.)

Suppose the s is a pseudoscalar a and that there is a new physics interaction involving heavy vector-like quarks of form:

$$\mathcal{L} \ni \overline{\psi}(M + i\gamma_5 \lambda a)\psi \tag{16}$$

Integrating out the heavy ψ gives loop diagrams that generate effective $a \to \gamma\gamma, gg$ couplings. The result in one particular model with a bunch of ψ's is

$$B(h \to 4\gamma) \sim 1.4 \times 10^{-5} \,, \quad B(h \to 2g2\gamma) \sim 7.6 \times 10^{-3} \,. \tag{17}$$

The one loop generation of these a couplings imply the possibility of non-prompt a decay:

$$c\tau_a \sim \frac{1}{\Gamma_{a\to gg}} = 1\,\mathrm{cm}\left(\frac{30\ \mathrm{GeV}}{m_a}\right)^3 \left(\frac{M}{450\ \mathrm{TeV}}\right)^2 \left(\frac{0.1}{\lambda\, b_3}\right)^2 \tag{18}$$

At LEP, an h with $m_h = 100\ \mathrm{GeV}$ decaying mainly via $h \to aa$ with a decaying in this way would have been missed.
However, even if the a is light, a B factory is not useful since there is no $a \to b\bar{b}$ coupling.

(b) If Z_2 is not imposed and/or $m_S^2 < 0$, there is SH mixing and the s Higgs boson (which is the quantum dof of S) will decay via this mixing to SM particles (remember, no SUSY yet).

- Even if $\alpha = \kappa = \delta = 0$, if $m_S^2 < 0$ ($h > 0$ by stability) $\langle S\rangle = x \neq 0$ and we get mixing from the k term which reduces to

$$\frac{1}{2}kv(xhs + hs^2)\,. \tag{19}$$

  $x \neq 0$ implies that the Higgs mass eigenstates will be neither purely h nor purely s.
  Further, decays of a heavier Higgs to a pair of lighter Higgs are a

general possibility due to the k term and the xS^3 component of the h term (after going to the mass eigenstate basis).

- If one or more of α, δ, κ are non-zero, hs mixing arises from the $\delta(H^\dagger H)S$ term even if $\langle S\rangle = 0$ (since $\langle H^0\rangle = \frac{v}{\sqrt{2}} \neq 0$). Note: $\langle S\rangle = 0$ is not guaranteed in this case even if $m_S^2 > 0$.

  The k, δ and κ terms all give rise to triple (mixed) Higgs couplings that could be responsible for Higgs to Higgs-pair decays.

Phenomenological Possibilities for hs mixing

Call the mass eigenstates h_{mix} and s_{mix}.

s_{mix} couples proportionally to the h couplings which means the $s_{mix} \to b\overline{b}$ coupling exists and $s_{mix} \to b\overline{b}$ decays would dominate if allowed.

Two attractive scenarios:

- One can set parameters so that $m_{h_{mix}} \sim 98~\mathrm{GeV}$ and $g^2_{ZZh_{mix}} B(h_{mix} \to b\overline{b}) \sim 0.1$ relative to SM, while $m_{s_{mix}} \sim 115~\mathrm{GeV}$ and $g^2_{ZZh_{mix}} B(s_{mix} \to b\overline{b}) \sim 0.9$.

  This would very nicely explain the two LEP excesses at $98~\mathrm{GeV}$ and $115~\mathrm{GeV}$.

But, both h_{mix} and s_{mix} are too heavy for a B factory.

- If $m_{s_{mix}} < 2m_b$ then even an h_{mix} with more or less SM couplings (somewhat reduced by mixing, but not necessarily very much) would have been difficult to see at LEP even if $m_{h_{mix}} \sim 100~\mathrm{GeV}$ by virtue of $h_{mix} \to s_{mix}s_{mix}$ decays being dominant (doesn't take much $h_{mix} \to s_{mix}s_{mix}$ coupling).

  A very attractive possibility is to explain the 2.3σ $98~\mathrm{GeV}$ LEP bump using $g^2_{ZZh_{mix}}B(h_{mix} \to b\bar{b}) \sim 0.1$ (but the model would not give the more marginal excess at $115~\mathrm{GeV}$).

  For this scenario a B factory is an extremely important tool: look for $\Upsilon \to \gamma s_{mix} \to \gamma\tau^+\tau^-$ ($m_{s_{mix}} > 2m_\tau$) or $\Upsilon \to \gamma s_{mix} \to \gamma 2j$ ($m_{s_{mix}} < 2m_\tau$).

Note that these last two scenarios both give excellent precision electroweak agreement since the Higgs bosons are light.

Non-SUSY Summary H denotes a $\sim 100~\mathrm{GeV}$ SM-like Higgs and h denotes a light Higgs with weak couplings to SM particles.

- Even outside the superstring and/or supersymmetry contexts, there is a lot of general theoretical and model-building motivation for light Higgs bosons.

- So long as there is some mixing with the Higgs that acquires a vev, all the light Higgs bosons, h, will couple to quarks, leptons and gauge bosons with strength most probably much below the SM level.

- In this case, the cross section for direct h production at a hadron or electron collider will be quite suppressed and detection very difficult.

- If $H \to hh$ decays are important (as is quite likely for at least some of the h's), the H will be hard to see at a hadron collider.

  We might well have to wait for the ILC $e^+e^- \to Z + X$ mode, with $Z \to e^+e^-$ or $\mu^+\mu^-$, where a Higgs bump in M_X can be detected for SM-like ZZH coupling regardless of how the H decays. Once the H is found, an h that is present in $H \to hh$ decay can be found.

  Unfortunately, it appears that this could be a long wait.

  And, not all light h's need appear in H decays.

  It would be much better to search directly for all light h's, at least those with with $m_h < 2m_b$, in $\Upsilon \to \gamma h$ at a B factory in the near term.

- For those that don't appear in the decays of a heavier Higgs, current B factories may be our **only** chance to find them.

- **Since the $b\bar{b}$ coupling of the light h is likely to be small, we must be able to probe very small $B(\Upsilon \to \gamma h)$.**

- **If light weakly coupled Higgs bosons exist, and we don't know it, then our collider observations could be sufficiently incomplete and therefore sufficiently misleading that we will never correctly identify the ultimate model chosen by nature.**

- **In any case, it would be really helpful to know ahead of time if the $H \to$ Higgs-pair channel can be present so that if we don't see a SM-like Higgs in the usual ways we will know where to focus.**

- **At the LHC, if we don't see the H but strongly suspect it is there (e.g. WW scattering is nice and perturbative and we don't see extra dimensions), Υ decays might provide the only Higgs signal before the ILC.**

Supersymmetric Models

We (R. Dermisek and J. Gunion, hep-ph/0510322, hep-ph/0502105, hep-ph/0611142 and hep-ph/0611197) **compare the MSSM to the NMSSM.**

The NMSSM is defined as the extension of the MSSM in which one supplements the $\widehat{H}_u$ and $\widehat{H}_d$ doublet Higgs superfields by adding exactly one singlet superfield $\widehat{S}$. There are then 3 CP-even Higgs bosons, $h_{1,2,3}$ and two CP-odd Higgs bosons, $a_{1,2}$.

There are excellent reasons to focus on SUSY and the NMSSM.

- **None of the previous Higgs-only models solve the naturalness hierarchy problem completely — at best they can delay the quadratic divergence problem to say 5 TeV or 10 TeV.**

- **SUSY remains the most attractive way to completely solve the hierarchy problem (cancellation of quadratic divergences in one-loop Higgs mass-squared corrections between sparticle and particle loops).**

- **Further, SUSY with exactly two-doublets (and any number of singlets) gives gauge coupling unification and renormalization group evolution symmetry breaking.**

- **There are two particularly important reasons to focus on the NMSSM rather than the MSSM.**

  1. **The μ problem**
     In the MSSM, the μ parameter in $W \ni \mu \widehat{H}_u \widehat{H}_d$,[1] is dimensionful, unlike all other superpotential parameters.
     A big question is why is it $\mathcal{O}(1~\mathrm{TeV})$ (as required for EWSB) and not zero (too low $m_{\widetilde{\chi}_1^\pm}$) or $\mathcal{O}(M_U, M_{\mathrm{Planck}})$ (huge tree-level Higgs mass, e.g.).
     Currently no satisfactory approach within the MSSM context.
     In the NMSSM, the μ-term component of the superpotential is replaced by $W \ni \lambda \widehat{S} \widehat{H}_u \widehat{H}_d$.
     The μ parameter is then automatically generated by $\langle S \rangle$ leading to $\mu_{eff} \widehat{H}_u \widehat{H}_d$ with $\mu_{eff} = \lambda \langle S \rangle$.
     The only requirement is that $\langle S \rangle$ be of order the SUSY-breaking scale at $\sim 1~\mathrm{TeV}$. This is almost automatically the case.
     Additional NMSSM features are:
     - **To avoid a massless axion, we require an additional superpotential term, $W \ni \frac{1}{3} \kappa \widehat{S}^3$.**
     - **Other possible superpotential terms with dimensionful parameters are**

[1] Hatted (unhatted) capital letters denote superfields (scalar superfield components).

absent if one demands that the superpotential be invariant under a Z_3 symmetry.

– If the Z_3 is applied also to soft SUSY breaking terms, only $\frac{1}{3}\kappa A_\kappa S^3$ is allowed in addition to $\lambda A_\lambda S H_u H_d$.

Net Result

Since the only *relevant* superpotential terms that are introduced have dimensionless couplings, the scale of the vevs (i.e. the scale of EWSB) is determined by the scale of SUSY-breaking.

Further, all the good properties of the MSSM (coupling unification and RGE EWSB, in particular) are preserved under singlet addition.

2. Weak-scale fine-tuning
   - In low-scale supersymmetry, the previous fine-tuning associated with the naturalness / hierarchy problem is reduced to a little-hierarchy problem.
   - The next level of fine-tuning has to do with the fact that m_Z^2 is very sensitive to GUT scale parameters. This sensitivity is unacceptably large if sparticle masses, especially those of the stops and the gluino, are large.

     We need a fairly light gluino and rather light stops to avoid m_Z^2 fine-tuning.
   - But, light stop masses are problem in the context of the CP-conserving

Minimal Supersymmetric Model (MSSM) as they would lead to a low mass for the SM-like Higgs boson, h^0.
In the presence of soft-SUSY-breaking, $(\tan\beta = h_u/h_d)$

$$m_{h^0}^2 \sim m_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} v^2 y_t^4 \sin^4\beta \log\left(\frac{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}{m_t^2}\right) + \ldots$$
$$\overset{\text{large}\tan\beta}{\sim} (91 \text{ GeV})^2 + (38 \text{ GeV})^2 \log\left(\frac{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}{m_t^2}\right). \quad (20)$$

Unless $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}} \gtrsim 1 \text{ TeV}$, $m_{h^0} < 114 \text{ GeV}$ and LEP limits become an issue.

– In the NMSSM, a SM-like h_1 with $m_{h_1} \sim 100 \text{ GeV}$ (which implies small F) is easily possible because $h_1 \to a_1 a_1$ (a_1 being the lightest CP-odd Higgs boson) obscures the LEP signal if $m_{a_1} < 2m_b$.

3. An additional reason to think of the NMSSM or a model with still more singlets is that extra singlet superfields are common in string models.
If we make use of singlets in the simplest possible way (i.e. no associated gauge group and no dimensionful superpotential parameters) $\Rightarrow$ the NMSSM.

We now pursue these issues in a bit more detail.

- A rigorous measure of fine-tuning is

$$F = \mathrm{Max}_p \left| \frac{p}{m_Z} \frac{\partial m_Z}{\partial p} \right| , \qquad (21)$$

where p runs over all GUT scale parameters. We will quote F values for $\tan\beta = 10$ and $M_{1,2,3} = 100, 200, 300~\mathrm{GeV}$.

$F > 20$ means worse than 5% fine tuning = bad.

In the MSSM, if $m_{h^0} \sim 100~\mathrm{GeV}$ were not LEP excluded for a SM-like h^0, then small $F < 10$ would be possible.

However, the best one can do in the MSSM is to have a mixed Higgs scenario (reduced h^0ZZ couplings) with m_{h^0} near $100~\mathrm{GeV}$, as possible if stop mixing is large and optimally chosen. $F \sim 15$ can be achieved for carefully adjusted A_t.

- The NMSSM can have $F \sim 5$ (no m_Z tuning) if the h_1 is SM-like and $m_{h_1} \sim 100~\mathrm{GeV}$.

  The preferred GUT scale soft-SUSY-breaking parameters turn out to be generically "no-scale" in nature (*i.e.* small). No special relations among substantial non-zero values are needed.

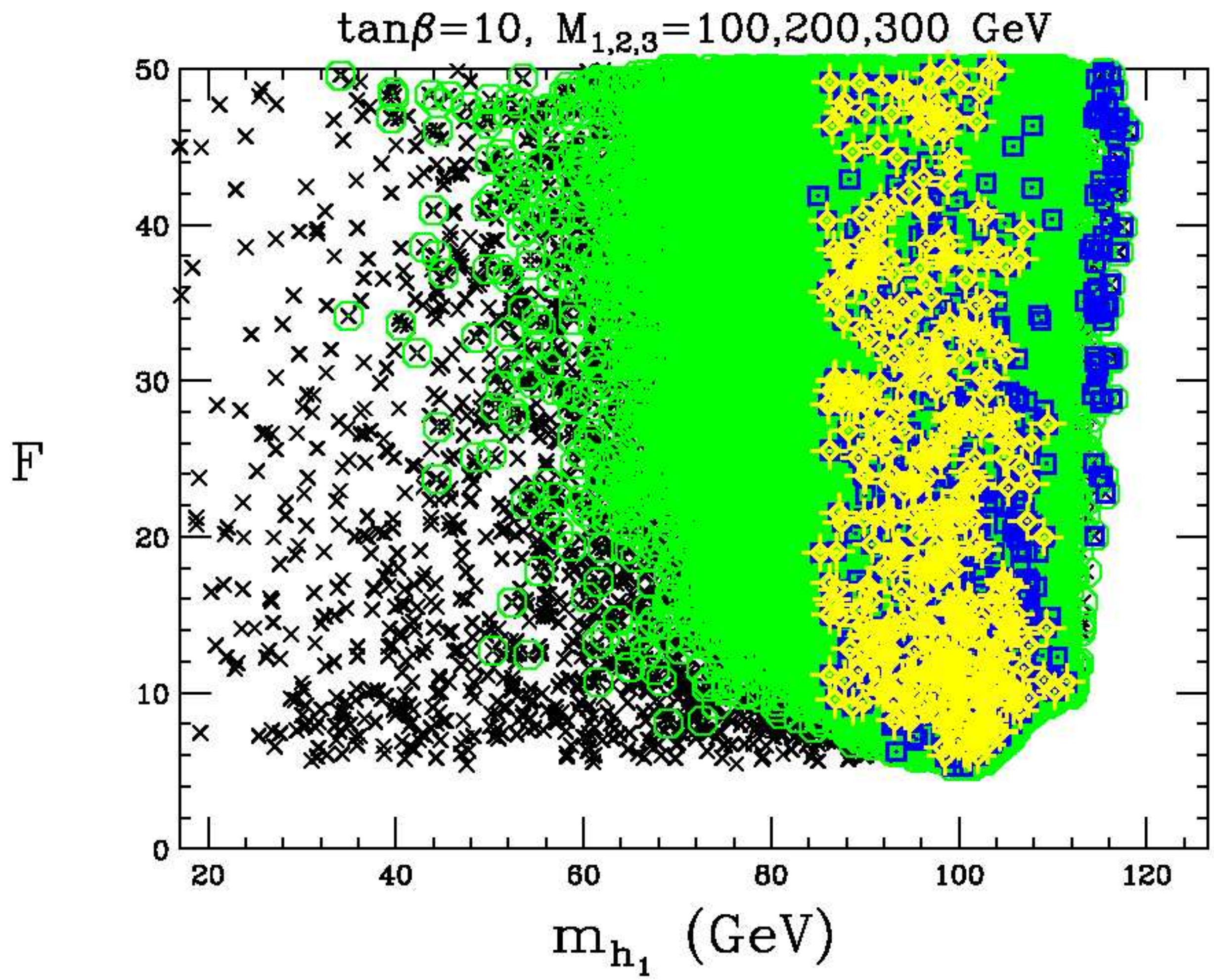

Figure 2: F vs. m_{h_1} for $M_{1,2,3} = 100, 200, 300$ GeV and $\tan\beta = 10$. Small $\times$ = no constraints other than global and local minimum, no Landau pole before M_U and neutralino LSP. The O's = stop and chargino limits imposed, but NO Higgs limits. The □'s = all LEP single channel, in particular $Z + 2b$, Higgs limits imposed. The large FANCY CROSSES are after requiring $m_{a_1} < 2m_b$, so that LEP limits on $Z + b's$, where $b's = 2b + 4b$, are not violated.

- Low-fine-tuning NMSSM models with $m_{h_1} \sim 100~\mathrm{GeV}$ require $B(h_1 \to a_1 a_1) > 0.7$ with $m_{a_1} < 2m_b$ to avoid LEP limits.

  Of course, $m_{h_1} \sim 100~\mathrm{GeV}$ is perfect for precision electroweak and $B(h_1 \to b\bar{b}) \sim 0.1$ is typical, which would explain the 2.3σ LEP excess near $m_{b\bar{b}} \sim 98~\mathrm{GeV}$ in $e^+e^- \to Z + b's$.

  $m_{h_1} \sim 100~\mathrm{GeV}$ requires $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}} \sim 350~\mathrm{GeV} \Rightarrow$ stops easily found at the LHC.

  It turns out that the appropriate a_1 properties also do not require fine-tuning of the GUT scale parameters, especially if $m_{a_1} > 2m_\tau$.

  This is the topic of hep-ph/0611142.

  – The a_1 properties depend on $A_\kappa(m_Z), A_\lambda(m_Z)$. If these are zero, then don't get $B(h_1 \to a_1 a_1) > 0.7$ as needed to give $B(h_1 \to b\bar{b}) \lesssim 0.2$ as required for $m_{h_1} \sim 100~\mathrm{GeV}$ to escape LEP limits.
  – However, if $A_\kappa(M_U), A_\lambda(M_U) \sim 0$ (possibly a nice model choice), then renormalization group equations (RGE's) generate

$$|A_\lambda(m_Z)| \sim 100 - 200~\mathrm{GeV}\,, \quad |A_\kappa(m_Z)| \sim few~~\mathrm{GeV}\,. \qquad (22)$$

  This is just what is needed to get large $B(h_1 \to a_1 a_1)$.

- A crucial a_1 property is its composition in terms of how much singlet Higgs, a_S, and how much doublet Higgs, A_{MSSM}:

$$a_1 \equiv \cos\theta_A A_{MSSM} + \sin\theta_A a_S \,. \tag{23}$$

  To get $B(h_1 \to a_1 a_1) > 0.7$, as required by LEP limits for $m_{h_1} \sim 100~\mathrm{GeV}$, requires $|\cos\theta_A| \gtrsim 0.05$ (at $\tan\beta = 10$), and this can only be achieved for $|A_\lambda| \gtrsim 100~\mathrm{GeV}$ and $|A_\kappa| \gtrsim few~\mathrm{GeV}$, as predicted by RGE, as noted in Eq. (22).

- To more precisely measure the GUT-scale tuning needed to achieve $m_{a_1} < 2m_b$ with sufficiently large $|A_\lambda|, |A_\kappa|$ to have $|\cos\theta_A| \gtrsim 0.05$, we defined a measure called G.

  Small G implies it is quite natural to get small m_{a_1} even for fairly general M_U-scale boundary conditions.

  We plot G as a function of $\cos\theta_A$ for various bins of m_{a_1} on the next page. All plotted points have $B(h_1 \to a_1 a_1) > 0.7$.

  We see that small G is only achieved for the black, green and red points, and not for the blue points. The blue points have $m_{a_1} < 2m_\tau$.

Net Result: Small G requires $m_{a_1} > 2m_\tau$ and $\cos\theta_A \sim -0.1$, (at $\tan\beta = 10$).

This $\cos\theta_A$ value is just fine for large $B(h_1 \to a_1 a_1)$.

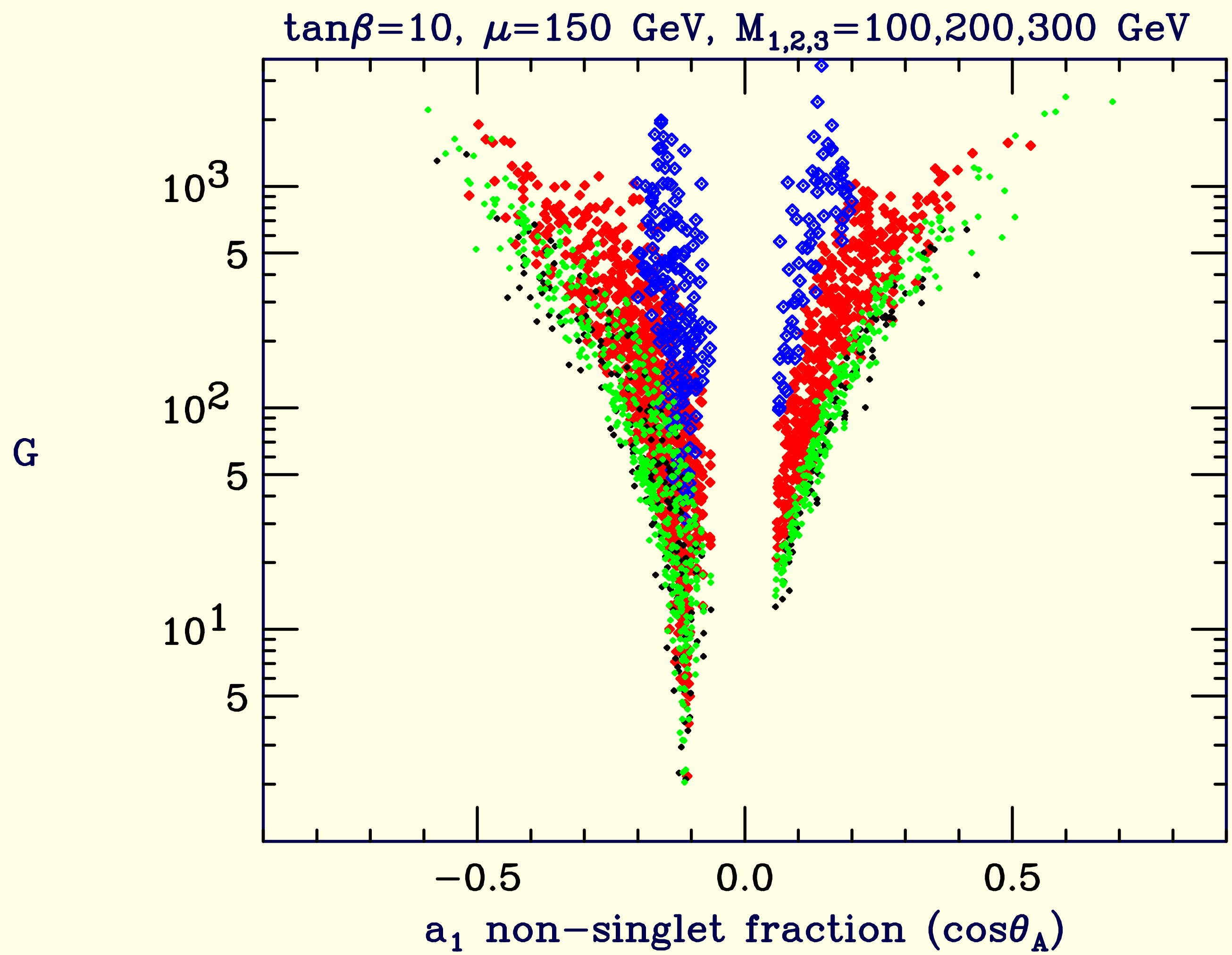

Figure 3: G vs. $\cos\theta_A$ for $M_{1,2,3} = 100, 200, 300$ GeV, $\mu_{\rm eff} = 150$ GeV and $\tan\beta = 10$ for a selection of scenarios with $B(h_1 \to a_1 a_1) > 0.7$ and $m_{a_1} < 2m_b$. The color coding is: blue $= m_{a_1} < 2m_\tau$; red $= 2m_\tau < m_{a_1} < 7.5$ GeV; green $= 7.5$ GeV $< m_{a_1} < 8.8$ GeV; and black $= 8.8$ GeV $< m_{a_1} < 9.2$ GeV. The plot is for $\tan\beta = 10$ and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV.

Summary to this point:

- **The NMSSM is intrinsically a beautiful model, better than the MSSM theoretically even.**
- $F < 10 - 15$ **requires** $m_{h_1} \sim 100~\mathrm{GeV}$, $m_{a_1} < 2m_b$ **and** $|\cos\theta_A| > 0.06$ ($\tan\beta = 10$).
- **LEP excess at** $M_{2b} \sim 100~\mathrm{GeV}$ **is often perfectly described, since** $B(h_1 \to a_1 a_1) > 0.7$ **typically implies** $B(h_1 \to b\bar{b}) \sim 0.1$.
- $m_{h_1} \sim 100~\mathrm{GeV}$ **is perfect for precision electroweak.**

The question is, how to find the h_1 **and/or the** a_1**?**

- **There is no time for details, but at the moment there is no proven way at the LHC. All standard channels fail:** *e.g.* $B(h_1 \to \gamma\gamma)$ **is much too small because of large** $B(h_1 \to a_1 a_1)$**. The possible new channels include:**

  1. $WW \to h_1 \to a_1 a_1 \to 4\tau$.
     Looks moderately promising but far from definitive results at this time.
  2. $t\bar{t}h_1 \to t\bar{t}a_1 a_1 \to t\bar{t}\tau^+\tau^-\tau^+\tau^-$.
     Study begun.

3. **A third possibility:** $\widetilde{\chi}_2^0 \to h_1 \widetilde{\chi}_1^0$ with $h_1 \to a_1 a_1 \to 4\tau$.
(Recall that the $\widetilde{\chi}_2^0 \to h_1 \widetilde{\chi}_1^0$ channel provides a signal in the MSSM when $h_1 \to b\overline{b}$ decays are dominant.)
4. **Last, but definitely not least: diffractive production** $pp \to pph_1 \to ppX$.
The mass M_X can be reconstructed with roughly a $1-2~\mathrm{GeV}$ resolution, potentially revealing a Higgs peak, independent of the decay of the Higgs. Preliminary results are that one expects about 3 clean, i.e. reconstructed and tagged, events per $30~\mathrm{fb}^{-1}$ of luminosity. $\Rightarrow$ clearly a high luminosity game.
5. The rather singlet nature of the a_1 and its low mass, imply no direct production/detection possible at the LHC.

- At the ILC, there is no problem since $e^+e^- \to ZX$ will reveal the $M_X \sim m_{h_1} \sim 100~\mathrm{GeV}$ peak no matter how the h_1 decays.

  But the ILC is more than a decade and > 6.7 billion dollars away.

- As it turns out $\Upsilon \to \gamma a_1$ decays hold great promise for a_1 discovery (or exclusion) as we now outline. **They should be pushed to the limit.** The signal may even be present in the data available now! The only issue will be reaching quite small $B(\Upsilon \to \gamma a_1)$.

Predictions for $B(\Upsilon \to \gamma a_1)$ for small F scenarios

(R. Dermisek, J. Gunion and B. McElrath, hep-ph/0612031)

- One begins with the Wilczek formula:

$$R \equiv \frac{\Gamma_0(V(1^{--}) \to \gamma a)}{\Gamma_0(V(1^{--}) \to \mu^+\mu^-)} = \frac{G_F m_Q^2}{\sqrt{2}\pi\alpha}\left(1 - \frac{m_a^2}{M^2}\right) \quad (24)$$

which assumes 'standard' γ_5 Yukawa coupling of the a with SM-like analogue strength. The $_0$ means tree-level.

- Various corrections must then be made.

1. QCD radiative corrections ala Vysotsky and Nason:

$$\Gamma(V \to \gamma a) = \Gamma_0(V \to \gamma a)\left[1 - \frac{\alpha_s C_F}{\pi} a_P(z) + \mathcal{O}(\alpha_s^2)\right] \quad (25)$$

where $z \equiv 1 - m_a^2/M^2$. $a_P(z)$ ranges from ~ 2 at $z = 0$ ($m_{a_1} = M$) to ~ 6 at $z = 1$ ($m_a = 0$). In relating to experimentally measured

$\Gamma(V \to \mu^+\mu^-)$ from PDG, we must also include radiative corrections for this mode:

$$\Gamma(V \to \mu^+\mu^-) = \Gamma_0(V \to \mu^+\mu^-)\left[1 - \frac{4\alpha_s C_F}{\pi}\right]. \qquad (26)$$

2. Bound state corrections: we employ the calculation of Pantaleone, Peskin, and Tye.
   These corrections, while yielding a big suppression for a scalar Higgs, yield a very modest enhancement for a pseudoscalar. The enhancement is typically $< 1\%$ for small m_a, rising to $\sim 10\%$ for $8 \lesssim m_a \lesssim 9.2~\mathrm{GeV}$. For $m_a \gtrsim 9.2~\mathrm{GeV}$ the a starts to mix with the η_b. We do not present results in this region. The recent work of Esteban Fullana and Miguel Sanchis-Lozano, hep-ph/0702190, attempts to get a reliable prediction in this region. They find that the photon is not very chromatic and that the best signal would be apparent lepton non-universality.
3. Relativistic Corrections: we use those from Aznauryan, Grigoryan, and Matinyan.

$$R_{rel} = R_0\left(\frac{M_\Upsilon^2 - m_a^2}{4m_b^2 - m_a^2}\right)^2\left[1 - \tfrac{1}{3}\Delta\left(\frac{36m_b^2 + m_a^2}{4m_b^2 - m_a^2}\right)\right] \qquad (27)$$

where

$$m_b^2\Delta \equiv \frac{\int \psi(p^2)p^4dp}{\int \psi(p^2)p^2dp}, \qquad (28)$$

with $\psi(p^2)$ being the radial part of the wave function of the b quarks in the Υ. We employ a value of Δ that fits well their plots. The result is substantial suppression. For example, $R_{rel} \sim \frac{1}{2}R_0$ for $m_a < 4~\mathrm{GeV}$. Suppression is even larger at large m_a.

4. Coupling Correction: The a_1 does not couple with SM-analogue strength. Its coupling is enhanced by $\tan\beta$ and suppressed by the smallness of its A_{MSSM} component fraction $\cos\theta_A$ — the a_S singlet component does not couple to $b\bar{b}$. The precise coupling correction factor is then:

$$R_{a_1} = R_{\text{'SM-like'}\, a} \times (\tan\beta\cos\theta_A)^2 . \qquad (29)$$

- **THE RESULTS**

  We will not plot points that violate the $B(\Upsilon \to \gamma a_1)$ limits of Fig. 3 of [1], Fig. 4 of [2], and Fig. 7b of [3].

  The first two limit $B(\Upsilon \to \gamma X)$, where X is any visible state.

  The first provides the only strong constraint on the $m_{a_1} < 2m_\tau$ region.

The third gives limits on $B(\Upsilon \to \gamma X)B(X \to \tau^+\tau^-)$ that eliminate $2m_\tau < m_{a_1} < 8.8~\mathrm{GeV}$ points with too high $B(\Upsilon \to \gamma a_1)$ (for $m_{a_1} > 2m_\tau$, $B(a_1 \to \tau^+\tau^-) \sim 0.9$).

Since the inclusive photon spectrum from Υ decays falls as E_γ increases, the strongest constraints are obtained for small m_{a_1}.

References

[1] P. Franzini *et al.*, Phys. Rev. D 35, 2883 (1987).
[2] H. Albrecht *et al.* [ARGUS Collaboration], Phys. Lett. B 154, 452 (1985).
[3] H. Albrecht *et al.* [ARGUS Collaboration], Z. Phys. C 29, 167 (1985).

In the first figure, we focus on the $\tan\beta = 10$ case with $M_{1,2,3}(m_Z) = 100, 200, 300~\mathrm{GeV}$ for which our previous plots were given. Two types of plot are shown:

1. A scan in A_λ, A_κ space at fixed $\mu_{\mathrm{eff}} = 150~\mathrm{GeV}$, requiring $B(h_1 \to a_1 a_1) > 0.7$ and $m_{a_1} < 9.2~\mathrm{GeV}$.
2. Results for the $F < 15$ points of Fig. 2.

In the second figure, we look at results for the A_λ, A_κ scans for $\tan\beta = 3$ and $\tan\beta = 50$.

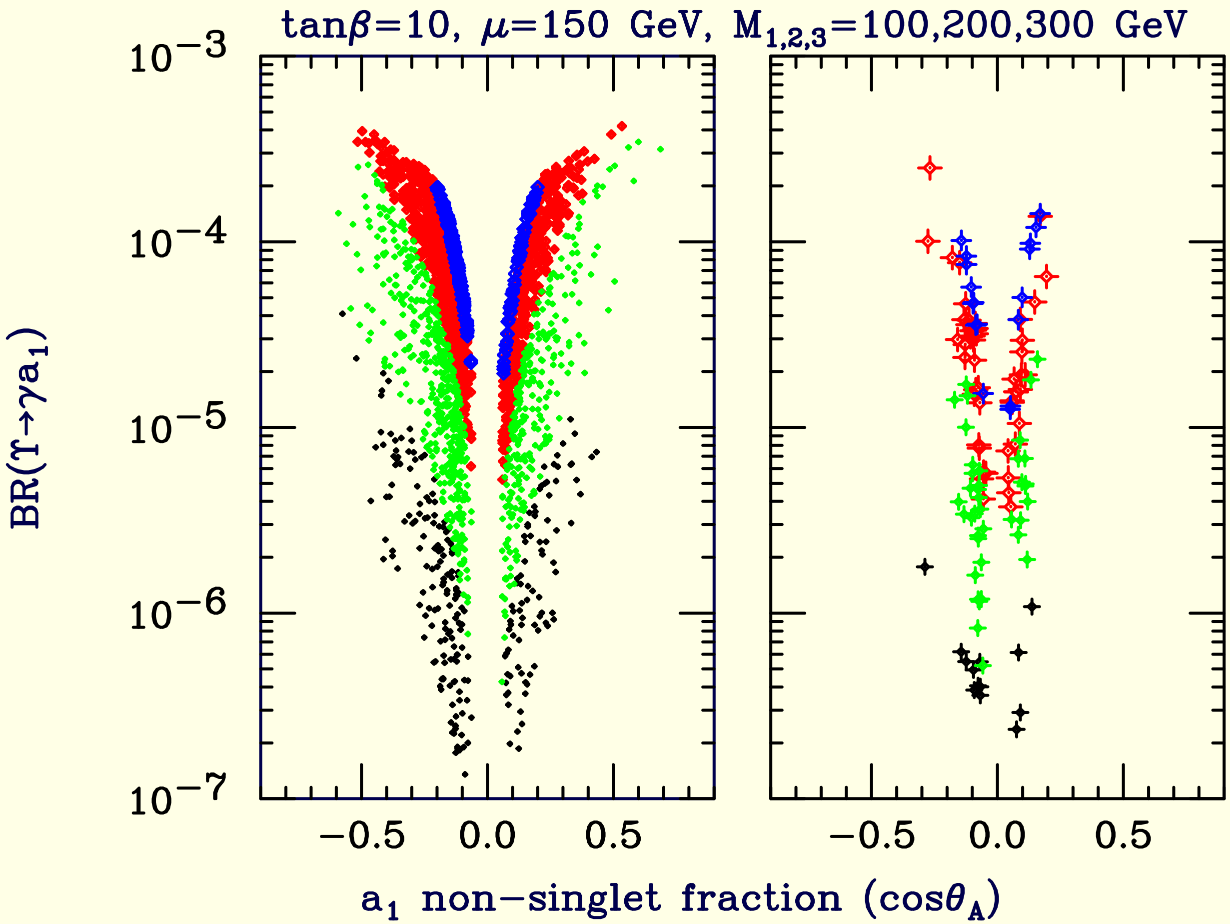

Figure 4: $B(\Upsilon \to \gamma a_1)$ for NMSSM scenarios with various ranges for m_{a_1} using color scheme of Fig. 3 (blue= $< 2m_\tau$, red= $[2m_\tau, 7.5]$, green= $[7.5, 8.8]$, black= $[8.8, 9.2]$). The left plot comes from the A_λ, A_κ scan described in the text, holding $\mu_{\rm eff}(m_Z) = 150$ GeV fixed. The right plot shows results for $F < 15$ scenarios with $m_{a_1} < 9.2$ GeV found in a general scan over all NMSSM parameters holding $\tan\beta$ and $M_{1,2,3}$ fixed as stated. The lower bound on $B(\Upsilon \to \gamma a_1)$ arises basically from the LEP requirement of $B(h_1 \to a_1 a_1) > 0.7$.

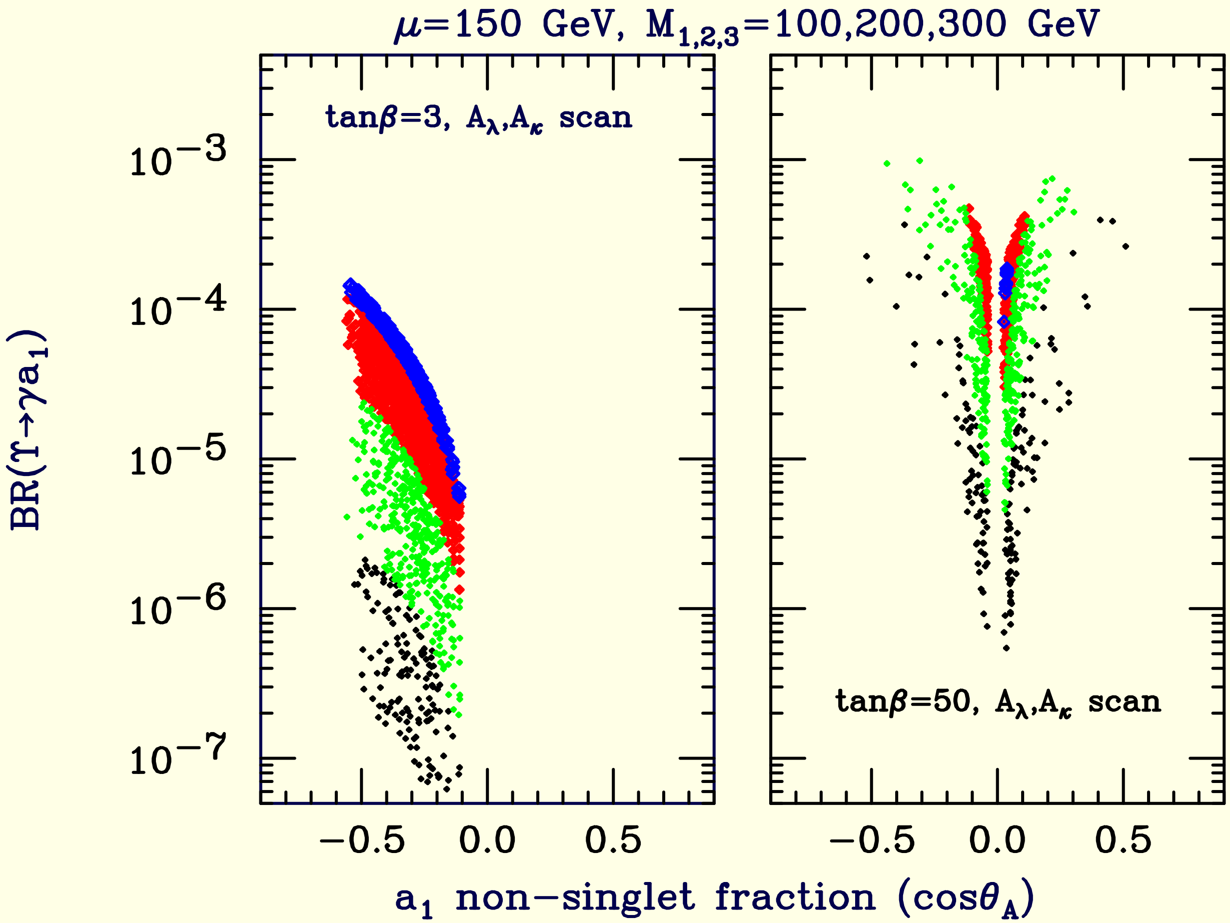

Figure 5: We plot $B(\Upsilon \to \gamma a_1)$ as a function of $\cos\theta_A$ for the A_λ, A_κ scan, taking $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV, $\mu_{\rm eff}(m_Z) = 150$ GeV with $\tan\beta = 3$ (left) and $\tan\beta = 50$ (right). The point notation is as before: blue= $< 2m_\tau$, red=$[2m_\tau, 7.5]$, green=$[7.5, 8.8]$, black=$[8.8, 9.2]$.

Some new limits from CLEO eliminate points in these figures between 2×10^{-4} and 4×10^{-5} for $2m_\tau < m_{a_1} < 7.5$ GeV, with weaker (and not terribly useful) limits for $m_{a_1} > 7.5$ GeV. However, the a_1-tuning measure G prefers still lower $B(\Upsilon \to \gamma a_1)$.

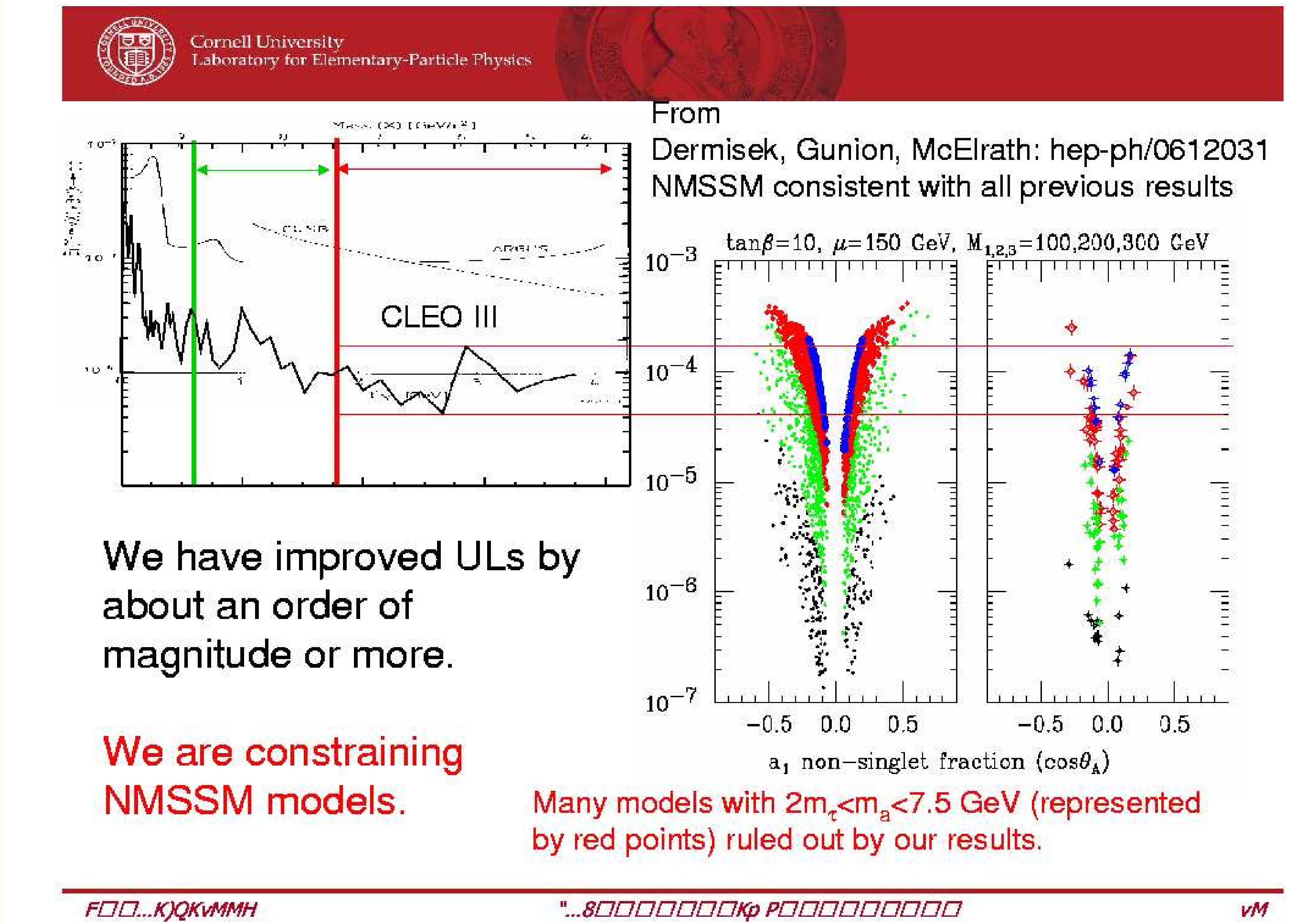

Figure 6: *New Limits from CLEO III (Krenick, Bottomonia, August 6, 2007) from* $\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)$*, which eliminates* $e^+e^- \to \gamma\tau^+\tau^-$ *background. Tag=2 prong (1 lepton)+* $\not{E}_T$*. Total of* 9 *Million* $\Upsilon(2S)$ *events.*

- **Summary of $B(\Upsilon \to \gamma a_1)$ results:**
  1. There are some large $B(\Upsilon \to \gamma a_1)$'s that might or might not have A_λ, A_κ fine-tuning issues (large G) waiting to be excluded by existing data.
  2. At $\tan\beta = 10$, small G points with $\cos\theta_A \sim -0.1$ (red, green and black) have $B(\Upsilon \to \gamma a_1)$ ranging from $\lesssim few \times 10^{-5}$ for $2m_\tau < m_{a_1} < 7.5~\mathrm{GeV}$ (red) to $\sim few \times 10^{-7}$ for $8.8~\mathrm{GeV} < m_{a_1} < 9.2~\mathrm{GeV}$ (black).
  3. At $\tan\beta = 3$, the $B(\Upsilon \to \gamma a_1)$ range is suppressed further.
  4. At $\tan\beta = 50$, $B(\Upsilon \to \gamma a_1) \gtrsim 10^{-6}$ for all points with $m_{a_1} < 9.2~\mathrm{GeV}$.
  5. We stress again that the lower bounds on $B(\Upsilon \to \gamma a_1)$ arise from the LEP requirement that $B(h_1 \to a_1 a_1) > 0.7$.
  6. Of course, we cannot exclude the possibility that $9.2~\mathrm{GeV} < m_{a_1} < 2m_b$.
     - For $9.2 \lesssim m_{a_1} \lesssim M_\Upsilon$, phase space for the decay causes increasingly severe suppression. Further, mixing of the a_1 with the η_b is generally present and smears out the photon spectrum (which is soft anyway) $\Rightarrow$ look for lepton non-universality.
     - And, there is the small region of $M_\Upsilon < m_{a_1} < 2m_b$ that cannot be covered by Υ decays.
  7. However, if $B(\Upsilon \to \gamma a_1)$ sensitivity can be pushed down to the 10^{-7} level, you may well discover the a_1.

Conclusions

- **The NMSSM can naturally have small fine-tuning with respect to GUT-scale parameters for both:**

  1) Electroweak Symmetry Breaking, *i.e.* getting the measured value of m_Z^2;

  2) Small $m_{a_1} < 2m_b$ and (simultaneously) large $B(h_1 \to a_1 a_1)$, both of which are needed to escape LEP limits for $m_{h_1} \sim 100~\mathrm{GeV}$ [the latter being required for 1)].

  $m_{a_1} > 2m_\tau$ is somewhat preferred by this latter fine-tuning issue.

- **If low EWSB fine-tuning is imposed for an acceptable SUSY model, we should expect:**

  – **a h_1 with $m_{h_1} \sim 100~\mathrm{GeV}$ and SM-like couplings to SM particles but with primary decays $h_1 \to a_1 a_1$ with $m_{a_1} < 2m_b$, where the a_1 is mainly singlet. The collider implications are:**

    Higgs detection will be quite challenging at a hadron collider.
    Higgs detection at the ILC is easy using the missing mass $e^+e^- \to ZX$ method of looking for a peak in M_X.
    Higgs detection in $\gamma\gamma \to h_1 \to a_1 a_1$ will be easy.

Detection of the a_1 could easily result from pushing on $\Upsilon \to \gamma a_1$.

– the stops and other squarks are light;
– the gluino, and, by implication assuming conventional mass orderings, the wino and bino all have modest mass;

- Even if the LHC sees the Higgs $h_1 \to a_1 a_1$ directly, it will not be able to get much detail. Only the ILC and possibly B-factory results for $\Upsilon \to \gamma a_1$ can provide the details needed to verify the model.

- It is likely that other models in which the MSSM μ parameter is generated using additional scalar fields can achieve small fine-tuning in a manner similar to the NMSSM.

  Low fine-tuning typically requires low SUSY masses which in turn typically imply $m_{h_1} \sim 100~\mathrm{GeV}$.

  And, to escape LEP limits in the $Z + b's$ channel, large $B(h_1 \to a_1 a_1)$ with $m_{a_1} < 2m_b$ would be needed.

  In general, the a_1 might not need to be so singlet as in the NMSSM and would then have larger $B(\Upsilon \to \gamma a_1)$.

- Although SUSY will be easily seen at the LHC, Higgs detection at the LHC may prove to be a real challenge.

Ability to check perturbativity of $WW \to WW$ at the LHC might prove to be very crucial to make sure that there really is a light Higgs accompanying light SUSY.

- A light a_1 allows for a light $\widetilde{\chi}_1^0$ to be responsible for dark matter of correct relic density: annihilation would be via $\widetilde{\chi}_1^0 \widetilde{\chi}_1^0 \to a_1$. To check the details, properties of the a_1 will need to be known fairly precisely

  The ILC might (but might not) be able to measure the properties of the very light $\widetilde{\chi}_1^0$ and of the a_1 in sufficient detail to verify that it all fits together.

  But, also $\Upsilon \to \gamma a_1$ decay information would help tremendously.

- Thus,

  Large B-factory data sets, optimally using $\Upsilon(3S) \to \pi^+\pi^-\Upsilon(1S)$ to tag the $1S$ state, should be pursued with great vigor.

  Unless, of course, it is found that backgrounds (most notably from $\Upsilon \to \gamma\tau^+\tau^-$) are insurmountable at the needed level.

  What are the limits? We have had a brief look, but clearly this is a job for experimentalists.

- In the $\gamma\tau^+\tau^-$ final state, the direct $\gamma\tau^+\tau^-$ production cross section is 61 pb.

  Using signal=background as the criterion, this becomes the limiting factor for branching ratios below the 4×10^{-5} level when running on the $\Upsilon(1S)$, and below the 2×10^{-4} level when running on the $\Upsilon(3S)$.
- To improve upon the latter, one can select a sample of known $\Upsilon(1S)$ events by looking for dipion transitions from the higher resonances.

  The dipion transition gives a strong kinematic constraint on the mass difference between the two Υ's.

  When running on the $\Upsilon(3S)$, the effective cross section in $\Upsilon(3S) \to \pi^+\pi^-\Upsilon(1S)$ is 179 pb (see archive Glenn:1998bd)

  To limit $B(\Upsilon \to \gamma a_1) \lesssim 10^{-6}$, $5.6~\mathrm{fb}^{-1}/\epsilon$ would need to be collected on the $\Upsilon(3S)$, where ϵ is the experimental efficiency for isolating the relevant events.
- This analysis can also be done on the $\Upsilon(4S)$, where the $\Upsilon(3S)$ is produced via ISR. The effective $\gamma_{ISR}\Upsilon(3S) \to \gamma_{ISR}\pi^+\pi^-\Upsilon(1S)$ cross section is 0.78 fb.

  To limit $B(\Upsilon \to \gamma a_1) \lesssim 10^{-6}$, $1.3~\mathrm{ab}^{-1}/\epsilon$ would need to be collected.
- These integrated luminosities needed to probe $B(\Upsilon \to \gamma a_1) \sim 10^{-6}$ would appear to be within reach at existing facilities and would allow discovery of the a_1 for many of the favored NMSSM scenarios.

- **Further Comments**

  – Of course, one should consider $b \to sa_1$ inclusive decays (also exclusive). We are working on this and have some preliminary results based on the formulas given by Hiller.
  These results suggest that $b \to sa_1 \to s\mu^+\mu^-$ limits may exclude most of the $m_{a_1} < m_b$ scenarios, which in any case are less preferred by A_λ, A_κ tuning issues.

  – $a_1 \to \gamma\gamma$ branching ratios remain very small in our scenarios because of the lower bound on $\cos\theta_A$, which implies that the a_1 has a minimum non-singlet component, in particular sufficient that a_1 decays to SM fermions dominate.
  For the general A_λ, A_κ scans with $B(h_1 \to a_1a_1) > 0.7$ and $m_{a_1} < 2m_b$ imposed, $B(a_1 \to \gamma\gamma) < 4 \times 10^{-4}$ with values near $few \times 10^{-5}$ being very common.
  $\Rightarrow$ the a_1 search strategies suggested by Cheung and collaborators will not work for these scenarios.
  Is it conceivable that a super-B factory could detect a signal for $\Upsilon \to \gamma a_1 \to \gamma\gamma\gamma$ with branching ratio at the 10^{-10} level?
  The needed number of Υ's is a stretch to say the least. But, presumably

backgrounds for three monochromatic photons are very tiny.
Certainly detection in this channel would provide a very interesting discovery and/or check on the consistency of the model.

– Could the $\zeta(8.3)$ have been real?
Obviously not at the level originally seen, but the mass fits perfectly with our scenarios.

In any case, we really hope that you will take the problem of Electroweak fine-tuning, and the NMSSM solution thereto, seriously.

After all, it fits perfectly with precision electroweak preference for a $m_{h_1} \sim 100~\mathrm{GeV}$ Higgs and with the $Z + 2b$ signal in the $M_{2b} \sim 100~\mathrm{GeV}$ region.

If you do, there is a very compelling case for pushing $\Upsilon \to \gamma a_1$ searches to the absolute extreme.