Exploring the CP Properties of the Higgs Sector at the Higgs Factory

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Outline

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• **General Outline of Techniques at LHC, LC and MUC**

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- Details on the MUC
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  – Isolated Higgs Eigenstate of Unknown CP Nature
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• Details on the MUC
  – Isolated Higgs Eigenstate of Unknown CP Nature
  – Overlapping $H^0$ and $A^0$ SUSY Higgs States
CP DETERMINATIONS

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Techniques based on production mechanism

• At LC there are many techniques based on $WW$ and/or $ZZ$ couplings for verifying a substantial $\text{CP}=+$ component.

  But such couplings only sensitive to $\text{CP}=−$ component at loop level in Higgs models. $\Rightarrow$ very hard to see $\text{CP}=−$ coupling even if there.

• Since $\text{CP}=+$ and $\text{CP}=−$ couplings to $t\bar{t}$ of any $h$ are both tree-level ($\bar{t}(a + ib\gamma_5)t$), $t\bar{t}h$ angular distributions allow CP determination for lighter $h$’s. Use optimal observables.

  – At the LC, as long as there is reasonable event rate ($\sqrt{s} > 800$ GeV), this is straightforward. (JFG, Grzadkowski, He), (carried on by TESLA TDR, Reina, Dawson, ...).
  – At the LHC, there will be a high event rate, but reconstruction of $t$ and $\bar{t}$ (identification required) is trickier and backgrounds will be larger. Still, there is considerable promise. (JFG, He; JFG, Pliszka, Sapinski).
LHC experimentalists must convince themselves they can do this.

- CP=+ and CP=− components also couple with similar magnitude but different structure to $\gamma\gamma$ (via 1-loop diagrams),

At the LC, ⇒ use $\gamma\gamma$ collisions. (JFG, Grzadkowski; JFG, Kelly; Djouadi etal, ..)

$$A_{CP=+} \propto \vec{\epsilon}_1 \cdot \vec{\epsilon}_2, \quad A_{CP=-} \propto (\vec{\epsilon}_1 \times \vec{\epsilon}_2) \cdot \hat{p}_{beam}. \quad (1)$$

- For pure CP states, maximize linear polarization and adjust orientation (⊥ for CP odd dominance, || for CP even dominance) to determine CP nature of any Higgs by using appropriate linearly polarized laser photons.

  In particular, can separate $A^0$ from $H^0$ when these are closely degenerate (as typical for $\tan \beta \gtrsim 4$ and $m_{A^0} > 2m_Z$).

- For mixed CP states, can use circularly polarized photons (better luminosity, reduced background) and employ helicity asymmetries to determine CP mixture.

  Recent estimate (JFG+Asner) for SM-like Higgs, ⇒ can demonstrate that CP-even asymmetry = 1 to within $\pm 20\%$ for $L = 500 fb^{-1}$ at the LC.

- At the LHC, can used polarized protons which transmit polarization to the gluons (substantially, according to many estimates) and can then proceed as in $\gamma\gamma$
collisions. (JFG, Yuan) **Backgrounds/sensitivity need experimental study.**

- At a muon collider Higgs factory there is a particularly appealing approach. For resonance, $R$, production at a MUC with $\bar{\mu}(a + ib\gamma_5)\mu$ coupling to the muon,

$$
\bar{\sigma}_S(\zeta) = \bar{\sigma}_S^0 \left( 1 + P_L^+ P_L^- + P_T^+ P_T^- \left[ \frac{a^2 - b^2}{a^2 + b^2} \cos \zeta - \frac{2ab}{a^2 + b^2} \sin \zeta \right] \right)
$$

- $\delta \equiv \tan^{-1} \frac{b}{a}$,

- $P_T$ ($P_L$) is the degree of transverse (longitudinal) polarization: no $P_T \Rightarrow$ sensitivity to $\bar{\sigma}_S^0 \propto a^2 + b^2$ only.

- $\zeta = \text{angle of the } \mu^+ \text{ transverse polarization relative to that of the } \mu^- \text{ as measured using the the direction of the } \mu^- \text{'s momentum as the } \hat{z} \text{ axis.}$

- Only the $\sin \zeta$ term is truly CP-violating, but $\cos \zeta$ also $\Rightarrow$ significant sensitivity to $a/b$.

Ideal = isolate $\frac{a^2 - b^2}{a^2 + b^2}$ and $\frac{-2ab}{a^2 + b^2}$ via the asymmetries (take $P_T^+ = P_T^- \equiv P_T$ and $P_L^\pm = 0$)

$$
\mathcal{A}_I \equiv \frac{\bar{\sigma}_S(\zeta = 0) - \bar{\sigma}_S(\zeta = \pi)}{\bar{\sigma}_S(\zeta = 0) + \bar{\sigma}_S(\zeta = \pi)} = P_T^2 \frac{a^2 - b^2}{a^2 + b^2} = P_T^2 \cos 2\delta,
$$
\[ A_{II} \equiv \frac{\bar{\sigma}_S(\zeta = \pi/2) - \bar{\sigma}_S(\zeta = -\pi/2)}{\bar{\sigma}_S(\zeta = \pi/2) + \bar{\sigma}_S(\zeta = -\pi/2)} = -P_T^2 \frac{2ab}{a^2 + b^2} = -P_T^2 \sin 2\delta. \]

If \( a^2 + b^2 \) already well determined, background is known, and \( A^2 \leq P_T^4 \), and \( P_T < 0.5 \Rightarrow \frac{A^2}{[\delta A]^2} \propto P_T^4 L. \)

But, must account for polarization precession: \( \Rightarrow \) can’t fix polarization directions. But, precession can be easily incorporated (Grzadkowski, JFG, Pliszka – advice from Raja)

A good determination (comparable to LC \( \gamma\gamma \)) of \( b \) and \( a \) is possible if luminosity can be upgraded from SM96 or higher proton source intensity is available.

An important experimental note: We will always imagine Higgs mass is known and take \( \sqrt{s} = \text{mass} \). \( \Rightarrow \) only \( b\bar{b} \) rate (not \( m_{b\bar{b}} \) resolution) matters in issues of \( S \) vs. \( B \).
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Techniques based on self-analyzing Higgs decays

To illustrate, consider \( h \to \tau^+\tau^- \) and \( \tau^\pm \to \pi^\pm\nu \) decays (JFG+Grzadkowski; also Soni and collaborators).

Imagine a general coupling \( \bar{\tau}(a + ib\gamma_5)\tau \): \( a = \text{CP-even}, \ b = \text{CP-odd} \).
Assume we know the Higgs rest frame \((e^+e^- \rightarrow Zh\) or \(\mu^+\mu^- \rightarrow h\) are ideal). ⇒ enough constraints to determine \(\pi^\pm\) directions in \(\tau^\pm\) rest frames.

Define \(\theta, \phi\) and \(\bar{\theta}, \bar{\phi}\) as the angles of \(\pi^-\) and \(\pi^+\) in the \(\tau^-\) and \(\tau^+\) rest frames, respectively, employing the direction of \(\tau^-\) in the \(h\) rest frame as the coordinate-system-defining \(z\) axis. ⇒

\[
dN \propto \left[ (b^2 + a^2\beta^2) (1 + \cos \theta \cos \bar{\theta}) + (b^2 - a^2\beta^2) \sin \theta \sin \bar{\theta} \cos (\phi - \bar{\phi}) 
- 2ab\beta \sin \theta \sin \bar{\theta} \sin (\phi - \bar{\phi}) \right] d\cos \theta d\cos \bar{\theta} d\phi d\bar{\phi},
\]

(3)

The idea is to use the above dependencies to isolate

\[
\rho_1 \equiv \frac{2ab\beta}{(b^2 + a^2\beta^2)}, \quad \rho_2 \equiv \frac{(b^2 - a^2\beta^2)}{(b^2 + a^2\beta^2)}.
\]

(4)

This is most ideally accomplished using projection operators and optimal observable techniques.

Note: For pure CP-even (odd), \(\rho_1 = 0\) and \(\rho_2 = -1\) (+1).

Note: \(\rho_1 \neq 0\) is signal for CP-violating Higgs sector.

What are the errors on these quantities?

\[
\delta \rho_1 = \left[ \frac{9}{2} - \rho_1^2 + (B/S)(9/2 + \rho_1^2) \right]^{1/2} / \sqrt{S},
\]

(5)
\[ \delta \rho_2 = \left[ \frac{9}{2} - \rho_2^2 + (B/S)(\frac{9}{2} + \rho_2^2 - 2\rho_2\rho_B^B) \right]^{1/2} / \sqrt{S}. \] (6)

where \( S \) and \( B \) are the signal and background rates and \( \rho_B^B \) is the \( \rho_2 \) value for the background, which can usually be neglected.

If we assume a modest value for \( S/\sqrt{B} = N_{SD} \), the dominate term is the \( B/S \) term and we have

\[ \delta \rho_i \sim \frac{[\frac{9}{2} + \rho_i^2]^{1/2}}{N_{SD}}. \] (7)

Consider a light SM-like \( h \rightarrow \rho_1 = 0 \) and \( \rho_2 = 1 \).

At the LC or the MUC, for the SM-like case we typically expect 1000's of \( h \)'s to be produced. \( B(h \rightarrow \tau^+\tau^-) \) can be significant, perhaps 10%.

With \( L = 500fb^{-1} \) at LC, \( N_{SD} \geq 15 \) in the \( Zh \rightarrow Z\tau^+\tau^- \) mode for light \( h \).

At muon collider, \( N_{SD} \sim 5 - 10 \) for light \( h \) using pessimistic \( L \) estimates.

As a result, we can expect reasonable accuracies from

\[ \delta \rho_1 \sim 2.2/N_{SD}, \quad \delta \rho_2 \sim 2.3/N_{SD}. \] (8)

The LC accuracy for \( \delta \rho_2/\rho_2 \) of order 15\% is quite good.
Still, the transverse polarization asymmetries at a MUC might do better, as we shall see. They might be only option for $H^0, A^0$ of SUSY!

→ Very important to outline what is necessary to perform the latter measurement and build it into the collider design.
IMPLICATIONS OF PRECESSION FOR CP DETERMINATION

Referring to earlier overview,

- Ideal would be run at fixed $\zeta = 0, \pi/4, \pi/2, 3\pi/2$ and obtain asymmetries, $A_{I,II}$.

- Precession makes this impossible.

Typical configuration: a $\mu^-$ or $\mu^+$ bunch enters the ring with a component $\vec{P}_V$ ($\vec{P}_H$) of polarization vertical (horizontal) with respect to the plane of the storage ring. [$\vec{P}_V$ and $\vec{P}_H$ are defined in the muon rest frame.]

$\vec{P}_H^-$ and $\vec{P}_H^+$ will rotate in the same directions as the $\mu^-$ and $\mu^+$ themselves (that is in opposite directions).

The rate of rotation of the $\vec{P}_H$'s as viewed from the laboratory frame is somewhat different than the rate at which the bunches themselves rotate.

The mismatch means that if, for instance, $\vec{P}_H^-$ were longitudinal at the time the $\mu^-$ bunch first enters the storage ring, it will not remain so but rather it will precess.
into the transverse direction and then back to the opposite longitudinal direction, and so forth.

- Take $\mathbf{B} = -B\hat{y}$, and write

\[
s_{\mu^-} = P_H^- \left[ \gamma(\beta, \hat{z}) \cos \theta^- - (0, \hat{x}) \sin \theta^- \right] + P_V^- (0, \hat{y}). \tag{9}
\]

\[
s_{\mu^+} = P_H^+ \left[ \gamma(\beta, -\hat{z}) \cos \theta^+ - (0, \hat{x}) \sin \theta^+ \right] + P_V^+ (0, \hat{y}). \tag{10}
\]

- $\hat{z}$ is direction of $\mu^-$ instantaneous momentum;
- $P_H$ ($P_V$) is horizontal (vertical, i.e. $\hat{y}$) degree of polarization;
- $P_H^\pm \cos \theta^\pm = P_L^\pm; \sqrt{[P_H^\pm \sin \theta^\pm]^2 + [P_V^\pm]^2} = P_T$
- If $\mu^-$ beam enters the storage ring with $\hat{P}_H^- = \hat{p}_{\mu^-}$, $\theta^-(N_T) = \omega(N_T - 1/2)$, where $N_T$ is the number of turns during storage, counted starting with $N_T = 1$ the first time the bunch passes the IP, and $\omega = 2\pi \gamma g_{\mu^-}^2$, with $\gamma = E/m_\mu$.

As a function of $\theta^-$ and $\theta^+$, defining $c_- \equiv \cos \theta^-$ etc.,

\[
\frac{\bar{\sigma}_S(\theta^+, \theta^-)}{\bar{\sigma}_S^0} = (1 + P_H^+ P_H^- c_+ c_-) + \cos 2\delta(P_V^+ P_V^- + P_H^+ P_H^- s_+ s_-)
+ \sin 2\delta(P_H^- P_V^+ s_- - P_H^+ P_V^- s_+). \tag{11}
\]
I: To approximate the $\zeta = 0$ configuration, we choose $P_H^+ = P_H^- = P_H = 0.05, \theta^- = \theta^+, P_V^+ = P_V^- = \sqrt{P^2 - P_H^2}$.

II: To approximate the $\zeta = \pi$ configuration, we choose $P_H^+ = P_H^- = P_H = 0.05, \theta^- = \theta^+ + \pi, P_V^- = -P_V^+ = -\sqrt{P^2 - P_H^2}$.

III: To emphasize the $\zeta = \pi/2$ and $\zeta = 3\pi/2$ configurations over many turns of the bunches, we choose $P_H^- = P (P_V^- = 0), P_H^+ = P_H = 0.05$ and $P_V^+ = \sqrt{P^2 - P_H^2}$.

Optimal luminosity allocation: $L/6$ to I, $L/6$ to II, $2L/3$ to III.

Note: $P_H = 0.05$ is chosen as small as possible consistent with being large enough to get precision beam energy measurement.

To determine how well one can perform the measurements, must develop strategy for maximizing $\langle P_T^4 \rangle L$ by selecting only energetic muons to accelerate and combining bunches.

- Define $(\hat{a}, \hat{b}) = (a, b) / (g m_\mu / 2 m_W)$.
- Give contours at $\Delta \chi^2 = 1, 4, 6.635, 9$ in the $\delta = \tan^{-1} \frac{\hat{b}}{\hat{a}}, r = \sqrt{\hat{a}^2 + \hat{b}^2}$ parameter space.
- Define $I =$ proton source intensity enhancement.
– Compare 4 cases
(i) $P = 0.2$, $L = 0.15\text{fb}^{-1}$ (which is the nominal situation, defined to have $I = 1$);
(ii) maintain same proton intensity, $I = 1$, but select only more energetic muons to the extent that it becomes possible to merge the two bunches to just one ‘full’ bunch.
$$\Rightarrow P^m(I = 1) \sim 0.39 \text{ and } L = 0.075\text{fb}^{-1}.$$ 
This is the case to pay most attention to since $P^2\sqrt{L}$ is a factor of 2.7 larger than in case (i).
(iii) increasing the proton source intensity by a factor of two, $I = 2$, while selecting only energetic muons such that merging is again just possible.
$$\Rightarrow P^m(I = 2) \sim 0.48 \text{ and } L = 0.075\text{fb}^{-1};$$
(iv) Assume very intense proton source, $I = 3$. It is then best to use just-full bunches (no merging).
$$\Rightarrow P^f(I = 3) \sim 0.45 \text{ and } L = 0.15\text{fb}^{-1}.$$ 

First, consider how well we can determine $\hat{a}$ and $\hat{b}$ for a light Higgs boson with SM-like production rate.
Contours at $\Delta \chi^2 = 1, 4(95.45\% CL), 6.635(99\% CL), 9$:

Note: $1\sigma (68.27\% CL)$ is $\Delta \chi^2 = 2.3$ for 2 parameters.

$\begin{align*}
m_h & = 110 \text{ GeV}, \hat{a} = 1, \hat{b} = 0, \\
& \text{CP-even, SM-like couplings.} \\
m_h & = 110 \text{ GeV}, \hat{a} = \hat{b} = 1/\sqrt{2}, \\
& \text{CP-mixed, SM rate.}
\end{align*}$
How well can we determine that we have a degenerate heavy $H^0$ and $A^0$ pair of the MSSM vs. a single pure CP-even or CP-odd Higgs with same event rate?

Naively, i.e. assuming you could keep $\zeta$ fixed and, taking $P_L^\pm = 0$ and $P_T^\pm = P$, you would use the pattern:

<table>
<thead>
<tr>
<th>$(\hat{a}, \hat{b})$</th>
<th>$\zeta = 0$</th>
<th>$\zeta = \pi/2$</th>
<th>$\zeta = \pi$</th>
<th>$\zeta = 3\pi/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1, 0)$</td>
<td>$1 + P^2$</td>
<td>$1$</td>
<td>$1 - P^2$</td>
<td>$1$</td>
</tr>
<tr>
<td>$(1/\sqrt{2}, 1/\sqrt{2})$</td>
<td>$1$</td>
<td>$1 - P^2$</td>
<td>$1$</td>
<td>$1 + P^2$</td>
</tr>
<tr>
<td>$(0, 1)$</td>
<td>$1 - P^2$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$(1/\sqrt{2}, 0) + (0, 1/\sqrt{2})$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

which clearly separates a degenerate pair from a single Higgs of given type.

In practice, we must use our earlier configurations I, II and III.

Let $\hat{N}_{A^0} \ (\hat{N}_{H^0}) = \text{number of } A^0 \ (H^0) \text{ events at the chosen } \sqrt{s} \text{ for a possible model divided by } (N_{A^0}^2 + N_{H^0}^2)^{1/2}_{MSSM}.$

Define $\Omega = \sqrt{\hat{N}_{A^0}^2 + \hat{N}_{H^0}^2}$ and $\Delta = \tan^{-1} \frac{\hat{N}_{A^0}}{\hat{N}_{H^0}}$
Summary plots, assuming $R = 0.1\%$ beam energy width, for which we take $L = 7 \text{fb}^{-1}$ as a reasonable 1 to 2 year integrated luminosity at $\sqrt{s} = 400 \text{ GeV}$.

Ability to measure $H^0$ and $A^0$ rates for $\sqrt{s} = m_{A^0} = 400 \text{ GeV}$.

$\Delta \chi^2$ for discriminating MSSM $H^0 + A^0$ from same rate pure CP-even (solid) or CP-odd (dashes) vs. $\tan \beta$.

$m_{A^0} = 400 \text{ GeV}$, no-mixing, $m_{\text{SUSY}} = 1 \text{ TeV}$

Luminosity-merging case (ii) does good at 68.7% CL.
CONCLUSIONS

• It will be crucial to have CP analysis of a Higgs signal.

• There are not many good techniques.

• It is very possible that the muon collider can provide the best CP analysis.

• It will be crucial to pay attention to:
  – controlling spin orientations and precessions of $\mu^+$ and $\mu^-$.
  – retaining ability to select high momentum muons initially and later using merged bunches with high polarization.