$e^- e^-$ Collisions and Generalized Higgs Boson Scenarios

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Even within SM context, should consider extended Higgs sector possibilities.

- Add triplets or higher reps. is a possibility.

If neutral vev ≠ 0, ⇒ ρ is no longer computable (even if representations and vevs are chosen so that ρ = 1 at tree level); ρ becomes another input parameter to the theory; is this so bad?

If neutral vev = 0, then no EWSB impact and ρ = 1 is natural.

$T = 3, |Y| = 4$ representations ⇒ ρ = 1 + finite loop correction for vev ≠ 0
Recall 1-loop results (notation used is $N_{T,Y}$):

$$\alpha_s(m_Z) = \alpha_{QED}(m_Z) \frac{5(b_1 - b_2)}{\sin^2 \theta_W (5b_1 + 3b_2 - 8b_3) - 3(b_2 - b_3)}$$

$$M_U = \exp(2\pi t_U) \text{ with } t_U = \frac{3 - 8\sin^2 \theta_W}{5(b_1 - b_2)\alpha_{QED}(m_Z)}$$

$$b_1 - b_2 \quad \text{SM} = \frac{1}{5}(N_{0,2} + 4N_{0,4}) - \frac{1}{15}(N_{1/2,1} + N_{1,2}) + \frac{11}{15}N_{1,3} - \frac{2}{3}N_{1,0} + \ldots + \frac{616}{135}N_{3,4}$$

High-scale coupling unification (want small $b_1 - b_2$):

Without SUSY

e.g. $N_{1/2,1} = 2$, $N_{1,0} = 1 \Rightarrow \alpha_s(m_Z) = 0.115$, $M_U = 1.6 \times 10^{14}$ GeV.

e.g. $N_{1/2,1} = 1$, $N_{1,2} = 2 \Rightarrow M_U \sim 1.5 \times 10^{13}$ GeV.

e.g. $N_{1/2,3} \neq 0$ solutions $\Rightarrow M_U \lesssim 10^{13}$ GeV.
With SUSY,

The $N_{\frac{1}{2},1} = 2$ (only) solution with $M_U \gtrsim 2 \times 10^{16}$ GeV is far and away the solution with highest $M_U$.

The solution with next highest $M_U$ is $N_{\frac{1}{2},3} = 4$, $N_{\frac{1}{2},1} = N_{1,2} = 2 \Rightarrow M_U \sim 5 \times 10^7$ GeV.

Low-scale (extra dimensions . . .) unification (want big $b_1 - b_2$)

Keeping all SM particles and Higgs on the brane:

e.g. SM case: $N_{\frac{1}{2},1} = N_{\frac{1}{2},3} = N_{1,2} = N_{1,0} = 4, N_{3,4} = 3 \Rightarrow \alpha_s(m_Z) = 0.112, M_U = 1000$ TeV, $\alpha_U = 0.04$.

e.g. SUSY case: $N_{\frac{1}{2},1} = N_{1,2} = N_{1,0} = 4, N_{3,4} = 4 \Rightarrow \alpha_s(m_Z) = 0.114, M_U = 4$ TeV, $\alpha_U = 0.07$.

Current Constraints

- Mass limits from LEP are model dependent, but certainly pair production pretty much excludes masses below 100 GeV.
• \( \rho \) at one-loop.

In models with \( \rho = 1 \) at tree level and for which \( \rho \) is finitely calculable, parameters must still be chosen so that Higgs loop corrections to \( \rho \) are not too large. This is usually automatic in the SUSY models, but in general models, too large a mass separation between neutral and charged Higgs with \( W, Z \) couplings is problematical.

• \( b \rightarrow s\gamma \).

In non-SUSY models, the charged Higgs for a type-II doublet allows a graph which adds to the already excessive SM prediction, so \( m_{H^\pm} \) must be large. Charged Higgs that do not couple to quarks (of which there are many in the models outlined earlier) are no problem.

In SUSY models, a stop-chargino graph can cancel an excessive \( H^\pm \) graph if \( m_{\tilde{t}} \) and \( m_{\tilde{\chi}_1^\pm} \) are small enough.

**Couplings**

This is a very complex topic and very model-dependent. I will simply outline some of the important issues and processes, focusing on triplets, for which I use the generic \( 2 \times 2 \) notation:

\[
\Delta = \begin{pmatrix}
\Delta^++/\sqrt{2} & \Delta^{++} \\
\Delta^0 & -\Delta^+/\sqrt{2}
\end{pmatrix}.
\]
Some reminders:

- There is never a $\gamma W^\pm \Delta^\mp$ vertex. There is generally a non-zero $ZW^\pm \Delta^\mp$ vertex if $v_\Delta = \langle \Delta^0 \rangle \neq 0$ (for $|Y| \neq 0$ rep), even if $\rho$ is tuned to $\simeq 1$.

- Charged and neutral (non-singlet) Higgs, triplet members or otherwise, have diagonal pair couplings to $\gamma \propto Q$ and $Z \propto A = T_3 - \sin^2 \theta_W Q$.

- $W_L W_L$ couplings to Higgs bosons of triplet reps requires a neutral member of (L) triplet with non-zero $v_\Delta$, which leads to need to fine-tune $\rho$.

- More generally, for $\Delta$'s in triplet representations, $\Delta V V$ couplings are proportional to the triplet vev (or zero if $\Delta$ is CP-odd in nature).

- Most $\Delta \Delta^{(i)} V$ couplings are non-zero even if the triplet vev is zero.

- Non-zero trilinear couplings of three $\Delta$'s require a triplet vev.

- $H \Delta \Delta$ couplings involving one doublet $H$ and two triplet $\Delta$'s are non-zero if the doublet vev is non-zero, even if the triplet vev is zero.
• When quantum-number allowed, $\Delta$-triplet couplings to $f^{(*)}\bar{f}$ require a non-zero triplet vev.

• There is a possibility of non-zero bi-lepton couplings of Higgs bosons. For example, for the standard SU(2)$_L$ case, with $Q = T_3 + \frac{Y}{2} = -2$, the allowed doubly-charged cases are:

\begin{align*}
e^{-}_Re^{-}_R & \to \Delta^{--}(T = 0, T_3 = 0, Y = -4), \\
e^{-}_Le^{-}_R & \to \Delta^{--}(T = \frac{1}{2}, T_3 = -\frac{1}{2}, Y = -3), \\
e^{-}_Le^{-}_L & \to \Delta^{--}(T = 1, T_3 = -1, Y = -2).
\end{align*}

Note that the above cases do not include the $T = 3, Y = -4$ representation that yields $\rho = 1$, nor the $T = 1, Y = -4$ triplet with no neutral member, but do include the $T = 1/2, Y = -3$ doublet representation with no neutral member, and the $T = 1, Y = -2$ triplet representation. A $\nu_L\nu_L \to \Delta^0(T = 1, T_3 = +1, Y = -2)$ coupling also exists, but does not lead to neutrino mass if $v_\Delta = 0$ (as preferred for $\rho = 1$ to be natural).

In the case of a $|Y| = 2$ triplet representation the lepton-number-violating coupling to (left-handed) leptons is specified in the lepton-number violating Lagrangian form:

$$\mathcal{L}_Y = \imath h_{ij} \psi^T_i C\tau_2 \Delta \psi_j + \text{h.c.}, \quad i, j = e, \mu, \tau.$$  

(2)
Limits on the $h_{ij}$ by virtue of the $\Delta^{--} \rightarrow \ell^-\ell^-$ couplings: writing $|h_{\ell\ell}^{\Delta^{--}}|^2 \equiv c_{\ell\ell} m_{\Delta^{--}}^2 (\text{GeV})$, strongest limits (no limits on $c_{\tau\tau}$) are:

- $c_{ee} < 10^{-5}$ (Bhabha),
- $c_{\mu\mu} < 5 \times 10^{-7}$ ($(g - 2)_{\mu}$ – predicted contribution has wrong sign) and
- $\sqrt{c_{ee}c_{\mu\mu}} < 10^{-7}$ (muonium-antimuonium).

If $\langle \Delta^0 \rangle = 0$ (for $\rho = 1 = \text{natural}$), $\Gamma_{\Delta^{--}}^T$ would be small. $\Rightarrow$ possibly very large $s$-channel $e^-e^-$ and $\mu^-\mu^-$ production rates.

**Decays**

- If the triplet vev is non-zero, then $\Delta \rightarrow VV$ (possibly virtual) decays and/or $\Delta \rightarrow f'f$ decays are usually most significant.

- If the triplet vevs are zero, then many channels are eliminated; dominant modes are $\Delta \rightarrow \Delta'V$; since many of the $\Delta$’s of typical model are approximately degenerate, many of these modes will be virtual.

$\Rightarrow$ dominance of bi-lepton modes is possible if bi-lepton couplings are non-zero.
For example, in the case of a $T = 1, |Y| = 2 \Delta^{--}$, we have

$$\Gamma^{\Delta^{--}W^-}_{\Delta^{--}} = \frac{g^2 m_{\Delta^{--}}^3}{16\pi m_W^2} \beta^3 \sim (1.3 \text{ GeV}) \left( \frac{m_{\Delta^{--}}}{100 \text{ GeV}} \right)^3 \beta^3,$$

$$\Gamma^{\ell^-\ell^-}_{\Delta^{--}} = \frac{|h_{\ell\ell}^{\Delta^{--}}|^2}{8\pi} m_{\Delta^{--}} \sim (0.4 \text{ GeV}) \left( \frac{c_{\ell\ell}}{10^{-5}} \right) \left( \frac{m_{\Delta^{--}}}{100 \text{ GeV}} \right)^3.$$

where $\beta$ is the usual phase space suppression factor.

- For example, if $m_{\Delta^{--}} = 360$ GeV, $m_{\Delta^-} = 250$ GeV we find $\Gamma(\Delta^{--} \rightarrow \Delta^- W^-) \sim 2 \text{ GeV}$ and $\Gamma(\Delta^{--} \rightarrow \ell^- \ell^-) = 19 \text{ GeV} \left( \frac{c_{\ell\ell}}{10^{-5}} \right)$. If any $c_{\ell\ell}$ is near $10^{-5}$ then $\Gamma^{\ell^-\ell^-}_{\Delta^{--}} > \Gamma^{\Delta^{--}W^-}_{\Delta^{--}}$ is likely.
- Since there are currently no limits on $c_{\tau\tau}$, the $\tau^- \tau^-$ channel could easily have the largest partial width and be the dominant decay of the $\Delta^{--}$.
- On the other hand, if all the $c_{\ell\ell}$ are very small then the $\Delta^- W^-$ mode is quite likely to be dominant if it is kinematically allowed.
- For zero triplet vevs, the $\Delta^0$ (generally the lightest) will be stable unless it has bi-lepton couplings to $\nu_L \nu_L$.

Such a $\Delta^0$ would be a candidate for the dark matter.
• Discover $\Delta^{--}$ in $p\bar{p} \rightarrow \Delta^{--}\Delta^{++}$ with $\Delta^{--} \rightarrow \ell^-\ell^-, \Delta^{++} \rightarrow \ell^+\ell^+$ ($\ell = e, \mu, \tau$) at TeV33 or LHC (J.G., Loomis, Pitts: hep-ph/9610237). LHC reach up to $m_{\Delta^{--}} \sim 1$ TeV.

⇒ TeV33 + LHC will tell us if such a $\Delta^{--}$ exists in the mass range accessible to NLC and FMC and how it decays.

– If no $W^-W^-$ decays are detected, it is very probable that $v_\Delta = 0$.
– If it decays to $\ell^-\ell^-$ ($\ell = e, \mu, \tau$), then $\Rightarrow$ the $\ell^-\ell^-$ coupling clearly exists.
– For any observed $\ell^-\ell^-$ mode, you know the $\Delta^{--} \rightarrow \ell^-\ell^-$ coupling is non-zero, but you do not know that the others are zero — only that they are relatively smaller.
– Even if a $\Delta^-W^-$ mode is dominant, that still does not mean that the $c_{\ell\ell}$’s are zero, only that they are quite small.
⇒ can we probe $c_{\ell\ell}$’s that are too small for $\ell^-\ell^-$ to appear in decays? Answer=yes.
– If the $\Delta^-W^-$ mode is disallowed and the $c_{\ell\ell}$’s are all very small ($\lesssim 10^{-16}$), the $\Delta^{--}$ could be sufficiently long-lived to escape the detector.
The hadron colliders would see the corresponding ‘stable’ particle highly ionizing tracks.

• $\Rightarrow$ Look for (and study if found) in $e^-e^- \rightarrow \Delta^{--}$ s-channel collisions.
This should be done no matter how the $\Delta^{--}$ is seen to decay at the hadron colliders.

Event rates can be enormous (see JFG, hep-ph/9803222 and hep-ph/9510350): equivalently can probe to very small $c_{\ell\ell}$.

– Using the Gaussian approximation, the effective cross section for $\Delta^{--}$ production in the $s$-channel is obtained by convoluting the standard $s$-channel pole form with the Gaussian distribution in $\sqrt{s}$ of rms width $\sigma_{\sqrt{s}}$.

A useful mnemonic for $\sigma_{\sqrt{s}}$ is

$$\sigma_{\sqrt{s}} \sim 0.2 \text{ GeV} \left( \frac{m_{\Delta^{--}}}{100 \text{ GeV}} \right) \left( \frac{R}{0.2\%} \right),$$

where $R$ is the beam energy resolution in percent. The crucial issue is how $\sigma_{\sqrt{s}}$ compares to $\Gamma^T_{\Delta^{--}}$.

The resulting cross section is denoted by $\bar{\sigma}_{\Delta^{--}}$. For $\Gamma^T_{\Delta^{--}} \gg \sigma_{\sqrt{s}}, \Gamma^T_{\Delta^{--}} \ll \sigma_{\sqrt{s}},$
\( \bar{\sigma}_{\Delta^{--}} \) at \( \sqrt{s} = m_{\Delta^{--}} \) is given by:

\[
\bar{\sigma}_{\Delta^{--}} = \begin{cases} 
\frac{4\pi BR(\Delta^{--} \rightarrow e^- e^-)}{m_{\Delta^{--}}^2}, & \Gamma_T^{\Delta^{--}} \gg \sigma \sqrt{s}; \\
\frac{\sqrt{\pi}}{2\sqrt{2}} \frac{4\pi \Gamma(\Delta^{--} \rightarrow e^- e^-)}{m_{\Delta^{--}}^2 \sigma \sqrt{s}}, & \Gamma_T^{\Delta^{--}} \ll \sigma \sqrt{s} .
\end{cases}
\]  

(5)

We compute rates as \( L \bar{\sigma}_{\Delta^{--}} \) with \( L = 50 \text{fb}^{-1} \) assumed under Gaussian peak (after accounting for losses in radiative tail).

Consider \( \Gamma_T^{\Delta^{--}} \ll \sigma \sqrt{s} \).

This can occur if \( \Delta^{--} \rightarrow \Delta^- W^- \) is either highly suppressed or forbidden, as is likely since the \( \Delta \)'s are typically rather degenerate, and all of the \( c_{\ell\ell} \)'s are relatively small, the most likely case.

Taking \( L = 50 \text{fb}^{-1} \), and using Eq. (4) for \( \sigma \sqrt{s} \) and the earlier result for \( \Gamma(\Delta^{--} \rightarrow e^- e^-) \), we find from Eq. (5) an event rate of

\[
N(\Delta^{--}) \sim 3 \times 10^{10} \left( \frac{c_{ee}}{10^{-5}} \right) \left( \frac{0.2\%}{R} \right) .
\]  

(6)

An enormous event rate results if \( c_{ee} \) is within a few orders of magnitude of its upper bound.
For 100 events, Eq. (6) ⇒ we probe

\[ c_{ee\mid 100 \text{ events}} \sim 3.3 \times 10^{-14} \left( \frac{R}{0.2\%} \right) \left( \frac{50 \text{fb}^{-1}}{L} \right), \quad \Gamma_{\Delta--}^T \ll \sigma \sqrt{s}, \quad (7) \]

independent of \( m_{\Delta--} \).

⇒ dramatic sensitivity — at least factor of \( 10^8 - 10^9 \) improvement over current limits. Observation ⇒ actual measurement of \( c_{ee} \) at level relevant to neutrino mass generation by a right-handed partner \( \Delta_R \) representation of the left-handed \( \Delta_{(L)} \) representation (with \( \langle \Delta_0^R \rangle \neq 0 \)).

If \( \Delta-- \to \mu^-\mu^- \) primarily, 10 events might → a viable signal.

If \( \Delta-- \to e^-e^- \) or \( \Delta--W^- \), 1000 events might be needed because of backgrounds. A better study is needed.

Other processes:

• \( \Delta--Z \) and \( \Delta--\gamma \) production in \( e^-e^- \) collisions.

This is essentially equivalent to using the bremsstrahlung tail in \( \sqrt{s} \) at the \( e^-e^- \) collider to self-scan for the \( \Delta-- \). Depending on \( c_{ee} \), it can allow discovery and
mass measurement, which in turn allows centering in $\sqrt{s}$ for direct resonance production as above.

The total number of events is proportional to $c_{ee}$ as in the $\Gamma^{T}_{\Delta^{-\rightarrow}} \ll \sigma \sqrt{s}$ limit of on-resonance production, but observable rates are only possible if $c_{ee}$ is not too far below current bounds.

- $\Delta^{-W^{-}}$ production in $e^{-e^{-}}$ collisions.

This process relies on a $\Delta^{- \rightarrow e^{-}\nu_{e}}$ coupling, and would be an interesting way of both observing any $\Delta^{-}$ with such a coupling and a way of determining the magnitude of the coupling.

Observable rates require substantial coupling.
CONCLUSIONS

- Exotic Higgs representations, e.g. triplet as motivated by seesaw approach to neutrino masses, will lead to exotic collider signals of doubly charged Higgs pair production at hadron colliders.

The kinematic reach of such signals at the LHC will be up to $m_{\Delta^{--}} \sim 1$ TeV, which means that for any lepton collider with $\sqrt{s} \lesssim 1$ TeV, we will know ahead of time what $e^-e^- \sqrt{s}$ will be required for $\Delta^{--}$ production in the $s$-channel.

- Even if the $\Delta^{--}$ is not seen to decay to $e^-e^-$, it will be essentially mandatory to have $e^-e^-$ collisions to produce the $\Delta^{--}$ as a probe of the possibly quite small (but immensely important) coupling $c_{ee}$, since very small values of $c_{ee}$ can be probed.