Probing the NMSSM $h \to aa$ Decay Scenario in Upsilon Decays

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Based largely on:
R. Dermisek and J. Gunion, hep-ph/0510322
R. Dermisek and J. Gunion, hep-ph/0502105
J. Gunion, D. Hooper and B. McElrath, hep-ph/0509024
R. Dermisek and J. Gunion, hep-ph/0611142 and hep-ph/0611197
R. Dermisek, J. Gunion and B. McElrath, hep-ph/0612031 * today's focus**

See also: J. Gunion, D. Miller, A. Pilaftsis, in CPNSH (CP-violating and Non-Standard Higgses) CERN Yellowbook Report.

Thanks to Miguel Sanchis-Lozano who stressed the importance of this study.

Outline

1. The LHC is coming, but is it guaranteed to find the Higgs of any SUSY model? Or, might Υ decays provide the first (and only?) Higgs signal before the ILC?
2. Why focus on SUSY?$\Rightarrow$ It remains the most attractive way to solve the hierarchy problem.
3. The Minimal SUSY Model (MSSM) is very attractive, but LEP limits on the lightest Higgs and the gluino imply that it is highly fine-tuned.
4. The Next to Minimal Supersymmetric Model (NMSSM) maintains all the attractive features of the MSSM while avoiding fine tuning, especially if $m_{h_1} \sim 100~\mathrm{GeV}$.
5. Low-fine-tuning NMSSM models with $m_{h_1} \sim 100~\mathrm{GeV}$ require $B(h_1 \to a_1 a_1) > 0.7$, with $m_{a_1} < 2m_b$ to avoid LEP limits.
6. $m_{h_1} \sim 100~\mathrm{GeV}$ is perfect for precision electroweak and, with $B(h_1 \to b\bar{b}) \sim 0.1$, explains the 2.3σ LEP excess near $m_{b\bar{b}} \sim 98~\mathrm{GeV}$ in $e^+e^- \to Z + b's$.

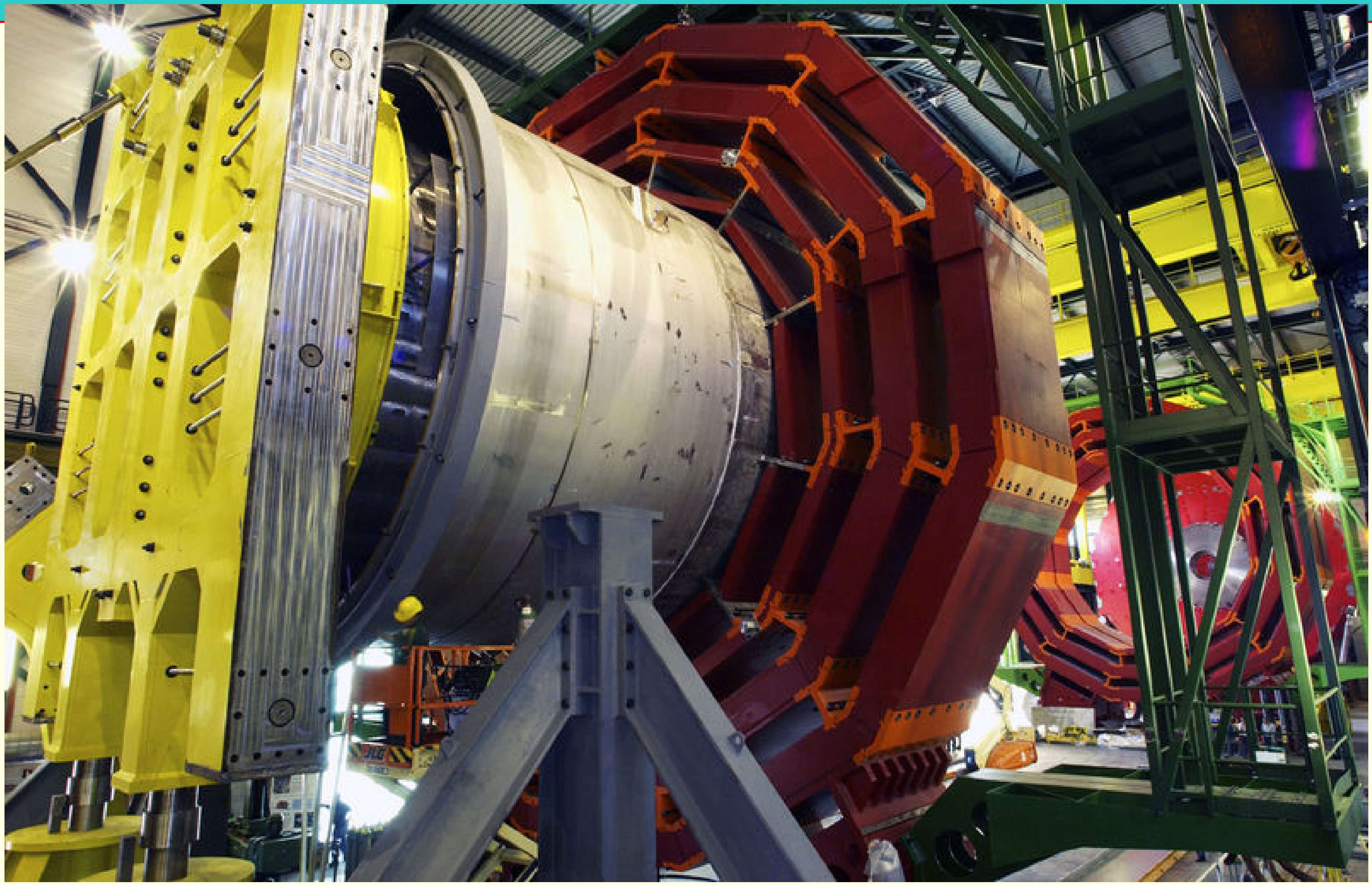
7. Collider Implications

One should look again at the LEP data for $h \to aa$ Higgs signals, especially with $aa \to 4\tau$.

Higgs discovery at the LHC will be essentially impossible in all the standard modes explored to date. New modes based on $h \to aa \to 4\tau$ or $4j$ must be proven.

Sensitivity to $B(\Upsilon \to \gamma a_1)$ down to 10^{-6} (maybe 10^{-7} would be needed) could find the a_1 in the near future!

The LHC is at hand



The CMS Detector

But will the LHC detectors detect the Higgs boson. The very attractive NMSSM SUSY scenario suggests we may have to work hard.

The Beauty of Supersymmetry

- SUSY is mathematically intriguing.
- SUSY is naturally incorporated in string theory.
- Scalar fields have a natural place in SUSY, and so there are candidates for the spin-0 fields needed for electroweak symmetry breaking and Higgs bosons.
- SUSY cures the naturalness / hierarchy problem (quadratic divergences are largely canceled) provided the SUSY breaking scale is of order $\sim 1~\mathrm{TeV}$.
- The MSSM comes close to being very nice.

  If we assume that all sparticles reside at or below TeV scale and that μ is also $\mathcal{O}(1~\mathrm{TeV})$, then, the MSSM has two particularly wonderful properties.

1. **Gauge Coupling Unification**

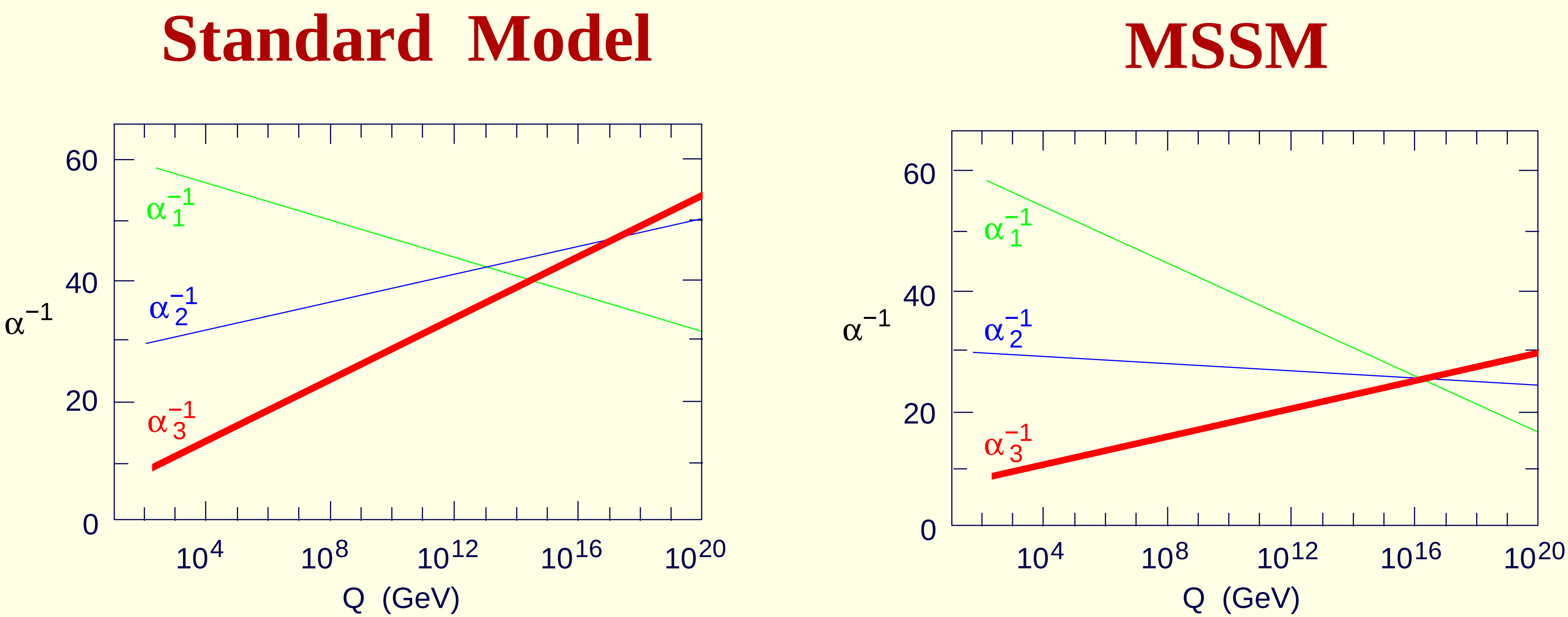


Figure 1: Unification of couplings constants ($\alpha_i = g_i^2/(4\pi)$) in the minimal supersymmetric model (MSSM) as compared to failure without supersymmetry.

The MSSM sparticle content + two-doublet Higgs sector ⇒ gauge coupling unification at $M_U \sim few \times 10^{16}\,\mathrm{GeV}$, close to M_P. High-scale unification correlates well with the attractive idea of gravity-mediated SUSY breaking.

2. RGE EWSB

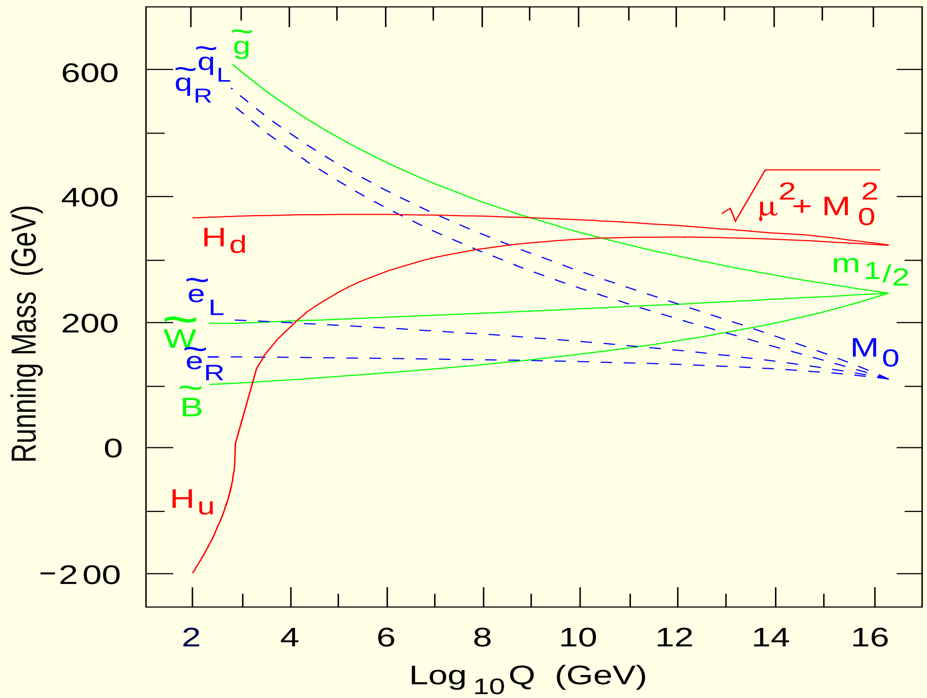


Figure 2: Evolution of SUSY-breaking masses or masses-squared, showing how $m_{H_u}^2$ is driven < 0 at low $Q \sim \mathcal{O}(m_Z)$.

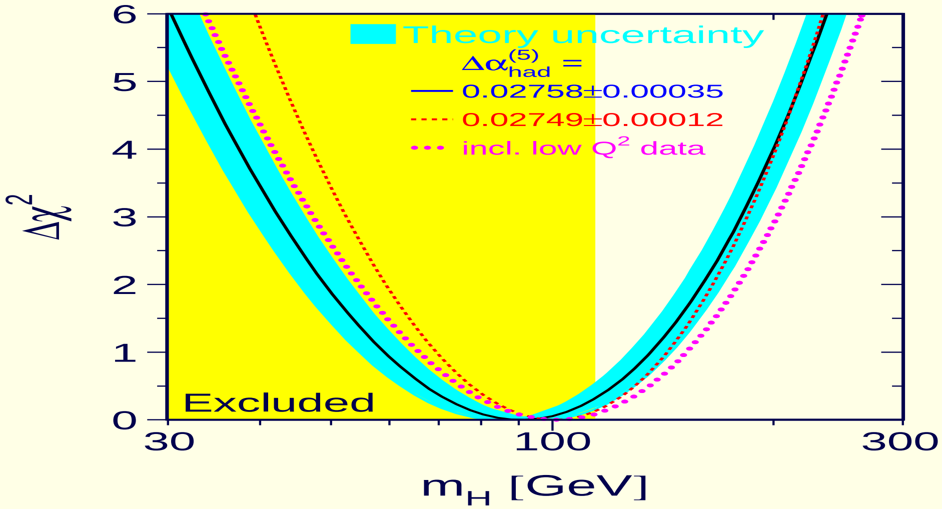
Starting with universal soft-SUSY-breaking masses-squared at M_U, the RGE's predict that the top quark Yukawa coupling will drive one of the soft-SUSY-breaking Higgs masses squared ($m_{H_u}^2$) negative at a scale of order $Q \sim m_Z$, thereby automatically generating electroweak symmetry breaking ($\langle H_u \rangle = h_u, \langle H_d \rangle = h_d$), BUT MAYBE m_Z IS FINE-TUNED.

- **The Higgs Mass**

In the presence of soft-SUSY-breaking, the light Higgs has ($\tan\beta = h_u/h_d$)

$$\begin{aligned} m_h^2 &\sim m_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} v^2 y_t^4 \sin^4\beta \log\left(\frac{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}{m_t^2}\right) + \ldots \\ &\overset{\text{large} \tan\beta}{\sim} (91~\mathrm{GeV})^2 + (38~\mathrm{GeV})^2 \log\left(\frac{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}{m_t^2}\right). \end{aligned} \quad (1)$$

A Higgs mass of order $100~\mathrm{GeV}$, as predicted for stop masses $\sim 2m_t$, is in wonderful accord with precision electroweak data.



So, why haven't we seen the Higgs? Is SUSY wrong, or is the MSSM too simple? We argue for the latter!

MSSM Problems

- **The μ parameter** in $W \ni \mu \widehat{H}_u \widehat{H}_d$,[1] is dimensionful, unlike all other superpotential parameters. A **big question** is why is it $\mathcal{O}(1~\mathrm{TeV})$ (as required for EWSB and $m_{\widetilde{\chi}_1^\pm}$ lower bound), rather than $\mathcal{O}(M_U, M_\mathrm{P})$ or 0.

  Currently no satisfactory approach within the MSSM context.

- **LEP limits:**

  In the MSSM, the lightest Higgs boson is typically very SM-like (no time to discuss exceptions such as CP violation in the Higgs sector).

  The LEP limit for a SM-like Higgs boson is $m_h > 114.4~\mathrm{GeV}$.

  In contrast, if sparticles have masses below a TeV, $m_h \sim 100~\mathrm{GeV}$ is the natural prediction.

  From earlier formula, we need $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}} > 900~\mathrm{GeV}$ for $m_h > 114.4~\mathrm{GeV}$. This leads to a new problem.

[1] Hatted (unhatted) capital letters denote superfields (scalar superfield components).

- **Fine-tuning**

  m_Z^2 is very sensitive to GUT scale parameters if sparticles masses, especially those of the stops and the gluino, are large.

  We need a very light gluino and a rather light stop to avoid fine-tuning.

  A rigorous measure of fine-tuning is

$$F = \mathrm{Max}_p \left| \frac{p}{m_Z} \frac{\partial m_Z}{\partial p} \right| , \tag{2}$$

  where p runs over all GUT scale parameters.

  $F > 20$ means worse than 5% fine tuning = bad.

  In the MSSM, if $m_h \sim 100~\mathrm{GeV}$ was ok (*i.e.* not LEP excluded), then small $F < 20$ would be possible.

  Instead, for heavy enough sparticles that $m_h > 114.4~\mathrm{GeV}$ the smallest value of F (avoiding extreme stop mixing) is $F \sim 125$ which is equivalent to worse than 1% fine tuning.

  One can do somewhat better if stop mixing is very large — but never as good as the NMSSM scenario we describe, which has $F < 15$, even as small as $F \sim 5$, without any extreme parameter choices.

- **So, what direction should one head in?**
  - CP-violating MSSM, e.g. CPX-like scenarios?
    These don't solve the μ issue, and nature has shown very little inclination for CP-violation as large as that needed to significantly alter the CP-conserving situation.
  - Large extra dimensions, little Higgs, Higgsless, ...
    All worth exploring, but these models are complicated and typically have problems of one kind or another, especially precision EW data.
  - Hints from string theory.
    Extra singlet superfields are common in string models.
    If we make use of singlets in the simplest possible way (i.e. no associated gauge group and no dimensionful superpotential parameters) $\Rightarrow$ the NMSSM.

The NMSSM

- **The NMSSM introduces just one extra singlet superfield, with superpotential $W \ni \lambda \widehat{S}\widehat{H}_u\widehat{H}_d$. The μ parameter is then automatically generated by $\langle S\rangle$ leading to $\mu_{eff}\widehat{H}_u\widehat{H}_d$ with $\mu_{eff} = \lambda\langle S\rangle$. The only requirement is that $\langle S\rangle$ be of order the SUSY-breaking scale at $\sim 1~\mathrm{TeV}$.**

- **However, $\lambda \widehat{S}\widehat{H}_u\widehat{H}_d$ cannot be the end.**

  To avoid a massless axion, we require an additional superpotential term, $W \ni \frac{1}{3}\kappa\widehat{S}^3$.

  Other possible superpotential terms with dimensionful parameters are absent if one demands that the superpotential be invariant under a Z_3 symmetry.

  If the Z_3 is applied also to soft SUSY breaking terms, only $\frac{1}{3}\kappa A_\kappa S^3$ is allowed in addition to $\lambda A_\lambda S H_u H_d$.

- **Net Result**

Since the only *relevant* superpotential terms that are introduced have dimensionless couplings, the scale of the vevs (i.e. the scale of EWSB) is determined by the scale of SUSY-breaking.

- Further, all the good properties of the MSSM (coupling unification and RGE EWSB, in particular) are preserved under singlet addition.

- New Particles

  The single extra singlet superfield of the NMSSM contains an extra neutral gaugino (the singlino) ($\Rightarrow \widetilde{\chi}^0_{1,2,3,4,5}$), an extra CP-even Higgs boson ($\Rightarrow h_{1,2,3}$) and an extra CP-odd Higgs boson ($\Rightarrow a_{1,2}$).

- The parameters of the NMSSM

  Apart from λ, κ, A_λ and A_κ the other two crucial Higgs sector parameters are

$$\tan\beta = h_u/h_d\,, \quad \mu_{\rm eff} = \lambda s\,, \tag{3}$$

  where $h_u \equiv \langle H_u \rangle$, $h_d \equiv \langle H_d \rangle$ and $s \equiv \langle S \rangle$.

  In addition, values must be input for the gaugino masses and for the soft terms related to the (third generation) squarks and sleptons that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths.

- To further study of the NMSSM Higgs sector, Ellwanger, Hugonie and I constructed NMHDECAY

  `http://www.th.u-psud.fr/NMHDECAY/nmssmtools.html`

  It computes all aspects of the Higgs sector and checks against most LEP limits and various other constraints.

- We also developed a program to examine the LHC observability of Higgs signals in the NMSSM.

  In a series of papers (beginning with JFG+Haber+Moroi at Snowmass 1996 and continued by JFG, Ellwanger, Hugonie, Moretti, Miller, .. .) we have demonstrated a significant "hole" in the LHC no-lose theorem for Higgs discovery.

  Only if we avoid that part of parameter space for which $h \to aa$ and similar decays are present is there a guarantee for finding a Higgs boson at the LHC in one of the nine "standard" channels (e.g. $h \to \gamma\gamma$, $t\bar{t}h, a \to t\bar{t}b\bar{b}$, $t\bar{t}h, a \to t\bar{t}\gamma\gamma$, $b\bar{b}h, a \to b\bar{b}\tau^+\tau^-$, $WW \to h \to \tau^+\tau^-$, ...

- **The portion of parameter space with $h \to aa, \ldots$ is small $\Rightarrow$ one is tempted to ignore it were it not for the fact that it is where fine-tuning can be absent.**

  As before, the canonical measure of fine-tuning employed is

$$F = \mathrm{Max}_p F_p \equiv \mathrm{Max}_p \left| \frac{d \log m_Z}{d \log p} \right| , \quad (4)$$

  where the parameters p comprise the GUT-scale values of λ, κ, A_λ, A_κ, and the usual soft-SUSY-breaking gaugino, squark, slepton, . . . masses.

- **How do we get small fine-tuning?**

  1. **F can be small ($F < 7$, $\Rightarrow$ fine-tuning no worse than 15%) for $m_{h_1} \sim 100 \div 104$ GeV (in a totally unconstrained scan of parameter space this is just what one finds for moderate $\tan\beta$). Neither lower nor higher! See figure on next page.**
     $m_{h_1} \sim 100$ GeV requires $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}} \sim 350$ GeV, as expected.
  2. **$m_{h_1} \sim 100$ GeV is only LEP-allowed if the main decay is $h_1 \to a_1 a_1$ and if $a_1 \to \tau^+\tau^-$ ($2m_\tau < m_{a_1} < 2m_b$) or $gg, q\overline{q}$ ($m_{a_1} < 2m_\tau$) so that $h_1 \to a_1 a_1 \to 4b's$ does not contribute to the strongly limited (for**

$m_{h_1} < 110\ \mathrm{GeV}$) $Z + b's$ final state at LEP.

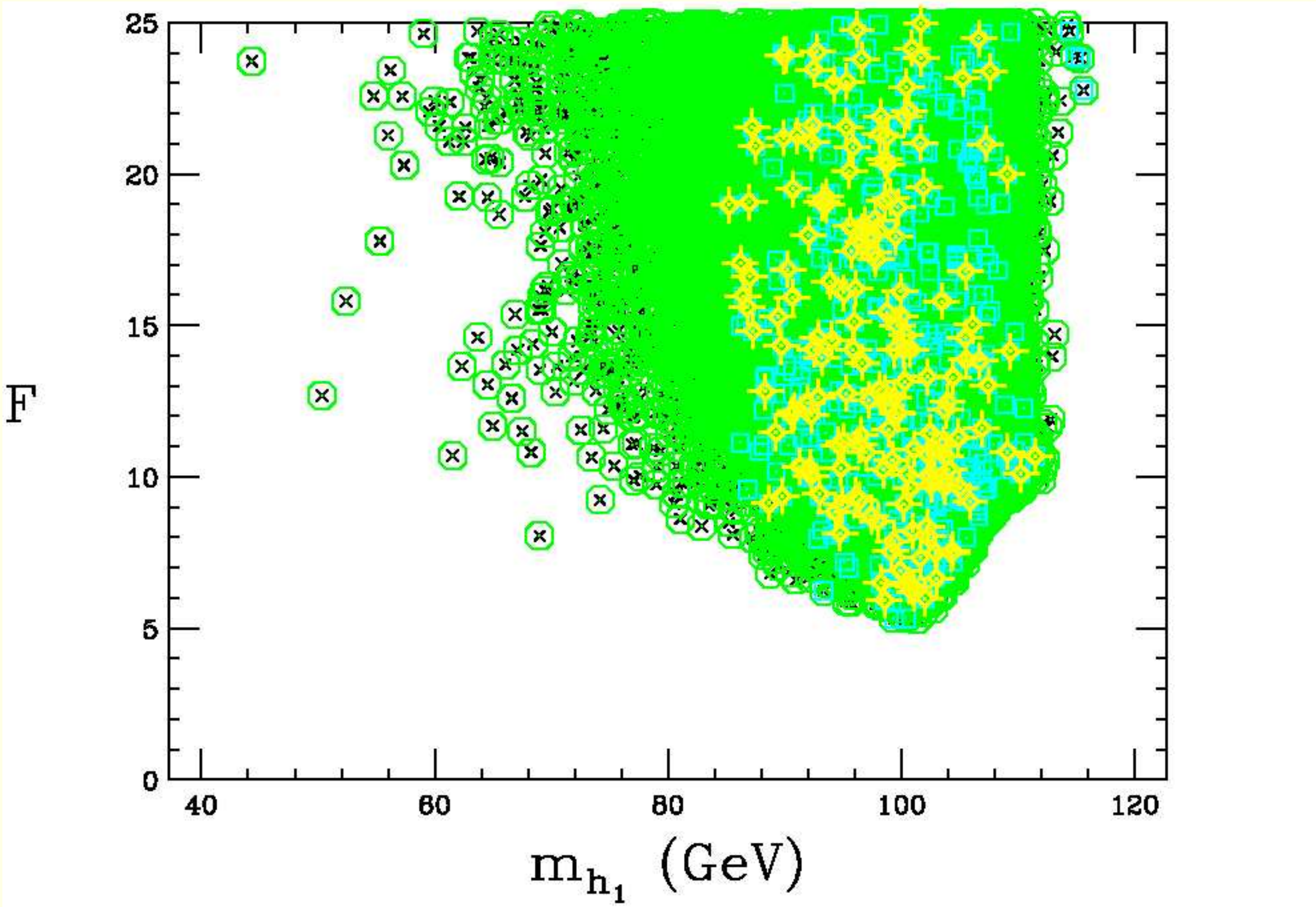


Figure 3: F vs. m_{h_1} for $M_{1,2,3} = 100, 200, 300\ \mathrm{GeV}$ and $\tan\beta = 10$. Small $\times$ = no constraints other than global and local minimum, no Landau pole before M_U and neutralino LSP. The O's = stop and chargino limits imposed, but NO Higgs limits. The □'s = all LEP single channel, in particular $Z + 2b$, Higgs limits imposed. The large FANCY CROSSES are after requiring $m_{a_1} < 2m_b$, so that LEP limits on $Z + b's$, where $b's = 2b + 4b$, are not violated.

3. An important issue: How natural is a light a_1 with $B(h_1 \to a_1 a_1)$ large? This is the topic of hep-ph/0611142. We only state some results. First some preliminaries.

- $m_{a_1} \to 0$ if $A_\kappa(m_Z), A_\lambda(m_Z) \to 0$ (associated with a $U(1)_R$ symmetry limit).
- However, in this limit $B(h_1 \to a_1 a_1) \lesssim 0.2$, which is insufficient to decrease $B(h_1 \to b\bar{b})$ to the $\lesssim 0.2$ level needed for $m_{h_1} \sim 100$ GeV to escape LEP limits.
- If $A_\kappa(M_U), A_\lambda(M_U) \sim 0$ (possibly a nice model choice), then renormalization group equations (RGE's) generate

$$|A_\lambda(m_Z)| \sim 100 - 200 \text{ GeV}, \quad |A_\kappa(m_Z)| \sim few \text{ GeV}. \tag{5}$$

  This is just what is needed to get large $B(h_1 \to a_1 a_1)$; see below.
- What is actually crucial is the composition of a_1 in terms of how much singlet Higgs, a_S, and how much doublet Higgs, A_{MSSM}:

$$a_1 \equiv \cos\theta_A A_{MSSM} + \sin\theta_A a_S . \tag{6}$$

  To get $B(h_1 \to a_1 a_1) > 0.7$, as required by LEP limits for $m_{h_1} \sim 100$ GeV, requires $|\cos\theta_A| \gtrsim 0.05$ (at $\tan\beta = 10$), and this can only be achieved for $|A_\lambda| \gtrsim 100~\mathrm{GeV}$ and $|A_\kappa| \gtrsim few~\mathrm{GeV}$, as predicted by RGE, as noted in Eq. (5).

4. So, is $m_{a_1} < 2m_b$ with sufficiently large $|A_\lambda|, |A_\kappa|$ to have $|\cos\theta_A| \gtrsim 0.05$ achievable without fine tuning the GUT scale boundary conditions?
   Answer 1: For certain types of M_U-scale boundary conditions, this can be totally automatic.
   Answer 2: More generally, there is a measure called G. Small G implies it is quite natural to get small m_{a_1} even for fairly general M_U-scale boundary conditions.
   We plot G as a function of $\cos\theta_A$ for various bins of m_{a_1} on the next page. All plotted points have $B(h_1 \to a_1 a_1) > 0.7$.
   We see that small G is only achieved for the black, green and red points, and not for the blue points. The blue points have $m_{a_1} < 2m_\tau$.

Net Result: Small G requires $m_{a_1} > 2m_\tau$ and $\cos\theta_A \sim -0.1$, (at $\tan\beta = 10$).
This $\cos\theta_A$ value is just fine for large $B(h_1 \to a_1 a_1)$.

As stated earlier, an appropriate a_1 scenario can be achieved without small G for some GUT boundary conditions. Thus, the above value of $\cos\theta_A$ and mass region $m_{a_1} > 2m_\tau$ should not be overly weighted in designing experiments.

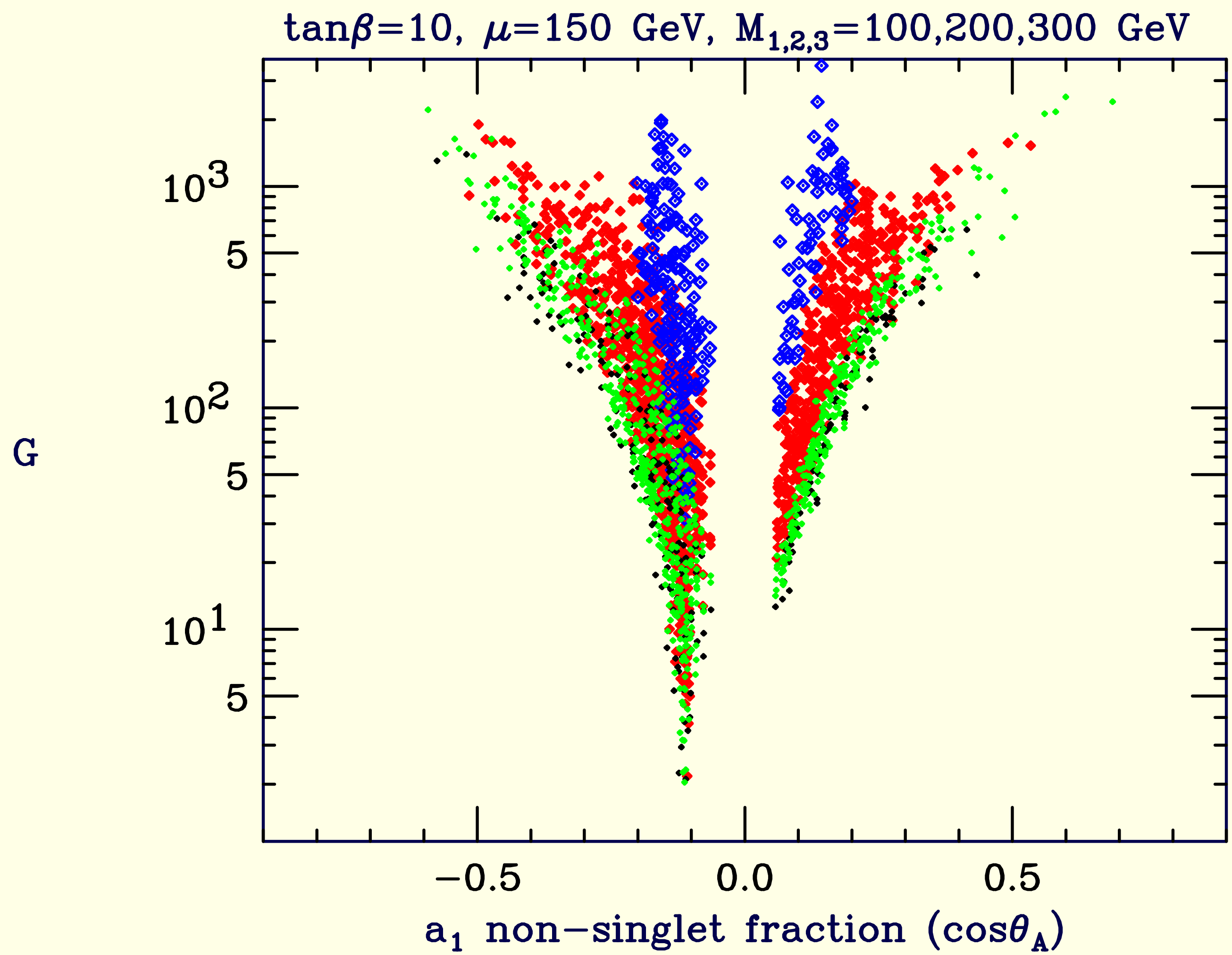


Figure 4: G vs. $\cos\theta_A$ for $M_{1,2,3} = 100, 200, 300$ GeV, $\mu_{\rm eff} = 150$ GeV and $\tan\beta = 10$ for a selection of scenarios with $B(h_1 \to a_1 a_1) > 0.7$ and $m_{a_1} < 2m_b$. The color coding is: blue $= m_{a_1} < 2m_\tau$; red $= 2m_\tau < m_{a_1} < 7.5$ GeV; green $= 7.5$ GeV $< m_{a_1} < 8.8$ GeV; and black $= 8.8$ GeV $< m_{a_1} < 9.2$ GeV. The plot is for $\tan\beta = 10$ and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV.

More Details on EWSB Fine-Tuning and LEP limits

- All low F scenarios have a rather SM-like h_1 as regards WW, ZZ and $f\overline{f}$ couplings, it is just LEP limits require that the primary decay is $h_1 \to a_1 a_1$, leaving $B(h_1 \to b\overline{b}) < 0.2$.

- Putting the ($m_{h_1} \sim 100~\mathrm{GeV}$) $F < 10$ scenarios with $m_{a_1} > 2m_b$ through the full LEP LHWG analysis, one finds that all are excluded at somewhat more than the 99% CL since the combined $Z + 2b$ and $Z + 4b$ signals contribute too large a net $Z + b's$ signal. $m_{h_1} \gtrsim 108~\mathrm{GeV}$ is needed to escape the $Z + b's$ constraint.

- The only way to achieve really low F, which comes with low $m_{h_1} \sim 100~\mathrm{GeV}$, and remain consistent with LEP is to have $m_{a_1} < 2m_b$, so that $h_1 \to a_1 a_1 \to 4\tau$ or $4j$.

  In this regard, it is important to note that LEP has never placed limits on the $e^+e^- \to Zh \to Z4\tau$ channel for h masses larger than about $87~\mathrm{GeV}$.

- What about the remainder in the $Zh \to Z + 2b$ channel. In fact, there

has been a long-standing excess in $Z + 2b$ for $M_{2b} \sim 100$ GeV.

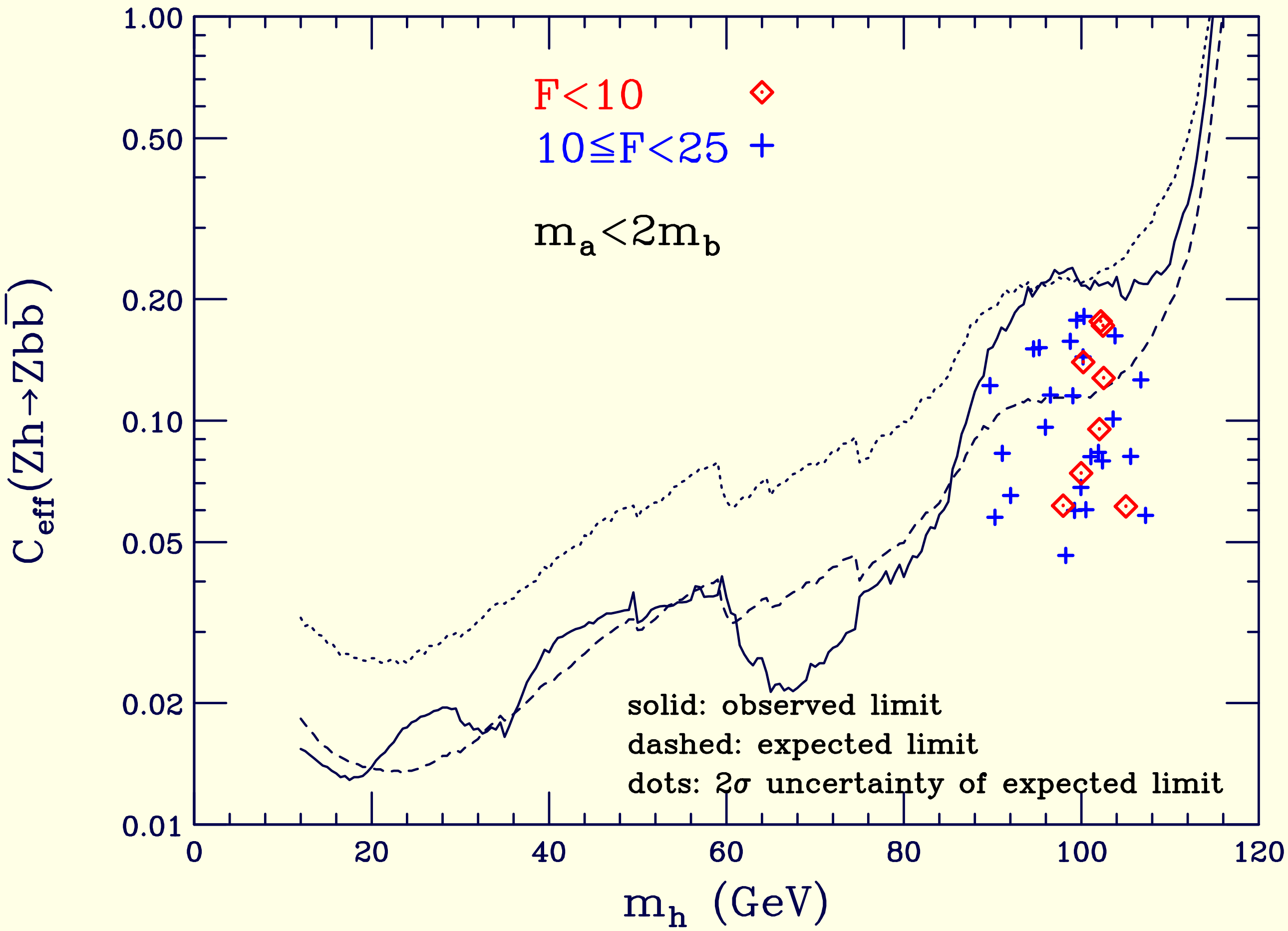


Figure 5: Observed LEP limits on C_{eff}^{2b} for the low-F points with $m_{a_1} < 2m_b$. Note observed limit is far above expected limit (*i.e.* there is an excess of events) near 100 GeV.

Many of the plotted low-F points can describe the excess perfectly.

Another view is to recall the famous $1 - CL_b$ plot for the $Z2b$ channel. (Recall: the smaller $1 - CL_b$ the less consistent is the data with expected background only.)

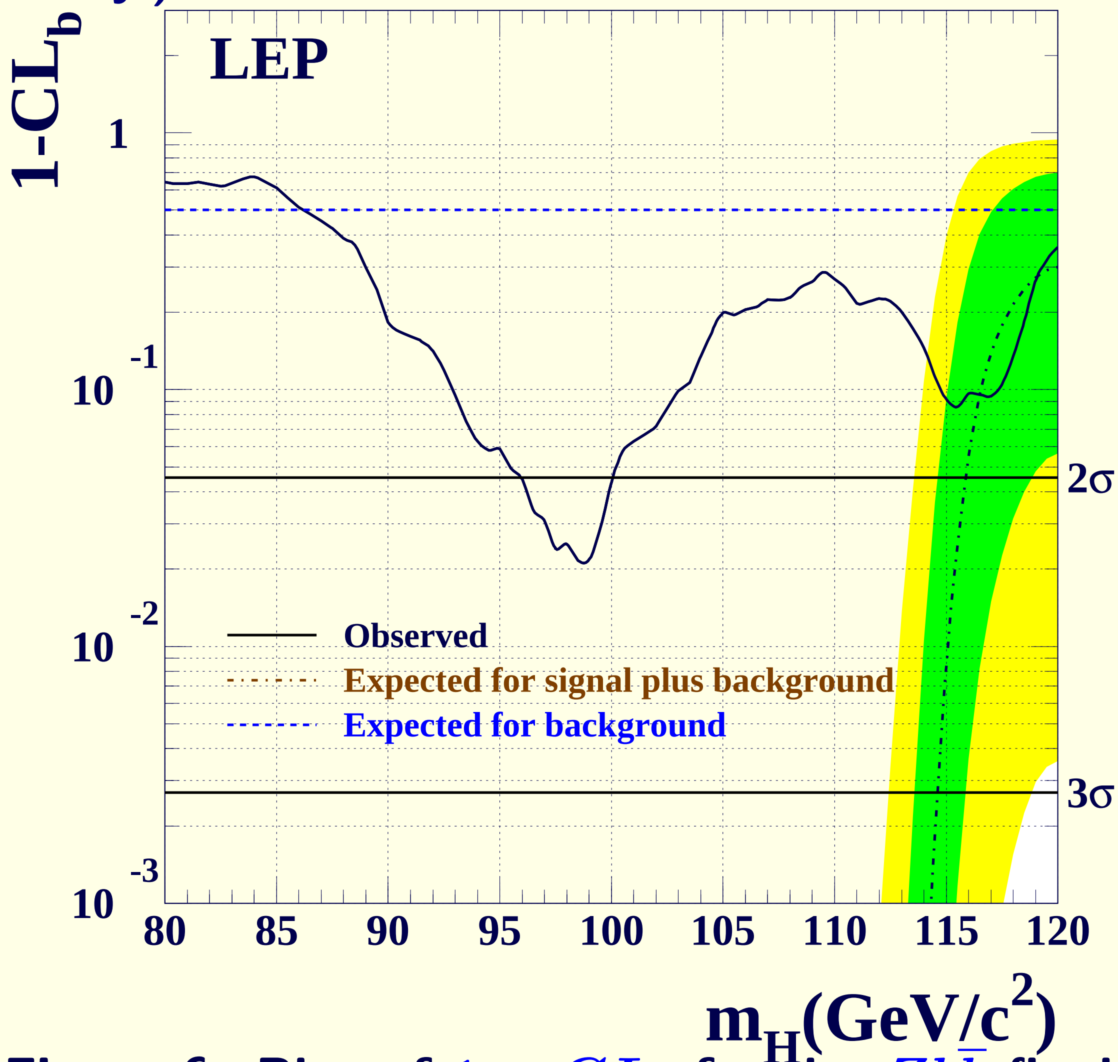


Figure 6: Plot of $1 - CL_b$ for the $Zb\bar{b}$ final state.

- There is an observed vs. expected discrepancy exactly where we want it at

$m_h \sim 100$ GeV! And because $B(h_1 \to b\bar{b})$ is $\sim 1/10$ the SM value for large $B(h_1 \to a_1 a_1)$, the discrepancy is of about the right size.

- **Are there other relevant limits on the kind of scenario we envision?**

  As stated earlier, if the $a_1 a_1 \to 4\tau$ decay is the relevant scenario, the LEP limits run out for $m_h > 87$ GeV.

  If $m_{a_1} < 2m_\tau$ and the $a_1 a_1 \to (gg, q\bar{q}) + (gg, q\bar{q})$ decay is relevant, then we have the hadronic decay limits. They run out for $m_h > 80$ GeV.

- **To see how well the $F < 10$, $m_{a_1} < 2m_b$ points describe the LEP excesses we have to run them through the full LHWG code. Well, we didn't do it, but Philip Bechtle did it for us.**

  The result: Although in our scan there are many, many points that satisfy all constraints and have $m_{a_1} < 2m_b$, the remarkable result is that those with $F < 10$ have a substantial probability that they predict the Higgs boson properties that would imply a LEP $Zh \to Z + b$'s excess of the sort seen.

Summary to this point:

- **The NMSSM is intrinsically a beautiful model, better than the MSSM theoretically even.**
- $F < 10 - 15$ **requires** $m_{h_1} \sim 100~\mathrm{GeV}$, $m_{a_1} < 2m_b$ **and** $|\cos\theta_A| > 0.06$ $(\tan\beta = 10)$.
- **LEP excess at** $M_{2b} \sim 100~\mathrm{GeV}$ **is often perfectly described, since** $B(h_1 \to a_1 a_1) > 0.7$ **typically implies** $B(h_1 \to b\bar{b}) \sim 0.1$.
- $m_{h_1} \sim 100~\mathrm{GeV}$ **is perfect for precision electroweak.**

The question is, how to find the h_1 **and/or the** a_1**?**

- **There is no time for details, but at the moment there is no proven way at the LHC. All standard channels fail:** *e.g.* $B(h_1 \to \gamma\gamma)$ **is much too small because of large** $B(h_1 \to a_1 a_1)$**. The possible new channels include:**

  1. $WW \to h_1 \to a_1 a_1 \to 4\tau$.
     Looks moderately promising but far from definitive results at this time.
  2. $t\bar{t}h_1 \to t\bar{t}a_1 a_1 \to t\bar{t}\tau^+\tau^-\tau^+\tau^-$.
     Study begun.

3. **A third possibility:** $\widetilde{\chi}_2^0 \to h_1 \widetilde{\chi}_1^0$ with $h_1 \to a_1 a_1 \to 4\tau$.
   (Recall that the $\widetilde{\chi}_2^0 \to h_1 \widetilde{\chi}_1^0$ channel provides a signal in the MSSM when $h_1 \to b\overline{b}$ decays are dominant.)
4. **Last, but definitely not least: diffractive production** $pp \to pph_1 \to ppX$.
   The mass M_X can be reconstructed with roughly a $1-2~\mathrm{GeV}$ resolution, potentially revealing a Higgs peak, independent of the decay of the Higgs. Preliminary results are that one expects about 3 clean, i.e. reconstructed and tagged, events per $30~\mathrm{fb}^{-1}$ of luminosity. $\Rightarrow$ clearly a high luminosity game.
5. The rather singlet nature of the a_1 and its low mass, imply no direct production/detection possible at the LHC.

- At the ILC, there is no problem since $e^+e^- \to ZX$ will reveal the $M_X \sim m_{h_1} \sim 100~\mathrm{GeV}$ peak no matter how the h_1 decays.

  But the ILC is decades away.

- As it turns out $\Upsilon \to \gamma a_1$ decays hold great promise for a_1 discovery (or exclusion) as we now outline. **They should be pushed to the limit.** The signal may even be present in the data available now! The only issue will be reaching quite small $B(\Upsilon \to \gamma a_1)$.

Predictions for $B(\Upsilon \to \gamma a_1)$ for small F scenarios

- **One begins with the Wilczek formula:**

$$R \equiv \frac{\Gamma_0(V(1^{--}) \to \gamma a)}{\Gamma_0(V(1^{--}) \to \mu^+\mu^-)} = \frac{G_F m_Q^2}{\sqrt{2}\pi\alpha}\left(1 - \frac{m_a^2}{M^2}\right) \quad (7)$$

which assumes 'standard' γ_5 Yukawa coupling of the a with SM-like analogue strength. The $_0$ means tree-level.

- **Various corrections must then be made.**

1. **QCD radiative corrections ala Vysotsky and Nason:**

$$\Gamma(V \to \gamma a) = \Gamma_0(V \to \gamma a)\left[1 - \frac{\alpha_s C_F}{\pi} a_P(z) + \mathcal{O}(\alpha_s^2)\right] \quad (8)$$

where $z \equiv 1 - m_a^2/M^2$. $a_P(z)$ ranges from ~ 2 at $z = 0$ ($m_{a_1} = M$) to ~ 6 at $z = 1$ ($m_a = 0$). In relating to experimentally measured

$\Gamma(V \to \mu^+\mu^-)$ from PDG, we must also include radiative corrections for this mode:

$$\Gamma(V \to \mu^+\mu^-) = \Gamma_0(V \to \mu^+\mu^-)\left[1 - \frac{4\alpha_s C_F}{\pi}\right]. \tag{9}$$

2. Bound state corrections: we employ the calculation of Pantaleone, Peskin, and Tye.
   These corrections, while yielding a big suppression for a scalar Higgs, yield a very modest enhancement for a pseudoscalar. The enhancement is typically $< 1\%$ for small m_a, rising to $\sim 10\%$ for $8 \lesssim m_a \lesssim 9.2~\mathrm{GeV}$. For $m_a \gtrsim 9.2~\mathrm{GeV}$ the a starts to mix with the η_b. We do not present results in this region since we doubt the reliability of available computations there.
3. Relativistic Corrections: we use those from Aznauryan, Grigoryan, and Matinyan.

$$R_{rel} = R_0\left(\frac{M_\Upsilon^2 - m_a^2}{4m_b^2 - m_a^2}\right)^2\left[1 - \tfrac{1}{3}\Delta\left(\frac{36m_b^2 + m_a^2}{4m_b^2 - m_a^2}\right)\right] \tag{10}$$

where

$$m_b^2\Delta \equiv \frac{\int \psi(p^2)p^4 dp}{\int \psi(p^2)p^2 dp}, \tag{11}$$

with $\psi(p^2)$ being the radial part of the wave function of the b quarks in the Υ. We employ a value of Δ that fits well their plots. The result is substantial suppression. For example, $R_{rel} \sim \frac{1}{2}R_0$ for $m_a < 4~\mathrm{GeV}$. Suppression is even larger at large m_a.

4. Coupling Correction: The a_1 does not couple with SM-analogue strength. Its coupling is enhanced by $\tan\beta$ and suppressed by the smallness of its A_{MSSM} component fraction $\cos\theta_A$ — the a_S singlet component does not couple to $b\bar{b}$. The precise coupling correction factor is then:

$$R_{a_1} = R_{\text{'SM-like'}\, a} \times (\tan\beta\cos\theta_A)^2\,. \tag{12}$$

- **THE RESULTS**

We will not plot points that violate the $B(\Upsilon \to \gamma a_1)$ limits of Fig. 3 of [1], Fig. 4 of [2], and Fig. 7b of [3].

The first two limit $B(\Upsilon \to \gamma X)$, where X is any visible state.

The first provides the only strong constraint on the $m_{a_1} < 2m_\tau$ region.

The third gives limits on $B(\Upsilon \to \gamma X)B(X \to \tau^+\tau^-)$ that eliminate $2m_\tau < m_{a_1} < 8.8~\mathrm{GeV}$ points with too high $B(\Upsilon \to \gamma a_1)$ (for $m_{a_1} > 2m_\tau$, $B(a_1 \to \tau^+\tau^-) \sim 0.9$).

Since the inclusive photon spectrum from Υ decays falls as E_γ increases, the strongest constraints are obtained for small m_{a_1}.

References

[1] P. Franzini *et al.*, Phys. Rev. D 35, 2883 (1987).

[2] H. Albrecht *et al.* [ARGUS Collaboration], Phys. Lett. B 154, 452 (1985).

[3] H. Albrecht *et al.* [ARGUS Collaboration], Z. Phys. C 29, 167 (1985).

In the first figure, we focus on the $\tan\beta = 10$ case with $M_{1,2,3}(m_Z) = 100, 200, 300~\mathrm{GeV}$ for which our previous plots were given. Two types of plot are shown:

1. A scan in A_λ, A_κ space at fixed $\mu_{\mathrm{eff}} = 150~\mathrm{GeV}$, requiring $B(h_1 \to a_1 a_1) > 0.7$ and $m_{a_1} < 9.2~\mathrm{GeV}$.
2. Results for the $F < 15$ points of Fig. 3.

In the second figure, we look at results for the A_λ, A_κ scans for $\tan\beta = 3$ and $\tan\beta = 50$.

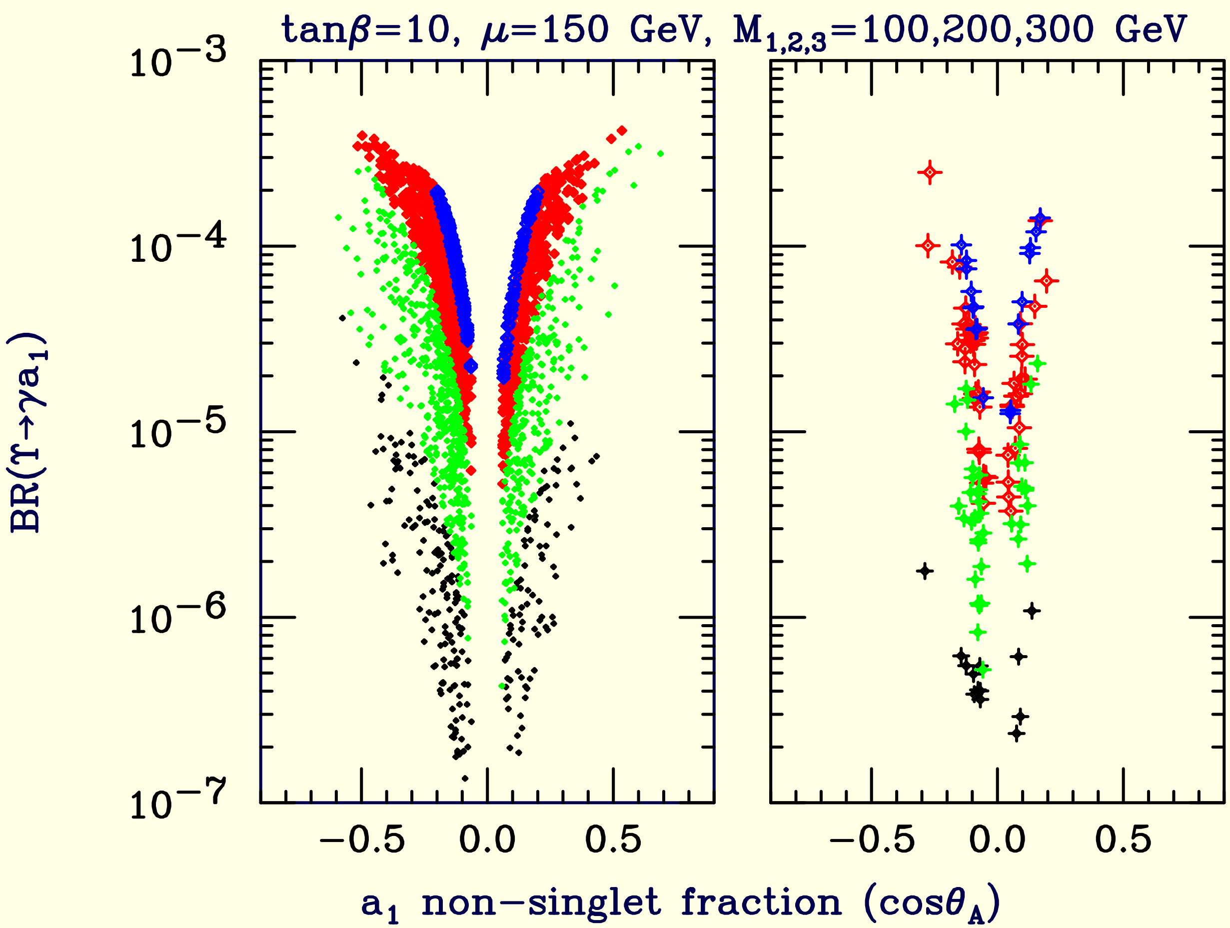


Figure 7: $B(\Upsilon \to \gamma a_1)$ for NMSSM scenarios with various ranges for m_{a_1} using color scheme of Fig. 4 (blue= $< 2m_\tau$, red=$[2m_\tau, 7.5]$, green=$[7.5, 8.8]$, black=$[8.8, 9.2]$). The left plot comes from the A_λ, A_κ scan described in the text, holding $\mu_{\rm eff}(m_Z) = 150$ GeV fixed. The right plot shows results for $F < 15$ scenarios with $m_{a_1} < 9.2$ GeV found in a general scan over all NMSSM parameters holding $\tan\beta$ and $M_{1,2,3}$ fixed as stated. The lower bound on $B(\Upsilon \to \gamma a_1)$ arises basically from the LEP requirement of $B(h_1 \to a_1 a_1) > 0.7$.

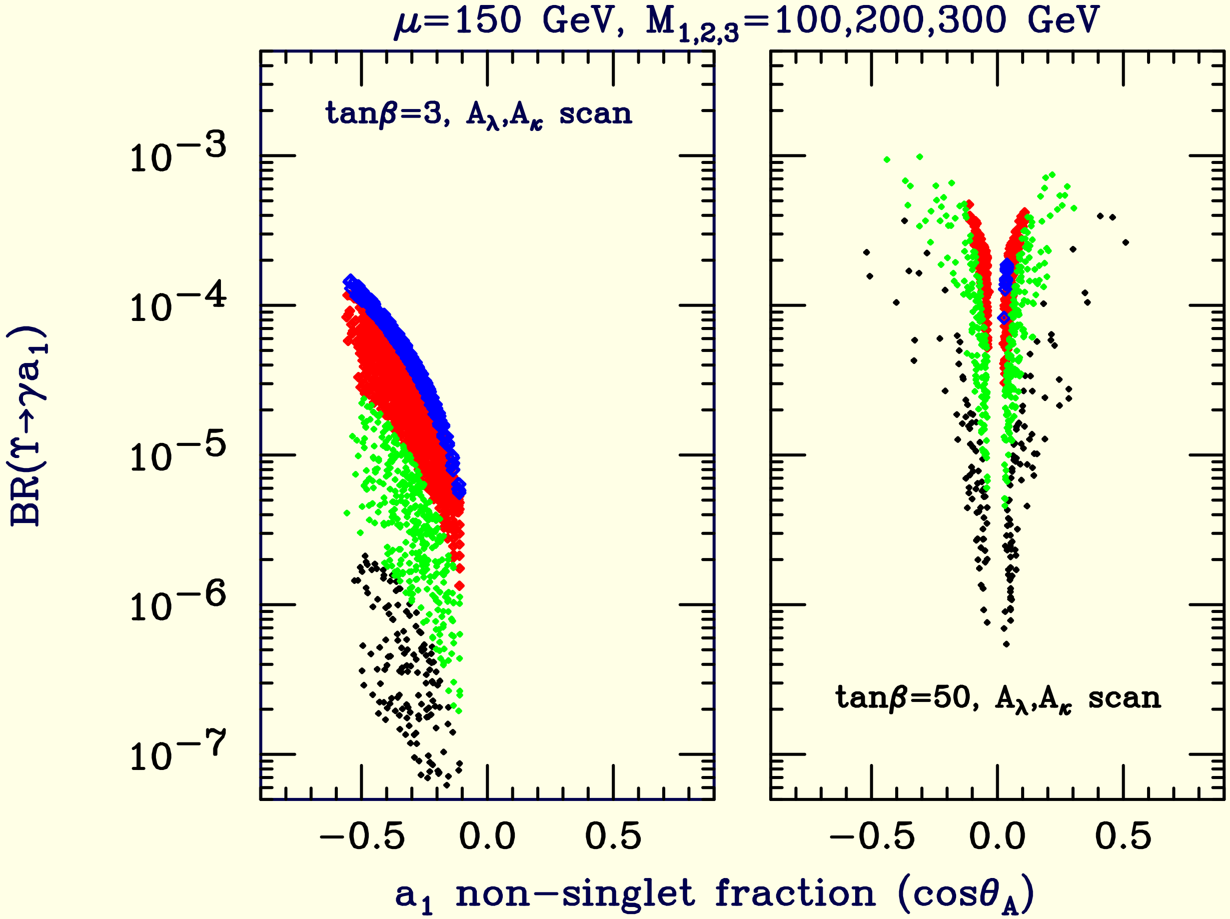


Figure 8: We plot $B(\Upsilon \to \gamma a_1)$ as a function of $\cos\theta_A$ for the A_λ, A_κ scan, taking $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV, $\mu_{\rm eff}(m_Z) = 150$ GeV with $\tan\beta = 3$ (left) and $\tan\beta = 50$ (right). The point notation is as before: blue= $< 2m_\tau$, red=$[2m_\tau, 7.5]$, green=$[7.5, 8.8]$, black=$[8.8, 9.2]$.

- **Summary of $B(\Upsilon \to \gamma a_1)$ results:**

1. There are some large $B(\Upsilon \to \gamma a_1)$'s that might or might not have A_λ, A_κ fine-tuning issues (large G) waiting to be excluded by existing data.
2. At $\tan\beta = 10$, small G points with $\cos\theta_A \sim -0.1$ (red, green and black) have $B(\Upsilon \to \gamma a_1)$ ranging from $\lesssim few \times 10^{-5}$ for $2m_\tau < m_{a_1} < 7.5~\mathrm{GeV}$ (red) to $\sim few \times 10^{-7}$ for $8.8~\mathrm{GeV} < m_{a_1} < 9.2~\mathrm{GeV}$ (black).
3. At $\tan\beta = 3$, the $B(\Upsilon \to \gamma a_1)$ range is suppressed further.
4. At $\tan\beta = 50$, $B(\Upsilon \to \gamma a_1) \gtrsim 10^{-6}$ for all points with $m_{a_1} < 9.2~\mathrm{GeV}$.
5. We stress again that the lower bounds on $B(\Upsilon \to \gamma a_1)$ arise from the LEP requirement that $B(h_1 \to a_1 a_1) > 0.7$.
6. Of course, we cannot exclude the possibility that $9.2~\mathrm{GeV} < m_{a_1} < 2m_b$. Phase space for the decay causes increasingly severe suppression.
   And, there is the small region of $M_\Upsilon < m_{a_1} < 2m_b$ that cannot be covered by Υ decays.
7. However, if $B(\Upsilon \to \gamma a_1)$ sensitivity can be pushed down to the 10^{-7} level, you may well discover the a_1.

Conclusions

- **The NMSSM can naturally have small tine-tuning of GUT-scale pamameters for both:**

  1) Electroweak Symmetry Breaking, $i.e.$ **getting the measured value of** m_Z^2**;**

  2) Small $m_{a_1} < 2m_b$ **and (simultaneously) large** $B(h_1 \to a_1 a_1)$**, both of which are needed to escape LEP limits for** $m_{h_1} \sim 100~\mathrm{GeV}$ **[the latter being required for 1)].**

  $m_{a_1} > 2m_\tau$ **is somewhat preferred by this latter fine-tuning issue.**

- **If low EWSB fine-tuning is imposed for an acceptable SUSY model, we should expect:**

  – **a** h_1 **with** $m_{h_1} \sim 100~\mathrm{GeV}$ **and SM-like couplings to SM particles but with primary decays** $h_1 \to a_1 a_1$ **with** $m_{a_1} < 2m_b$**, where the** a_1 **is mainly singlet. The collider implications are:**

    Higgs detection will be quite challenging at a hadron collider.
    Higgs detection at the ILC is easy using the missing mass $e^+e^- \to ZX$ **method of looking for a peak in** M_X**.**
    Higgs detection in $\gamma\gamma \to h_1 \to a_1 a_1$ **will be easy.**

Detection of the a_1 could easily result from pushing on $\Upsilon \to \gamma a_1$.

– the stops and other squarks are light;

– the gluino, and, by implication assuming conventional mass orderings, the wino and bino all have modest mass;

- Even if the LHC sees the Higgs $h_1 \to a_1 a_1$ directly, it will not be able to get much detail. Only the ILC and possibly B-factory results for $\Upsilon \to \gamma a_1$ can provide the details needed to verify the model.
- It is likely that other models in which the MSSM μ parameter is generated using additional scalar fields can achieve small fine-tuning in a manner similar to the NMSSM.

  Low fine-tuning typically requires low SUSY masses which in turn typically imply $m_{h_1} \sim 100~\mathrm{GeV}$.

  And, to escape LEP limits in the $Z + b's$ channel, large $B(h_1 \to a_1 a_1)$ with $m_{a_1} < 2m_b$ would be needed.

  In general, the a_1 might not need to be so singlet as in the NMSSM and would then have larger $B(\Upsilon \to \gamma a_1)$.
- Although SUSY will be easily seen at the LHC, Higgs detection at the LHC

may prove to be a real challenge.

Ability to check perturbativity of $WW \to WW$ at the LHC might prove to be very crucial to make sure that there really is a light Higgs accompanying light SUSY.

- A light a_1 allows for a light $\widetilde{\chi}_1^0$ to be responsible for dark matter of correct relic density: annihilation would be via $\widetilde{\chi}_1^0\widetilde{\chi}_1^0 \to a_1$. To check the details, properties of the a_1 will need to be known fairly precisely

  The ILC might (but might not) be able to measure the properties of the very light $\widetilde{\chi}_1^0$ and of the a_1 in sufficient detail to verify that it all fits together.

  But, also $\Upsilon \to \gamma a_1$ decay information would help tremendously.

- Thus,

  Large B-factory data sets, optimally using $\Upsilon(3S) \to \pi^+\pi^-\Upsilon(1S)$ to tag the $1S$ state, should be pursued with great vigor.

  Unless, of course, it is found that backgrounds (most notably from $\Upsilon \to \gamma\tau^+\tau^-$) are insurmountable at the needed level.

  What are the limits? We have had a brief look, but clearly this is a job for experimentalists.

- In the $\gamma\tau^+\tau^-$ final state, the direct $\gamma\tau^+\tau^-$ production cross section is 61 pb.

  Using signal=background as the criterion, this becomes the limiting factor for branching ratios below the 4×10^{-5} level when running on the $\Upsilon(1S)$, and below the 2×10^{-4} level when running on the $\Upsilon(3S)$.
- To improve upon the latter, one can select a sample of known $\Upsilon(1S)$ events by looking for dipion transitions from the higher resonances.

  The dipion transition gives a strong kinematic constraint on the mass difference between the two Υ's.

  When running on the $\Upsilon(3S)$, the effective cross section in $\Upsilon(3S) \to \pi^+\pi^-\Upsilon(1S)$ is 179 pb (see archive Glenn:1998bd)

  To limit $B(\Upsilon \to \gamma a_1) \lesssim 10^{-6}$, $5.6~\mathrm{fb}^{-1}/\epsilon$ would need to be collected on the $\Upsilon(3S)$, where ϵ is the experimental efficiency for isolating the relevant events.
- This analysis can also be done on the $\Upsilon(4S)$, where the $\Upsilon(3S)$ is produced via ISR. The effective $\gamma_{ISR}\Upsilon(3S) \to \gamma_{ISR}\pi^+\pi^-\Upsilon(1S)$ cross section is 0.78 fb.

  To limit $B(\Upsilon \to \gamma a_1) \lesssim 10^{-6}$, $1.3~\mathrm{ab}^{-1}/\epsilon$ would need to be collected.
- These integrated luminosities needed to probe $B(\Upsilon \to \gamma a_1) \sim 10^{-6}$ would appear to be within reach at existing facilities and would allow discovery of the a_1 for many of the favored NMSSM scenarios.

- **Further Comments**

  – Of course, one should consider $b \to sa_1$ inclusive decays (also exclusive). We are working on this and have some preliminary results based on the formulas given by Hiller.
  These results suggest that $b \to sa_1 \to s\mu^+\mu^-$ limits may exclude most of the $m_{a_1} < m_b$ scenarios, which in any case are less preferred by A_λ, A_κ tuning issues.

  – $a_1 \to \gamma\gamma$ branching ratios remain very small in our scenarios because of the lower bound on $\cos\theta_A$, which implies that the a_1 has a minimum non-singlet component, in particular sufficient that a_1 decays to SM fermions dominate.
  For the general A_λ, A_κ scans with $B(h_1 \to a_1 a_1) > 0.7$ and $m_{a_1} < 2m_b$ imposed, $B(a_1 \to \gamma\gamma) < 4 \times 10^{-4}$ with values near $few \times 10^{-5}$ being very common.
  $\Rightarrow$ the a_1 search strategies suggested by Cheung and collaborators will not work for these scenarios.
  Is it conceivable that a super-B factory could detect a signal for $\Upsilon \to \gamma a_1 \to \gamma\gamma\gamma$ with branching ratio at the 10^{-10} level?
  The needed number of Υ's is a stretch to say the least. But, presumably

backgrounds for three monochromatic photons are very tiny.

Certainly detection in this channel would provide a very interesting discovery and/or check on the consistency of the model.

– Could the $\zeta(8.3)$ have been real?

Obviously not at the level originally seen, but the mass fits perfectly with our scenarios.

In any case, we really hope that you will take the problem of Electroweak fine-tuning, and the NMSSM solution thereto, seriously.

After all, it fits perfectly with precision electroweak preference for a $m_{h_1} \sim 100~\mathrm{GeV}$ Higgs and with the $Z + 2b$ signal in the $M_{2b} \sim 100~\mathrm{GeV}$ region.

If you do, there is a very compelling case for pushing $\Upsilon \to \gamma a_1$ searches to the absolute extreme.