Motivations and Discovery Prospects for Elusive Higgs Boson(s)

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There are excellent motivations for an $m_H \lesssim 105$ GeV SUSY Higgs with SM-like couplings to SM particles but elusive decays.

- Precision Electroweak (PEW) data prefer a Higgs boson with SM-like $g_{WWH}, ZZH$ and $m_h \lesssim 105$ GeV
- The simplest solution to the hierarchy problem is SUSY.
- Gauge coupling unification prefers something close to the MSSM.
- Absence of EWSB fine-tuning requires a light SUSY spectrum (in particular, a light $\tilde{t}$) and a light $\tilde{t}$ implies that the SM-like Higgs of SUSY is light.
- MSSM scenarios having a Higgs with SM-like properties that is light, i.e. $m_h \lesssim 105$ GeV (for PEW perfection) are excluded by LEP.
- Extended SUSY models, including the NMSSM (which preserves all good MSSM features and solves the $\mu$ problem) give elusive decay scenarios not ruled out by LEP for $m_h < 105$ GeV.
- LHC strategies for Higgs searches will need to be expanded.
- Higgs cross sections (initiated by SM particles with SM-like \( h \) couplings) are determined. Main processes are \( gg \rightarrow h \) and \( qq \rightarrow q'q'WW \) with \( WW \rightarrow h \).
In the absence of new physics, Higgs decays are also determined by these same couplings.
However, Beyond the SM physics could completely alter the Higgs decay patterns.

This may make it hard to get our hands on the Higgs boson at the LHC.

If you are too impatient to wait to find a Higgs at the LHC, you can buy one online. Of all known and hypothesized particles the Higgs is the most popular.

The **Higgs Boson** is the theoretical particle of the Higgs mechanism, which physicists believe will reveal how all matter in the universe gets its mass. Many scientists hope that the Large Hadron Collider in Geneva, Switzerland will detect the elusive Higgs Boson when it begins colliding particles at 99.99% the speed of light.

Wool felt with gravel fill for maximum mass.

$9.75 plus shipping
Or, you could write a letter to the Higgs boson:

Dear Higgs Boson,

We know you're out there. We can feel you now. We know that you're afraid. You're afraid of us; you're afraid of change. We don't know the future. We didn't write this to tell you how this is going to end. We wrote this to tell you how it's going to begin.

As you know, our Large Hadron Collider has had some setbacks due to a.... uh... "transformer malfunction" but we know it was you. You sabotaged our machine. We hope you've been enjoying your vacation because we're scheduled to restart in September 2009 and we're pissed.

....so run and hide, asshole. Run and hide. If you should get careless and allow yourself to get detected by the Tevatron, we are going to be supremely disappointed; because we want to find you first, and when we do, rest assured we are not going to publish right away. We're going to teach you some manners first.

Love,

CERN

We really should not count on knowing what the Higgs "looks like". It could be ...
Priestly, highly orthodox
Ornery/ mean, highly heretical

singer Daniel Higgs
Beautiful but unorthodox
Or, will the LHC bury the Higgs?
Motivation for Non-Standard Decays — single $H$

- A fairly recent plot of $\Delta \chi^2(PEW)$ vs. $m_H$ is:

At 95% CL, $m_{h_{SM}} < 157$ GeV and the $\Delta \chi^2$ minimum is near 85 GeV when all data are included.
The latest $m_W$ and $m_t$ measurements also prefer $m_{h_{SM}} < 100$ GeV.

Further, the blue-band plot may be misleading due to the discrepancy between the "leptonic" and "hadronic" measurements of $\sin^2 \theta_W^{eff}$, which yield $\sin^2 \theta_W^{eff} = 0.23113(21)$ and $\sin^2 \theta_W^{eff} = 0.23222(27)$, respectively. The SM has a CL of only 0.14 when all data are included.

If only the leptonic $\sin^2 \theta_W^{eff}$ measurements are included, the SM gives a fit with CL near 0.78. However, the central value of $m_{h_{SM}}$ is then near 50 GeV with a 95% CL upper limit of $\sim 105$ GeV (Chanowitz, xarXiv:0806.0890).
Figure 1: $\chi^2$ distributions as a function of $m_H$ from the combination of the three leptonic asymmetries $A_{LR}$, $A^\ell_{FB}$, $A_\ell(P_\tau)$ (solid line); the three hadronic asymmetries $A^b_{FB}$, $A^c_{FB}$, and $Q_{FB}$ (dashed line); and the three $m_H$-sensitive, nonasymmetry measurements, $m_W, \Gamma_Z$, and $R_l$ (dot-dashed line). The horizontal lines indicate the respective 90% symmetric confidence intervals.

- Thus, in an ideal model, a Higgs with SM-like $ZZ$ coupling should have mass no larger than 105 GeV.
But, at the same time, the $H$ must escape LEP and CDF/D0 limits on $m_H$. In the case of a completely SM-like Higgs they are summarized as

![Search for the Higgs Particle](image)


| Mode Limit (GeV) | SM modes 114.4 | $2\tau$ or $2b$ only 115 | $2j$ 113 | $WW^* + ZZ^*$ 100.7 | $\gamma\gamma$ 117 | $E$ 114 | 4e, 4μ, 4γ 114?
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<td>$4b$ 110</td>
<td>pure $4\tau$</td>
<td>any (e.g. $4j^1$) 82</td>
<td>$2f + E$ 90?</td>
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1. Latest ALEPH result.

To have $m_H \leq 105$ GeV requires one of the final three modes.
• Perhaps the ideal Higgs should be such as to predict the $2.3\sigma$ excess at $M_{bb} \sim 98$ GeV seen in the $Z + b\bar{b}$ final state.

![Graph](image)

**Figure 2**: Plots for the $Zb\bar{b}$ final state. $F$ is the $m_Z$-fine-tuning measure for the NMSSM.

The simplest possibility for the excess is to have $m_H \sim 100$ GeV and $B(H \rightarrow b\bar{b}) \sim (0.1 - 0.2) \times B(H \rightarrow b\bar{b})_{SM}$ (assuming $H$ has SM $ZZ$ coupling as desired for precision electroweak) with the remaining $H$ decays being to one or more of the poorly constrained channels.
One generic way of having a low LEP limit on $m_H$ is to suppress the $H \to b\bar{b}$ branching ratio by having a light $a$ (or $h$) with $B(H \to aa) > 0.7$ and $m_a < 2m_b$ (to avoid LEP $Z + 4b$ limit at 110 GeV, i.e. above ideal). For $2m_\tau < m_a < 2m_b$, $a \to \tau^+\tau^-$. For $m_a < 2m_\tau$, $a \to jj$.


Since the $Hb\bar{b}$ coupling is so small, very modest $Haa$ coupling suffices.

Higgs pair modes can easily dominate until we pass above the $WW$ threshold.

So, let us suppose that we want $m_H < 105$ GeV. We should then recall the triviality and global minimum constraints on the scale $\Lambda$ of new physics.
Figure 3: Triviality and global minimum constraints on $m_{h_{\text{SM}}}$ vs. $\Lambda$.

The implication is that some new physics should arise for $\Lambda < 10^4(10^3)$ GeV if $m_h \sim 100$ GeV ($\sim 50$ GeV). A wonderful choice would be SUSY.

- SUSY does many wonderful things. In particular, SUSY cures the naturalness / hierarchy problem.
• Indeed, the MSSM comes close to being very nice.

If we assume that all sparticles reside at the $O(1 \text{ TeV})$ scale and that $\mu$ is also $O(1 \text{ TeV})$, then, the MSSM has two particularly wonderful properties.

1. **Gauge Coupling Unification**

![Figure 4: Unification of couplings constants ($\alpha_i = g_i^2/(4\pi)$) in the minimal supersymmetric model (MSSM) as compared to failure without supersymmetry.](image)

$\alpha_i$ are the coupling constants for the Standard Model, $\alpha_1$, $\alpha_2$, and $\alpha_3$, which correspond to the electromagnetic, weak, and strong forces, respectively. In the MSSM, these coupling constants unify at a high scale, demonstrating the predictive power of supersymmetry.
Figure 5: Evolution of the (soft) SUSY-breaking masses or masses-squared, showing how $m_{H_u}^2$ is driven < 0 at low $Q \sim \mathcal{O}(m_Z)$.

But, must one fine-tune the GUT scale parameters to get correct $Z$ mass? $F$ measures the degree to which GUT parameters must be tuned. Want $F < 10$. This requires $m_{\tilde{t}} \lesssim 400$ GeV and a relatively light gluino.

For such $m_{\tilde{t}}$ SUSY predicts $m_h < 110$ GeV. This is a problem for
the MSSM for which the $h$ is typically SM-like in its decays. To get $m_h > 114$ GeV requires $m_{\tilde{t}} > 800$ GeV and then $F > 50$.

- What is needed is a SUSY model for which the stop mass can be low but for which the resulting light $\lesssim 105$ GeV Higgs is not excluded by LEP. LEP exclusion can be avoided by having unusual decays as seen earlier.

- The NMSSM is perfect

It is the $h_1$ that is light and SM-like and the $a_1$ is mainly singlet and has a small mass that is protected by a $U(1)_R$ symmetry. Large $B(h_1 \rightarrow a_1a_1)$ is easy to achieve. We will simplify and denote for the most part $h_1 \rightarrow h$ and $a_1 \rightarrow a$.

The many attractive features of the NMSSM are well known:

1. Solves $\mu$ problem: $W \ni \lambda \hat{S} \hat{H}_u \hat{H}_d \Rightarrow \mu_{\text{eff}} = \lambda \langle S \rangle$.

2. Preserves MSSM gauge coupling unification.
3. Preserves radiative EWSB.

4. Preserves dark matter (assuming $R$-parity is preserved).

5. Like any SUSY model, solves quadratic divergence hierarchy problem.

6. Has additional attractive features when $m_h \sim 90 - 105$ GeV is allowed because of $h \rightarrow aa$ decays with $m_a < 2m_b$:

   (a) Allows minimal fine-tuning for getting $m_Z$ (i.e. $v$) correct after evolving from GUT scale $M_U$. (R. Dermisek and J. F. Gunion, Phys. Rev. D 73, 111701 (2006) [arXiv:hep-ph/0510322])
   
   This is because $\tilde{t}_1, \tilde{t}_2$ can be light ($\sim 350$ GeV is just right). Also need $m_{\tilde{g}}$ not too far above 300 GeV.

   (In MSSM, such low stop masses are not acceptable since $m_{h^0}$ would be below LEP limits; large $m_\tilde{t}$ ⇒ $m_Z$ fine tuning would be large, especially if $m_h$ is SM-like.)

   (b) An $a$ with large $B(h \rightarrow aa)$ and $m_a < 2m_b$ can be achieved without fine-tuning of the $A_\lambda$ and $A_\kappa$ soft-SUSY breaking parameters ($V \ni$
\( A_\lambda S H_u H_d + \frac{1}{3} A_\kappa S^3 \) that control the \( a \) properties. (R. Dermisek and J. F. Gunion, Phys. Rev. D 75, 075019 (2007) [arXiv:hep-ph/0611142].)

The \( a \) is largely singlet (\( e.g. \) 10\% at amplitude level if \( \tan \beta \sim 10 \)) and \( \sim 7.5 \text{ GeV} \lesssim m_a \) (but below \( 2m_b \)) in the best cases.

7. Of course, multi-singlet extensions of the NMSSM will expand the possibilities.

Indeed, typical string models predict a plethora of light \( a \)’s, light \( h \)’s and light \( \tilde{\chi} \)’s.

8. Many other non-Higgs decay modes of the \( h \) or \( h_1 \) have been proposed. Even sticking to SUSY, we have lots.

Models which preserve \( R \)-parity and thus dark matter possibility include:

(a) \( h \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_1^0 \) followed by \( \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 f \bar{f} \) (S. Chang and T. Gregoire, arXiv:09030403):

Turns out to be hard to accommodate given LEP constraints.

(b) \( h \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow \tilde{G} \tilde{G} \gamma \gamma \rightarrow E_T \gamma \gamma \): Can’t recall others who have worked on this, but I consider it likely that LEP would have seen such decays for a light \( h \) in the mass range of interest for PEW perfection.
(c) $h \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow \not{E}_T$: perfectly viable for non-unified gaugino masses, but LEP limit for invisibly decaying $h$ is 114 GeV which is too heavy for PEW perfection.

Many other models also have dominant invisible $h$ decay, but all suffer from the $m_h > 114$ GeV LEP limit for this mode that is less than ideal for PEW.

$$\xi^2 = \left[ \frac{\sigma(Zh)}{\sigma(Zh_{SM})} \right] \times B(h \rightarrow \not{E})$$ curve for invisible mode is very steep, $\Rightarrow$ allowed $m_h$ does not decrease much until $\xi^2$ is quite small.

Models which violate $R$ parity (and therefore require an alternative DM candidate than the $\tilde{\chi}_1^0$):

(a) There are too many to list systematically. A particularly nasty one is baryon-violating $R$-parity decays (L.M. Carpenter, D.E. Kaplan and E-J Rhee, arXiv:hep-ph/0607204) $h \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow (3j)(3j)$.

Such a multi-jet mode is least constrained by LEP ($m_h > 82$ GeV is the limit) and the lighter the $h$ the better the agreement with precision data (especially dropping hadronic asymmetries).
Predictions regarding a light $a$ and the NMSSM $a$

- Define the mass eigenstate: $a = \cos \theta_A a_{MSSM} + \sin \theta_A a_S$.

**Figure 6:** $G$ vs. $\cos \theta_A$ for $M_{1,2,3} = 100, 200, 300$ GeV and $\tan \beta = 10$ from $\mu_{\text{eff}} = 150$ GeV scan (left) and for points with $F < 15$ (right) having $m_a < 2m_b$ and large enough $B(h \rightarrow aa)$ to escape LEP limits. The color coding is: blue = $m_a < 2m_\tau$; red = $2m_\tau < m_a < 7.5$ GeV; green = $7.5$ GeV $< m_a < 8.8$ GeV; and black = $8.8$ GeV $< m_a < 9.2$ GeV.

- In the figure, $G$ is a measure (Dermisek+JFG: hep-ph/0611142) of the degree
to which $A_{\lambda}$ and $A_{\kappa}$ have to be fine tuned ("light-\(a\)" fine tuning) in order to achieve required \(a\) properties of \(m_a < 2m_b\) and \(B(h \to aa) > 0.7\).

The plot of \(G\) vs. \(\cos \theta_A\) shows a strong preference for \(m_a > 7.5\) GeV and \(\cos \theta_A \lesssim 0.1\) (for \(\tan \beta = 10\)). Note the strict lower bound on \(\cos \theta_A\) needed for \(B(h \to aa) > 0.7\).

- Define a generic coupling to fermions by

\[
\mathcal{L}_{af\bar{f}} \equiv iC_{af\bar{f}} \frac{ig_2m_f}{2m_W} \bar{f} \gamma_5fa, \quad \text{then} \quad C_{\bar{a}b\bar{b}} = \cos \theta_A \tan \beta \quad (1)
\]

At large \(\tan \beta\), SUSY corrections \(C_{\bar{a}b\bar{b}} = C_{ab\bar{b}}^{\text{tree}}[1/(1 + \Delta_{b}^{\text{SUSY}})]\) can be large and either suppress or enhance \(C_{\bar{a}b\bar{b}}\) relative to \(C_{a\tau-\tau+}\). Will ignore.

- The extracted \(C_{\bar{a}b\bar{b}}\) limits (JFG, arXiv:0808.2509 and JFG+Dermisek, in preparation; see also Ellwanger and Domingo, arXiv:0810.4736) appear in Fig. 7.

- The most unconstrained region is that with \(m_a > 8\) GeV, especially \(9\) GeV \(< m_a \leq 12\) GeV.

The \(9\) GeV \(m_a < 2m_B\) portion of the latter is the same as the region with least "light-\(a\)" fine-tuning in the NMSSM.
- One needs to achieve limits of $C_{ab\bar{b}} < 0.3$ to rule out the $a$ of the $C_{ab\bar{b}} = \cos\theta_A \tan\beta \lesssim 1$ (a number which applies for $\tan\beta > 3$) scenarios preferred to achieve small light-$a$ finetuning.

Figure 7: Limits on $C_{ab\bar{b}}$ from JFG, arXiv:0808.2509 and JFG+Dermisek, in preparation. These limits include recent BaBar $\Upsilon_{3S} \rightarrow \gamma\mu^+\mu^-$ and $\gamma\tau^+\tau^-$ limits. Color code: $\tan\beta = 0.5$; $\tan\beta = 1$; $\tan\beta = 2$; $\tan\beta \geq 3$. 
In the $\sim 9 \text{ GeV} \lesssim m_a \lesssim 12 \text{ GeV}$ region only the OPAL limits are relevant. Those presented depend upon how the $a \leftrightarrow \eta_b$ states mixing is modeled. A particular model (Drees+Hikasa: Phys.Rev.D41:1547,1990) is employed. Perhaps now that the first $\eta_b$ state has been observed, this region can be better pinned down. I have not incorporated recent work by Domingo et al. (arXiv:0810.4736) which models this mixing in a manner consistent with the available information. In any case, models predict many $\eta$-type states in this region, not just the one that has been observed.

Given $C_{abb}$ limits, an interesting question is whether there is any possibility that a light $a$ could be responsible for the observed $a_\mu$ discrepancy which is of order $\Delta a_\mu \sim 30 \times 10^{-10}$.

For this, large $C_{abb}$ is needed.

The plotted limits (mainly BaBar at up near $m_a \sim 9 \text{ GeV}$) suggest that it is generically possible from $C_{abb}$ limits if $m_a > 9 \text{ GeV}$, but is not possible in the NMSSM scenarios with small light-$a$ fine-tuning since they do not have large $C_{abb}$. 

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We will see that $B(a \rightarrow \mu^+\mu^-)$ is an interesting quantity.

Figure 8: $B(a \rightarrow \mu^+\mu^-)$ for various $\tan \beta$ values.
It will also become important to know about $B(a \rightarrow \tau^+\tau^-)$. Note values at high $\tan \beta$ of $\sim 0.75$ for $m_a \gtrsim 10$.

**Figure 9:** $B(a \rightarrow \tau^+\tau^-)$ for various $\tan \beta$ values.
Both are influenced by the structures in $B(a \rightarrow gg)$, which in particular gets substantial at high $m_a$ where the $b$-quarks of the internal $b$-quark loop can be approximately on-shell.

Figure 10: $B(a \rightarrow gg)$ for various $\tan \beta$ values.
More on the strong BaBar limits on $B(\Upsilon_{3S} \rightarrow a\gamma)B(a \rightarrow \mu^+\mu^-)$ that become very constraining for $m_a < 2m_\tau$.

**Figure 11:** BaBar limits on $B(\Upsilon_{3S} \rightarrow \gamma a)B(a \rightarrow \mu^+\mu^-)$.

For $m_a < 2m_\tau$, the limits are below $2 \times 10^{-6}$ except for very low $m_a$. 
A comparison to NMSSM predictions ⇒ most NMSSM scenarios with $B(h \to aa) > 0.7$ and $m_a < 2m_\tau$ are eliminated; only a few at $\tan \beta \lesssim 3$ survive.

**Figure 12:** For $\tan \beta = 3$, we plot $B(\Upsilon_{3S} \to \gamma a) \times B(a \to \mu^+\mu^-)$ for NMSSM scenarios with various ranges for $m_a$. Color code: $m_a < 2m_\tau$; $2m_\tau < m_a < 7.5$ GeV; $7.5$ GeV $< m_a < 8.8$ GeV; $8.8$ GeV $< m_a < 2m_B$ GeV. The left plot comes from an $A_\lambda$, $A_\kappa$ scan holding $\mu_{\text{eff}}(m_Z) = 152$ GeV fixed. The right plot shows results for $F < 15$ scenarios with $m_a < 2m_B$. 

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Figure 13: For $\tan \beta = 10$ we plot $B(\Upsilon_{3S} \rightarrow \gamma a) \times B(a \rightarrow \mu^+ \mu^-)$ for NMSSM scenarios with various ranges for $m_a$. Color code: $m_a < 2m_\tau$; $2m_\tau < m_a < 7.5$ GeV; $7.5$ GeV $< m_a < 8.8$ GeV; $8.8$ GeV $< m_a < 2m_B$ GeV. The left plot comes from an $A_\lambda, A_\kappa$ scan holding $\mu_{eff}(m_Z) = 150$ GeV fixed. The right plot shows results for $F < 15$. 

For $\tan \beta = 10$ we plot $B(\Upsilon_{3S} \rightarrow \gamma a) \times B(a \rightarrow \mu^+ \mu^-)$ for NMSSM scenarios with various ranges for $m_a$. Color code: $m_a < 2m_\tau$; $2m_\tau < m_a < 7.5$ GeV; $7.5$ GeV $< m_a < 8.8$ GeV; $8.8$ GeV $< m_a < 2m_B$ GeV. The left plot comes from an $A_\lambda, A_\kappa$ scan holding $\mu_{eff}(m_Z) = 150$ GeV fixed. The right plot shows results for $F < 15$.
Thus, we have a convergence whereby low “light-a” fine tuning in the NMSSM and direct $\Upsilon_{3S} \rightarrow \gamma \mu^+ \mu^-$ limits single out the $m_a > 2m_\tau$ part of parameter space.

LHC studies of light $h$ NMSSM scenarios should (and have) focused on this case.

With regard to the $a$ itself, we should focus on Tevatron and LHC probes of a light $a$ with $2m_\tau < m_a < 2m_B$.

This is not to say that the Tevatron and LHC cannot be sensitive to $m_a < 2m_\tau$:

1. $B(a \rightarrow \mu^+ \mu^-)$ is much larger. **BUT**
2. Acceptance is presumably considerably smaller because of $p_T$ distributions for the $\mu$’s shifting down.
3. Backgrounds are presumably larger.

Studies of $m_a < 2m_\tau$ cases at hadron colliders are worth pursuing since they might completely eliminate all such NMSSM ideal Higgs scenarios.

Here we will focus on $m_a > 2m_\tau$. 
In fact, results from ALEPH that came out last week (Kyle Cranmer, Nov. 3 seminar) further shift the focus to high $m_a$ in the NMSSM context.

**Expected limits @ $m_a = 4,10$ GeV**

Seeing no sign of excess, we proceed to set limits

> Here, we make reference to background acceptance uncertainties in MSSM Higgs analysis. (Statistical errors dominate, systematics make little difference in result)
• Comparison to NMSSM ideal scenarios:

1. $m_h \sim 95 \text{ GeV} - 103 \text{ GeV}$ to minimize electroweak $m_Z$ finetuning.
2. Large enough $B(h \to b\bar{b}) \sim 0.15 - 0.2$ to explain $2.3\sigma$ LEP excess.
3. $9 \lesssim m_a \lesssim 2m_B$ to fully minimize light-$a$ finetuning.

In this case, we typically have (see plots)

1. $\sigma(h)/\sigma(h_{\text{SM}}) \sim 0.92 - 1.0$
2. $B(h \to aa) \sim 0.8 - 0.85$
3. $B(a \to 2\tau) \sim 0.75 - 0.8$

Together, these yield $\xi^2$ as low as $\xi^2 \sim 0.43$ at $\tan \beta = 10$ and as low as $\xi^2 \sim 0.35$ at $\tan \beta = 3$.

• Thus, while $m_a \sim 4$ GeV is still ok if $m_h \sim 105$ GeV and $\xi^2$ is in the above range, ALEPH limits tend to push into the higher $m_a$ part of model space that is most ideal with respect to light-$a$ finetuning perspective and explaining the LEP $2.3\sigma$ excess near 100 GeV.

If all LEP experiments perform this kind of analysis and combine results will they rule out this corner?
Figure 14: $\xi^2$ vs. $m_a$ for $\tan \beta = 10$. 

$\tan \beta = 10$, $M_{1,2,3} = 100, 200, 300$ GeV

$F < 15$, $G < 20$
Figure 15: $\xi^2$ vs. $m_h$ for $\tan \beta = 10$. 

$\tan \beta = 10, M_{1,2,3} = 100, 200, 300$ GeV 

- $\blacklozenge$ F<15, G<20
- $\blacklozenge$ F<15, 20< G<30
Figure 16: $\xi^2$ vs. $m_a$ for $\tan \beta = 3$. 
Figure 17: $\xi^2$ vs. $m_h$ for $\tan \beta = 3$. 

$\tan \beta = 3$, $M_{1,2,3} = 100,200,300$ GeV

F<15, G< 20
Hadron collider constraints on a light $a$

- As we have seen, the Upsilon constraints on a light $a$ run out for $m_a > M_{\Upsilon_{3S}} - \delta$. This leaves open the possibility that $\Delta a_\mu$ could be explained by a light $a$ if $C_{ab\bar{b}}$ is big in this region. Remarkably, existing Tevatron data rule out this possibility (JFG+Dermisek, in preparation). And LHC constraints on the $a$ or $\tilde{a}$ are likely to be even stronger.

- At a hadron collider, one studies $\mu^+\mu^-$ pair production and tries to reduce the heavy flavor background by isolation cuts on the muons. Various studies of $\Upsilon$ production have been performed and CDF has even done an analysis in which they look for a very narrow $\epsilon$ (a hypothesized particle of a non-SUSY model) over the region $6.3 < m_\epsilon < 9$ GeV. The latest CDF limits from $L = 630\text{ pb}^{-1}$ of data on $R \equiv \sigma(\epsilon)B(\epsilon \rightarrow \mu^+\mu^-)/\sigma(\Upsilon_{1S})B(\Upsilon_{1S} \rightarrow \mu^+\mu^-)$ rule out the old peak at $m_\epsilon = 7.2$ GeV and can be adopted to limit this same ratio for a general $a$ or the NMSSM $a$. 
Ingredients:

- First, we need the cross sections. These are basically from $gg$ fusion with $gga$ coupling induced by quark loops. Higher order corrections, both virtual and real (e.g. for the latter $gg \rightarrow ag$) are, however, quite significant.

Main points are:

1. Isolation cuts on $\mu$’s do not seem to exclude NLO real radiation diagrams (based on CDF, ATLAS, CMS $\Upsilon$ efficiencies and fact that $\sigma(\Upsilon)$ has many components involving one or more extra final state $g$ or $q$).

2. Slow energy variation. At $m_a = 10$ GeV and $\tan\beta = 10$, one finds $\sigma_{NLO}(1.96, 7, 10, 14 \text{ TeV}) \sim 1.5 \times 10^5, 5 \times 10^5, 7 \times 10^5, 9 \times 10^5 \text{ pb}$.

3. For NMSSM, multiply by $(\cos \theta_A)^2$.

- Then, we must know $B(a \rightarrow \mu^+\mu^-)$, which we plotted earlier, a rough value being 0.003 for $m_a > 2m_\tau$ and $\tan\beta > 2$.

- We need efficiencies for detecting the $\mu^+$ and $\mu^-$ at given $m_a$.

- We must know the background, which mainly derives from heavy flavor production, especially $b\bar{b}$ where the $b$’s decay semi-leptonically.
Figure 18: Tevatron cross sections for $\tan \beta = 1, 2, 3, 10$ (lowest to highest point sets). For each $m_a$ and $\tan \beta$ value, the lower (higher) point is the cross section without (with) resolvable parton final state contributions.
For later reference when we discuss LHC:

Figure 19: LHC, $\sqrt{s} = 10$ TeV cross sections for $\tan \beta = 1, 2, 3, 10$ (lowest to highest point sets). Factor of about $7 \times$ Tevatron at higher $m_\alpha$. 
Putting it all together gives:

Figure 20: Tevatron limits compared to previous plot limits for $\tan \beta = 0.5, 1, 2, \geq 3$. Tevatron at $L = 10 \text{ fb}^{-1}$ competes with BaBar for $m_\alpha \sim 9 \text{ GeV}$ and would win above that. Indeed, the $L = 10 \text{ fb}^{-1}$ statistically extrapolated limits are approaching the $C_{ab\bar{b}} = \tan \beta \cos \theta_A \sim 1$ level that impacts the most preferred NMSSM scenarios.
For $M_{\mu^+\mu^-} > 9$ GeV, CDF did not perform the $R$ analysis. Instead, we use the event number plots that extend to larger $M_{\mu^+\mu^-}$. We ask for the $|C_{ab\bar{b}}|$ limits assuming no 90% CL (1.686$\sigma$) fluctuation in $S/\sqrt{B}$-optimized $m_\alpha$ interval of $2\sqrt{2}\sigma_r$, where $\sigma_r$ is the $M_{\mu^+\mu^-}$ resolution.

Figure 21: $L = 630$ pb$^{-1}$ and 10 fb$^{-1}$ limits based on no 1.686$\sigma$ excess in optimal interval.
We see that in the region below 12 GeV where a light $a$ might have explained $\Delta a_\mu$ if $C_{ab\bar{b}} \gtrsim 32$, current Tevatron data forbids such a large $C_{ab\bar{b}}$. One can finally conclude that $\Delta a_\mu$ cannot be due to a light $a$.

What about the LHC? There have been studies by CMS and ATLAS, and for reasons that I am still trying to explore with the experimentalists the di-muon background in the CMS studies is larger than that in the ATLAS studies. Also, only ATLAS has presented public results — see Fig. 22.

Figure 22: ATLAS dimuon spectrum prediction after corrections for acceptance and efficiencies (D. D. Price, arXiv:0808.3367 [hep-ex].).
Consider $\tan \beta = 10$ and $\cos \theta_A = 0.1$ (middle range of most preferred NMSSM models).

- After accounting for efficiency tracking factor of $\sim 50\%$ (vs. CDF 6%), the need to double plotted continuum background which was only from $b\bar{b}$ (in particular, did not include $c\bar{c}$), and the resolutions $\sigma_r(M_{\mu^+\mu^-})$ (54 MeV at $J/\psi$ and 170 MeV at $\Upsilon_{1S}$), we compute the number, $N_{\Delta M_{\mu^+\mu^-}}$, of events in an interval of total width $\Delta M_{\mu^+\mu^-} = 2\sqrt{2}\sigma_r$ (the interval that maximizes $S/\sqrt{B}$).

We obtain background levels of 2121, 23519, and 4819 at 8 GeV, $M_{\Upsilon_{1S}}$ and 10.5 GeV, respectively. Note: at $\Upsilon_{1S}$ peak can use $\Upsilon_{1S} \to e^+e^-$ to independently measure this background.

We compute $\sqrt{N_{\Delta M_{\mu^+\mu^-}}} 1\sigma$ errors of 45, 153 and 69 at 8 GeV, $M_{\Upsilon_{1S}}$ and 10.5 GeV, respectively.

- We now consider the $a \to \mu^+\mu^-$ signal rates.

From Fig. 19, we see that at $\tan \beta = 10$ the total $a$ cross section ranges...
from about \(4.2 \times 10^5 \text{ pb} (\cos \theta_A)^2 \sim 4200 \text{ pb}\) at \(m_a = 8 \text{ GeV}\) to \(\sim 8500 \text{ pb}\) at \(m_a \lesssim 2m_B\) for \(\sqrt{s} = 14 \text{ TeV}\). Including \(B(a \to \mu^+\mu^-) \sim 0.003\) we get \(\sigma(gg \to a \to \mu^+\mu^-) \sim 12 - 25 \text{ pb}\) in \(m_a \in [8 \text{ GeV} - 2m_B]\)

Multiplying by the \(Erf(1) = 0.8427\) acceptance factor for the ideal interval being employed and using \(L = 10 \text{ pb}^{-1}\), we obtain \(a\) event numbers of 101, 185 and 211 at \(m_a = 8 \text{ GeV}, M_{\Upsilon_{1S}}\) and 10.5 GeV, respectively.

The statistical significances of the \(a\) peaks for \(L = 10 \text{ pb}^{-1}\) are then 2.2\(\sigma\), 1.2\(\sigma\) and 3.0\(\sigma\), respectively. But, small \(S/B\) values esp. at \(\Upsilon_{1S}\) peak.

- Of course, we currently expect that substantial early running will mostly take place at \(\sqrt{s} = 7 \text{ TeV}\) and \(\sqrt{s} = 10 \text{ TeV}\).

As noted earlier, lower \(\sqrt{s}\) implies a somewhat smaller \(a\) cross section in the \([8 \text{ GeV}, 2m_B]\) mass interval on which we are focusing. Roughly, relative to \(\sqrt{s} = 14 \text{ TeV}\), the \(a\) cross section decreases by a factor of \(\sim 1.3\) at \(\sqrt{s} = 10 \text{ TeV}\) and a factor of \(\sim 1.7\) at \(\sqrt{s} = 7 \text{ TeV}\) in this mass interval.

Since the backgrounds are also basically \(gg\) fusion induced, we presume that these same factors will apply to them. At \(\sqrt{s} = 10 \text{ TeV} (\sqrt{s} = 7 \text{ TeV})\)
this then will reduce the statistical significances given above by a factor of $1/\sqrt{1.3}$ ($1/\sqrt{1.7}$).

The statistical significances at $m_a = 8$ GeV, $M_{Y_1S}$ and $10.5$ GeV are, respectively, then $2.0\sigma$, $1.1\sigma$, $2.7\sigma$ at 10 TeV and $1.7\sigma$, $0.9\sigma$, $2.3\sigma$ at 7 TeV.

At 10 TeV (7 TeV), to reach the $5\sigma$ signal level for $\tan\beta \cos\theta_A = 1$ at $m_a = M_{Y_1S}$ would require only $L = 207$ pb$^{-1}$ ($L = 309$ pb$^{-1}$).

Such integrated luminosities are quite likely to be achieved after a year or two of LHC early operation.

- We are somewhat surprised by the small integrated luminosities that we estimate for $5\sigma$ effects at ATLAS.

For example, at $m_a = 10.5$ GeV, about $L = 90$ fb$^{-1}$ is needed at Tevatron energies vs. only $34$ pb$^{-1}$ required at 10 TeV at ATLAS.

Obvious factors causing increased or decreased significance at ATLAS include: $\sim 5$ in increase in $gg$ induced xsecs due to larger $\sqrt{s}$; factor of $\sim 8$
increase in detection efficiency (50% at ATLAS vs. 6% at CDF); factor of \(\sim 3\) decrease due to worse ATLAS resolution (\(\sim 170\) MeV at ATLAS vs. \(\sim 52\) MeV at CDF). Net factor of \(\sim 13\) would imply that 90 \(\text{fb}^{-1}\) at CDF would be equivalent to \(\sim 6.6\) \(\text{fb}^{-1}\) at ATLAS. But this is still a factor of about 180 more than the \(L = 34\) \(\text{pb}^{-1}\) estimate above.

- One can repeat this kind of analysis using CMS inputs. We obtain the following comparisons.

Table 2: Comparison of statistical significances for \(C_{ab\bar{b}} = \cos \theta_A \tan \beta = 1\)

<table>
<thead>
<tr>
<th>Case</th>
<th>(m_a = 8) GeV</th>
<th>(m_a = M_{\Upsilon_1S})</th>
<th>(m_a \lesssim 2m_B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tevatron, (L = 10) (\text{fb}^{-1})</td>
<td>0.9</td>
<td>0.7</td>
<td>1.7</td>
</tr>
<tr>
<td>CMS, LHC7, (L = 10) (\text{pb}^{-1})</td>
<td>0.27</td>
<td>0.18</td>
<td>0.57</td>
</tr>
<tr>
<td>CMS, LHC10, (L = 10) (\text{pb}^{-1})</td>
<td>0.31</td>
<td>0.21</td>
<td>0.65</td>
</tr>
<tr>
<td>CMS, LHC14, (L = 10) (\text{pb}^{-1})</td>
<td>0.36</td>
<td>0.24</td>
<td>0.75</td>
</tr>
<tr>
<td>ATLAS LHC7, (L = 10) (\text{pb}^{-1})</td>
<td>1.7</td>
<td>0.9</td>
<td>2.3</td>
</tr>
<tr>
<td>ATLAS LHC10, (L = 10) (\text{pb}^{-1})</td>
<td>2.0</td>
<td>1.1</td>
<td>2.7</td>
</tr>
<tr>
<td>ATLAS LHC14, (L = 10) (\text{pb}^{-1})</td>
<td>2.2</td>
<td>1.2</td>
<td>3.0</td>
</tr>
</tbody>
</table>

\(C_{ab\bar{b}} \sim 0.2\) requires \([(1/0.2)^2]^2 \sim 625 \times\) more \(L\) to reach same levels.
NMSSM models in which several, perhaps many, Higgses carry the $ZZ$ coupling

These arise for $\tan \beta < 3$. (R. Dermisek and J. F. Gunion, arXiv:0811.3537 [hep-ph].)

- It is possible to have $h_1, h_2, h^+$ all light but escaping LEP and Tevatron detection by virtue of decays to $a$ with $m_a < 2m_b$.

- $h_1$ need not be exactly SM-like — $h_2$ can be light enough ($\sim 100$ GeV) for precision electroweak when $g_{h_2WW}^2$ is substantial.

- Relevant scenarios often arise for $C_{ab\bar{b}} \gtrsim 1$, especially if $\tan \beta = 2$. Current limits imply that $m_a \gtrsim 9$ GeV is needed for $C_{ab\bar{b}} \sim 2$ to be ok. However, low $\tan \beta$ scenarios also arise for very small $C_{ab\bar{b}} \sim 0.2$, for which exclusion via direct $a$ searches is very hard.

- The multiple LEP (and Tevatron) escapes:
  1. $B(h_1 \rightarrow aa)$ is large, and $e^+e^- \rightarrow Zh_1 \rightarrow Zaa \rightarrow Z4\tau$ is only constrained for $m_{4\tau} < 85$ GeV (recall decreased $B(a \rightarrow 2\tau)$ at low $\tan \beta$). Limit is lower if $ZZh_1$ coupling is somewhat suppressed.
2. $B(h^+ \rightarrow W^+a)$ is often large, and $e^+e^- \rightarrow h^+h^- \rightarrow W^+W^-aa$ with $a \rightarrow 2\tau$ was not directly searched for.

3. $B(h^+ \rightarrow \tau^+\nu)$ is often significant (but never dominant) and for cases with $m_{h^\pm}$ close to $m_W$, $e^+e^- \rightarrow h^+h^- \rightarrow \tau^+\tau^\nu\tau$ could explain the $2.8\sigma$ deviation from lepton universality in $W$ decays measured at LEP.

4. $B(h_2 \rightarrow aa)$ and/or $B(h_2 \rightarrow Za)$ are large.

   Thus, even if $e^+e^- \rightarrow Zh_2$ has large $\sigma$ (which is often the case since $m_{h_2}$ is not large), would not have seen it since the $h_2 \rightarrow Za$ decay was never looked for and an incomplete job was done on $h_2 \rightarrow aa \rightarrow 4\tau$.

5. For $\tan\beta = 1.7$ it is easy to find cases where $e^+e^- \rightarrow Zh_1 \rightarrow Zb\bar{b}$ and $e^+e^- \rightarrow Zh_2 \rightarrow Zb\bar{b}$ would yield a substantial contribution to the LEP $(0.1 - 0.2) \times SM$ excess near $m_{b\bar{b}} \sim 98$ GeV.

6. To observe or constrain the $a$ for larger (light-$a$ finetuning preferred) $m_a \lesssim 2m_B$, will require Tevatron high luminosity data or LHC. Still lots of models, even if not all, can be probed in this way.

7. High Tevatron $L$ would also better limit $B(t \rightarrow h^+b)$ which at the moment is allowed up to the 40% level as these decays are included in the way CDF and D0 determine the $t\bar{t}$ cross section for the $h^+ \rightarrow W^+a$. 

J. Gunion, Univ. of Freiburg, Nov. 27, 2009 52
Detecting the light $h$ of the NMSSM

All standard LHC channels fail: e.g. $B(h \rightarrow \gamma\gamma)$ is much too small because of large $B(h \rightarrow aa)$.

The possible new LHC channels include:

1. $gg \rightarrow h \rightarrow aa \rightarrow 4\tau$ and $2\tau + \mu^+\mu^-$

Always use $\mu$ tag for accepted events. $2\tau + 2\mu$ is main signal source after cuts.

There is an actual D0 analysis (A. Haas et. al.) of this mode using about $L \sim 4 \text{ fb}^{-1}$ of data. There are even small $\sim 1\sigma$ excesses for $m_a \sim 4$ and $10 - 11$ GeV consistent with predicted signal. About $L \sim 40 \text{ fb}^{-1}$ would
be needed for a $3\sigma$ signal.


At the LHC? Studied by Wacker et al.

- $\sigma(gg \rightarrow h) \sim 50$ pb for $m_h \sim 100$ GeV.
- $B(h \rightarrow aa) \sim 0.8 - 0.9$.
- $B(a \rightarrow \mu^+\mu^-) \sim 0.0035 - 0.004$ and $B(a \rightarrow \tau^+\tau^-) \sim 0.95 - 0.98$
- Useful branching ratio product is $2 \times B(a \rightarrow \mu^+\mu^-)B(a \rightarrow \tau^+\tau^-) \sim 0.0075$.
- Cut efficiencies $\epsilon \sim 0.018$. 
Net useful cross section:

\[ \sigma(gg \rightarrow h)B(h \rightarrow aa)[2B(a \rightarrow \mu^+\mu^-)B(a \rightarrow \tau^+\tau^-)]\epsilon \sim 4 - 7 \text{ fb}. \]  

(2)

Backgrounds are small so perhaps 10 events in a single \(\mu^+\mu^-\) bin would be convincing \(\Rightarrow\) need about \(L = 2\ \text{fb}^{-1}\).

Note: If \(m_a < 2m_\tau\), then \(B(a \rightarrow \mu^+\mu^-) > 0.06\) and

\[ \sigma(gg \rightarrow h)B(h \rightarrow aa)[B(a \rightarrow \mu^+\mu^-)^2\epsilon > (153 \ \text{fb}) \times \epsilon. \]  

(3)

If \(\epsilon > 0.02\) (seems likely) then \(\Rightarrow \sigma_{eff} > 3\ \text{fb}\). This should be really background free and would close the \(m_a < 2m_\tau\) ”window of worry”.

2. \(WW \rightarrow h \rightarrow aa \rightarrow \tau^+\tau^- + \tau^+\tau^-\).

Key will be to tag relevant events using spectator quarks and require very little activity in the central region by keeping only events with 4 or 6 tracks.

Looks moderately promising but far from definitive results at this time (see, A. Belyaev et al., arXiv:0805.3505 [hep-ph] and our work, JFG+Tait+Z. Han, below). More shortly.
3. $t\bar{t}h \rightarrow t\bar{t}aa \rightarrow t\bar{t} + \tau^+\tau^- + \tau^+\tau^-$.  
   No study yet. Would isolated tracks/leptons from $\tau$’s make this easier than $t\bar{t}h \rightarrow t\bar{t}b\bar{b}$?

4. $W, Z + h \rightarrow W, Z + aa \rightarrow W, Z + \tau^+\tau^- + \tau^+\tau^-$.  
   Leptons from $W, Z$ and isolated tracks/leptons from $\tau$’s would provide a clean signal. No study yet.

5. $\tilde{\chi}_2^0 \rightarrow h\tilde{\chi}_1^0$ with $h \rightarrow aa \rightarrow 4\tau$.  
   (Recall that the $\tilde{\chi}_2^0 \rightarrow h\tilde{\chi}_1^0$ channel provides a signal in the MSSM when $h \rightarrow b\bar{b}$ decays are dominant.)

6. Last, but definitely not least: diffractive production $pp \rightarrow pph \rightarrow ppX$.  
   The mass $M_X$ can be reconstructed with roughly a $1 - 2$ GeV resolution, potentially revealing a Higgs peak, independent of the decay of the Higgs.  
   The event is quiet so that the tracks from the $\tau$’s appear in a relatively clean environment, allowing track counting and associated cuts.
Signal significances from JFG, Forshaw, Pilkington, Hodgkinson, Papaefstathiou: arXiv:0712.3510 are plotted in Fig. 23 for a variety of luminosity and triggering assumptions.

Figure 23: (a) The significance for three years of data acquisition at each luminosity. (b) Same as (a) but with twice the data. Different lines represent different $\mu$ trigger thresholds and different forward detector timing. Some experimentalists say more efficient triggering is possible, doubling the number of events at given luminosity.

CMS folk claim we can increase our rates by about a factor of 2 to 3 using additional triggering techniques.
The Collinearity Trick

• Since $m_a \ll m_h$, the $a$’s in $h \rightarrow aa$ are highly boosted.

$\Rightarrow$ the $a$ decay products will travel along the direction of the source $a$.

$\Rightarrow p_a \propto \sum$ visible 4-momentum of the charged tracks in its decay.

Labeling the two $a$’s with indices 1 and 2 we have

$$p_{i,vis} = f_i p_{a,i}$$

(4)

where $1 - f_i$ is the fraction of the $a$ momentum carried away by neutrals.

• $pp \rightarrow pph$ case

The accuracy of this has now been tested in the $pp \rightarrow pph$ case, and gives an error for $m_h$ of order 5 GeV, but this is less accurate than $m_h$ determination from the tagged protons and so is not used.

However, we are able to make four $m_a$ determinations per event.
Figure 24: (a) A typical $\alpha$ mass measurement. (b) The same content as (a) but with the breakdown showing the 4 Higgs mass measurements for each of the 6 events, labeled 1 — 6 in the histogram.

Figure 24 shows the distribution of masses obtained for 180 fb$^{-1}$ of data collected at $3 \times 10^{33}$ cm$^{-2}$s$^{-1}$, corresponding to about 6 Higgs events and therefore 24 $m_{\alpha}$ entries.

By considering many pseudo-data sets, we conclude that a typical experiment would yield $m_{\alpha} = 9.3 \pm 2.3$ GeV, which is in re-assuringly good agreement with the input value of 9.7 GeV.
For $m_h = 100$ GeV and SM-like $WWh$ coupling, $\sigma(WW \rightarrow h) \sim 7$ pb, implying $7 \times 10^5$ events before cuts for $L = 100$ fb$^{-1}$.

In this case, we do not know the longitudinal momentum of the $h$, but we should have a good measurement of its transverse momentum from the tagging jets and other recoil jets.

This gives two equations in the two unknown $f_{1,2}$ and allows us to solve and construct mass peaks.

Figure 25: (a) A typical $h$ mass distribution. (b) A typical $a$ mass distribution. No cuts imposed; signal only.
Other related scenarios

- Low $\tan \beta$ NMSSM scenarios in which the first two CP-even Higgs bosons both have mass in the $\lesssim 100$ GeV region and decay so as to escape LEP (and Tevatron) limits. See earlier section.

- Drop dark matter requirement: $\Rightarrow$ huge plethora of possibilities in SUSY.

  Includes ”hidden valley” decays, $R$-parity violating decays, . . . .

- A string of Higgs, as possibly hinted at by the CDF multi-muon events.

  The SM-like Higgs could then decay into a string of Higgs bosons: e.g.
  
  $h \rightarrow h_1 h_1 \rightarrow (h_2 h_2)(h_2 h_2) \rightarrow ((h_3 h_3)(h_3 h_3))((h_3 h_3)(h_3 h_3)) \rightarrow \ldots$

  (Any of the $h_i$’s could be $a$’s and then $a_i \rightarrow a_j h_k$ would follow.)

  (Ellwanger et al have an NMSSM model that gives CDF multi-muon, but implications for unusual $h$ decays are unclear.)
Many singlets, as generically possible in string models, could mix with the doublet Higgs and create a series of Higgs eigenstates (with mass weight in the \(< 100 \text{ GeV}\) region for good PEW).

It can be arranged that these eigenstates decay in complex ways that would have escaped LEP limits.

In fact, one can get really low "effective" Higgs mass from PEW point of view while fitting under LEP constraint curve.

This is the "worst case" scenario envisioned long ago in JFG, Espinosa: hep-ph/9807275.

A true Higgs continuum as in the model of J. Van der Bij and collaborators and in the “unhiggs” models of Georgi and others.

These models rely on extra-dimensional concepts.

The hierarchy problem remains unless the ultraviolet completion scale / extra-dimension cutoff scale is low.
There would be only one narrow Higgs-like resonance and it would be impossible to see at the LHC since it would have only 10% of the usual $g_{ZZh}^2$ (to explain $2.3\sigma$ LEP excess near 98 GeV).

The many $a_i$ or $h_i$ of the preceding models would be replaced by a continuum and a search for narrow resonances as I have discussed would no longer work: ⇒ LHC won’t be able to detect such a continuum.

• Would a continuation of LEP/LEP2, especially up to $\sqrt{s} = 250$ GeV have seen a Higgs with unusual decays / less than expected $ZZh$ coupling?

And could it detect a series of Higgs bosons or even a continuum?

Recall that precision electroweak favors placing all the excess below about 157 GeV or perhaps even below 105 GeV (in a $g_{ZZh}^2$-weighted sense) as I have argued.

At LEP or any $e^+e^-$ collider the process $e^+e^- \rightarrow ZX$ will reveal a $M_X \sim m_h \sim 90 - 100$ GeV peak no matter how the $h$ decays so long as $g_{ZZh}^2 \gtrsim 0.05g_{ZZhSM}^2$, provided $L$ is adequate.
If there are many Higgs or even a continuum of Higgs, then the excesses in various bins of $M_X$ will be apparent even if there is a broad sort of spectrum and $X$ has a mixture of decays, provided the integrated $L$ is large.

- **ILC**

At the ILC, there is no problem: for planned $\sqrt{s}$ and $L$, $e^+e^- \rightarrow ZX$ is guaranteed to reveal the excess just as LEP might have.

But the ILC is decades away.
Conclusions

In case you hadn’t noticed, theorists have been going a bit crazy waiting for the Higgs.

"Unfortunately", a lot of the theories developed make sense, but I remain enamored of the NMSSM scenarios and hope for eventual verification that nature has chosen "wisely".
Meanwhile, all I can do is watch and wait (but perhaps not from quite so close a viewpoint).