The Importance of $\gamma C$ Probes in addition to LHC/LC for Unravelling the Scalar Sector of the Randall-Sundrum Model

Jack Gunion
Davis Institute for High Energy Physics, U.C. Davis


$\gamma C$ Collaborators: D. Asner, J. Gronberg, M. Velasco

LCWS 2004, 4/21/2004
References:


3. B. Grzadkowski and J. F. Gunion, “Bulk scalar stabilization of the radion without metric 
ph/0304241].


ph/0110320].


7. J. F. Gunion, M. Toharia and J. D. Wells, “Precision electroweak data and the mixed 
ph/0311219].
Previous work:

- $\xi = 0$:

- $\xi \neq 0$:
Presuming the new physics scale to be close to the TeV scale, there can be a rich new phenomenology in which Higgs and radion physics intermingle if the $\xi R \hat{H}^\dagger \hat{H}$ mixing term is present in $\mathcal{L}$. 
Some possibly very dramatic changes in phenomenology.

- There are two branes, separated in the 5th dimension ($y$) and $y \rightarrow -y$ symmetry is imposed. With appropriate boundary conditions, the 5D Einstein equations \( \Rightarrow \)

\[
\begin{align*}
    ds^2 &= e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - b_0^2 dy^2, \\
    \sigma(y) &\sim m_0 b_0 |y|.
\end{align*}
\]

- \( e^{-2\sigma(y)} \) is the warp factor; scales at \( y = 0 \) of order \( M_{Pl} \) on the hidden brane are reduced to scales at \( y = 1/2 \) of order TeV on the visible brane.

- Fluctuations of \( g_{\mu\nu} \) relative to \( \eta_{\mu\nu} \) are the KK excitations \( h_{n,\mu\nu} \).

- Fluctuations of \( b(x) \) relative to \( b_0 \) define the radion field.

- In addition, we place a Higgs doublet \( \widehat{H} \) on the visible brane. After various rescalings, the properly normalized quantum fluctuation field is called \( h_0 \).
Including the $\xi$ mixing term

- We begin with

$$S_\xi = \xi \int d^4x \sqrt{g_{\text{vis}}} R(g_{\text{vis}}) \hat{H}^\dagger \hat{H},$$  \hspace{1cm} (2)

where $R(g_{\text{vis}})$ is the Ricci scalar for the metric induced on the visible brane.

- A crucial parameter is the ratio

$$\gamma \equiv v_0/\Lambda_\phi.$$  \hspace{1cm} (3)

where $\Lambda_\phi$ is vacuum expectation value of the radion field.

- After writing out the full quadratic structure of the Lagrangian, including $\xi \neq 0$ mixing, we obtain a form in which the $h_0$ and $\phi_0$ fields for $\xi = 0$ are mixed and have complicated kinetic energy normalization.

We must diagonalize the kinetic energy and rescale to get canonical
normalization.

\[ h_0 = \left( \cos \theta - \frac{6\xi \gamma}{Z} \sin \theta \right) h + \left( \sin \theta + \frac{6\xi \gamma}{Z} \cos \theta \right) \phi \]
\[ \equiv dh + c\phi \]

\[ \phi_0 = -\cos \theta \frac{\phi}{Z} + \sin \theta \frac{h}{Z} \equiv a\phi + bh . \]  

\[ (4) \]

\[ (5) \]

• In the above equations

\[ Z^2 \equiv 1 + 6\xi \gamma^2 (1 - 6\xi) . \]

\[ (6) \]

\( Z^2 > 0 \) is required to avoid tachyonic situation.

This \( \Rightarrow \) constraint on maximum neg. and pos. \( \xi \) values.

• The process of inversion is very critical to the phenomenology and somewhat delicate.
The result found is that the physical mass eigenstates $h$ and $\phi$ cannot be too close to being degenerate in mass, depending on the precise values of $\xi$ and $\gamma$; extreme degeneracy is allowed only for small $\xi$ and/or $\gamma$.

Using this inversion, for given $\xi$, $\gamma$, $m_h$ and $m_\phi$ we compute $Z^2$, $m_{h_0}^2$ and $m_{\phi_0}^2$, $\theta$ to obtain $a$, $b$, $c$, $d$ in Eqs. (4) and (5).

- **Net result**

  4 independent parameters to completely fix the mass diagonalization of the scalar sector when $\xi \neq 0$. These are:

  $$\xi, \ \gamma, \ m_h, \ m_\phi, \ (7)$$

  where we recall that $\gamma \equiv v_0/\Lambda_\phi$ with $v_0 = 246$ GeV.

  The quantity $\hat{\Lambda}_W = \frac{1}{\sqrt{3}} \Lambda_\phi$ fixes the KK-graviton couplings to the $h$ and $\phi$ and

  $$m_1 = x_1 \frac{m_0 \Lambda_\phi}{M_{Pl} \sqrt{6}} \ (8)$$

  is the mass of the first KK graviton excitation ($x_1$ is the first zero of the Bessel function $J_1$ ($x_1 \sim 3.8$))
\( m_0/M_{Pl} \) is related to the curvature of the brane and should be a relatively small number for consistency of the RS scenario.

- Sample parameters that are safe from precision EW data and RunI Tevatron constraints are \( \Lambda_\phi = 5 \) TeV (\( \Rightarrow \hat{\Lambda}_W \sim 3 \) TeV) and \( m_0/M_{Pl} = 0.1 \).

The latter \( \Rightarrow m_1 \sim 780 \) GeV; i.e. \( m_1 \) is typically too large for KK graviton excitations to be present, or if present, important, in \( h, \phi \) decays.

But, KK excitations in this mass range (and much higher) will be observed and well measured at the LHC.

This will provide important information.

1. Mass gives \( m_1 \) in above notation.
2. Excitation spectrum as a function of \( m_{jj} \) determines \( m_0/M_{Pl} \).
3. Combine ala Eq. (8) to get \( \Lambda_\phi \).

Thus, the observation of the first KK excitation and its \( m_{jj} \) spectrum determines 1 of 4 Higgs-sector parameters as well as \( m_0/M_{Pl} \), leaving \( \xi \), \( m_h \) and \( m_\phi \) to be sorted out by Higgs/radion sector.

- For given \( \Lambda_\phi \) and \( m_0/M_{Pl} \), we complete the inversion by writing out the kinetic energy terms of the complete Lagrangian using the substitutions of
Eqs. (4) and (5) and demanding that the coefficients of $-\frac{1}{2} h^2$ and $-\frac{1}{2} \phi^2$ agree with the given input values for $m_H^2$ and $m_{\phi}^2$.

Results shown take $m_0/M_{Pl} = 0.1$. 
The Couplings

The $f \bar{f}$ and $VV$ couplings

For $V = W, Z$ and all $f$, the $h$ and $\phi$ couplings are rescaled relative to SM couplings by the universal factors:

$$g_{fVh} \equiv (d + \gamma b), \quad g_{fV\phi} \equiv (c + \gamma a). \quad (9)$$

The $gg$ and $\gamma\gamma$ couplings

- There are the standard loop contributions, except rescaled by $f \bar{f} / V V$ strength factors $g_{fVh}$ or $g_{fV\phi}$.

- In addition, there are anomalous contributions, which are expressed in terms of the $SU(3) \times SU(2) \times U(1)$ $\beta$ function coefficients $b_3 = 7$, $b_2 = 19/6$ and $b_Y = -41/6$.

- The anomalous couplings of $h$ and $\phi$ enter only through their radion admixtures, $g_h = \gamma b$ for the $h$, and $g_\phi = \gamma a$ for the $\phi$. 
• First, consider the $f\bar{f}/VV$ couplings of $h$ and $\phi$ relative to SM, taking $m_h = 120$ GeV and $\Lambda_{\phi} = 5$ TeV.

For the chosen value of $m_0/M_{Pl} = 0.1$, once $m_h$ and $\Lambda_{\phi}$ are fixed, the remaining free parameters are $m_{\phi}$ and $\xi$. The plots give the couplings in the $m_{\phi}, \xi$ parameter space.

Note the hourglass shape that defines the theoretically allowed region.

• The most important points

If $g_{fVh}^2 < 1$ is observed then $m_{\phi} > m_h$, and vice versa, except for small region near $\xi = 0$.

In cases where $g_{fV_{\phi}}$ is small, prior indirect knowledge of, or constraints on, $m_{\phi}$ could be crucial.

At large $|\xi|$, if $m_{\phi} > m_h$ the $ZZ\phi$ couplings can become sort of SM strength, implying SM type discovery modes could become relevant.
Figure 1: Contours of $g^2_{fVh} = (d + \gamma b)^2$ (relative to SM) for $\Lambda_\phi = 5$ TeV, $m_h = 120$ GeV.

- Observe suppression if $m_\phi > m_h$ and vice versa.
Figure 2: $\frac{g_{ZZh}^2}{g_{ZZh}^2_{SM}} = \frac{g_{f\phi h}^2}{g_{f\phi h}^2_{SM}}$ as a function of $\xi$ for several $m_\phi$ values.
Figure 3: Contours of $g_{fV\phi}^2 = (c + \gamma a)^2$ for $\Lambda_\phi = 5$ TeV, $m_h = 120$ GeV

- Substantial $g_{fV\phi}^2$ is possible if $m_\phi > m_h$ and $\xi$ is not too small.
Figure 4: \( \frac{g^2_{ZZ\phi}}{g^2_{ZZh}}_{\text{SM}} = \frac{g^2_{ff\phi}}{g^2_{ffh}}_{\text{SM}} \) as a function of \( \xi \) for several \( m_\phi \) values.
Some important points are:

- $h$ branching ratios are quite SM-like (even if partial widths are different) except that $h \rightarrow gg$ can be bigger than normal, especially when $g_{fVh}^2$ is suppressed.

- For $m_\phi < 2m_W$, $\phi \rightarrow gg$ is very possibly the dominant mode in the substantial regions near zeroes of $g_{fV\phi}^2$.

For $m_\phi > 2m_W$, $\phi$ branching ratios are sort of SM-like (except at $\xi \simeq 0$) but total and partial widths are rescaled.
At the LHC, we (Ref. [4]) focused on the case of a relatively light Higgs boson, $m_h = 120$ GeV for example.

- The precision EW studies (Ref. [7]) suggest that some of the larger $|\xi|$ range is excluded, but we studied the whole range just in case.

- We rescaled the statistical significances predicted for the SM Higgs boson at the LHC using the $h$ and $\phi$ couplings predicted relative to the $h_{\text{SM}}$.

A modified version of HDECAY was employed.

- The most important modes are $gg \rightarrow h \rightarrow \gamma\gamma$ and $gg \rightarrow \phi \rightarrow ZZ^{(*)} \rightarrow 4\ell$.

Also useful are $\tilde{t}\tilde{t}h$ with $h \rightarrow b\bar{b}$ and $h \rightarrow ZZ^* \rightarrow 4\ell$. 
Figure 5: SM Higgs search capabilities at the LHC for ATLAS and CMS.

- An example of the type of effect that will be observed is that the $h \rightarrow \gamma\gamma$ mode becomes unobservable if $|\xi|$ is large and $m_{\phi} > m_h$ (which together imply suppressed $hWW$ coupling and hence suppressed $W$-loop
contribution to the $\gamma\gamma h$ couplings).

One interesting graph is below (left). Note how we lose the $h \rightarrow \gamma\gamma$ mode if $m_\phi > m_h$, especially if $\xi < 0$. If $m_\phi < m_h$, $h \rightarrow \gamma\gamma$ will be strong if $\xi < 0$, but can be considerably weakened if $\xi > 0$.

**Figure 6:** $gg \xrightarrow{\xi} h \rightarrow \gamma\gamma/gg \xrightarrow{\xi} h_{SM} \rightarrow \gamma\gamma$ and $WW \rightarrow h \rightarrow \tau^+\tau^-/WW \rightarrow h_{SM} \rightarrow \tau^+\tau^-$ (same as for $gg \rightarrow t\bar{t}h \rightarrow t\bar{t}b\bar{b}$) for $m_{h_{SM}} = m_h; \Lambda_\phi = 5$ TeV.
Figure 7: The ratio of the rate for $gg \rightarrow \phi \rightarrow ZZ$ to the corresponding rate for a SM Higgs boson with mass $m_\phi$ assuming $m_h = 120$ GeV and $\Lambda_\phi = 5$ TeV as a function of $\xi$ for $m_\phi = 110, 140$ and 200 GeV. Recall that the $\xi$ range is increasingly restricted as $m_\phi$ becomes more degenerate with $m_h$. Note: for $m_\phi > m_h$ the mode approaches SM strength if $\xi < 0$ and is nearing SM strength if $\xi > 0$ and near maximal.
**Figure 8:** $L = 30 \text{ fb}^{-1}$ illustration of mode complementarity at the LHC for $m_h = 120$ GeV. Outer black lines show theoretically consistent (hour-glass shaped) parameter region. The blank (white) regions within the hour glass show the regions where neither the $gg \rightarrow h \rightarrow \gamma\gamma$ mode nor the (not very important at this $m_h$ value) $gg \rightarrow h \rightarrow 4\ell$ mode yields a $> 5\sigma$ signal. The regions between dark blue curves define the regions where $gg \rightarrow \phi \rightarrow 4\ell$ is $> 5\sigma$. The graphs are for $\Lambda_\phi = 2.5$ TeV (left) $\Lambda_\phi = 5$ TeV (center) and $\Lambda_\phi = 7.5$ TeV (right).

The LHC can find either the $h$ or $\phi$ except for the $m_\phi < m_h$, $\xi > 0$ and large, region.
But, some portion of this difficult region is disfavored by the precision electroweak data — e.g. $|\xi| \lesssim 1.5$ is preferred in the $\Lambda_\phi = 5$ TeV case.

The region where neither the $h$ nor the $\phi$ can be detected grows (decreases) as $m_h$ decreases (increases). It diminishes as $m_h$ increases since the $gg \rightarrow h \rightarrow 4\ell$ increases in strength at higher $m_h$.

Luminosity helps:

![Graph](image)

Figure 9: In this figure, the cyan (not the white) regions are those where $h$ discovery is not possible for $\Lambda_\phi = 5$ TeV and $m_h = 120$ GeV case assuming LHC $L = 30\text{fb}^{-1}$ (left) or $L = 100\text{fb}^{-1}$ (right).
The regions where the $h$ is not observable are reduced by considering either a larger data set or $qqh$ Higgs production, in association with forward jets. An integrated luminosity of 100 fb$^{-1}$ would remove the regions at large positive $\xi$ in the $\Lambda_\phi = 5$ and 7.5 TeV plots of Fig. 8. Similarly, including the $qqh$, $h \rightarrow WW^* \rightarrow \ell\ell\nu\bar{\nu}$ channel in the list of the discovery modes removes the same two regions and reduces the large region of $h$ non-observability at negative $\xi$ values.

- Figure 8 also exhibits regions of $(m_h, \xi)$ parameter space in which both the $h$ and $\phi$ mass eigenstates will be detectable.

In these regions, the LHC will observe two scalar bosons somewhat separated in mass, with the lighter (heavier) having a non-SM-like rate for the $gg$-induced $\gamma\gamma$ ($Z^0Z^0$) final state.

Additional information will be required to ascertain whether these two Higgs bosons derive from a multi-doublet or other type of extended Higgs sector or from the present type of model with Higgs-radion mixing.
• An $e^+e^-$ LC should guarantee observation of a light $h$ throughout all of the allowed parameter region by virtue of the fact that $g^2_{fVh}$ is not all that suppressed anywhere. (See earlier coupling figures.) Indeed, any light scalar, $s$ will be detected at the LC in the $Z^* \to Zs$ mode if $g^2_{ZZs} \gtrsim 0.01$.

• Unfortunately, the $\phi$ can have quite suppressed couplings and $g^2_{fV\phi}$ can fall below 0.01 for a significant part of parameter space. See Fig. 10. Unfortunately, this is also the region where precision measurements of the $h$ properties at the LC will have $\lesssim 2.5\sigma$ deviations from SM expectations, implying that we could conclude that we had a simple SM Higgs sector.
Figure 10: The dark blue region is that where $g_{fV\phi}^2 \lesssim 0.01$.

Can a $\gamma\gamma$ collider help? Also, what if there is no LC, but only a low energy $\gamma\gamma$ collider based on a few CLIC modules.
Let’s remind ourselves about the results for the SM Higgs boson obtained in the CLIC study of hep-ex/0111056.

There, a SM Higgs boson with $m_{h_{\text{SM}}} = 115$ GeV was examined.

After the cuts, one obtains about $S = 3280$ and $B = 1660$ in the $\gamma\gamma \rightarrow h_{\text{SM}} \rightarrow b\bar{b}$ channel, corresponding to $S/\sqrt{B} \sim 80$ !!!

We will assume that these numbers do not change significantly for a Higgs mass of 120 GeV by slightly increasing the operating energy.

- After mixing, the $S$ rate for the $h$ will be rescaled relative to that for the $h_{\text{SM}}$. Of course, $B$ will not change.

  The rescaling is shown in the figure.

- The $S$ for the $\phi$ can also be obtained by rescaling if $m_{\phi} \sim 115$ GeV.
Figure 11: The rates for $\gamma\gamma \rightarrow h \rightarrow b\bar{b}$ and $\gamma\gamma \rightarrow \phi \rightarrow b\bar{b}$ relative to the corresponding rate for a SM Higgs boson of the same mass. Results are shown for $m_h = 120$ GeV and $\Lambda_\phi = 5$ TeV as functions of $\xi$ for $m_\phi = 20$, 55 and 200 GeV.
Expectations for the $h$

Observe that for $m_\phi < m_h$ we have either little change or enhancement, whereas significant suppression of the LHC $gg \to h \to \gamma \gamma$ rate was possible in this case for positive $\xi$.

Also note that for $m_\phi > m_h$ and large $\xi < 0$ (where the LHC could not see the $h$) there is much less suppression of $\gamma \gamma \to h \to b\bar{b}$ than for $gg \to h \to \gamma \gamma$ — at most a factor of 2 vs a factor of 8 (at $m_\phi = 200$ GeV). This is no problem since $S/\sqrt{B} \sim \frac{1}{2}80 \sim 40$ is still a very strong signal.

- In fact, we can afford a reduction by a factor of 16 before we hit the 5$\sigma$ level!

- Thus, the $\gamma \gamma$ collider will allow $h$ discovery (for $m_h = 120$) throughout the entire hourglass, which is something the LHC cannot absolutely do.

Expectations for the $\phi$

- For $m_\phi < 120$ GeV, the $\phi \to b\bar{b}$ channel will continue to be the most relevant for $\phi$ discovery, but studies have not yet been performed to obtain the $S$ and $B$ rates for low masses.
• Using the factor of 16 mentioned above, the \( \phi \) with \( m_\phi < 120 \text{ GeV} \) is very likely to elude discovery at the \( \gamma \gamma \) collider. (Recall that it also eludes discovery at the LHC for this region.)

The only exceptions to this statement occur at the very largest \( |\xi| \) values for \( m_\phi \geq 55 \text{ GeV} \) where \( S_\phi > S_{h_{\text{SM}}}/16 \).

• Of course, we need to have signal and background results after cuts for these lower masses to know if the factor of 16 is actually the correct factor to use.

To get the best signal to background ratio we would want to lower the machine energy (relatively easy for CLIC case) and readjust cuts and so forth.

This study should be done.

• For the \( m_\phi > m_h \) region, we will need results for the \( W W \) and \( ZZ \) modes. Our current results are not encouraging.
A special role for the $\gamma C$ if we already have LHC and possibly LC results

Case 1: Suppose the $\phi$ is not seen at any of the three colliders.

- For $L \gtrsim 100 \text{fb}^{-1}$, the $h$ is very likely to be seen at the LHC as well as at a $\gamma C$ and the LC.

- Since $m_h$ will be well-measured, we are dealing with just 2 parameters, $m_\phi$ and $\xi$, to be determined. This requires 2 measurements to determine the parameters and 3 or more measurements to test the model.

- If we could trust LHC and $\gamma C$ and LC absolute rates (systematics being the question), their different dependencies on the parameters imply that we could then determine $m_\phi$ and $\xi$ and test the model even if we don’t see the $\phi$.

- An interesting way to phrase the LHC and $\gamma C$ rate measurements is in
terms of the ratio of the rates:

\[
\frac{\frac{\Gamma(gg\to h)\Gamma(h\to \gamma\gamma)}{\Gamma_{h}^{\text{tot}}}}{\frac{\Gamma(\gamma\gamma\to h)\Gamma(h\to b\bar{b})}{\Gamma_{h}^{\text{tot}}}} = \frac{\Gamma(h\to gg)}{\Gamma(h\to b\bar{b})}.
\] (10)

Using this result, we may compute

\[
R_{h_{gg}} \equiv \left[ \frac{\Gamma(h\to gg)}{\Gamma(h\to b\bar{b})} \right] \left[ \frac{\Gamma(h\to gg)}{\Gamma(h\to b\bar{b})} \right]^{-1}_{SM}.
\] (11)

This is a very!!! interesting number since it directly probes for the presence of the anomalous $ggh$ coupling.

In particular, $R_{h_{gg}} = 1$ if the only contributions to $\Gamma(h\to gg)$ come from quark loops and all quark couplings scale in the same way. However, the RS model predicts anomalous $gg$ coupling contributions in addition to rescaled standard loop contributions.
As a result, substantial deviations from $R_{hgg} = 1$ are predicted.

Figure 12: In the left two plots, we give the ratios $R_{hgg}$ and $R_{\phi gg}$ of the $hgg$ and $\phi gg$ couplings-squared including the anomalous contribution to the corresponding values expected in its absence. Results for the analogous ratios $R_{h\gamma\gamma}$ and $R_{\phi\gamma\gamma}$ are presented in the two plots on the right. Results are shown for $m_h = 120$ GeV and $\Lambda_\phi = 5$ TeV as functions of $\xi$ for $m_\phi = 20$, 55 and 200 GeV. (The same type of line is used for a given $m_\phi$ in the right-hand figure as is used in the left-hand figure.)

- The ratio $R_{hgg}$ is the only direct probe of the anomalous $ggh$ coupling.
We can estimate the accuracy with which $R_{hgg}$ can be measured as follows.

Assuming the maximal reduction of $1/2$ for the $S$ rescaling at the $\gamma\gamma$ CLIC collider, we find that

$$\frac{\Gamma(h \rightarrow \gamma\gamma)\Gamma(h \rightarrow b\bar{b})}{\Gamma_{tot}^h}$$

can be measured with an accuracy of about $\sqrt{S + B}/S \sim \sqrt{3200/1600} \sim 0.035$.

The dominant error will then be from the LHC which will typically measure

$$\frac{\Gamma(h \rightarrow gg)\Gamma(h \rightarrow \gamma\gamma)}{\Gamma_{tot}^h}$$

with an accuracy of between 0.1 and 0.2 (depending on parameter choices and available $L$).

From Fig. 12, we see that 0.2 fractional accuracy will reveal deviations of $R_{hgg}$ from 1 for all but the smallest $\xi$ values.

The direction and magnitude of those deviations will give strong constraints on $m_\phi$ relative to $m_h$ and $\xi$ (although, for instance, you can’t tell if $m_\phi < m_h$ and $\xi < 0$ or $m_\phi > m_h$ and $\xi > 0$).

In any case, $R_{hgg}$ alone gives a strong constraint on the 2 remaining parameters, $m_\phi$ and $\xi$. \Rightarrow need one more input to fix the parameters or two more inputs to over constrain.
Case 2: We also observe the $\phi$ at one or more machine.

- This is possible if $|\xi|$ is large, with the LHC giving probably the best overall chance — see Fig. 10 — although the LHC also has a good shot if $m_\phi > m_h$ — see Fig. 7.

The value of $R_{hgg}$ combined with knowing $m_\phi$ will then determine $\xi$ without relying on any absolute rates.

In addition, the $e^+e^- \rightarrow Z^* \rightarrow Z\phi$ rate in the inclusive mode is expected to be very reliable in an absolute sense. This rate determines directly

$$\frac{g_{Z\phi}^2}{g_{Zh_{\text{SM}}}^2} = g_f V_\phi,$$

(12)

a quantity that is wildly varying as a function of the model parameters, see earlier Fig. 4. This will over constrain and test the model.

- If the LHC also sees the $\phi$ we also get $m_\phi$ and another model-testing rate.

$\Rightarrow$ lots of cross checks on the model.
Conclusions

• Like the LC, the $\gamma C$ can see the $h$ (for the sort of mass studied here) for all of the $(\xi, m_\phi)$ RS parameter space.

  Both colliders can see the $h$ where the LHC can’t, although the “bad” LHC regions are not very big for full $L$.

• The ability to measure $R_{hgg}$ may be the strongest reason in the Higgs context for having the $\gamma C$ as well as the LHC and LC.

  Almost all non-SM Higgs theories predict $R_{hgg} \neq 1$ for one reason another, unless one is in the decoupling limit.

• If the LC can detect the $\phi$, the motivation for building the $\gamma C$ becomes even somewhat stronger since the measurement of $R_{hgg}$ becomes a very definitive test of the RS model.

• Don’t forget that the LHC can see the $\phi$ if $m_\phi > m_h$ and $|\xi|$ is large, implying that even if the LC is not available, we might get a definitive $(\xi, m_\phi)$ parameter determination using the measured $m_\phi$ and $R_{hgg}$. 
Further, the $\phi$ rate at the LHC would then test the model.

Further model tests would be possible if we could accurately measure the rate for $h$ production in other LHC and/or $\gamma C$ channels — something that is certainly possible, but not guaranteed (especially with high accuracy).

- Overall, there is a nice complementarity among the machines — each brings new abilities to probe and definitively test the model.

- Thus, there is a strong case for the $\gamma C$ in the RS model context!, especially if a Higgs boson is seen at the LHC that has non-SM-like rates, ...