MUON COLLIDER PHYSICS

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OUTLINE

The Machine

- Some key parameters.
- Some key machine features.

The Physics

- Front end physics.
- Z physics.
- Higgs bosons.
- Pseudo-Goldstone Bosons of technicolor.
- Precision m_W, m_t measurements.
- Leptoquarks
- R-parity violation in supersymmetry.
- Extra dimensions.
- Strong WW sector physics.
- New Z'
- High Mass Technicolor.
- Standard supersymmetry.

1 Key muon collider parameters and features

The many stages:

- Intense muon source.
- Cooled moderate energy muon beam for neutrino source, ...
- Low-energy Higgs factory collider: $\sqrt{s} \sim 100$ GeV.
- Intermediate-energy collider: $\sqrt{s} = 200 - 500$ GeV.
- High-energy collider: $\sqrt{s} > 3$ TeV.

For the three colliders, yearly integrated luminosities are still a question. Benchmarks are (using $\mathcal{L} = 1 \times 10^{32}\mathrm{cm}^{-2}\mathrm{s}^{-1} \Rightarrow L = 1\ \mathrm{fb}^{-1}/\mathrm{yr}$):

- $L_{\rm year} \gtrsim 0.1, 0.22, 1\,\mathrm{fb}^{-1}$ at $\sqrt{s} \sim 100\,\mathrm{GeV}$ for beam energy resolutions of $R = 0.003\%, 0.01\%, 0.1\%$, respectively;
- $L_{\rm year} \sim 2, 6, 10\ \mathrm{fb}^{-1}$ at $\sqrt{s} \sim 200, 350, 400\ \mathrm{GeV}$, respectively, for $R \sim 0.1\%$;
- $L_{\rm year} \sim 1000\ \mathrm{fb}^{-1}$ at $\sqrt{s} = 3 - 4$ TeV with $R \gtrsim 0.1\%$.

Key features of the muon collider:

- Beam energy spread of $\Delta E_{\rm beam}/E_{\rm beam} = 0.01R(\%)$ with $R = 0.003\%$ can be achieved at the low-energy Higgs factory so that the Gaussian spread in $\sqrt{s}$, given by

$$\sigma \sim 2\ \mathrm{MeV} \left(\frac{R}{0.003\%}\right) \left(\frac{\sqrt{s}}{100\ \mathrm{GeV}}\right) , \qquad (1.1)$$

  can be as small as the natural width of a light SM-like Higgs boson.
- For very narrow resonance scan:

- Need to know E_{beam} to few parts in 10^{-6}.
- Need stability in E at this level.
- Need to know σ

Measurements for each muon spill (10^8 spills per year) will be performed using muon spin precession and spectra of e's from the decays.

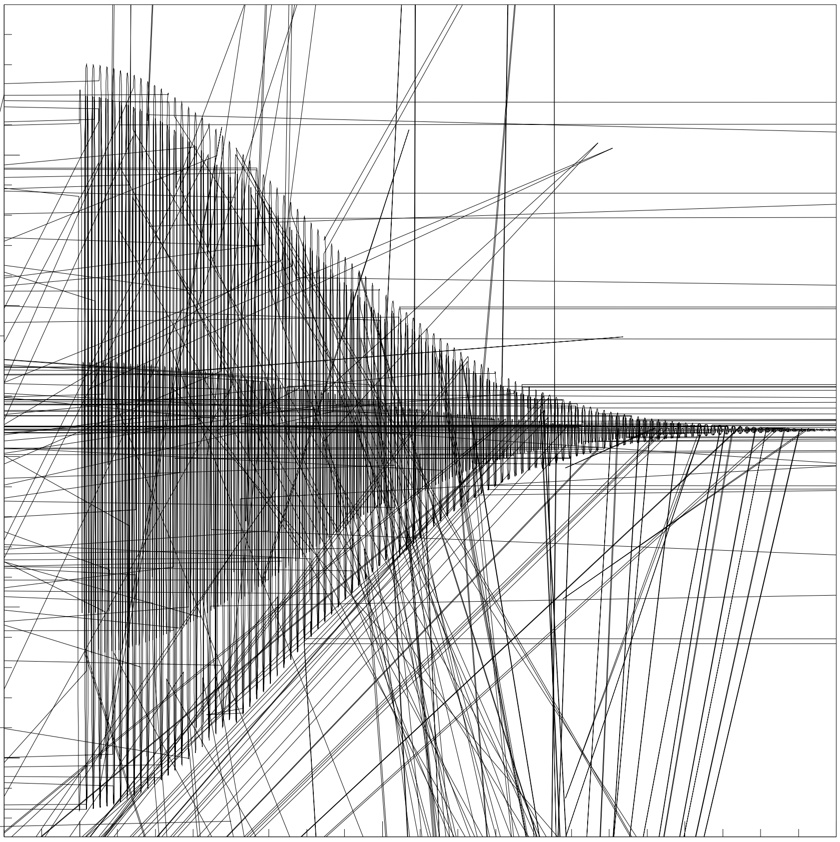

Figure 1: Oscillation with turn number for number of e's in $30-40$ GeV range from 50 GeV muons.

Defining $\nu = (g_\mu - 2)E_{\mathrm{beam}}/[2m_\mu]$, $N(T) \propto \int e^{i2\pi\nu T} S(\nu) d\nu$, where $S(\nu)$ is the distribution of μ's as a function of ν.

- Oscillation frequency → E_{beam}.
- Amplitude of envelope $\propto e^{-\frac{1}{2}(T 2\pi\nu \frac{\Delta E_{\mathrm{beam}}}{E_{\mathrm{beam}}})^2}$.

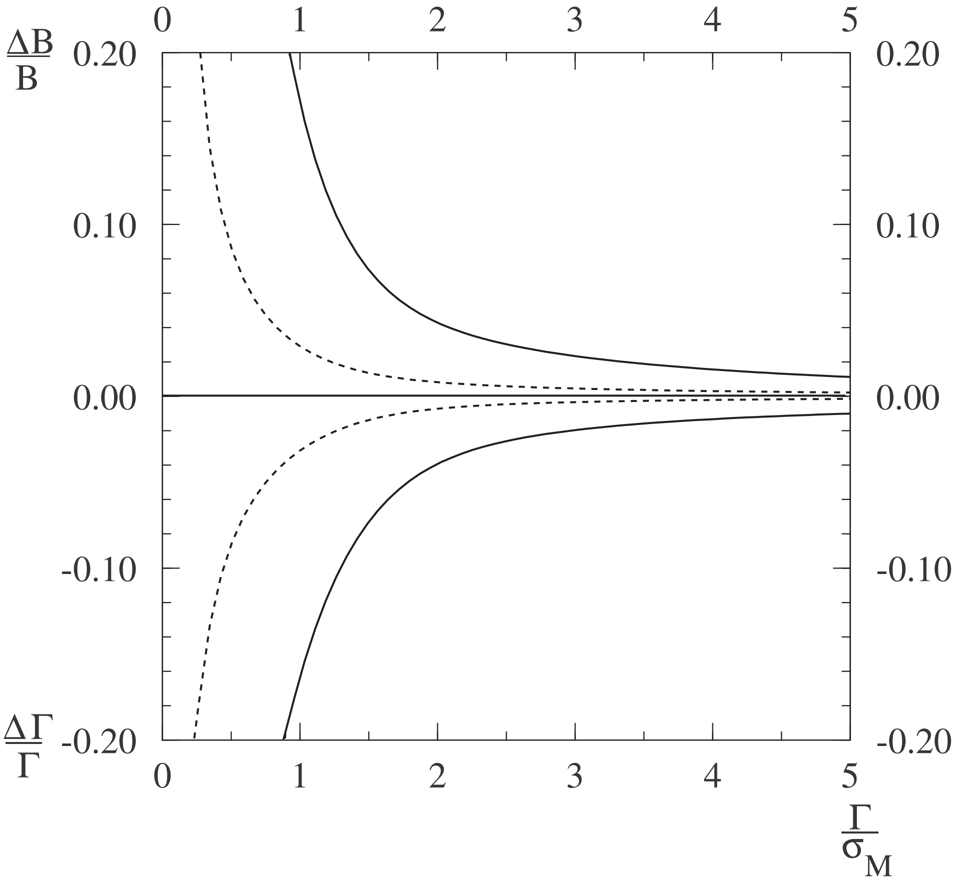

Figure 2: The fractional errors induced in B and Γ by $\Delta\sigma_M$ as functions of Γ/σ_M when employing the three-point scan measurement ($E = M$, $E = M \pm 2\Gamma$); results are shown for $\Delta\sigma_M/\sigma_M = 0.05$ (solid line) and $\Delta\sigma_M/\sigma_M = 0.01$ (dashed line).

$\Rightarrow$ For $R = 0.003\%$, $\Delta E_{\rm beam}/E_{\rm beam} \sim 10^{-7}$ ($\Delta E_{\rm beam} \sim 5$ keV), $\Delta\sigma/\sigma \sim 0.01$ *per spill*.

$\Rightarrow$ For each spill these errors are tiny compared to statistical errors for the channel rates. For 10^8 spills, only $1/N_{\rm channel}^{1/2}$ matters.

$\Rightarrow$ Main concern is systematic $\Delta\sigma/\sigma$ error.

How important might a systematic $\Delta\sigma$ systematic error be?

Fig. 2 $\Rightarrow \Delta\Gamma/\Gamma, \Delta B/B \sim 3\Delta\sigma/\sigma$ at $\Gamma/\sigma \sim 1$, for scan.

2 Front-end physics

- Physics with low-energy hadrons $(p, \overline{p}, K, \pi)$.
- Stopped muon physics and intense muon beam physics.

  Planned 10^{14} muons/sec vs. current 10^{8}/sec $\Rightarrow$ big increase in sensitivity to rare decays, etc.

  - Example 1: $\mu \to e\gamma$ — current limit $B < 0.5 \times 10^{-12}$ (MEGA-LAMPF might reach 6×10^{-13}) would certainly be pushed well into the $B \sim 0.1 - 1 \times 10^{-13}$ domain expected in not-too-heavy SUSY. New Z', leptoquarks and heavy neutrinos generically would predict similar.
  - Example 2: $\mu N \to eN$ conversion — current $< 8.4 \times 10^{-13}$ (SINDRUM II - PSI might reach 4×10^{-14}; MECO - AGS proposal, with $\sim 10^{11} \mu$/sec, $\to < 10^{-16}$) pushed to $< 10^{-18} - 10^{-19}$ for $10^{13} - 10^{14} \mu$/sec, before encountering new background.

    Arguably even more sensitive to new physics than $\mu \to e\gamma$ improvement would be: $1/200 \leq \frac{B(\mu \mathrm{Ti} \to e \mathrm{Ti})}{B(\mu \to e\gamma)} \leq 200$ or more $\Rightarrow$ $B(\mu N \to eN) \leq 10^{-18}$ is equivalent to $B(\mu \to e\gamma) \leq 2 \times 10^{-16} - 5 \times 10^{-21}$.

- Neutrino physics: $\mu^{-} \to \nu_{\mu}, \overline{\nu}_{e}$.

  - Neutrino oscillations: if special storage ring $\Rightarrow$ 10^{21} $\mu^{\pm}$ per year $\Rightarrow \sim 10^{20}$ neutrinos in beam; fluxes well known.

    No oscillation: CC events yield $\nu_{\mu} \to \mu^{-}$, $\overline{\nu}_{e} \to e^{+}$. Presence of e^{-} or μ^{+} $\Rightarrow$ oscillations are present.

Table 1: Neutrino oscillation studies a a neutrino factory. μ^- beam assumed. 'NC/CC' denotes ratio of 'short'/'long' events in a calorimetric detector: penetrating μ of ν_μ-induced CC event ⇒ long event; NC events and $\overline{\nu}_e$-induced CC events ⇒ short events.

Oscillation	Type	Measured Quantity
$\overline{\nu}_e \leftrightarrow \overline{\nu}_X$	disappearance	near/far, 'NC/CC'
$\overline{\nu}_e \leftrightarrow \overline{\nu}_\mu$	appearance	'wrong-sign' μ^+ CC events
$\overline{\nu}_e \leftrightarrow \overline{\nu}_\tau$	appearance	$\tau^+ \to \mu^+ \ldots$ CC events
$\nu_\mu \leftrightarrow \nu_X$	disappearance	near/far, 'NC/CC'
$\nu_\mu \leftrightarrow \nu_e$	appearance	'wrong-sign' e^- CC events
$\nu_\mu \leftrightarrow \nu_\tau$	appearance	$\tau^- \to e^- \ldots$ CC events

Example: an experiment capable of identifying muons and determining their charge (even if e's are not easily seen) is sensitive to:

- $\overline{\nu}_e \leftrightarrow \overline{\nu}_\mu$ and $\overline{\nu}_e \leftrightarrow \overline{\nu}_\tau$ oscillations (via the $\tau^+ \to \mu^+$ decay) in the μ^+ appearance channel – $N[\mu^+]$.
- $\nu_\mu \leftrightarrow \nu_X$ oscillations through disappearance = depletion of $N[\mu^- + \mu^+]$ – charge i.d. not required.
- Two independent 2×2 mixings for solar and atmospheric ⇒ must detect $\nu_\mu \leftrightarrow \nu_\tau$ mixing to corroborate atmospheric results.
- If solar + atmospheric analysed in 3-generation mixing case ⇒ $\overline{\nu}_e \leftrightarrow \overline{\nu}_\mu$ becomes interesting if $\theta_{13} \neq 0$.

$$P(\overline{\nu}_e \to \overline{\nu}_\mu) = \sin^2(\theta_{23}) \sin^2(2\theta_{13}) \sin^2\left(\frac{\Delta m^2_{23} L}{4E_\nu}\right) \tag{2.1}$$

$$P(\overline{\nu}_e \to \overline{\nu}_\tau) = \cos^2(\theta_{23})\sin^2(2\theta_{13})\sin^2\left(\frac{\Delta m^2_{23}L}{4E_\nu}\right) \tag{2.2}$$

$$P(\nu_\mu \to \nu_\tau) = \cos^4(\theta_{13})\sin^2(2\theta_{23})\sin^2\left(\frac{\Delta m^2_{23}L}{4E_\nu}\right) \tag{2.3}$$

$\frac{\Delta m^2 L}{4E_\nu} \to \frac{1.27\Delta m^2(\mathrm{eV}^2)L(\mathrm{km})}{E(\ \mathrm{GeV})}$ in meaningful units.

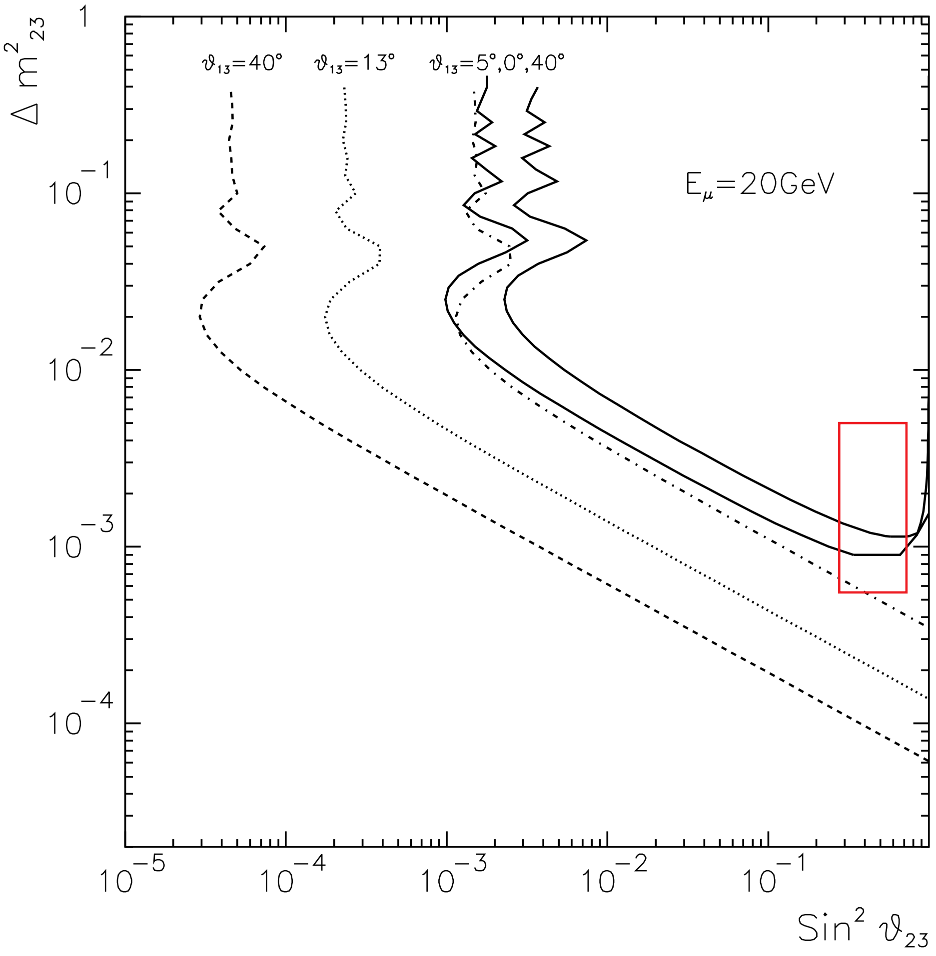

Figure 3: 90% CL reach for a 20 GeV μ^- beam and $L = 732$ km (CERN-Gran Sasso). Dotted lines = appearance, mainly $\overline{\nu}_e \leftrightarrow \overline{\nu}_\mu$ – $N[\mu^+]$; Solid lines = disappearance – $N[\mu^+ + \mu^-]$. Use energy integrated number of events.

∗ $N[\mu^+]$ (charge identification) buys a lot unless $\theta_{13} \sim 0$.

∗ $N[\mu^+]$ allows probe down to $\sin^2(\theta_{13}) \sim 10^{-2}$–$10^{-4}$ in SuperK-preferred region of Δm^2_{23}.

- μp collider.

Standard QCD studies, distribution functions, *etc.*: big improvement in kinematic coverage.

Leptoquarks and/or Contact Interactions: big improvement over HERA. Also, if something eventually seen in $eq \Rightarrow$ mandatory to search for analogous phenomenon in μq.

The $E_\mu = 50$ GeV and 200 GeV μp colliders would be crucial facilities for flavor dependence study if leptoquarks exist.

Conclusion: The front-end would provide opportunities in these areas that far exceed what is possible at currently available/planned facilities.

3 Z Physics

The muon collider could be run as a Z factory that would quickly exceed statistical levels achieved at LEP and SLC/SLD.

- $\sigma(Z)_{\rm peak} = \frac{12\pi}{m_Z^2} B(Z \to \mu^+\mu^-) \Sigma_X B(Z \to X) \sim 6 \times 10^7$ fb, for $\Gamma_Z^{\rm tot} \gg \sigma$.
- $\mathcal{L}_{\rm LEP} \sim 2.4 \times 10^{31} {\rm cm}^{-2}{\rm s}^{-1} \Rightarrow 1.5 \times 10^7$ Z's per year; no polarization.
- $\mathcal{L}_{\rm MC} \sim 1 \times 10^{32} {\rm cm}^{-2}{\rm s}^{-1}$ ($R \gtrsim 0.1\%$ perfectly ok) $\Rightarrow \sim 6 \times 10^7$ Z's per year; partial ($\sim 20-30\%$?) polarization for *both* beams.
- Important physics topics are many, including:
  - CP Violation, e.g. in $Z \to B_d \overline{B}_d \to \psi K_S$.
  - $B_s - \overline{B}_s$ mixing via $B_s(\overline{b}s) \to D_s^- \ell^+ \nu, \ldots$.
  - Best measurement of $\sin^2 \theta_W^{\rm eff} = \frac{1}{4}\left(1 - \frac{v_\mu}{a_\mu}\right)$ via $A_{LR} = \frac{2 v_\mu a_\mu}{v_\mu^2 + a_\mu^2}$.
  - Final resolution of R_b situation.
  - $Z \to \gamma X$. Probes many kinds of new physics.
    - ∗ Axions.

      $Z \to \gamma A$ (A=invisible) limits from LEP are few $\times 10^{-6}$. If axions exist, this decay could be observed with improved sensitivity. Stronger limits would constrain models.
    - ∗ Technicolor.

      $Z \to \gamma P^0$, where P^0 is the lightest (chargeless and colorless) pseudo-Nambu-Goldstone boson.

$$\Gamma(Z \to \gamma P^0) = \frac{\alpha^2 m_Z^3}{96\pi^3 v^2} N_{TC}^2 A_{P^0 Z\gamma}^2 \left(1 - \frac{m_{P^0}^2}{m_Z^2}\right)^3 , \qquad (3.1)$$

where $A_{P^0Z\gamma} = -\frac{4}{2\sqrt{6}}\left(\frac{1-4s_W^2}{4s_Wc_W} - \frac{t_W}{3}\right)$ and $P^0 \to b\bar{b}, \tau^+\tau^-, gg, \gamma\gamma$.

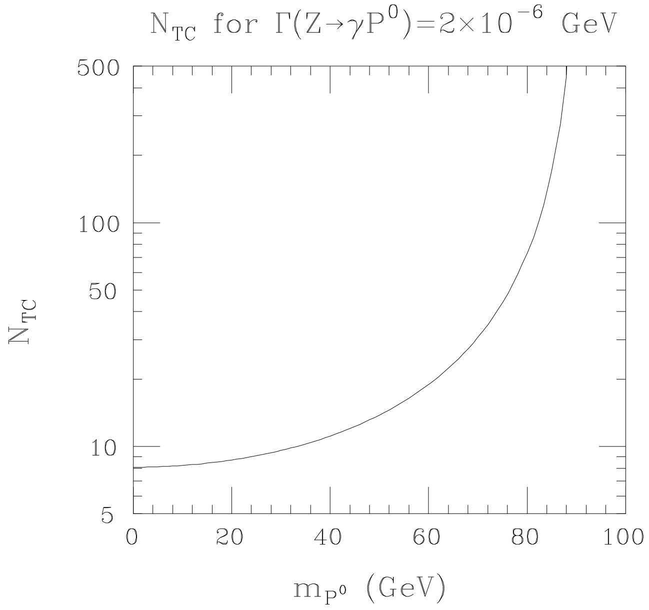

Figure 4: The value of N_{TC} required for $\Gamma(Z \to \gamma P^0) > 2 \times 10^{-6}$ GeV is plotted as a function of m_{P^0}.

Improving limits by a factor of 10 would rule out light P^0 in most technicolor models.

Conclusion: A muon collider would be a good machine for some important outstanding Z physics issues requiring high luminosity and significant polarization. It would provide increased sensitivity to rare processes that probe new physics.

4 Narrow Resonance Generalities

The totally unique feature of a muon collider is the possibility of s-channel $\mu^+\mu^- \to X$ with $\sigma < \Gamma \sim$ few MeV, assuming X has enough $\mu^+\mu^-$ coupling.

Compute $\overline{\sigma}$ by convoluting a Gaussian $\sqrt{s}$ distribution of width σ with the standard s-channel Breit Wigner Higgs resonance cross section. For $\sqrt{s} = M$, one obtains

$$\overline{\sigma} \simeq \frac{\pi\sqrt{2\pi}\Gamma(X \to \mu\mu)\, B(X \to F)}{M^2} \times \begin{cases} \frac{1}{\sigma}\,; & \sigma \gg \Gamma \\ \frac{2\sqrt{2}}{\Gamma\sqrt{\pi}}\,; & \sigma \ll \Gamma \end{cases} . \tag{4.1}$$

Eq. (4.1) $\Rightarrow$ for big $\overline{\sigma}$, need small Γ and $\sigma \lesssim \Gamma$.

Locate X and center on resonance by rough scan.

Determine width and rates. Two procedures:

- High precision scan. Three-point scan = optimal – total $L_{\rm total}$ distributed as $L_{\rm total}/5$ at M, and $2L_{\rm total}/5$ a $M \pm 2\sigma$ each.

  Although smaller R implies smaller $\mathcal{L}$, the correlation is such that for small Γ (*e.g.* a SM-like Higgs boson with $m_h \lesssim 2m_W$) it is best to use the smallest R that can be achieved.

  Statistics for Γ limited by small S/B on wings.

- New technique: two measurements at $\sqrt{s} = M$ — one with $\sigma^{\rm min} \ll \Gamma$ and one with $\sigma^{\rm max} \gg \Gamma$: $\quad \overline{\sigma}(\sigma^{\rm min})/\overline{\sigma}(\sigma^{\rm max}) \propto \sigma^{\rm max}/\Gamma$.

  Statistics for Γ limited by small S/B for larger $\sigma^{\rm max}$.

  Critical issues:

  - Sensitivity of Γ to $\overline{\sigma}(\sigma^{\rm min})/\overline{\sigma}(\sigma^{\rm max})$.

– Statistical error in $\overline{r} \equiv \overline{\sigma}(\sigma^{\rm min})/\overline{\sigma}(\sigma^{\rm max})$.

Need to know how $\mathcal{L}$ varies with R. Benchmark values given earlier $\Rightarrow \mathcal{L}(R=0.03\%) = 4.7\mathcal{L}(R=0.003\%)$, for example.

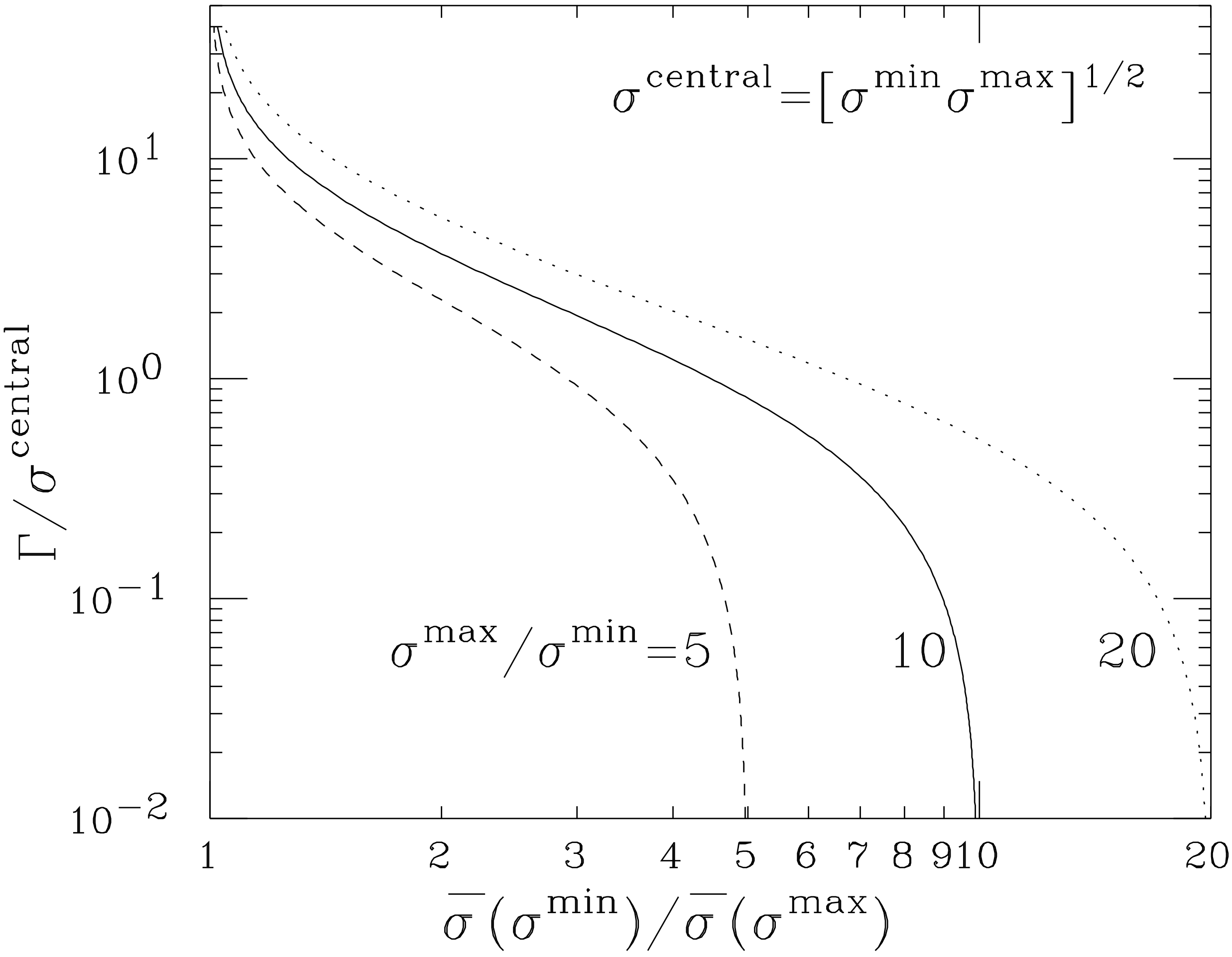

Figure 5: $\Gamma/\sigma^{\rm central}$ plotted as a function of the cross section ratio $\overline{\sigma}(\sigma^{\rm min})/\overline{\sigma}(\sigma^{\rm max})$ for the indicated values of $\sigma^{\rm max}/\sigma^{\rm min}$ keeping $\sigma^{\rm central} \equiv \sqrt{\sigma^{\rm max}\sigma^{\rm min}}$ fixed.

Note: $\Delta\Gamma/\Gamma = |s|\Delta\overline{r}/\overline{r}$, where $|s|$ is the slope in the log – log plot.

$|s| \sim 1.5$ (~ 2) for $\sigma^{\rm max}/\sigma^{\rm min} = 10$ and $\Gamma/\sigma^{\rm central} \sim 2-3$ (~ 1).

Note: Systematic errors from ratio technique are $\Delta\Gamma/\Gamma = \Delta\sigma/\sigma$ and $\Delta B = 0$.

5 Higgs physics

Standard Model-Like Higgs

$m_h \lesssim 2m_W$ is required for good s-channel cross section. Above that $\Gamma_h^{\rm tot}$ becomes big and $\overline{\sigma}_h \propto BF(h \to \mu^+\mu^-)$ is too small.

Strategy: $\Delta m_h \sim 100$ MeV from LHC if $m_h \lesssim 2m_W$ (2γ mode) $\Rightarrow$ First center on $\sqrt{s} \sim m_h$ via rough scan and then measure Higgs properties.

$m_h > 96$ GeV means no Z-pole problem; centering might take $L = 0.1~\mathrm{fb}^{-1}$ or about 1 yr.

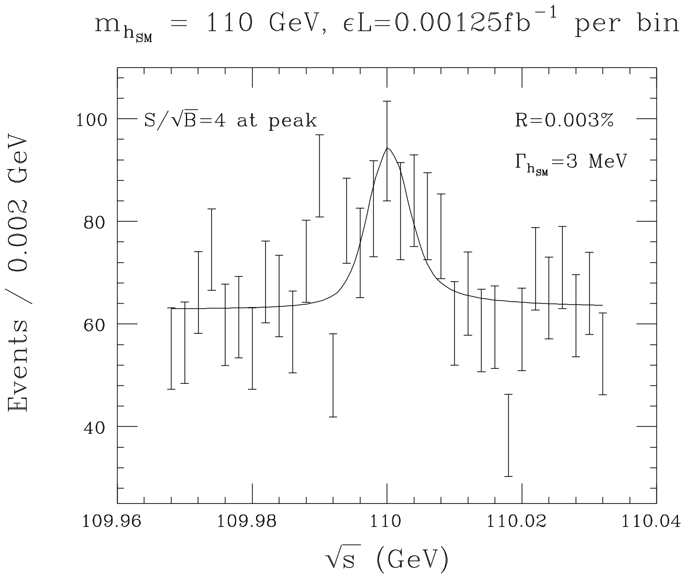

Figure 6: Initial scan for centering on $m_{h_{SM}} = 110$ GeV Higgs boson.

- The crucial measurements are then:
  - The very tiny Higgs width: $\Gamma_h^{\rm tot} = 1 - 10$ MeV for a SM-like Higgs with $m_h \lesssim 140$ GeV (i.e. mass as predicted for h^0 of SUSY).
  - $\sigma(\mu^+\mu^- \to h \to X)$ for $X = b\overline{b}, WW^\star, ZZ^\star, c\overline{c}, \tau^+\tau^-$.

Accuracy achievable? First consider optimized 3-point scan.

At $L_{\rm total} = 0.4~{\rm fb}^{-1}$ ($\geq$ 4 years at $R = 0.003\%$) the errors for $\sigma B(h_{SM} \to b\overline{b})$ are small, but those for $\Gamma^{\rm tot}_{h_{SM}}$ are bad if m_h large.

Table 2: Percentage errors (1σ) for $\Gamma_h^{\rm tot}$ and $\overline{\sigma}B(H \to b\overline{b}, WW^\star, ZZ^\star)$ for $\mu^+\mu^- \to H$. For the scan procedure we use $R = 0.003\%$ and 4-year luminosity of $L = 0.4~{\rm fb}^{-1}$, using $L/5$ at $E = m_h$, $2L/5$ at $E = m_h + 2\sigma$ and $2L/5$ at $E = m_h - 2\sigma$. For the $\overline{r}$-ratio procedure, we assume $E = m_h$ and accumulate $L = 0.2~{\rm fb}^{-1}$ at $R = 0.003\%$ and $L = 0.94~{\rm fb}^{-1}$ at $R = 0.03\%$, corresponding to two years of running at each R. Efficiencies, cuts and bremsstrahlung included.

Quantity	Errors for the scan procedure					
Mass (GeV)	**100**	**110**	**120**	**130**	**140**	**150**
$\overline{\sigma}B(b\overline{b})$	4%	3%	3%	5%	9%	28%
$\overline{\sigma}B(WW^\star)$	32%	15%	10%	8%	7%	9%
$\overline{\sigma}B(ZZ^\star)$	–	190%	50%	30%	26%	34%
$\Gamma_h^{\rm tot}$	30%	16%	16%	18%	29%	105%

Quantity	Errors for the $\overline{r}$-ratio procedure					
Mass (GeV)	**100**	**110**	**120**	**130**	**140**	**150**
$\overline{\sigma}B(b\overline{b})$	3.8%	2.8%	2.8%	4.4%	7.6%	21%
$\overline{\sigma}B(WW^\star)$	26%	12%	7.7%	5.7%	5.0%	5.6%
$\overline{\sigma}B(ZZ^\star)$	–	190%	46%	25%	20%	22%
$\Gamma_h^{\rm tot}$	45%	25%	20%	19%	17%	18%

What about the new ratio technique?

- $L_{\rm year}\overline{\sigma}_h$ $\sim$constant in going from $R = 0.003\%$ to $R = 0.03\%$, but S/B decreases.
- For same $L_{\rm total}$, $\overline{r}$ statistical Γ errors are smaller (larger) for larger (smaller) m_h. For $\overline{\sigma}_h B$, $\overline{r}$ errors smaller for all m_h.
- For $\Gamma_h^{\rm tot} \sim \sigma$ (as for $m_h \leq 120$ GeV), systematic errors for scan ($\Delta\Gamma_h^{\rm tot}/\Gamma_h^{\rm tot} \sim 3\Delta\sigma/\sigma$) are bigger than for $\overline{r}$-technique ($\Delta\Gamma_h^{\rm tot}/\Gamma_h^{\rm tot} = \Delta\sigma/\sigma$).
    - $\ast \Rightarrow$ if $\Delta\sigma/\sigma \sim 0.01$, $\overline{r}$ technique better for rates for all m_h.

    - ∗ ⇒ if $\Delta\sigma/\sigma \sim 0.05$, $\overline{r}$ technique better for $\Gamma_h^{\rm tot}$ for all m_h.

- Use these measurements to distinguish between h_{SM} and h^0 of the MSSM ⇒ constraints on H^0 and A^0.
  - – If only s-channel Higgs factory MC data are available (*i.e.* no Zh NLC or MC data) use σB ratios. At $m_{h_{SM}} = 110$ GeV, $L_{\rm tot} = 0.4~{\rm fb}^{-1}$ Table 2 scan errors ⇒

$$(WW^\star h_{SM})^2/(b\overline{b}h_{SM})^2 \to\sim \pm 15\%$$

$$(c\overline{c}h_{SM})^2/(b\overline{b}h_{SM})^2 \to\sim \pm 20\%$$

$$(WW^\star h_{SM})^2/(\tau^+\tau^- h_{SM})^2 \to\sim \pm 18\%$$

$$(c\overline{c}h_{SM})^2/(\tau^+\tau^- h_{SM})^2 \to\sim \pm 22\%$$

    Systematic errors of order ±5% − ±10% from uncertainty in the b quark mass will also enter. Combining these independent measurements ⇒ $> 2\sigma$ deviation for m_{A^0} up to ~ 400 GeV.

  - – What about absolute $\mu^+\mu^- \to h \to b\overline{b}$ rate?

    Analogous to $gg \to h \to \gamma\gamma$ rate at LHC and $e^+e^- \to hZ[h \to b\overline{b}]$ rate at NLC. The usual issue is theoretical systematics for such absolute rates.

    Can we disentangle effects from $\Gamma_h^{\rm tot}$, m_b, …? Consider SUSY.

    - ∗ $h \to \mu^+\mu^-$ coupling is determined by m_μ (known), $\tan\beta$ and Higgs mixing angle α as determined by m_{A^0}, squark mixing, ….
    - ∗ $\Gamma(h \to b\overline{b})$ is determined by m_b (somewhat uncertain), $\tan\beta$ and α . . .

- If no [redacted]s known function of all of abo[redacted] by above threshold SUSY mas[redacted]

[redacted]ill know [redacted]in terms [redacted]ng.

$B(h \to b\overline{b}) = \frac{\Gamma(h \to b\overline{b})}{\Gamma_h^{\rm tot}}$

varies slowly with m_b;

$\sigma \neq 0$ damps small $\Gamma_h^{\rm tot}$

more than large $\Gamma_h^{\rm tot} \Rightarrow$

little m_b dependence.

Figure 7: We plot as a function of m_b, $\mu^+\mu^- \to h$ for $\sigma = 0$ (top), $\mu^+\mu^- \to h \to b\overline{b}$ for $\sigma = 0$ (middle), $\mu^+\mu^- \to h \to b\overline{b}$ for $\sigma(R = 0.003\%)$. From Janot.

Now look at χ^2 of deviation from SM expectation. Claims:

- LHC and NLC rates, after including m_b systematics do not probe regions where direct A^0 discovery will not be possible.
- Muon Collider with $L_{\rm tot} = 0.2~{\rm fb}^{-1}$ probes $m_{A^0} \lesssim 600$ GeV at 3 sigma level.

Figure 8: (a) LHC + NLC deviations; (b) MC deviations for $L_{\rm total} = 0.2~{\rm fb}^{-1}$; (c) MC deviations for $L_{\rm tot} = 10~{\rm fb}^{-1}$. $3-4$ sigma region is 2nd darkest.

- Using NLC $\sqrt{s} = 500$ GeV ($L_{\rm tot} = 200~{\rm fb}^{-1}$) e^+e^- data there is a cleaner alternative.

  There are 4 ways to determine $\Gamma(h \to \mu^+\mu^-)$:

  1) $\Gamma(h_{SM} \to \mu^+\mu^-) = \frac{[\Gamma(h_{SM}\to\mu^+\mu^-)B(h_{SM}\to b\overline{b})]_{\rm MC}}{B(h_{SM}\to b\overline{b})_{\rm NLC}}$;

  2) $\Gamma(h_{SM} \to \mu^+\mu^-) = \frac{[\Gamma(h_{SM}\to\mu^+\mu^-)B(h_{SM}\to WW^\star)]_{\rm MC}}{B(h_{SM}\to WW^\star)_{\rm NLC}}$;

  3) $\Gamma(h_{SM} \to \mu^+\mu^-) = \frac{[\Gamma(h_{SM}\to\mu^+\mu^-)B(h_{SM}\to ZZ^\star)]_{\rm MC}\Gamma^{\rm tot}_{h_{SM}}}{\Gamma(h_{SM}\to ZZ^\star)_{\rm NLC}}$;

  4) $\Gamma(h_{SM} \to \mu^+\mu^-) = \frac{[\Gamma(h_{SM}\to\mu^+\mu^-)B(h_{SM}\to WW^\star)\Gamma^{\rm tot}_{h_{SM}}]_{\rm MC}}{\Gamma(h_{SM}\to WW^\star)_{\rm NLC}}$.

Typically, $\Gamma(h \to \mu^+\mu^-)$ error $\lesssim 5\%$ for MC $L_{\rm tot} = 0.4~{\rm fb}^{-1}$. $\Rightarrow$ probes at 3σ level out to $m_{A^0} \gtrsim 600$ GeV for all $m_h \lesssim 2m_W$. No systematics.

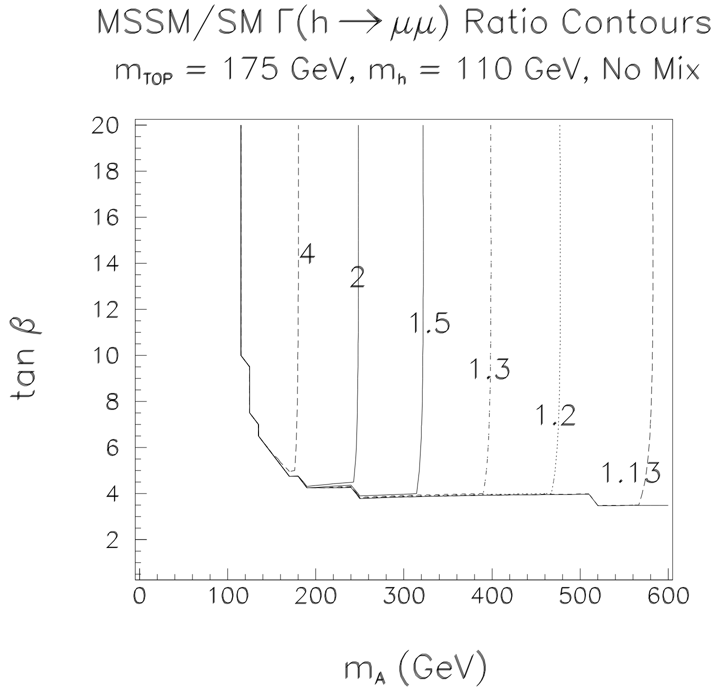

Figure 9: Contours in $(m_{A^0}, \tan\beta)$ parameter space for $\Gamma(h^0 \to \mu^+\mu^-)/\Gamma(h_{SM} \to \mu^+\mu^-)$. Assume “No mixing”; take $m_{h^0} = m_{h_{SM}} = 110$ GeV. For “maximal mixing”, little change — only size of allowed range is altered.

Determine to $\pm 4\%$ with $L = 0.4~{\rm fb}^{-1}$ scan and NLC ($200~{\rm fb}^{-1}$) and LHC ($300~{\rm fb}^{-1}$) data $\Rightarrow$ 3σ sensitivity for m_{A^0} up to 600 GeV.

SUSY H^0 and A^0

- Possibilities for H^0, A^0 are limited at other machines.
  - Discovery of H^0, A^0 not possible at LHC for all $(m_{A^0}, \tan\beta)$: e.g. if $m_{\tilde{t}} = 1$ TeV, $B(b \to s\gamma)$ requires $m_{A^0} > 350$ GeV, and then $\tan\beta \gtrsim 3$ is bad at LHC.
  - At $\sqrt{s} = 500$ GeV, $e^+e^- \to H^0 A^0$ pair production probes only to $m_{A^0} \sim m_{H^0} \lesssim 230 - 240$ GeV.
  - A $\gamma\gamma$ collider could potentially probe up to $m_{A^0} \sim m_{H^0} \sim 0.8\sqrt{s} \sim 400$ GeV with $L \gtrsim 150 - 200$ fb^{-1}.

- $\mu^+\mu^- \to H^0, A^0$ potentially allows production and study of H^0, A^0 up to $m_{A^0} \sim m_{H^0} \lesssim \sqrt{s}$. For $L = 50$ fb^{-1} (5 yrs running at $< \mathcal{L} >= 1 \times 10^{33}$, as possibly achievable for $R \gtrsim 0.1\%$ for $\sqrt{s} = 300 - 500$ GeV):
  - **with preknowledge/restrictions** on m_{A^0} (from m_{h^0} data, LHC or NLC discovery, *etc.*) $\mu^+\mu^- \to H^0$ and $\mu^+\mu^- \to A^0$ can be studied with precision for all $\tan\beta \gtrsim 1$.
  - **without preknowledge**, $\mu^+\mu^- \to H^0, A^0$ discovery by scanning from $250 - 500$ GeV at the MC will be possible for most of $(m_{A^0}, \tan\beta)$ parameter space such that they cannot be discovered at the LHC (in particular, if $m_{A^0} \gtrsim 250$ GeV and $\tan\beta \gtrsim 4$).
  - even if the MC is run at $\sqrt{s} = 500$ GeV, the H^0, A^0 can be discovered in the bremsstrahlung tail if the $b\bar{b}$ mass resolution (either by direct reconstruction or hard photon recoil) is good enough and $\tan\beta$ is large.

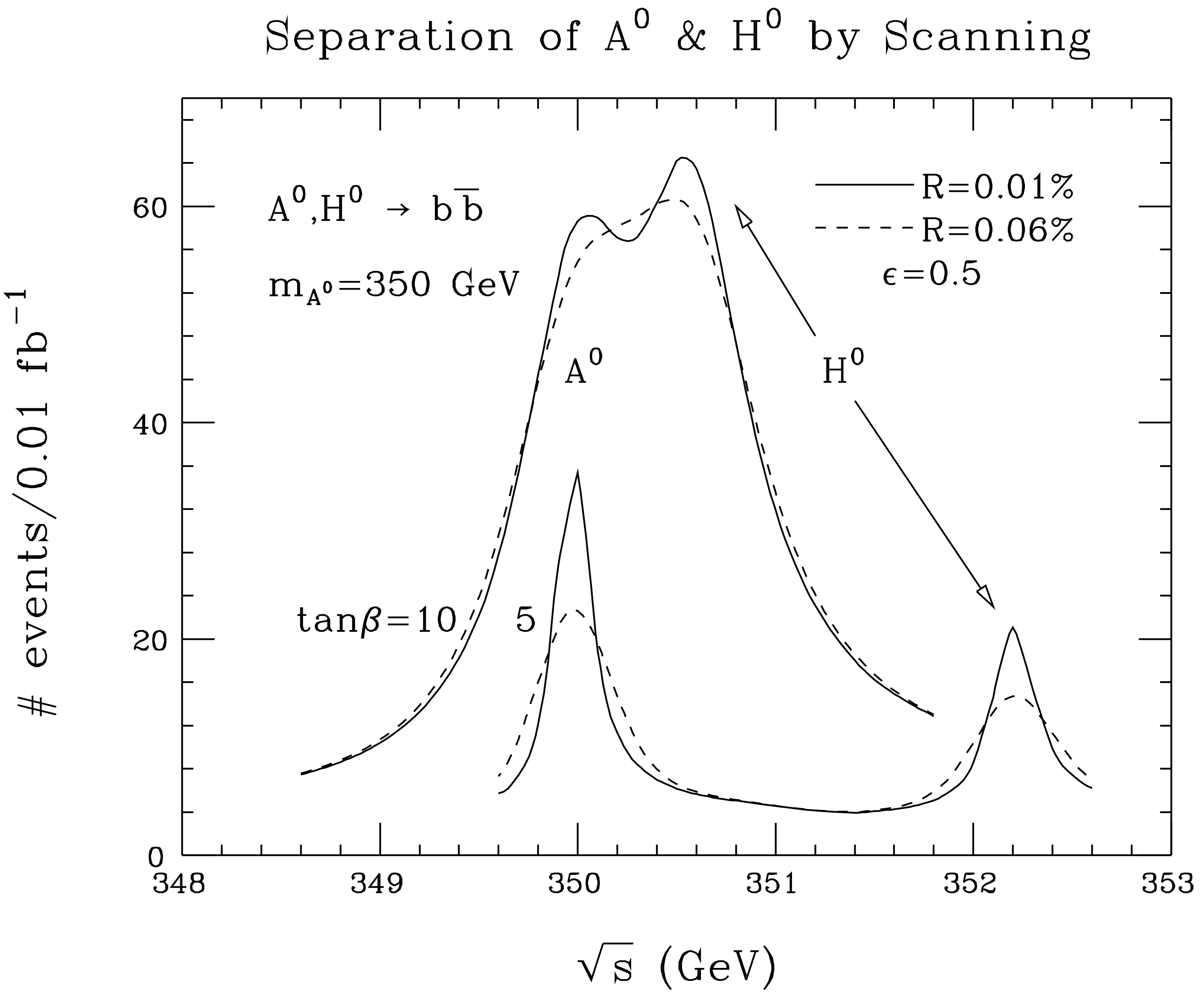

Figure 10: Plot of $b\bar{b}$ final state event rate as a function of $\sqrt{s}$ for $m_{A^0} = 350$ GeV, in the cases $\tan\beta = 5$ and 10, resulting from the H^0, A^0 resonances and the $b\bar{b}$ continuum background. We have taken $L = 0.01$ fb^{-1} (at any given $\sqrt{s}$), $\epsilon = 0.5$, $m_t = 175$ GeV, and included two-loop/RGE-improved radiative corrections to Higgs masses, mixing angles and self-couplings using $m_{\widetilde{t}} = 1$ TeV and neglecting squark mixing. SUSY decays are assumed to be absent. Curves are given for two resolution choices: $R = 0.01\%$ and $R = 0.06\%$

Only the muon collider can have the resolution σ needed to separate a very degenerate H^0 and A^0 pair.

Once separated, many tests of SUSY become possible.

Higgs pair production at higher energy.

- $m_{A^0} \gtrsim 1$ TeV cannot be ruled out simply on basis of hierarchy and naturalness (non-universality $\Rightarrow$ fine tuning not bad).

  $\Rightarrow$ energy reach for $H^0 A^0$ pair production of $\sqrt{s} > 2$ TeV would be very valuable.

  - Discovery of $H^0 A^0$ in their $b\overline{b}$ or $t\overline{t}$ decay modes and $H^+ H^-$ in their $t\overline{b}$ and $b\overline{t}$ decays will be easy for expected luminosities, even if SUSY decays are present.
  - Measurement of mass with reasonable accuracy will be possible.

- Measurement of branching ratios in combination with measurement of $m_{A^0} \sim m_{H^0} \sim m_{H^\pm}$ and, say, $m_{\widetilde{\chi}_1^\pm}$ (chargino) discriminates with incredible statistical significance between different closely similar GUT scenarios.

  Including other sparticle pair channels will further enhance discrimination between scenarios.

  Ultimate goal is determination of all soft-SUSY-breaking parameters.

6 Pseudo-Goldstone boson physics

- All dynamical symmetry breaking (DSB) theories (including technicolor) with symmetry in the DSB sector larger than $SU(2)_L \times SU(2)_R$ will contain PGB's.

- In large class of models, the lightest PGB is the colorless, neutral, $T_3 = -1/2$ mass eigenstate, P^0. It is much lighter than any other DSB state, since its mass is proportional to m_b.

- Because the P^0 is $T_3 = -1/2$ it couples to $\mu^+\mu^-$ with $\lambda_\mu \sim m_\mu/v$, similar to Higgs.

  Further, it is almost as narrow as Higgs: for 'typical' model parameter choices $\Gamma_{P^0}^{\rm tot} = 2~\text{MeV} - 20~\text{MeV}$ for $m_{P^0} = 50 - 200~\text{GeV}$.

- No current constraints on P^0 from LEP, LEP2, Tevatron RunI.

  Some hope at RunII.

  LHC will detect $gg \to P^0 \to \gamma\gamma$ Higgs like mode, with very high event rate, from $?? \leq m_{P^0} \leq 200~\text{GeV}$. ?? might be of order 30 GeV (uncertainty due to $\gamma - j$ discrimination level being uncertain); detailed study required.

- Because the P^0 has no tree-level coupling to ZZ, $\Rightarrow$ not easily studied at e^+e^- collider (best mode: $e^+e^- \to \gamma P^0$): only $\gamma\gamma$ collider provides some potential: $\gamma\gamma \to P^0 \to b\overline{b}$ robust; $\tau^+\tau^-$ and $\gamma\gamma$ final states uncertain.

- Only the muon collider could really study the P^0 in detail and have the ability to scan for its presence at low mass.

$\lesssim 1$ year would be required to scan $m_{P^0} < 80$ GeV.

LHC discovery certain above this and $\Rightarrow \sim 100$ MeV mass uncertainty; for m_{P^0} not close to m_Z MC can scan this in $\lesssim 1$ year and center on $\sqrt{s} \simeq m_{P^0}$.

- Once centered on $\sqrt{s} \sim m_{P^0}$, good measurements of rates in the $b\bar{b}$, $\tau^+\tau^-$, $c\bar{c}$ and gg s-channel production final states will be possible:

  An accurate measurement of the total width $\Gamma^{\rm tot}_{P^0}$ will also result.

Table 3: Fractional statistical errors (1σ) for $\overline{\sigma}B(P^0 \to {\rm all})$ (combining $b\bar{b}$, $\tau^+\tau^-$, $c\bar{c}$ and gg tagged-channel rates) and $\Gamma^{\rm tot}_{P^0}$ for $\mu^+\mu^- \to P^0$. We compare results for an $R = 0.003\%$ three-point scan with total integrated luminosity of $L = 0.4~{\rm fb}^{-1}$ to results obtained using the $\overline{r}$-ratio technique assuming accumulated luminosities at $E = m_{P^0}$ of $L = 0.2f~{\rm fb}^{-1}$ at $R = 0.003\%$ and $L = 0.94(2-f)~{\rm fb}^{-1}$ at $R = 0.03\%$ (corresponding to roughly $2f$ years of running at $R = 0.003\%$ and $(4-2f)$ years of running at $R = 0.03\%$ for pessimistic instantaneous luminosity assumptions). f (tabulated below) is chosen to minimize the error in $\Gamma^{\rm tot}_{P^0}$. Efficiencies, cuts, tagging and bremsstrahlung included.

Quantity	Errors for the scan procedure					
Mass (GeV)	**60**	**80**	m_Z	**110**	**150**	**200**
$\overline{\sigma}B$	0.0029	0.0054	0.043	0.0093	0.012	0.018
$\Gamma^{\rm tot}_{P^0}$	0.014	0.029	0.25	0.042	0.052	0.10
Quantity	Errors for the $\overline{r}$-ratio procedure					
Mass (GeV)	**60**	**80**	m_Z	**110**	**150**	**200**
f	0.8	0.7	0.6	0.8	0.9	1.0
$\overline{\sigma}B$	0.0029	0.0062	0.055	0.010	0.011	0.016
$\Gamma^{\rm tot}_{P^0}$	0.014	0.028	0.24	0.041	0.039	0.053

- With this muon collider information and the LHC measurement can determine many interesting parameters of the effective DSB Langrangian describing the P^0.

Conclusion If DSB is nature's choice and PGB's exist, there is no substitute for the muon collider.

7 Precision Measurements of m_W and m_t

- At NLC, m_W via $q\bar{q}$ mass reconstruction at $\sqrt{s} = 500$ GeV and m_t via $t\bar{t}$ threshold measurements with $L = 50\ \mathrm{fb}^{-1}$. $L > 50\ \mathrm{fb}^{-1} \Rightarrow$ systematics dominated.
- At MC, WW threshold and $t\bar{t}$ threshold measurements are the most accurate ways to get m_W and m_t. Small R and precise $E_{\mathrm{beam}} \Rightarrow$ errors are always statistics dominated.

  But, L_{year} ($R \sim 0.1\%$ ok) $\ll 50\ \mathrm{fb}^{-1}$: $L(\sqrt{s} = 2m_W) \sim 1\ \mathrm{fb}^{-1}/\mathrm{yr}$ and $L(\sqrt{s} = 2m_t) \sim 3\ \mathrm{fb}^{-1}/\mathrm{yr}$, vs. the NLC $L = 50\ \mathrm{fb}^{-1}/\mathrm{yr}$.

  1 year $\Rightarrow$ Table 4

Table 4: Comparison for the achievable precision in m_W and m_t measurement at NLC vs. MC after 1 year of optimized running.

	NLC	MC
Δm_W (MeV) / L (fb^{-1})	20 / 50	62 / 1
Δm_t (MeV) / L (fb^{-1})	200 / 50	360 / 3

Conclusion: Systematics from beam energy spread *etc.* are low enough at MC that NLC(1 year)=MC(2-3 years).

Since precision electroweak tests are optimized for $\Delta m_W/\Delta m_t \sim 0.02$, suggest NLC focus on $\sqrt{s} \sim 2m_W$ and MC focus on $\sqrt{s} \sim 2m_t$.

8 R-parity violating scenarios

Suppose there is $\not{R}$ of form (baryon number ok, but lepton number is violated):

$$\lambda_{ijk}\widehat{L}^i_L\widehat{L}^j_L\widehat{\overline{E}}^k_R + \lambda'_{ijk}\widehat{L}^i_L\widehat{Q}^j_L\widehat{\overline{D}}^k_R$$

- $\lambda' \neq 0 \Rightarrow$ LQ=squark events in μp collisions. The $\mu^+ s \to \tilde{c}$ (no family transition) and $\mu^+ s \to \tilde{t}$ couplings could be much larger than e^+ HERA analogue couplings.

- $\lambda \neq 0 \Rightarrow$ possibility of $e^+e^- \to \tilde{\nu}_\tau$ (λ_{131}) and/or $\tilde{\nu}_\mu$ (λ_{121}) and $\mu^+\mu^- \to \tilde{\nu}_\tau$ (λ_{232}) and/or $\tilde{\nu}_e$ (λ_{122}).

  $\tilde{\nu}_\tau$ is probably the lightest $\tilde{\nu}$ and λ_{232} is probably the largest λ's $\Rightarrow$ MC especially interesting. Small $\Delta E_{\rm beam}$ = big advantage over NLC.

  Some details for $\tilde{\nu}_\tau$ at muon collider.

  - For simplicity, assume only λ_{232} and λ'_{333} (surely the largest of the λ''s) non-zero.
  - Scenarios for $\tilde{\nu}_\tau$ decay are:
    - ∗ $\tilde{\nu}_\tau \to \nu_\tau\widetilde{\chi}^0_1$ if $m_{\widetilde{\chi}^0_1} < m_{\tilde{\nu}_\tau}$ ($\widetilde{\chi}^0_1$ decays via $\not{R}$ couplings);
    - ∗ $\tilde{\nu}_\tau \to \mu^+\mu^-$ (if $\lambda'_{333} \ll \lambda_{232}$);
    - ∗ $\tilde{\nu}_\tau \to b\overline{b}$ (if $\lambda_{232} \ll \lambda'_{333}$).

    $\Gamma_{\tilde{\nu}_\tau}$ tends to be large if $m_{\widetilde{\chi}^0_1} < m_{\tilde{\nu}_\tau}$, but can be very small if $\tilde{\nu}_\tau \to \nu_\tau\widetilde{\chi}^0_1$ is disallowed. **Let's assume latter.**
  - LHC and/or normal NLC data show $\not{R}$ and $\Rightarrow$ some knowledge of $m_{\tilde{\nu}_\tau}$: $\Delta m_{\tilde{\nu}_\tau} \sim 100$ MeV if $\mu^+\mu^-$ and or $b\overline{b}$ decays observable.

    But, *magnitude of λ_{232} and λ'_{333} not easily extracted.*

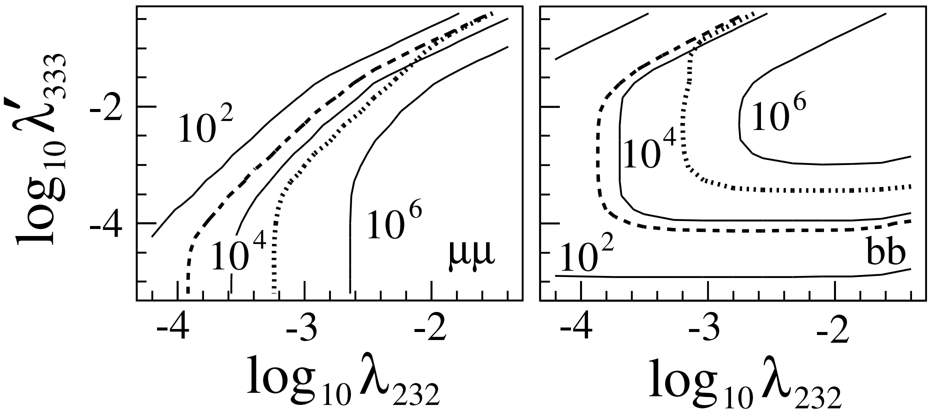

Figure 11: Contours at $\sqrt{s} = m_{\tilde{\nu}_\tau} = 100$ GeV for (i) $\sigma(\mu^+\mu^- \to \tilde{\nu}_\tau \to \mu^+\mu^-)$ and (ii) $\sigma(\mu^+\mu^- \to \tilde{\nu}_\tau \to b\bar{b})$ after cuts in fb in the $(\lambda_{232}, \lambda'_{333})$ plane for the $m_{\tilde{\nu}_\tau} < m_{\tilde{\chi}_1^0}$ scenario. The dashed (dotted) contour gives the optimistic (pessimistic/scan) 3σ discovery boundary for a total integrated luminosity of $L = 0.1\ \mathrm{fb}^{-1}$ and beam energy resolution of $R = 0.003\%$.

- Once seen, go to muon collider and scan for precise location.
- Once centered, high L measurements at and near $m_{\tilde{\nu}_\tau}$ allow determination of $\not{R}$ couplings.

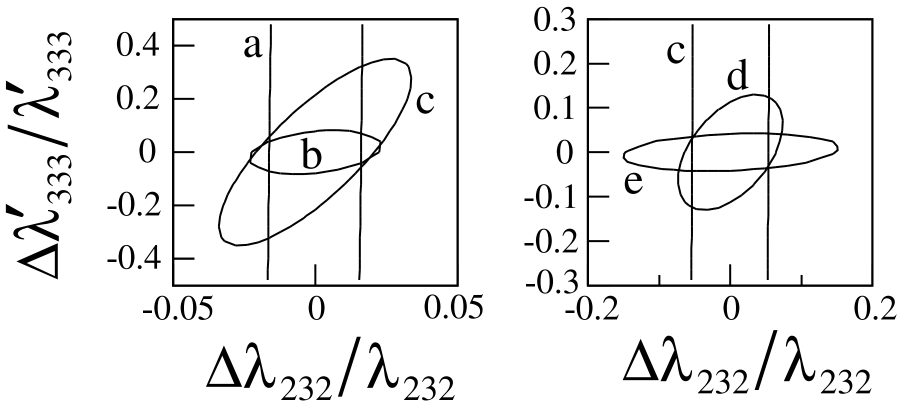

Figure 12: $\chi^2 = 1$ contours in the $(\Delta\lambda_{232}/\lambda_{232}, \Delta\lambda'_{333}/\lambda'_{333})$ plane for (i) the $m_{\tilde{\nu}_\tau} = 100\ \mathrm{GeV} < m_{\tilde{\chi}_1^0}$ scenario assuming $L = 0.3\ \mathrm{fb}^{-1}$, $R = 0.003\%$ and (ii) the $m_{\tilde{\nu}_\tau} = 150\ \mathrm{GeV} > m_{\tilde{\chi}_1^0} = 100$ GeV scenario for $L = 3\ \mathrm{fb}^{-1}$, $R = 0.1\%$. Contours are for $\lambda_{232} = 5 \times 10^{-4}$ and $\lambda'_{333} =$: (a) 10^{-5}; (b) 5×10^{-4}; (c) 10^{-2}; (d) 10^{-1}; (e) 0.5.

- $R = 0.003\%$ could resolve the $\mathcal{O}(1\ \mathrm{MeV})$ splitting between the CP-even and CP-odd components of the $\tilde{\nu}_\tau$.

9 Extra Dimensions

- Canonical signature: $\mu^+\mu^- \to \gamma G$.

$$\frac{d^2\sigma}{dx_\gamma d\cos\theta}(\mu^+\mu^- \to \gamma G) = \frac{\alpha\pi^{\delta/2}}{32\Gamma(\delta/2)s}\left(\frac{\sqrt{s}}{M_D}\right)^{\delta+2} f(x_\gamma, \cos\theta) \tag{9.1}$$

with $x_\gamma = 2E_\gamma/\sqrt{s}$, $\delta = \#$ extra dimensions, $M_D =$ quantum gravity scale.

Using continuum approximation with graviton mass $m = \sqrt{s(1-x_\gamma)}$.

$f(x,y) \propto (1-x)^{\delta/2-1}$ prefers low x_γ, *i.e.* high m enhanced by increasing G phase space.

- $\gamma Z \to \gamma\nu\overline{\nu}$ cut away by requiring $m \neq m_Z \Rightarrow \gamma\nu\overline{\nu}$ 'continuum' is main background.

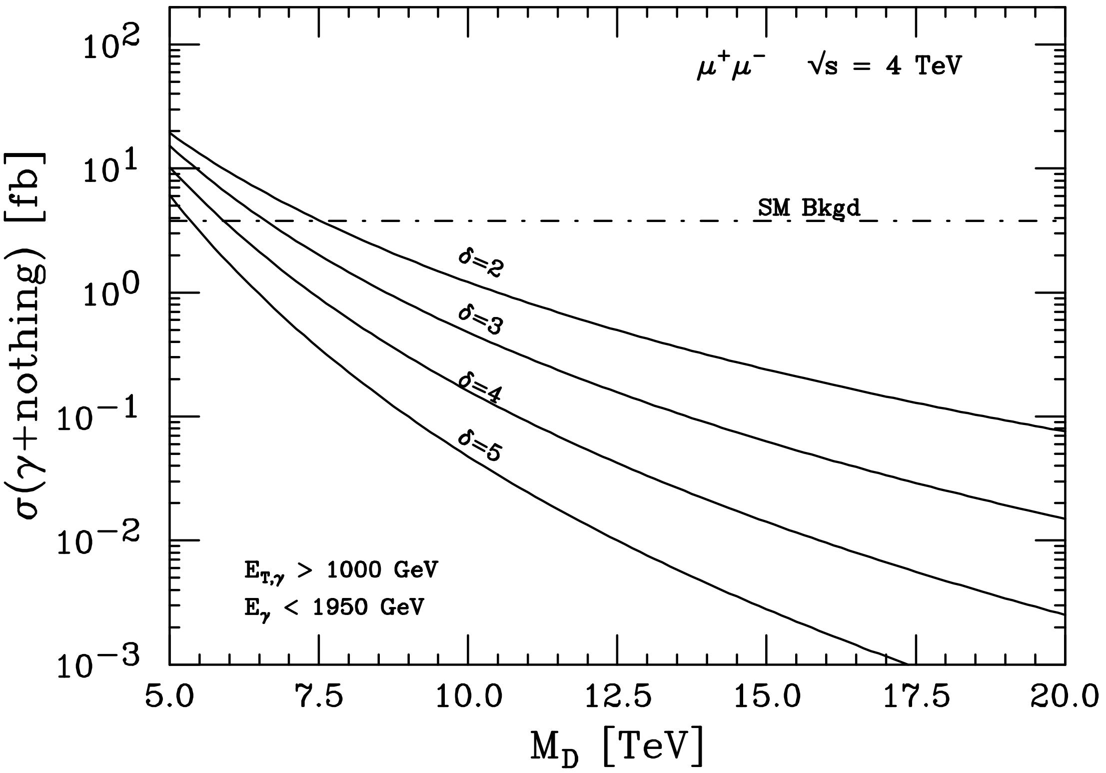

Figure 13: $E_\gamma > 1950$ GeV, $E_\gamma^T > 1$ TeV signal and background at $\sqrt{s} = 4$ TeV.

- Require $S/\sqrt{B} > 5 \Rightarrow$

Table 5: Maximum M_D that can be detected for $\sqrt{s} = 1, 2, 4$ TeV with $L_{\rm total} = 200, 100, 100$ fb^{-1} compared to LHC in jet + $\not\!\!E_T$.

δ $\sqrt{s}$(TeV) =	$M_D^{\rm max}$(TeV) at MC 1	 2	 4	$M_D^{\rm max}$ at LHC (100 fb^{-1}) 14
$\delta = 2$	4.1	6.5	10.6	8.5
$\delta = 3$	3.1	5.2	8.6	6.8
$\delta = 4$	2.5	4.3	7.3	5.8
$\delta = 5$	2.0	3.7	6.5	5.0

- Main issue is energy. Only a 4 TeV machine can exceed LHC reach, $\Rightarrow$ muon collider higher $\sqrt{s}$ vs. NLC might be crucial.

  Not clear that better energy resolution is useful for this signal.

- KK excitations γ^*, Z^*, μ^* would be possible $\Rightarrow$ excellent energy resolution a big bonus.

10 Strong WW interactions, Z', Technicolor

- If no light SM-like Higgs is found at LHC, NLC or MC, then WW sector will be strongly interacting.
- To fully explore, need energy: $\sqrt{s} \sim 3-4$ TeV, with appropriately matched luminosity, is very much better than $\sqrt{s} \sim 1.5-2$ TeV. At $\sqrt{s} \sim 3-4$ TeV, expected MC luminosities imply that statistics will be such that:
  - We can study all isospin channels, $I = 0, 1, 2$ *as a function of* WW *mass* $\Rightarrow$ close analogy to $\pi\pi$ scattering — $\mu^+\mu^+$ collisions are needed as well as $\mu^+\mu^-$.
  - We can separate $W_L W_L$, $W_L W_T$ and $W_T W_T$ polarization channels from one another and, thereby, clearly separate new physics in these different channels.

$\Rightarrow$ ability to lay out in detail the effective 'chiral' $\mathcal{L}$ for the WW sector.

  - Also $WW \to t\bar{t}$ will exhibit large enhancements if there is no light Higgs boson.

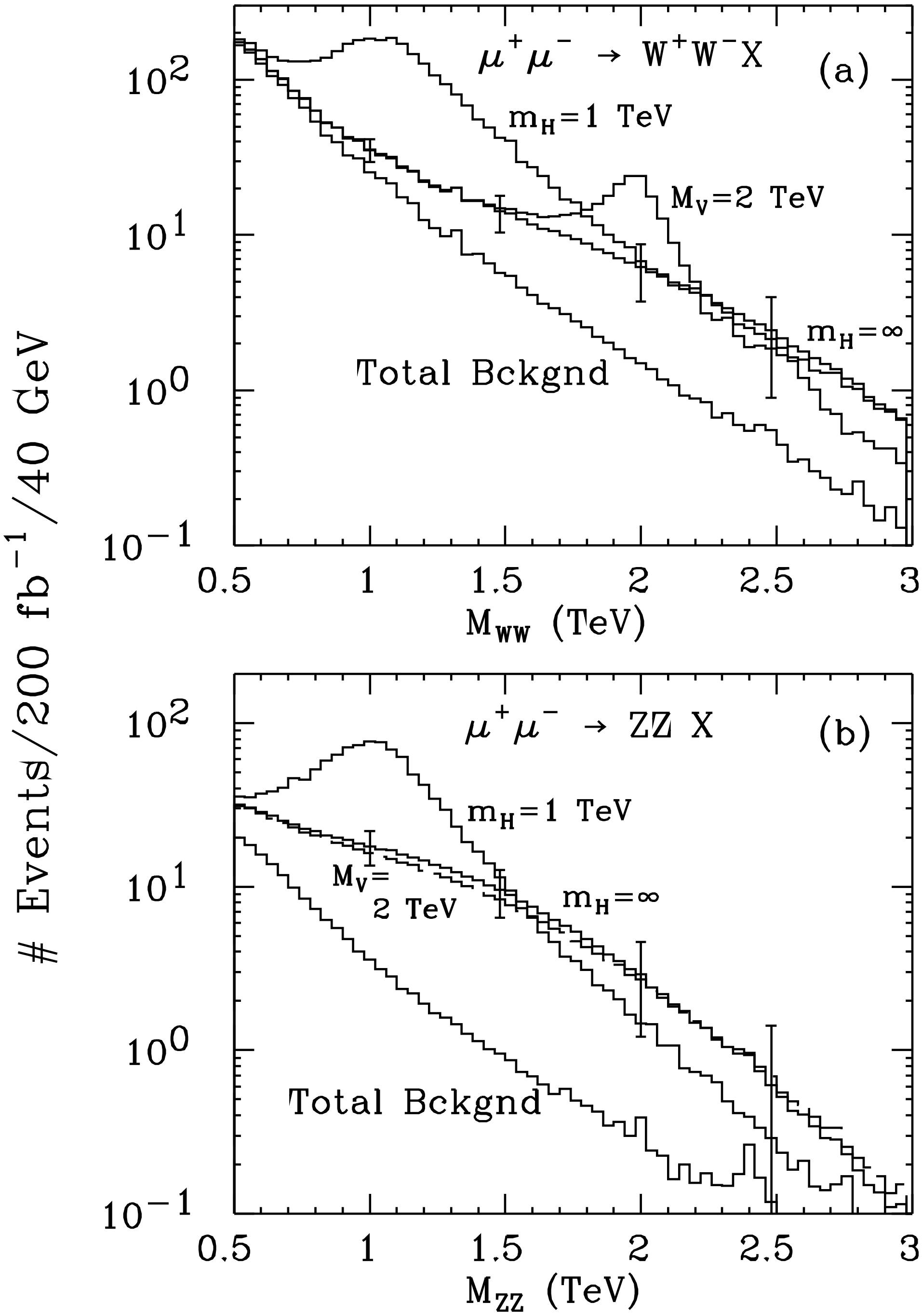

Figure 14: Events as a function of M_{VV} for sample SEWS models (including the combined backgrounds) and for the combined backgrounds alone in the (a) W^+W^- and (b) ZZ final states after imposing all cuts. Sample signals shown are: (i) the SM Higgs with $m_{h_{SM}} = 1$ TeV; (ii) the SM with $m_{h_{SM}} = \infty$ unitarized via K-matrix techniques (LET-K model); and (iii) the Vector model with $M_V = 2$ TeV and $\Gamma_V = 0.2$ TeV. (In the ZZ final state the histogram for (iii) lies slightly lower than than that for model (ii) at lower M_{VV}.) Results are for $L = 200$ fb^{-1} and $\sqrt{s} = 4$ TeV. Sample error bars for the illustrated 40 GeV bins are shown in the case of the $m_{h_{SM}} = \infty$ model.

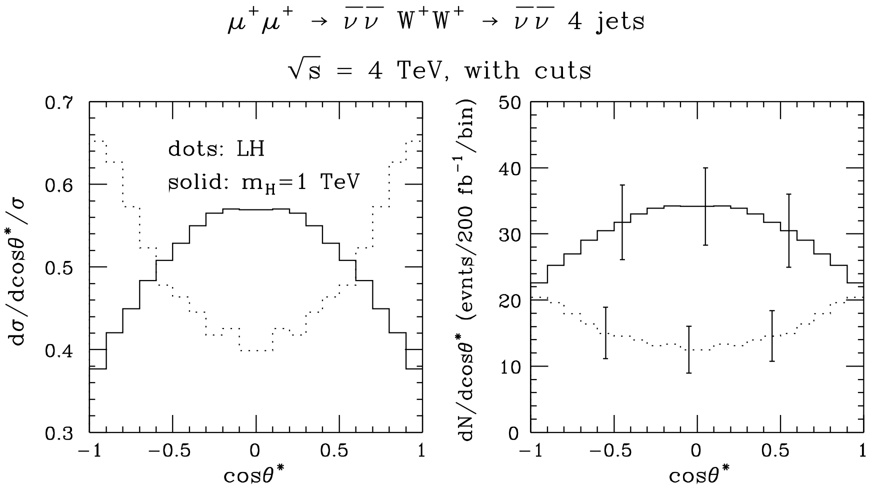

Figure 15: $dN/d\cos\theta^*$ (for $L = 200$ fb^{-1}) vs. $\cos\theta^*$ of the W^+ decays in the W^+W^+ final state. Typical error bars are displayed. Cuts: $M_{VV} \geq 500$ GeV, $p_T^V \geq 150$ GeV, $|\cos\theta_W^{\rm lab}| < 0.8$ and $30 \leq p_T^{VV} \leq 300$ GeV.

- A new Z' or ρ_T is easily discovered in the bremsstrahlung tail when running the muon collider at high $\sqrt{s}$.

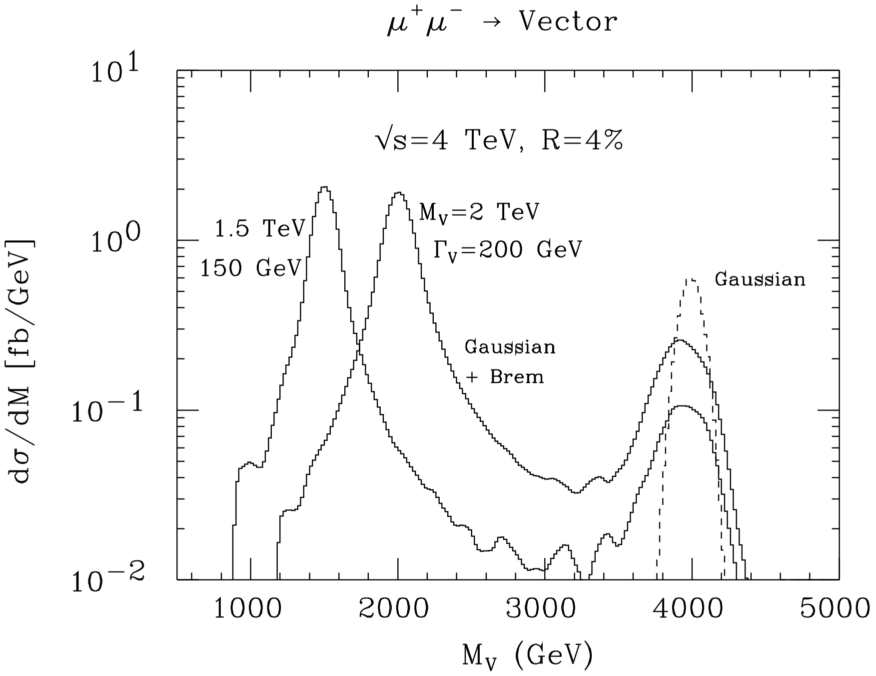

Figure 16: Appearance of a $M_V = 1.5$ TeV or 2 TeV, $\Gamma_V = 0.1 \times M_V$ heavy vector resonance in the bremsstrahlung tail of a $\mu^+\mu^-$ collider operating at 4 TeV.

- Reset $\sqrt{s}$ and possibly build cheap new storage ring for a V factory.
- For a small enough Γ the muon collider would be crucial. E.g. $M_V =$ 1 TeV and $\Gamma_V = 3$ GeV (as entirely possible in some technicolor models)

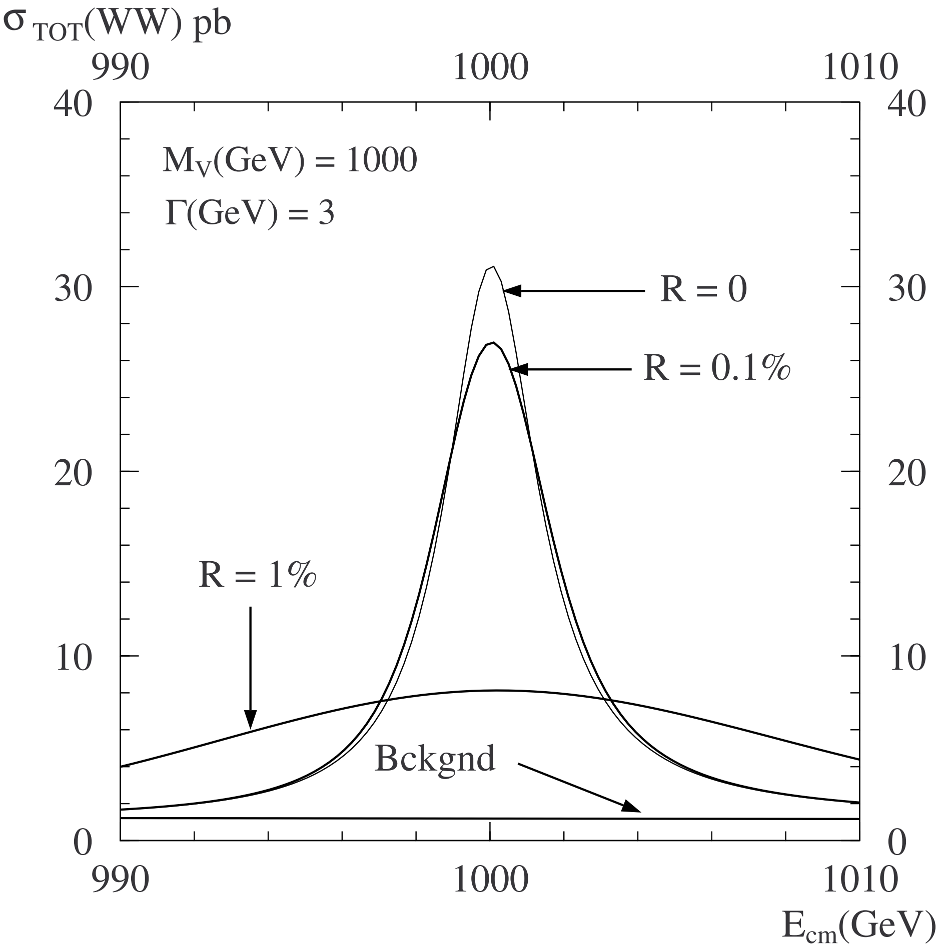

Figure 17: $M_V = 1$ TeV, $\Gamma_V = 3$ GeV: $\ell^+\ell^- \to V \to WW$ for $R = 0$, $R = 0.1\%$ and $R = 1\%$.

11 Standard SUSY studies

- Light SUSY particle example: $\mu^+\mu^- \to \widetilde{\chi}_1^+\widetilde{\chi}_1^-$.

  If $m_{\widetilde{\chi}_1^\pm} \lesssim 200$ GeV (needed to avoid fine-tuning in unified mass scenario), Tevatron and LHC will detect $\widetilde{\chi}_1^\pm\widetilde{\chi}_2^0$ pair production (e.g. tri-lepton channel) and measure $m_{\widetilde{\chi}_1^\pm} - m_{\widetilde{\chi}_1^0}$.

  Then, a threshold 2-point scan at MC using a total of $L = 50$ fb^{-1} (> **5 years** for $R = 0.1\%$ and pessimistic $L_{\rm year}$'s) $\Rightarrow \Delta m_{\widetilde{\chi}_1^\pm} \sim 150$ MeV for $m_{\widetilde{\chi}_1^+} = 200$ and $m_{\widetilde{\nu}_\mu} = 500$ GeV (worse for lower $m_{\widetilde{\nu}_\mu}$).

  Studying spectra of ℓ's and j's in $\mu^+\mu^- \to \widetilde{\chi}_1^+\widetilde{\chi}_1^-$ at $\sqrt{s} \sim 500$ GeV also $\Rightarrow$ small errors for $m_{\widetilde{\chi}_1^\pm}$ and $m_{\widetilde{\chi}_1^0}$.

- Heavy SUSY particles:

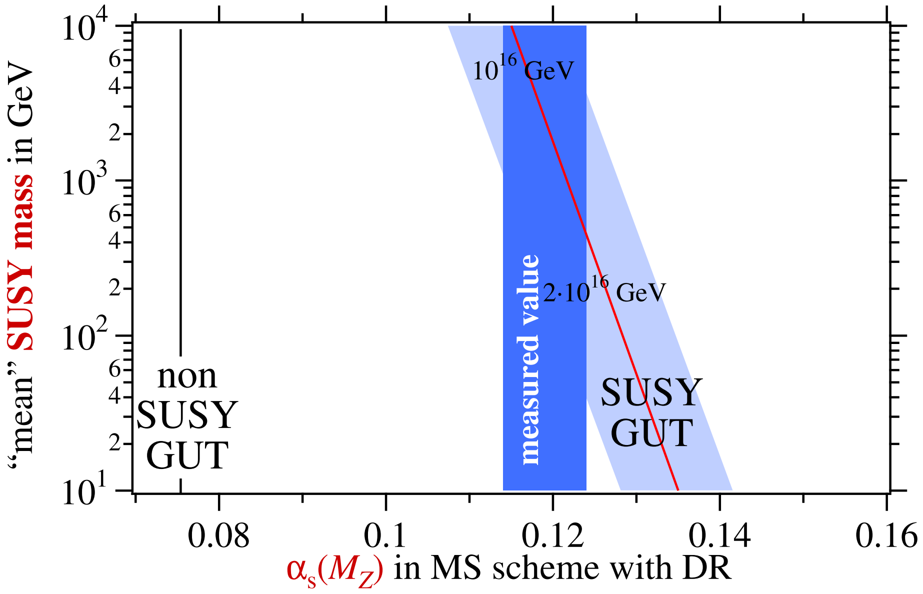

Figure 18: α_s prediction in supersymmetric GUT with minimal particle content in the Dimensional Regularization scheme.

  - Gauge unification 'best' (*i.e.* lower α_s at scale m_Z is possible) if there are SUSY particles above 1 TeV.

  - Heavy ($\gtrsim$ 1 TeV) masses for $\tilde{u}$ and $\tilde{d}$ preferred to suppress FCNC, and do not cause fine-tuning or naturalness/hierarchy problems.

- LHC will probably allow ‘detection’ of very heavy SUSY particles, but will not allowed detailed study of their properties (backgrounds + low event rates).

- $\Rightarrow$ for heavy sparticles, higher energy reach of muon collider would be crucial, especially for β^3 p-wave scalar pair production, for sorting out and study of their properties.

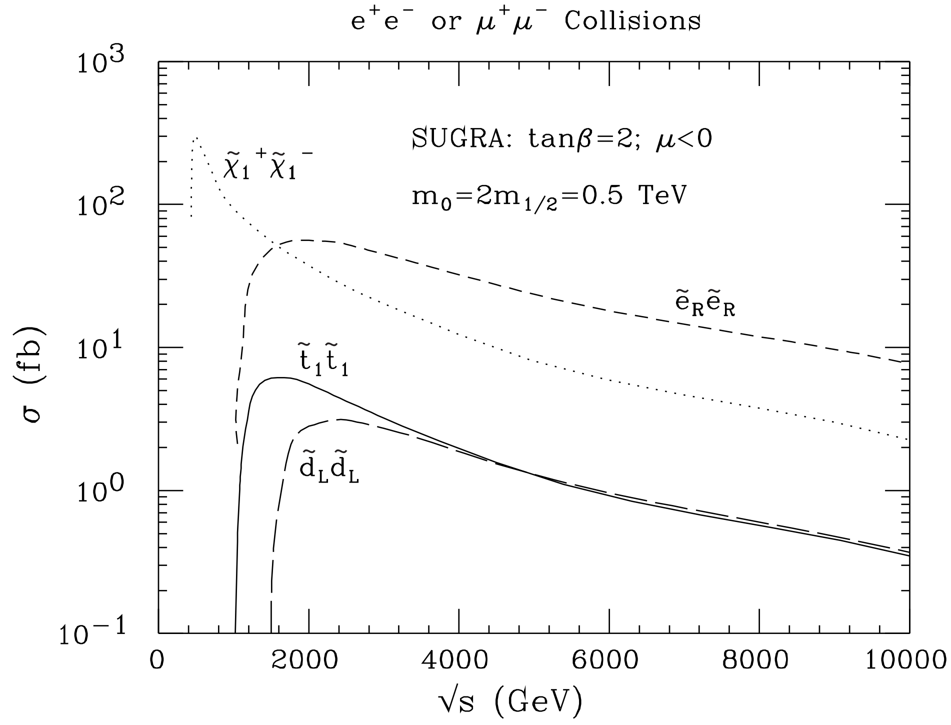

Figure 19: The production cross sections for SUSY particles in a supergravity model with heavy scalars.

Example: $m_0 = 2m_{1/2} = 500$ GeV , $\quad \tan\beta = 2, \quad A_0 = 0, \quad \mu < 0$.

General scaling law: in mSUGRA, $\sigma_{\max}$ is at $\sqrt{s} \sim 4m_0$ for scalars.

- How well could we determine soft-SUSY-breaking parameters at a very high $\sqrt{s}$ muon collider?
  - Many different sparticle pairs simultaneously produced.
  - $\Rightarrow$ contributions to many overlapping channels.
  - How to organize? = List of observables.

  (A) Look at the number of events in channels with large missing energy specified by the numbers of:

  $$(t, b, j, e^-, e^+, \mu^-, \mu^+, \tau^-, \tau^+, h^0, Z, W^-, W^+)$$

  where the t, Z, W is called a t, Z, W only if decay is hadronic.

  (B) End points of lepton spectra, and other kinematic distributions.

  - Parameters to be probed: The simplest interesting set is
    - M_1, M_2, $\tan\beta$, m_{A^0}, $\mu \Rightarrow 5$.
    - $A_t, A_b, A_\tau \Rightarrow 3$.
    - $m_{\tilde{\ell}_L}$, $m_{\tilde{\ell}_R}$, $m_{\tilde{q}_L}$, $m_{\tilde{u}_R}$, $m_{\tilde{d}_R}$ × 3 families $\Rightarrow 15$.

    $\Rightarrow$ 23 parameters. Typical scenario $\Rightarrow$ 200–300 channels with at least 5 events (some with 100's or 1000's of events).
  - Errors on parameters? Use channel N_i event rates and compute $V_{jk} = \Sigma_i \frac{\frac{\partial N_i}{\partial p_j}\frac{\partial N_i}{\partial p_k}}{N_i}$. 1σ errors on parameter p_k are given by $\Delta p_k = V^{-1}_{kk}$.
    - If backgrounds are suppressed with 40% efficient cuts $\Rightarrow$ very small statistical errors.
    - Systematics due to channel 'cross-talk' must be studied.

12 Conclusions

There will be an enormous number of very important physics studies that should be performed at the next generation of machines.

⇒ We will need many machines in order to have enough luminosity to complete these studies in a timely fashion.

A muon collider and associated intense muon beams would make major contributions to any foreseeable new physics study.

- The front-end physics of an intense muon beam would be very exciting.

  - Rare decays, EDM, $\mu \to e$ conversion, *etc.*
  - Neutrino beam physics including oscillation experiments.
  - μp collider physics, including contact interactions and leptoquarks.

- At lower $\sqrt{s}$ up to ~ 500 GeV:

  - It would prove of special value in studying the SM or MSSM Higgs sector or the physics associated with any narrow resonance ($\tilde{\nu}_\tau$, Δ^{--}, P^0, $\ldots$).
  - The MC would allow precision threshold studies, possible particularly interesting for SUSY $\widetilde{\chi}_1^+\widetilde{\chi}_1^-$ pair production.
  - Trying to improve L for small R at low $\sqrt{s}$ should be given attention to make these studies less costly in terms of time.

- At high $\sqrt{s} \gtrsim 3$ TeV:

  - It would be needed to study the heavy SUSY scalars expected.

  - It would be indispensable for studying strong WW dynamics if there is no light Higgs boson (which would mean no SUSY).
  - Especially good for heavy but narrow technicolor resonance or Z'.

- In general, the MC would be indispensable for determining the lepton flavor dependence expected for many types of new physics.

OVERVIEW TABLE: compare LHC, ≤ 2 TeV NLC, ≤ 4 TeV MC.

X = inaccessible, Y = accesible, F = μ flavor helps, P = polarization helps (NLC), R = MC energy resolution helps, E = high energy helps.

Physics	LHC	NLC	MC
Narrow Resonances			
Higgs	Y	Y	Y: F,R!!
PNGB's	Y	?: $\gamma\gamma$	Y: F,R!!
Supersymmetry			
Heavy H^0, A^0	X?	Y?; $\gamma\gamma$?	Y: E,F,R
Sfermions	$\tilde q$,$\tilde\ell$?	$\tilde\ell$,$\tilde q$?	$\tilde\ell$,$\tilde q$: E,F
Charginos	Y?	Y: P	Y: F,R,E?
R violation	$\tilde q$ decays	λ_{1i1}	λ_{2i2}: F,R!!,E
SUSY breaking	good	better	superb: F,R,E
Strong WW, TC			
Continuum	$\leq$ 1.5 TeV	$\leq$ 1.5 TeV	$\leq$ 2.5 TeV: E
Resonances	Y	Y	Y: R,E
Extra Dimensions			
$\not\!\!E_T$	Y	Y	Y: E,R?
Resonances	q^*, g^*	γ^*, Z^*, e^*	γ^*, Z^*, μ^*: R!!,E

The physics motivations for a MC are undeniable. We must continue with the R&D required to assess viability, at the most rapid possible 'natural' pace.