Remarks on Muon Collider Physics

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What New Ideas Could Impact Thinking About the Muon Collider

1. A light Higgs avoiding LEP limits via unusual decays that overwhelm $b\bar{b}$. Example, the NMSSM no-fine-tuning models of Dermisek and JFG.

2. Unparticle influences on the Higgs sector

3. Other light Higgs models that delay the fine-tuning problem by virtue of many doublets.

4. Flavor Higgs bosons

5. Flavor Gauge bosons

6. SUSY masses

7. Unparticles generally

Higgs scan

- The rms beam spread in $\sqrt{s}$ (denoted by $\sigma_{\sqrt{s}}$) prior to including bremsstrahlung is given by

$$\sigma_{\sqrt{s}} = R\sqrt{s}/\sqrt{2}\,, \qquad (1)$$

where R is the resolution in the energy of each beam. A convenient formula for $\sigma_{\sqrt{s}}$ is

$$\sigma_{\sqrt{s}} = (7 \text{ MeV}) \left(\frac{R}{0.01\%}\right) \left(\frac{\sqrt{s}}{100 \text{ GeV}}\right) . \qquad (2)$$

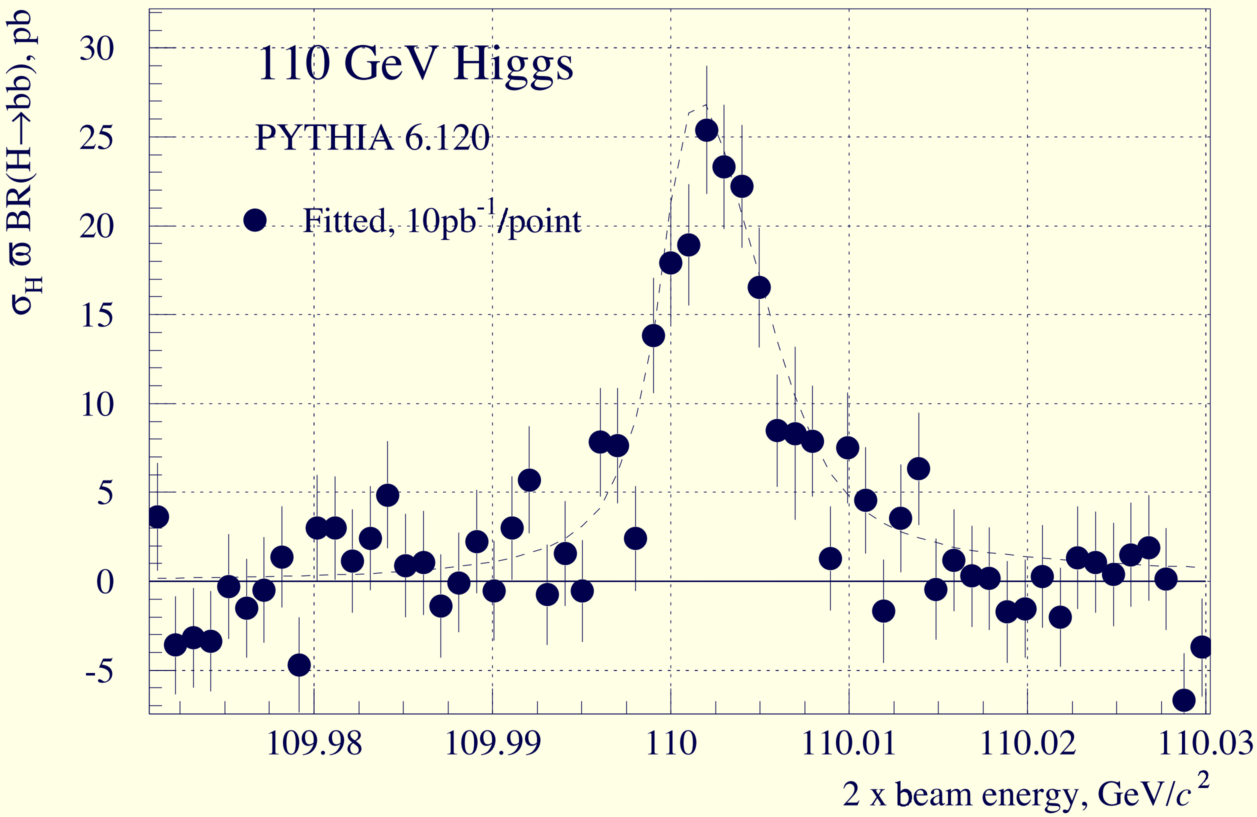
The critical issue is how this resolution compares to the calculated total widths of Higgs bosons when $\sqrt{s} = m_h$.

For a SM Higgs of mass $m_h = 100$ GeV, $\Gamma_h^{\text{tot}} = 3$ MeV.

Machine designs discussed in the past achieve $R \sim 0.03\%$ so that the energy resolution in Eq. (2) is not smaller than the SM Higgs width. But, it is not enormously larger either.

Thus, one will be able to scan the resonance shape as shown below.

- **The visualization of the SM Higgs scan is shown below, assuming $R = 0.03\%$. $L = 10\mathrm{pb}^{-1}$ per point has been assumed as appropriiate for $\mathcal{L} = 10^{31}\mathrm{cm}^{-2}\mathrm{sec}^{-1}$, which in turn is limited by the needed small R.**



One gets in the end a few thousand SM Higgs bosons per year.

What is important to note is that if the $m_h = 100~\mathrm{GeV}$ Higgs is broader than we think, this could have a dramatic impact on how to approach the

R choice, the related L question, and so forth.

We shall discuss a couple of models in which this would be the case.

First, it is useful to think of some limiting cases.

- The s-channel Higgs resonance cross section is

$$\sigma_h(\sqrt{\hat{s}}) = \frac{4\pi\Gamma(h\to\mu\mu)\,\Gamma(h\to X)}{\left(\hat{s}-m_h^2\right)^2 + m_h^2[\Gamma_h^{\rm tot}]^2}\,, \tag{3}$$

where $\hat{s}=(p_{\mu^+}+p_{\mu^-})^2$ is the c. m. energy squared of a given $\mu^+\mu^-$ annihilation, X denotes a final state and $\Gamma_h^{\rm tot}$ is the total width.

The sharpness of the resonance peak is determined by $\Gamma_h^{\rm tot}$.

Neglecting bremsstrahlung for the moment, the effective signal cross section is obtained by convoluting $\sigma_h(\hat{s})$ with the Gaussian distribution in $\sqrt{\hat{s}}$ centered at $\sqrt{\hat{s}}=\sqrt{s}$:

$$\overline{\sigma}_h(\sqrt{s}) = \int \sigma_h(\sqrt{\hat{s}})\,\frac{\exp\left[-(\sqrt{\hat{s}}-\sqrt{s})^2/(2\sigma_{\sqrt{s}}^2)\right]}{\sqrt{2\pi}\sigma_{\sqrt{s}}}\,d\sqrt{\hat{s}}\,. \tag{4}$$

- In the case where the Higgs width is much smaller than the Gaussian width $\sigma_{\sqrt{s}}$, the effective signal cross section result for $\sqrt{s} = m_h$, denoted by $\overline{\sigma}_h$, is

$$\overline{\sigma}_h = \frac{2\pi^2 \Gamma(h \to \mu\mu)\, B(h \to X)}{m_h^2} \times \frac{1}{\sigma_{\sqrt{s}}\sqrt{2\pi}} \qquad (\Gamma_h^{\rm tot} \ll \sigma_{\sqrt{s}}) \, . \tag{5}$$

  In the other extreme where the Higgs width is much broader than $\sigma_{\sqrt{s}}$, then at $\sqrt{s} = m_h$ we obtain

$$\overline{\sigma}_h = \frac{4\pi B(h \to \mu\mu) B(h \to X)}{m_h^2} \qquad (\Gamma_h^{\rm tot} \gg \sigma_{\sqrt{s}}) \, . \tag{6}$$

  Note that this equation implies that if there is a large contribution to the Higgs width from some channel other than $\mu\mu$, we will get a correspondingly smaller total event rate due to the small size of $B(h \to \mu\mu)$.

  The number of events is $N = L\overline{\sigma}_h$.

- In the case of $h \to aa \to 4\tau$, the Higgs width is 10 to 20 times larger than in the SM, but the coupling to $\mu^+\mu^-$ is more or less SM-like.

Given that we think we can design the machine to have $\sigma_{\sqrt{s}}$ of order the SM $\Gamma_h^{\rm tot}$, we would be able to operate in the $\Gamma_h^{\rm tot} \gg \sigma_{\sqrt{s}}$ limit.

But, is this optimal? We could probably get higher luminosity if relaxed R without leaving the desirable $\sigma_{\sqrt{s}} \ll \Gamma_h^{\rm tot}$ limit.

Then, the only issue will be the efficiency for tagging on the final 4τ final state, or perhaps $4b$ if we are not in absolutely minimal fine-tuning situation which requires $a \to 2\tau$ and $m_h \sim 100~\mathrm{GeV}$. This has not been studied, but it is hard to imagine one can't do it.

Note that in the $h \to aa \to 2\tau + 2\tau$ final state, the a's will be highly boosted. In this case, unless one were have have an asymmetric $\mu^+\mu^-$ collider, it is hard to directly reconstruct m_a since the missing momenta will be approximately back-to-back. How well one could do is a real question.

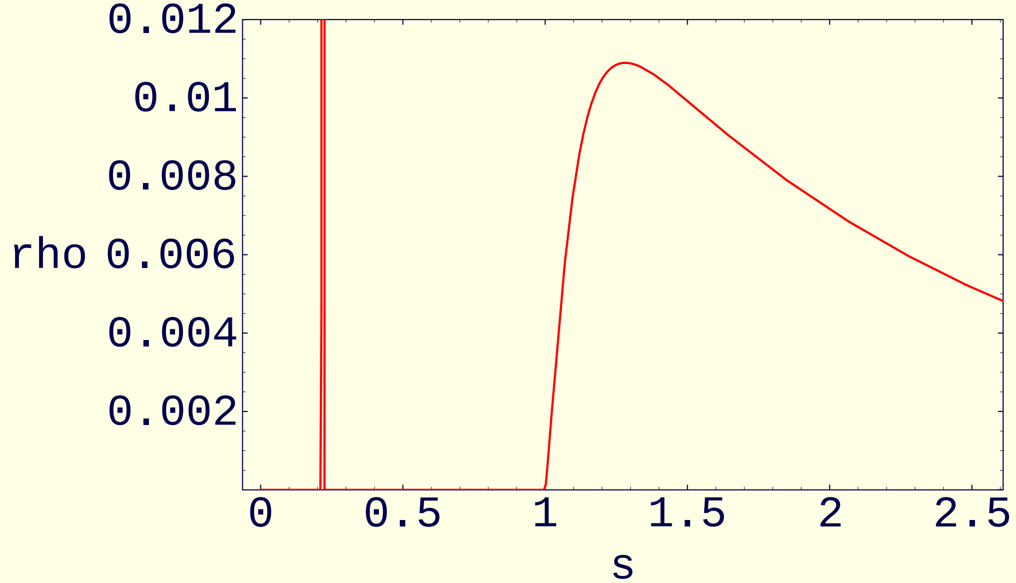
Of course, it would also be highly desirable to measure the now 10 times smaller $B(h \to b\overline{b})$ to check for completeness. Of course, you will want to look for the even smaller $B(h \to \tau^+\tau^-)$ as well.

- In the unparticle theory, the result depends very much upon whether the Higgs is above or below the "mass gap". If below, *i.e.* $m_h^2 < \zeta v^2$ in the language of arXiv:0707.4309, then the Higgs coupling to all SM particles is reduced by a factor of

$$R \sim \frac{1}{K(m_h)} \tag{7}$$

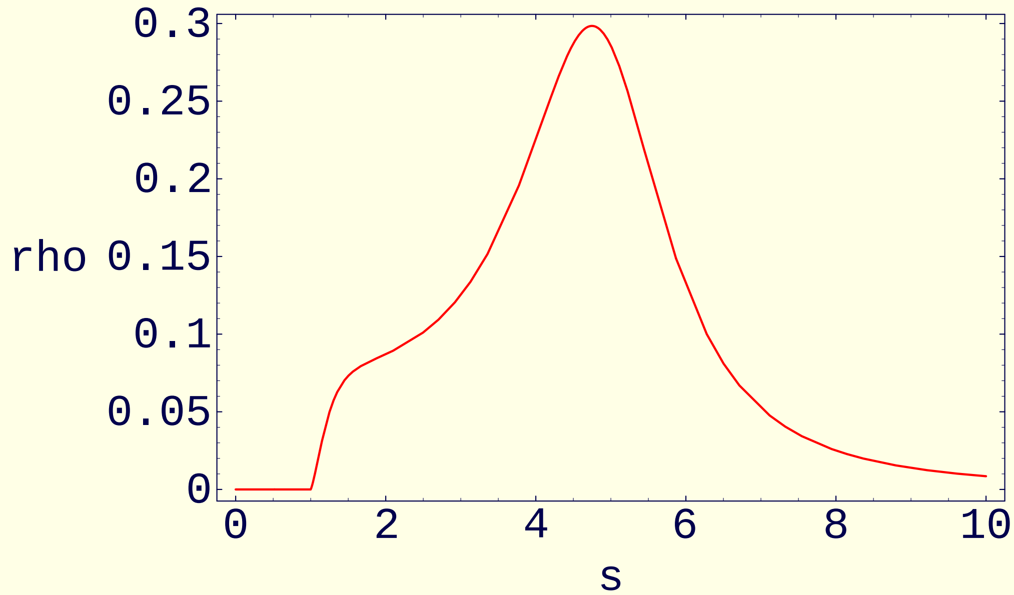
which can be a substantial reduction.

The spectral function showing the somewhat shifted Higgs pole and the unparticle continuum ($\int \rho = 1$) is shown below.



Of course, the challenge will be to measure this whole spectrum, since the continuum part is mainly unparticle and therefore mainly invisible.

In the case where $m_h^2 > \zeta v^2$, the Higgs pole sits in the middle of the continuum and things are even more difficult. The spectral function looks like



which still displays a peak in the vicinity of $s = m_h^2$, but the problem is that the Higgs is mainly decaying invisibly and that $\Gamma_h^{\rm tot}$ is typically quite large as illustrated below.

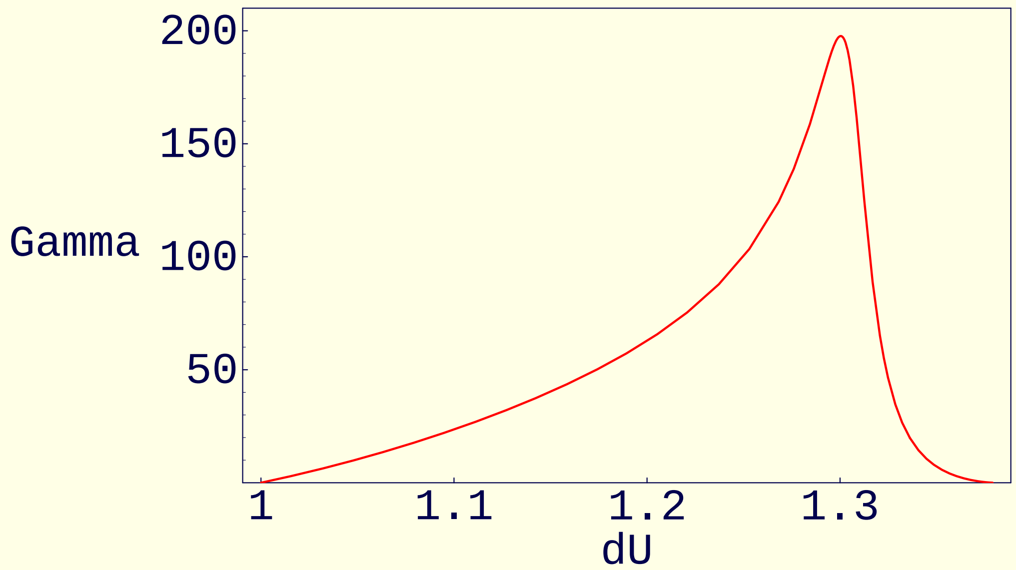


Figure 1: The effective Higgs width in units of GeV for a particular unparticle model with $m_h^2 > \zeta v^2$, for $\zeta = .2$.

Here, d_U is the parameter that specifies the unparticle model via the deconstruction of the unparticle operator:

$$\mathcal{O} = \sum_n F_n \phi_n \,, \quad F_n^2 = \frac{A_{d_U}}{2\pi} \Delta^2 \left(M_n^2\right)^{d_U - 2} \,. \tag{8}$$

Consistency of the theory requires that $1 < d_U < 2$.

How does one explore this spectral function with a scan? You need to have some trigger on the final state, and that would be tough given that $B(h \to \mu^+\mu^-)B(h \to b\overline{b})$ is very tiny.

The e^+e^- collider approach advocated by Espinosa and JFG would be to do $\mu^+\mu^- \to Zh$ and look at the event excess (a broad continuum in this case) recoiling against the Z. At first sight, the ability to sit on the h resonance would not be useful in this model.

Incidentally, van der Bij also has a model with many singlets that looks a lot like a generalized unparticle model of this latter type which would also yields these same kinds of features.

- **Many doublets**

  What is the motivation? Fine-tuning.

  The point is that the t-loop correction to the tree-level mass of Higgs i is proportional to

$$\Lambda^2 f_i^2 \tag{9}$$

  where f_i is the fraction of the SM vev, v, carried by ϕ_i. If there are many doublets, then f_i can be small and fine-tuning can be delayed to much higher Λ than the usual $\Lambda \sim 1~\mathrm{TeV}$ for the SM Higgs.

The implications for Higgs phenomenology are substantial.

Each h_i would then have much narrower width than the SM-Higgs if it only decayed to $b\bar{b}$, but Higgs to Higgs pair decays would probably dominated if kinematically allowed.

In the absence of Higgs pair final states, you would be talking about SM-like branching ratios, but a high likelihodd of $\Gamma_{h_i}^{\rm tot} \ll \sigma_{\sqrt{s}}$. The cross section goes like $\frac{\Gamma(h\to\mu^+\mu^-)}{\sigma_{\sqrt{s}}}$, where the numerator is now suppressed.

Well, there is clearly a whole range of possibilities. Again, $\mu^+\mu^- \to Zh_i$ would pick up the spectrum of Higgs bosons, although the rate for each would be suppressed. Still, JFG+Espinosa estimate that after putting in precisions electroweak constraints the spectrum is sufficiently confined that with high L you could dig out the signals.

Would it then be useful to go back and to the $\mu^+\mu^-$ scan? Very model dependent.

Flavor Higgs and Flavor Gauge Bosons

These appear in the RS1 model of Csaki, Grossman et.al. in which they purport to solve the flavor problem associated with the need to have a low Λ for a naturally light Higgs boson.

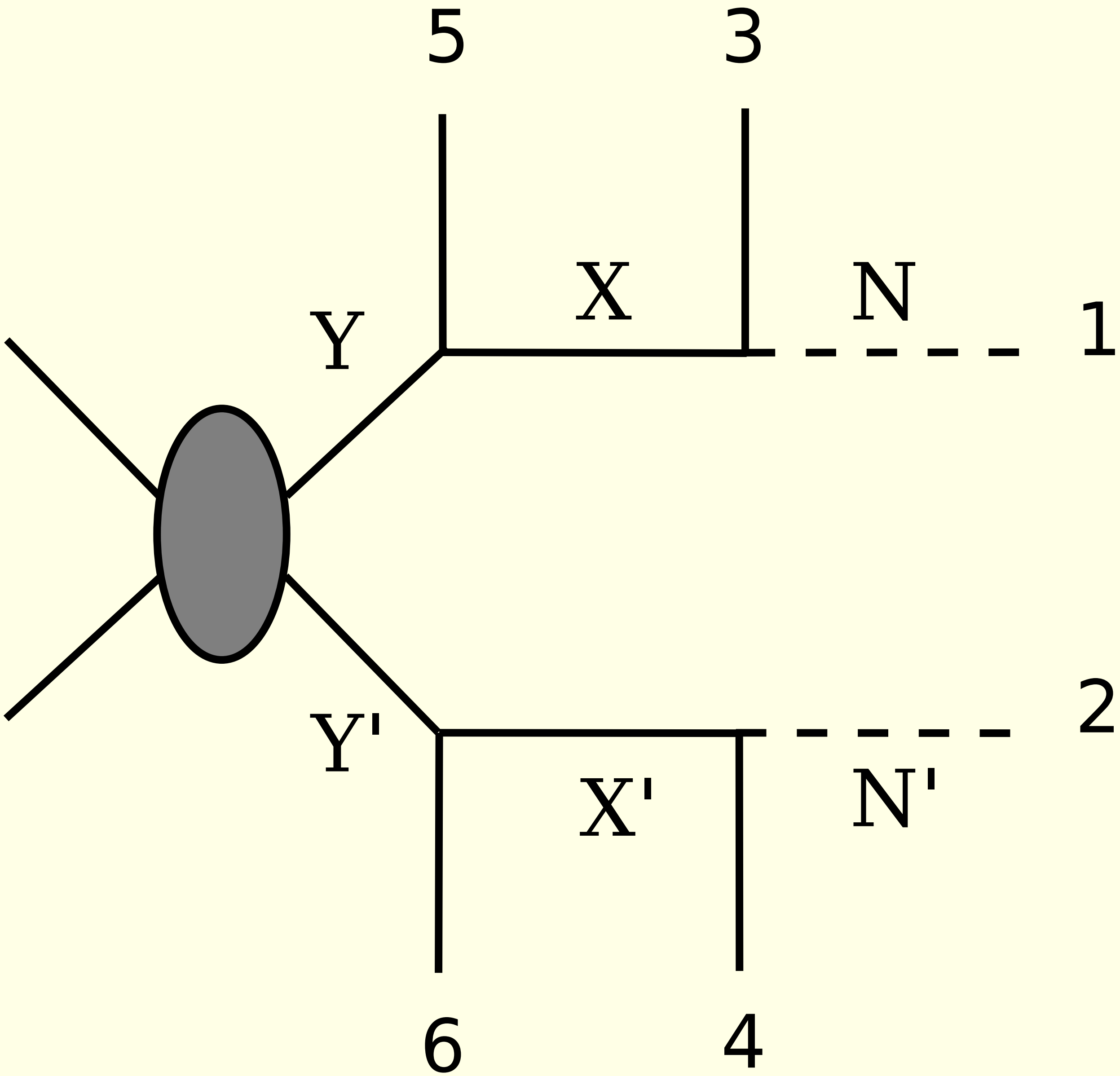
Recall that generically one expects operators at order $1/\Lambda^2$ to effectively arise from the theory that completes the model at scales above Λ. I was hoping to have a chance to look at what unique capabilities a $\mu^+\mu^-$ collider might have for exploring these flavorful objects. Generically, the adjective "flavor" would imply that a $\mu^+\mu^-$ colllider might have some capabillities beyond those of an e^+e^- collider.

SUSY Masses

I would like to point out a new technique for getting SUSY Masses at an e^+e^- or $\mu^+\mu^-$ collider. The point of this method is that it might get excellent masses without having to do the threshold scan. (Of course, the threshold scan would always do better, but you might want to run at the highest energy for full discovery and the question is then how well can you get SUSY masses in such a case.)

One production process will be $\mu^+\mu^- \to \widetilde{\chi}_2^0\widetilde{\chi}_2^0$ and (e.g. SPS1a) each $\widetilde{\chi}_2^0$ will decay via $\widetilde{\chi}_2^0 \to \widetilde{\ell}_R\ell \to \widetilde{\chi}_1^0\overline{\ell}\ell$, where we use $\ell = e, \mu$ (which can be well measured).

This has the general topology illustrated below,



where $N = N' = \widetilde{\chi}_1^0$, $X = X' = \widetilde{\ell}_R$, **and** $Y = Y' = \widetilde{\chi}_2^0$. **One can**

presumably isolate this topology by vetoing jets and any extra leptons, selecting events with exactly $2e$ and 2μ plus missing energy.

Let us now count constraints. There are three unknown masses, and for each event the 4-momentum of *one* of the $\widetilde{\chi}_1^0$'s is unknown, but, the 4-momentum of the other can be obtained by momentum balance given the very well known initial state momenta.

Thus, after n events we will have $3+4n$ unknowns. But, in terms of the 3 masses that are common to every event, we will have $6n$ constraints. We will be able to solve (subject to some combinatoric ambiguity — that is why $2e+2\mu$ is best) when

$$3+4n-6n<0, \quad \text{i.e.} \quad n=2\,. \tag{10}$$

So, one just puts together all event pairs, each one of which (aside from experimental resolutions) will give a solution for all the masses of the SUSY particles. There will be *many* pairs available and one can hope for excellent mass determinations.

I will discuss tomorrow, the analogue of this for the LHC. There, because we only know the transverse momentum balance, one must go to one more particle per chain to get to the point where each pair will give a solution. Resolution issues are more important because the 3rd visible particle in each chain is a quark, and also several production mechanisms leading to the same final state can be important. Nonetheless, even there one does quite well.

Unparticles Generally

Have not had time to think about this, but expect that a $\mu^+\mu^-$ collider would only have those additional capabilities that would come from the reduced beamsstrahlung. Not sure if this has a significant impact or not.