Brief Review of NMSSM Higgs and ADD-Invisible Higgs Phenomenologies

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The NMSSM

- NMHDECAY
- LHC and LC Phenomenology

References

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Since the MSSM is being pushed into parameter regions characterized by substantial fine tuning and a "little" hierarchy problem, and since no really attractive explanation for the μ parameter has emerged, it is time to adopt the NMSSM [1] as the baseline supersymmetric model.

The NMSSM phenomenology is considerably richer than that of the MSSM in many important ways. The focus here is on Higgs physics.

NMHDECAY

We (Ellwanger, Hugonie, JFG [11]) have developed the NMSSM analogue of HDECAY. Our conventions for parameters in the NMSSM are as follows.

a) Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

$$\lambda \ \widehat{S}\widehat{H_u}\widehat{H_d} + \frac{\kappa}{3} \ \widehat{S}^3 \tag{1}$$

depending on two dimensionless couplings λ, κ beyond the MSSM. (Hatted capital letters denote superfields, and unhatted capital letters will denote their scalar components).

b) The associated trilinear soft terms are

$$\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3 \,. \tag{2}$$

c) The final two input parameters are

$$\tan\beta = \langle H_u \rangle / \langle H_d \rangle \ , \ \mu_{\rm eff} = \lambda \langle S \rangle \ . \tag{3}$$

These, along with M_Z, can be viewed as determining the three SUSY breaking masses squared for H_u, H_d and S through the three minimization equations of the scalar potential.

Thus, as compared to two independent parameters in the Higgs sector of the MSSM (often chosen as $\tan\beta$ and M_A), the Higgs sector of the NMSSM is described by the six parameters

$$\lambda \ , \ \kappa \ , \ A_\lambda \ , \ A_\kappa, \ \tan\beta \ , \ \mu_{\rm eff} \ . \tag{4}$$

We will choose sign conventions for the fields such that λ and $\tan\beta$ are positive, while κ, A_λ, A_κ and $\mu_{\rm eff}$ should be allowed to have either sign.

In addition, values for the gaugino masses and of the soft terms related to the squarks and sleptons that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths must be input.

- We provide two forms of the NMHDECAY program:

– **NMHDECAY_SLHA.f — for study of one parameter point in the SLHA conventions for particle labeling etc. familiar to experimentalists;**
– **NMHDECAY_SCAN.f — designed for general phenomenological work including scanning over ranges of NMSSM parameters.**

The programs, and associated data files, can be downloaded from the two web pages:

`http://www.th.u-psud.fr/NMHDECAY/nmhdecay.html`

`http://higgs.ucdavis.edu/nmhdecay/nmhdecay.html`

The web pages provide simplified descriptions of the programs and instructions on how to use them. The programs will be updated to include additional features and refinements in subsequent versions. We welcome comments with regard to improvements that users would find helpful.

- **Input files are** `slhainp.dat` **and** `scaninp.dat`**, respectively. They are simple!**

```
#
#  Total number of points scanned
#
1000
#
#  Output format 0=short 1=long (not recommended for big scannings)
#
0
#
#  lambda
#
0.5
0.5
#
#  kappa
#
-0.15
-0.15
#
#  tan(beta)
#
3.5
3.5
#
#  mu
#
200.
200.
#
#  A_lambda
#
780.
780.
#
#  A_kappa
#
150.0
250.0
```

Table 1: Sample `scaninp.dat` file — 1st half for sample case #2.

```
#
#  Remaining soft terms (no scan)
#
mQ3=  1.D3
mU3=  1.D3
mD3=  1.D3
mL3=  1.D3
mE3=  1.D3
AU3=  1.5D3
AD3=  1.5D3
AE3=  1.5D3
mQ=   1.D3
mU=   1.D3
mD=   1.D3
mL=   1.D3
mE=   1.D3
M1=   5.D2
M2=   1.D3
M3=   3.D3
```

Table 2: The 2nd half of `scaninp.dat` file for sample case #2.

NMHDECAY performs the following tasks:

1. **It computes the masses and couplings of all physical states in the Higgs, chargino and neutralino sectors.**[1]
   Error messages are produced if a Higgs or squark mass squared is negative.
2. **It computes the branching ratios into two particle final states (including charginos and neutralinos — decays to squarks and sleptons will be implemented in a later release) of all Higgs particles.**
3. **It checks whether the Higgs masses and couplings violate any bounds from negative Higgs searches at LEP, including many quite unconventional channels that are relevant for the NMSSM Higgs sector.**
   It also checks the bound on the invisible Z width (possibly violated for light neutralinos).
   In addition, NMHDECAY checks the bounds on the lightest chargino and on neutralino pair production.
   Corresponding warnings are produced in case any of these phenomenological constraints are violated.

[1] For the Higgses, we have included the leading two-loop effects, but neglected subleading two-loop contributions and subleading one-loop purely electroweak contributions. In MSSM limit, our Higgs masses agree to within a few GeV with HDECAY.

4. It checks whether the running Yukawa couplings encounter a Landau singularity below the GUT scale.
   A warning is produced if this happens.
5. Finally, NMHDECAY checks whether the physical minimum (with all vevs non-zero) of the scalar potential is deeper than the local unphysical minima with vanishing $\langle H_u \rangle$ or $\langle H_d \rangle$.
   If this is not the case, a warning is produced.

- Below, I will discuss an example we employ to illustrate the use of these programs.

  It represents a scenario in which Higgs to Higgs decays make LHC Higgs detection very difficult.

- First, recall that normal MSSM Higgs detection at the LHC relies on:

  1) $gg \to h/a \to \gamma\gamma$;
  2) associated Wh/a or $t\bar{t}h/a$ production with $\gamma\gamma\ell^{\pm}$ in the final state;
  3) associated $t\bar{t}h/a$ production with $h/a \to b\bar{b}$;
  4) associated $b\bar{b}h/a$ production with $h/a \to \tau^+\tau^-$;
  5) $gg \to h \to ZZ^{(*)} \to$ 4 leptons;
  6) $gg \to h \to WW^{(*)} \to \ell^+\ell^-\nu\bar{\nu}$;

7) $WW \to h \to \tau^+\tau^-$;
8) $WW \to h \to WW^{(*)}$.

In supersymmetric models, it is also useful to include the mode

9) $WW \to h \to invisible$.

which, however, plays little role in the following. We also assume that $t \to H^\pm b$ will be observable for $m_{H^\pm} < 155~\mathrm{GeV}$ (could be raised).

- We estimate the expected statistical significances at the LHC in all Higgs boson detection modes 1) – 9) by rescaling results for the SM Higgs boson and/or the the MSSM h, H and/or A.

- $\boxed{\text{Scenarios for which LHC Higgs detection is “easy”, for } L = 300\mathrm{fb}^{-1}\text{!}}$

If Higgs decays to Higgs and/or SUSY are forbidden, then [9]: We can always detect at least one of the NMSSM Higgs bosons.

This was not the case [2] until the $t\bar{t}h \to t\bar{t}b\bar{b}$ mode [3, 4] (We have had the experimentalists extrapolate this beyond the usual SM mass range of interest.) and the WW fusion modes [5, 6, 7] were brought into play.

The point yielding the very lowest LHC statistical significance in an extensive scan over 10^9 points in parameter space had the following parameters:

$$\lambda = 0.0535; \quad \kappa = 0.0259; \quad \tan\beta = 5.42; \quad \mu_{\rm eff} = 145; \quad A_\lambda = -46 \text{ GeV}; \quad A_\kappa = -141 \text{ GeV}. \tag{5}$$

Properties of the Higgs bosons for this point are listed in table 3.

Other points with relatively weak LHC signals are similar in that:

1. **the Higgs masses are closely spaced and below or at least not far above the WW/ZZ decay thresholds,**
2. **the CP-even Higgs bosons tend to share the WW/ZZ coupling strength (indicated by R_i in the table),**
3. **couplings to $b\bar{b}$ of all Higgs bosons (the b_i or b_i' in the table) are not very enhanced,**
4. **and couplings to gg (the g_i or g_i' in the table) are suppressed relative to the SM Higgs comparison.**

The most visible process for this point was the $WW \to h_3 \to \tau^+\tau^-$ channel, but many other (notably $t\bar{t}h \to t\bar{t}b\bar{b}$) channels are also visible.

Overall, we have a quite robust LHC no-lose theorem for NMSSM parameters such that LEP constraints are passed and Higgs-to-Higgs decays are not allowed once full LHC luminosity is achieved.

Table 3: Properties of the neutral NMSSM Higgs bosons for the most difficult no-Higgs-to-Higgs-decays LHC point. In the table, $R_i = g_{h_i VV}/g_{h_{SM}VV}$, $t_i = g_{h_i t\bar{t}}/g_{h_{SM}t\bar{t}}$, $b_i = g_{h_i b\bar{b}}/g_{h_{SM}b\bar{b}}$ and $g_i = g_{h_i gg}/g_{h_{SM}gg}$ for $m_{h_{SM}} = m_{h_i}$. Similarly, t_i' and b_i' are the $i\gamma_5$ couplings of a_i to $t\bar{t}$ and $b\bar{b}$ normalized relative to the scalar $t\bar{t}$ and $b\bar{b}$ SM Higgs couplings and g_i' is the $a_i gg$ $\epsilon \times \epsilon'$ coupling relative to the $\epsilon \cdot \epsilon'$ coupling of the SM Higgs.

Higgs	h_1	h_2	h_3	a_1	a_2
Mass (GeV)	94	113	147	133	173
R_i	-0.440	-0.743	-0.505	0	0
t_i or t_i'	-0.421	-0.647	-0.662	-0.183	0.026
b_i or b_i'	-0.993	-3.55	4.10	-5.37	0.757
g_i or g_i'	0.470	0.554	0.435	0.139	0.021
$B(h_i\ or\ a_i \to b\bar{b})$	0.902	0.908	0.870	0.911	0.903
$B(h_i\ or\ a_i \to \tau^+\tau^-)$	0.081	0.085	0.086	0.088	0.095
Chan. 1) $S/\sqrt{B}$	0.00	0.20	0.26	0.00	0.00
Chan. 2) $S/\sqrt{B}$	0.83	0.76	0.22	0.00	0.00
Chan. 3) $S/\sqrt{B}$	3.03	6.28	5.64	5.64	0.00
Chan. 4) $S/\sqrt{B}$	0.00	0.88	3.24	3.24	0.04
Chan. 5) $S/\sqrt{B}$	0.00	0.12	1.59	–	–
Chan. 6) $S/\sqrt{B}$	0.00	0.00	1.26	–	–
Chan. 7) $S/\sqrt{B}$	0.00	6.88	6.96	–	–
Chan. 8) $S/\sqrt{B}$	0.00	0.17	0.44	–	–
All-channel $S/\sqrt{B}$	3.14	9.39	9.75	6.50	0.04

- **The difficult scenarios: Higgs to Higgs (or SUSY) decays**

  The importance of Higgs to Higgs decays was realized in [2, 8]. Detailed NMSSM scenarios were first studied in [9, 10].

  We have shown that (for relatively heavy squarks and gauginos) all scenarios of this type for which discovery is not possible in modes 1) – 9) are such that there is a SM-like Higgs h_H which decays to a pair of lighter Higgs, $h_L h_L$.

  In general, the h_L decays to $b\overline{b}$ and $\tau^+\tau^-$ (if $m_{h_L} > 2m_b$) or to jj and $\tau^+\tau^-$ (if $2m_\tau < m_{h_L} < 2m_b$).

  A possibly viable LHC signal then comes from $WW \to h_H \to h_L h_L \to jj\tau^+\tau^-$ in the form of a bump in the $M_{jj\tau^+\tau^-}$ reconstructed mass distribution. It is not a wonderful signal, but it is a signal.

  For most such cases, h_L is actually the lightest CP-odd scalar a_1 and h_H is the lightest or 2nd lightest CP-even scalar, h_1 or h_2.

- However, there are also cases not excluded by LEP (but we are pushing the LEP people for improvements) in which $m_{a_1} < 2m_\tau \Rightarrow a_1 \to c\overline{c}, s\overline{s}, gg$.

We believe it will be very difficult to find techniques that will allow extraction of a signal in these cases where neither b nor τ tagging is relevant. The only hope would be jet counting, but QCD backgrounds are probably enormous.

Since the $b\overline{b}$ coupling of these very light a_1's is not enhanced significantly (typically), there are no reliable exclusions coming from Υ or $B_{s,d}$ decays. We believe there is simply too much model dependence in the theory for such decays, although we would be happy to be persuaded otherwise.

- Incidentally, the MNMSSM ($\kappa = 0$ and $A_\kappa = 0$) also has this kind of case where LHC discovery is not possible. (I did not have time to review the Pilaftsis etal papers, but I suspect that Higgs-to-Higgs decays must have been left out to arrive at the opposite conclusion.)

- There are also cases in which $h_H = h_2$ and $h_L = h_1$, $m_{h_1} > 2m_b$, but yet $h_1 \to c\bar{c}, gg$ decays are completely dominant — parameters are chosen near a special region where the h_1 decouples from leptons and down-type quarks.

  Again, it is very hard to imagine a technique for extracting a signal at the LHC.

  One such case is illustrated below.

Sample case: no LHC signal

- For figs. 2–1, we take $\lambda = 0.5$, $\kappa = -0.15$, $\tan\beta = 3.5$, $\mu_{\rm eff} = 200~{\rm GeV}$, $A_\lambda = 780~{\rm GeV}$ and $A_\kappa \in [150~{\rm GeV}, 250~{\rm GeV}]$.

  The `scaninp.dat` file for this case was given in Table 1.

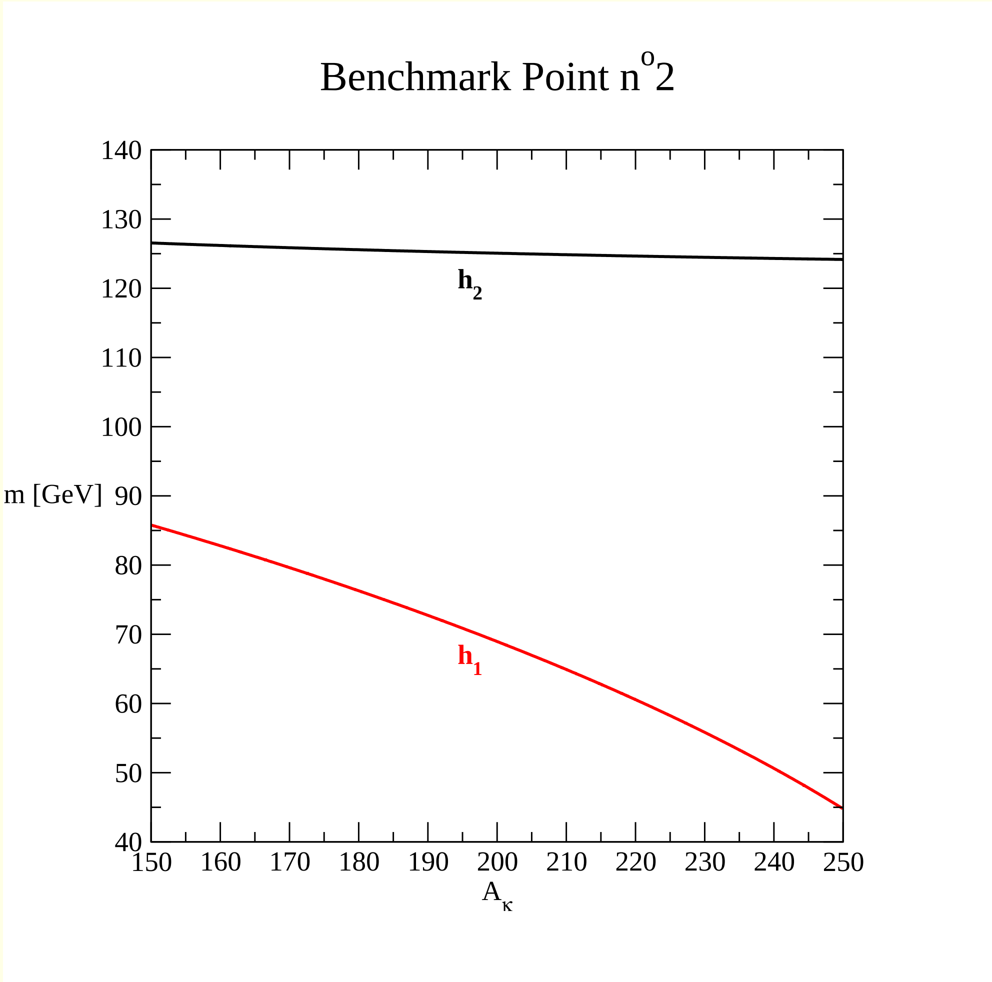
- For much of this parameter range, neither the h_1 nor the h_2 would have been observable at LEP.

  In particular, fig. 1–left shows that $m_{h_2} \gtrsim 120~{\rm GeV}$ implying that the h_2 is beyond the LEP kinematical reach.

  The h_1 is lighter, but $m_{h_1} > 2m_b$. However, this light Higgs is not excluded by LEP over most of the above A_κ range since: a) its reduced coupling to gauge bosons is small; and b) $h_1 \to b\bar{b}$ is suppressed so that $h_1 \to jj$ decays are dominant (see fig. 2–left).

  In fig. 1–right, we plot $\xi^2 = C_V(h_1)^2 \times BR(h_1 \to jj)$ for our selected points as well as the region excluded by LEP searches in this channel.

  We see that only if $m_{h_1} \lesssim 53~{\rm GeV}$, which corresponds to $A_\kappa \gtrsim 235~{\rm GeV}$, would the h_1 be excluded by LEP data.



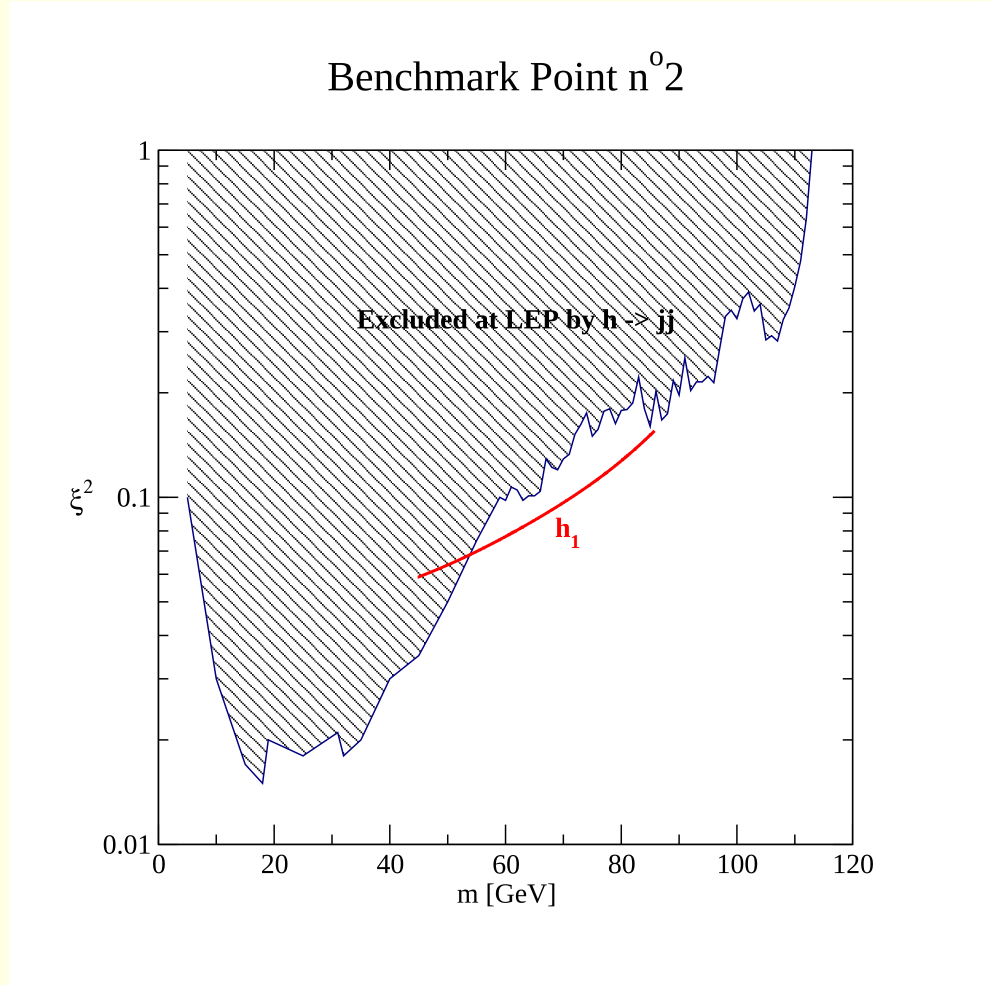


Figure 1: Left: m_{h_1} and m_{a_1} as a function of A_κ for the same parameters as in fig. 2. Right: LEP constraints in comparison to predictions for h_1 for these parameters. Note the correlation of $m = m_{h_1}$ with A_κ given in left-hand graph. New LEPHIGGS results may lower LEP exclusion curve in jj channel and make finding this kind of point more difficult.

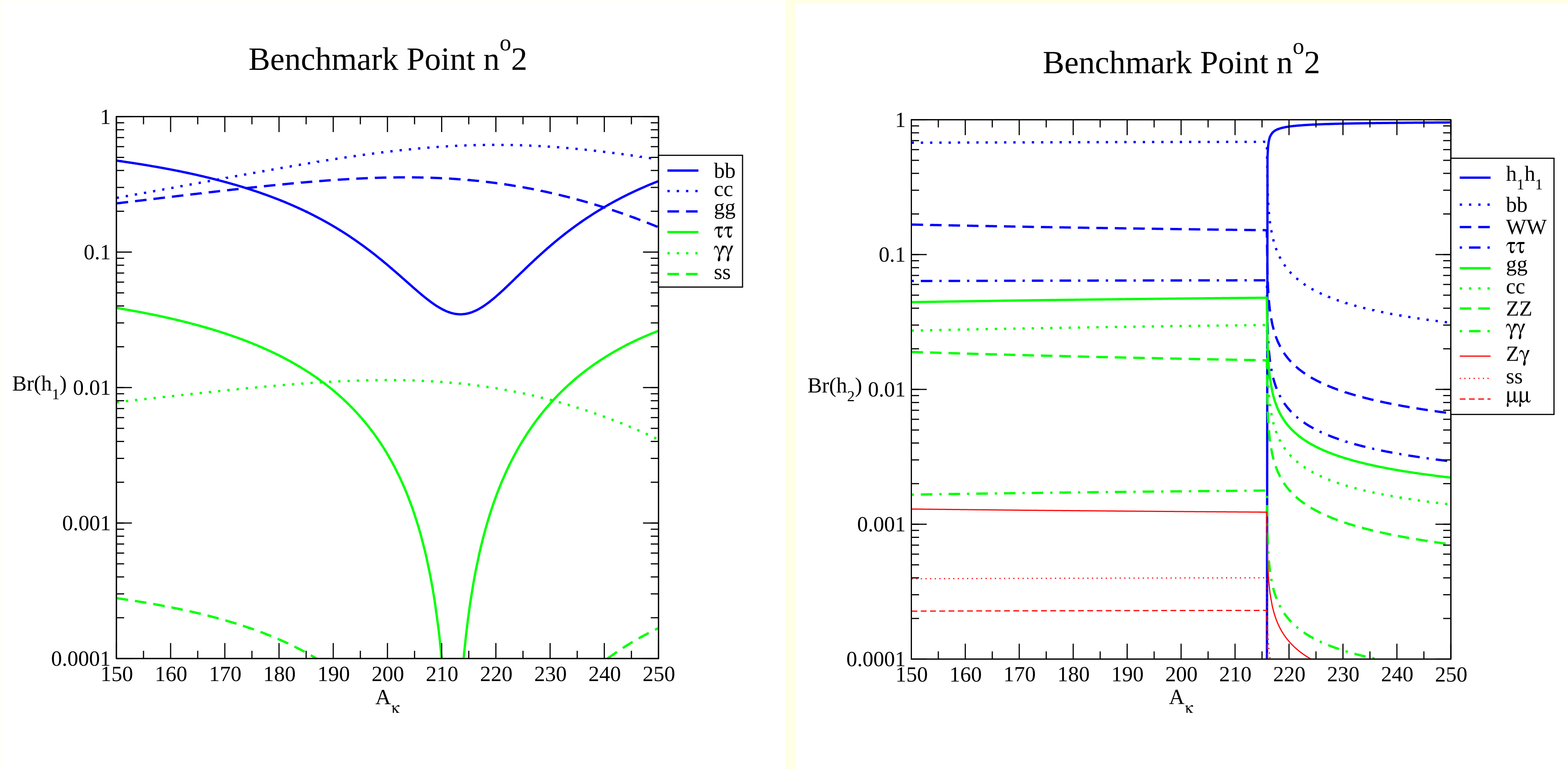


Figure 2: Left: Branching ratios of h_1 as a function of A_κ for $\lambda = 0.3$, $\kappa = -0.15$, $\tan\beta = 3.5$, $\mu_{\rm eff} = 200$ GeV, $A_\lambda = 780$ GeV, $m_{\rm squark} = 1$ TeV, and $A_t = 1.5$ TeV. Right: Branching ratios of h_2 as a function of A_κ for the same parameter choices.

- **Will these Higgs bosons be observable at the LHC?**

  In this regard, it is important to note from fig. 2–right that when $A_\kappa \gtrsim 215$ GeV, $h_2 \to h_1 h_1$ decays are dominant. This occurs because m_{h_1} decreases with A_κ, see fig. 1–left.

  Meanwhile, fig. 2–left shows that $BR(h_1 \to b\bar{b})$ and $BR(h_1 \to \tau^+\tau^-)$ are both small when $A_\kappa \in [205~\mathrm{GeV}, 220~\mathrm{GeV}]$; in this region of parameter space, the h_1 decays mainly to $c\bar{c}$ or gg.

  Thus, for $A_\kappa \sim 215 - 220$ GeV:

  - **The h_1 has a mass that lies below the mass range currently studied for Higgs detection at the LHC.**
    Further, the h_1 will be so weakly produced at the LHC (since $\xi^2 \lesssim 0.1$) that extensions to lower Higgs masses of the current LHC studies would probably conclude it was undetectable.
  - **Simultaneously, the strongly produced h_2 has decays dominated by $h_2 \to h_1 h_1$ with $h_1 \to c\bar{c}, gg$ (but not $b\bar{b}$ or $\tau^+\tau^-$).**
    As a result, the techniques for $h \to aa$ (which require a significant $a \to \tau^+\tau^-$ branching ratio) do not apply, and the h_2 would also appear to be very difficult to observe at the LHC.

How common are points that require the $aa \to jj\tau^+\tau^-$ mode at the LHC?

1. We scanned randomly over 10^8 points in the ranges:

$$10^{-4} \leq \lambda \leq 0.75; \quad -0.65 \leq \kappa \leq 0.65; \quad 1.6 \leq \tan\beta \leq 54;$$
$$-1 \text{ TeV} \leq \mu_{\rm eff}, A_\lambda, A_\kappa \leq +1 \text{ TeV}. \tag{6}$$

2. Of the 10^8 points, 86818793 yielded negative $m_{h_1}^2$, $m_{a_1}^2$ or $m_{H^\pm}^2$, implying that $\sim 13\%$ survive the basic requirements for a local minimum of the Higgs potential.

3. All points for which all Higgs masses-squared were positive also had positive $m_{\widetilde t_1}^2$ and $m_{\widetilde b_1}^2$.

4. Of the $\sim 1.32 \times 10^7$ remaining points, 1407077 would have resulted in an observable LEP signal as defined in NMHDECAY.

5. Of the remainder, 41306 are eliminated by the requirement of no $t \to H^\pm b$ decays and 576 are eliminated since there were no Higgs-to-Higgs decays.

   (Note how small the no-Higgs-to-Higgs fraction is.)

6. Of the remaining 11732824 points, 11726304 would yield 5σ signals in channels 1) – 9) and are not considered further.

7. This leaves 6520 points.

   Of these, 2198 have a Landau pole below M_U and 266 have an unphysical global minimum.

8. The result is 3480 points for which Higgs-to-Higgs decays are present, no Higgs would have been observed at LEP and no Higgs would be observable at the LHC in modes 1) – 9).

   This represents $\sim 0.026\%$ of the 13181207 points that have a proper local minimum.

   Thus, the standard LHC detection modes 1) – 9) suffice 99.974% of the time, for $L = 300\mathrm{fb}^{-1}$.

   Still, the parameter ranges associated with these points for which all NMSSM Higgs bosons escape LEP detection and LHC detection in modes 1) – 9) are broad:

$$0.0623 \leq \lambda \leq 0.7235; \quad -0.6230 \leq \kappa \leq 0.6331; \quad 1.65 \leq \tan\beta \leq 53.13;$$
$$-1 \text{ TeV} \leq \mu_{\mathrm{eff}}, A_\lambda \leq 1 \text{ TeV}; \quad -715 \text{ GeV} \leq A_\kappa \leq 502 \text{ GeV}\,. \tag{7}$$

9. The $jj\tau^+\tau^-$ detection mode works for all but 26 of the 3480 points.

   For some of the 26 points, $h_{1,2} \to a_1 a_1$ decays are prominent but $m_{a_1} \leq 2m_\tau$.

   For the remainder the a_1 or h_1 in the $a_1 a_1$ or $h_1 h_1$ pair final state simply has suppressed couplings to $b\overline{b}$ and $\tau^+\tau^-$. We saw an example of this earlier.

   In either case, τ triggering does not work and NMSSM Higgs detection at the LHC would probably be impossible.

   The following table illustrates 5 parameter space points having one or the other of the above characteristics.

10. We note that for all these 3480 points, the h_3 or a_2 will only be detectable if a super high energy LC is eventually built so that $e^+e^- \to Z \to h_3 a_2$ is possible.

Point Number	10	11	12	13	14
Bare Parameters					
λ	0.390	0.500	0.270	0.373	0.411
κ	0.183	$-.152$	0.147	0.243	$-.184$
$\tan\beta$	3.50	3.50	2.86	3.36	2.42
$\mu_{\rm eff}$	-245.0	200.0	-753.0	-315.0	184.0
A_λ	-230.0	780.0	312.0	171.0	626.0
A_κ	-5.0	230.0	8.4	52.1	32.8
CP-even Higgs Boson Masses and Couplings					
m_{h_1} (GeV)	94.1	57.3	95.4	88.0	113.8
R_1	0.945	-0.278	0.997	0.980	-0.992
t_1	0.949	-0.301	0.991	0.966	-0.989
b_1	0.890	0.015	1.047	1.135	-1.011
g_1	0.952	0.326	0.988	0.957	0.988
$B(h_1 \to b\bar{b})$	0.047	0.055	0.003	0.001	0.007
$B(h_1 \to \tau^+\tau^-)$	0.004	0.003	0.000	0.000	0.001
$B(h_1 \to c\bar{c} + s\bar{s} + gg)$	0.005	0.933	0.000	0.000	0.001
$B(h_1 \to a_1 a_1)$	0.943	0.000	0.996	0.999	0.991
m_{h_2} (GeV)	239.5	124.7	483.1	198.5	168.9
R_2	-0.327	-0.961	-0.014	-0.026	-0.122
t_2	-0.299	-0.952	-0.364	-0.321	-0.085
b_2	-0.669	-1.066	2.843	3.314	-0.339
g_2	0.295	0.948	0.366	0.384	0.080
$B(h_2 \to b\bar{b})$	0.002	0.048	0.020	0.060	0.004
$B(h_2 \to \tau^+\tau^-)$	0.000	0.004	0.002	0.007	0.000
$B(h_2 \to W^+W^- + ZZ)$	0.437	0.012	0.003	0.001	0.050
$B(h_2 \to a_1 a_1)$	0.246	0.000	0.002	0.079	0.944
$B(h_2 \to h_1 h_1)$	0.314	0.930	0.010	0.007	0.000
$B(h_2 \to a_1 Z)$	0.000	0.000	0.485	0.845	0.002
$B(h_2 \to \widetilde\chi_1^0 \widetilde\chi_1^0)$	0.000	0.000	0.000	0.000	0.000
$B(h_2 \to \chi_i^0 \widetilde\chi_j^0 + \widetilde\chi_i^+ \widetilde\chi_j^-)$	0.000	0.000	0.000	0.000	0.000

Table 4: Properties of points for which the $WW \to h_H \to h_L h_L \to jj\tau^+\tau^-$ modes don't work.

Point Number	10	11	12	13	14
m_{h_3} (GeV)	561.7	731.1	820.8	406.2	529.8
R_3	-0.017	-0.006	-0.079	-0.199	-0.019
t_3	-0.301	-0.290	-0.093	-0.228	-0.430
b_3	3.466	3.481	0.031	0.138	2.391
g_3	0.302	0.288	0.093	0.229	0.431
$B(h_3 \to b\bar{b})$	0.045	0.015	0.000	0.000	0.008
$B(h_3 \to \tau^+\tau^-)$	0.006	0.002	0.000	0.000	0.001
$B(h_3 \to t\bar{t})$	0.674	0.278	0.099	0.088	0.457
$B(h_3 \to W^+W^- + ZZ)$	0.009	0.001	0.500	0.365	0.003
$B(h_3 \to Higgses)$	0.127	0.360	0.401	0.546	0.318
$B(h_3 \to \chi_i^0\widetilde{\chi}_j^0 + \widetilde{\chi}_i^+\widetilde{\chi}_j^-)$	0.138	0.344	0.000	0.000	0.212
CP-odd Higgs Boson Masses and Couplings					
m_{a_1} (GeV)	40.0	188.2	1.3	3.4	1.9
t'_1	0.000	0.044	0.076	0.204	0.081
b'_1	0.000	0.534	0.624	2.303	0.473
g'_1	0.000	0.038	0.363	1.003	0.197
$B(a_1 \to b\bar{b})$	0.015	0.007	0.000	0.000	0.000
$B(a_1 \to \tau^+\tau^-)$	0.001	0.001	0.000	0.000	0.000
$B(a_1 \to c\bar{c} + s\bar{s} + gg)$	0.000	0.000	0.948	0.938	0.936
$B(a_1 \to \gamma\gamma)$	0.983	0.000	0.000	0.000	0.000
$B(a_1 \to \widetilde{\chi}_1^0\widetilde{\chi}_1^0)$	0.000	0.992	0.000	0.000	0.000
m_{a_2} (GeV)	557.9	735.5	492.7	270.6	535.1
$m_{h^\pm}$ (GeV)	559.6	726.9	485.1	202.7	526.1
Most Visible of the LHC Processes 1)-9)	5 (h_2)	2 (h_2)	5 (h_3)	5 (h_3)	2 (h_1)
$N_{SD} = S/\sqrt{B}$ of this process at L =300 fb^{-1}	1.5	1.3	0.1	0.9	0.2

Table 5: Properties (continued) of selected scenarios for which LHC Higgs detection would not even be possible in the $WW \to h_H \to h_L h_L \to jj\tau^+\tau^-$ modes.

Difficult scenarios at the LC

- Whether or not we have a good LHC signal if nature chooses a difficult point, ultimately, a means of confirmation and further study will be critical.

  Thus, it is important to summarize the prospects at the LC.

- For difficult scenarios, we always find that the h_H ($h_H = h_1$ or h_2) has reasonable WW, ZZ coupling and mass at most $\sim 150~\mathrm{GeV}$ (but possibly much lower).

  Discovery of the h_H will be very straightforward via $e^+e^- \to Zh_H$ using the $e^+e^- \to ZX$ reconstructed M_X technique which is independent of the "unexpected" complexity of the h_H decay to $h_L h_L$ ($h_L = a_1$, or h_1 for $h_H = h_2$).

  This will immediately provide a direct measurement of the ZZh_H coupling with very small error.

- The LC will find it quite easy to look for even a rather light h_H decaying to $h_L h_L$ in the ZX channel.

- Once it is found, then, look for different final states and check for Higgs-like coupling of the h_L to various final state fermions.

Perhaps one will find one of the special cases with h_L decoupled from $b\overline{b}$ and $\tau^+\tau^-$.

Perhaps the situation will be canonical with h_L having standard Higgs-like decays.

Either way, we will be able to pin down the nature of the Higgs sector and the parameters of the NMSSM.

NMSSM Summary

The LHC can detect at least one Higgs boson of the NMSSM for almost all of the model parameter space, assuming accumulated luminosity of $L = 300\mathrm{fb}^{-1}$.

However, there are residual corners of parameter space for which we would see supersymmetry (and see perturbative WW scattering), and yet have to wait until the LC to see a Higgs boson or (more optimistically) check that the observed $jj\tau^+\tau^-$ LHC signal was really a Higgs boson signal.

Once the LHC + LC really fix all 5 NMSSM parameters, we may find that $m_{h_3} + m_{a_2} > 1~\mathrm{TeV}$ (quite common in our scans). In this case, LHC might be able to find the heavy Higgs signal since the approximate masses will be known. Sample channel: $gg \to h_3 \to t\bar{t}$ with $m_{h_3} \gtrsim 900~\mathrm{GeV}$.

Invisible Higgs Decays in ADD

References

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- The simplest model of extra dimensions is the ADD [1] model in which only gravity propagates in the extra dimensions and all the extra dimensions have the same compactification radius.

  All SM particles live only on the 3-brane.

- In this model, the KK excitations (gravitensors and graviscalars) are invisible to 3-brane detectors since they propagate mainly in the bulk.

Hence, a canonical signature for ADD extra dimensions is substantial missing energy associated with various kinds of events.

- The generic signature involving $\not{E}_T$ is production of a $jets/\gamma + \not{E}_T$ final state, in which the KK gravitational excitations are radiated away into the extra dimension to create the $\not{E}_T$.

- However, it is also generically the case the there will be a mixing (on the 3-brane) between the Higgs boson of the SM and the curvature tensor.

  In ADD models, the interaction between the Higgs complex doublet field H and the Ricci scalar curvature R of the induced 4-dimensional metric g_{ind} is given by the following action

$$S = -\xi \int d^4x \sqrt{g_{ind}} R(g_{ind}) H^\dagger H \,. \tag{8}$$

  After the usual shift $H = (\frac{v+h}{\sqrt{2}}, 0)$, this interaction leads to the mixing term [2]

$$\mathcal{L}_{\text{mix}} = \epsilon h \sum_{\vec{n}>0} s_{\vec{n}} \tag{9}$$

with

$$\epsilon = -\frac{2\sqrt{2}}{M_{\rm P}}\xi v m_h^2 \sqrt{\frac{3(\delta-1)}{\delta+2}}\,. \tag{10}$$

Above, $M_{\rm P} = (8\pi G_N)^{-1/2}$ is the reduced Planck mass, δ is the number of extra dimensions, ξ is a dimensionless parameter and $s_{\vec{n}}$ is a graviscalar KK excitation with mass $m_{\vec{n}}^2 = 4\pi^2\vec{n}^2/L^2$, L being the size of each of the extra dimensions.

- The above mixing requires rediagonalizing to the physical eigenstates h' and $s'_{\vec{n}}$ (which are mixtures of the SM Higgs h and the graviscalars $s_{\vec{n}}$).

  The $s'_{\vec{n}}$ eigenvalues are nearly continuous and so those near in mass to the h' act coherently together with the h'

- Consider the amplitude for $I \to h' + \sum_{\vec{n}>0} s'_{\vec{n}} \to F$, where I and F are SM particle initial and final states (such as $I = W^*W^*$ and $F = b\bar{b}$).

  One finds via a very direct and brute force computation that $h' + \sum_{\vec{n}>0} s'_{\vec{n}}$ forms an "effective" coherent state, which we denote by h_{eff}, that has invisible and SM decays.

In the narrow width approximation, the main effect of the mixing is to add an (invisible) width (we give the expression later) to the SM Higgs width leaving the production rate for h_{eff} the same as for a SM Higgs boson.

This is the approximation of [2].

- For example, consider a WW fusion initial state and a SM final state F. The net is cross section given by (neglecting a small wave function renormalization correction)

$$\sigma(WW \to h' + \sum_{\vec{m}>0} s'_{\vec{m}} \to F) = \sigma_{SM}(WW \to h \to F) \left[\frac{\Gamma^{SM}_{h\to F}}{\Gamma^{SM}_h + \Gamma_{h_{eff}\to graviscalar}} \right] \quad (11)$$

Note the appearance of the unmixed SM cross section above.

- For the invisible graviscalar final states, $\sigma(WW \to h' + \sum_{\vec{n}>0} s'_{\vec{n}} \to graviscalar)$ is obtained by replacing $\Gamma^{SM}_{h\to F}$ by $\Gamma_{h_{eff}\to graviscalar}$ in Eq. (11) above.

- The net result of this discussion is that the coherently summed amplitude will give the SM cross section multiplied by a branching ratio to the final state that must be computed with the inclusion of the (invisible) $h_{eff} \to graviscalars$ width obtained above that arises from the mixing (or oscillation) of the Higgs itself into the closest KK graviscalar levels.

- We reemphasize that these graviscalars are invisible since they are weakly interacting and mainly reside in the extra dimensions whereas the Higgs resides on the brane.

  The invisible mixing width is given by [2, 3]

$$
\begin{aligned}
\Gamma_{h_{eff}\to graviscalar} &= 2\pi\xi^2 v^2 \frac{3(\delta-1)}{\delta+2}\frac{m_h^{1+\delta}}{M_D^{2+\delta}} S_{\delta-1} \\
&\sim (16\,MeV) 20^{2-\delta}\xi^2 S_{\delta-1}\frac{3(\delta-1)}{\delta+2}\left(\frac{m_h}{150\,GeV}\right)^{1+\delta} \\
&\quad \times \left(\frac{3\,TeV}{M_D}\right)^{2+\delta} . \qquad (12)
\end{aligned}
$$

Note: The result (12) is a factor of 2 larger than found in Refs. [2, 3].

As we shall see, $\Gamma_{h_{eff} \to graviscalar}$ is typically much larger than Γ_h^{SM} when m_h is small.

- Our parameters are M_D, δ and ξ (we assume that m_h will be well measured in some SM channel at the LHC or in $e^+e^- \to ZX$ at a future LC).

Prospects for Discovery of Higgs and Extra Dimensions at the LHC and a future LC

- For a Higgs boson with m_h below the WW threshold, the invisible width causes a significant suppression of the LHC Higgs rate in the standard visible channels.

  For example, for $\delta = 2$, $M_D = 500$ GeV and $m_h = 120$ GeV, $\Gamma_{h' \to graviscalar}$ is of order 50 GeV already by $\xi \sim 1$, *i.e.* far larger than the SM prediction of 3.6 MeV.

  Even when $m_h > 2m_W$, the branching ratio into invisible states can be substantial for M_D values as large as several TeV

  Therefore, we must search for the Higgs boson in both visible and invisible channels.

- **LHC: Higgs in Visible Channels**

  Detailed studies of the Higgs boson signal significance, with inclusive production, have been carried out by the ATLAS [4, 5] and CMS [6] experiments.

We will employ the results of [6]. These were obtained for $L = 30\mathrm{fb}^{-1}$. For $L = 100\mathrm{fb}^{-1}$, we will simply rescale the statistical significances in each channel by $\sqrt{100/30}$.

- $\boxed{\text{LHC: Higgs in Invisible Channel}}$

The LHC experiments will also be sensitive to an invisibly decaying Higgs boson produced via WW-fusion, as summarized in Ref. [6].

- $\boxed{jets/\gamma + \not{E}_T \text{ at the LHC}}$

The following figure shows that the prediction of the ADD model for $jets/\gamma + \not{E}_T$ is only reliable in a very limited range of M_D.

A signal at low M_D might or might not be present depending upon how the theoretical result behaves at subprocess energies above M_D, a region where the theory is not reliable.

Even more importantly, given an observed signal we cannot be sure how to interpret it. This makes parameter determination via this means at the LHC impossible.

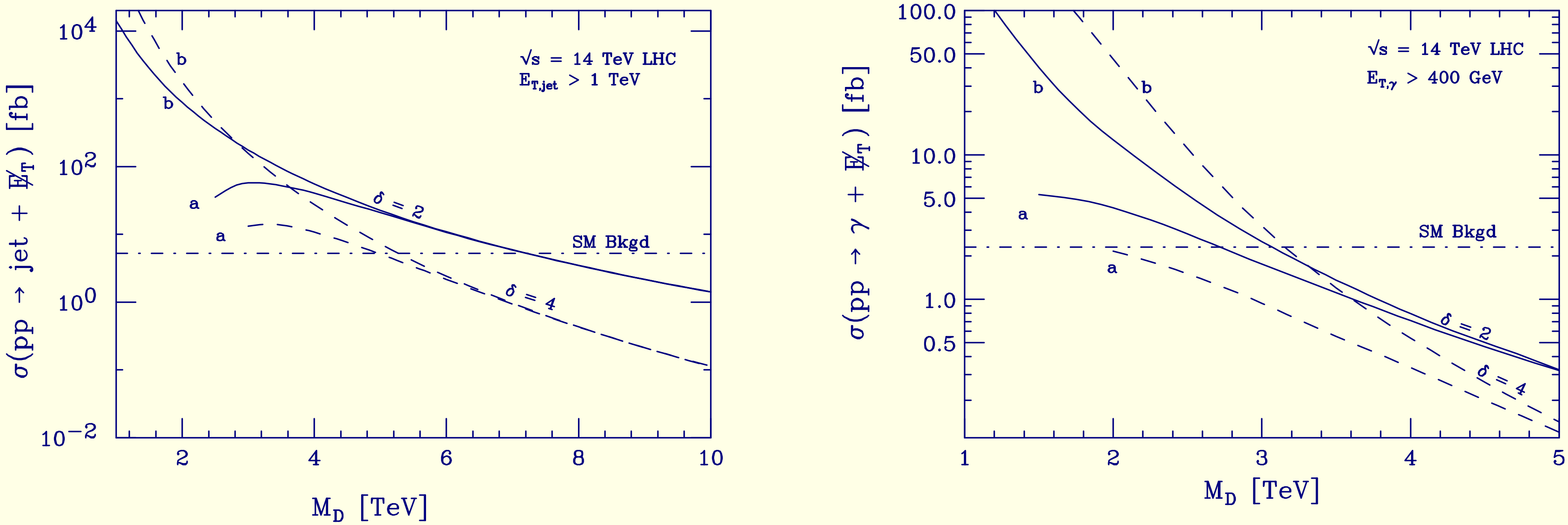

Figure 3: $jets + \not{E}_T$ **and** $\gamma + \not{E}_T$ **cross sections after integrating over a)** $\hat{s} < M_D^2$ **or b) all** $\hat{s}$**, where** $\hat{s}$ **is subprocess** s**.**

- **LC searches for invisible Higgs decays**

A TeV-class e^+e^- linear collider will be able to see the h_{eff} Higgs signal regardless of the magnitude of the invisible branching ratio simply by looking for a peak in the M_X mass spectrum in $e^+e^- \to ZX$ events.

We have employed the $\sqrt{s} = 350~\mathrm{GeV}$, $L = 500\mathrm{fb}^{-1}$ results of [7] to determine the portion of (M_D, ξ) parameter space for which the invisible Higgs signal will be observable at the LC at the $5~\sigma$ or better level.

- **Visible Higgs Channels at the LC**

We use the results of the many studies available and simply rescale according to the visible branching ratio. In effect, this amounts to using results in the $b\bar{b}$ final state when $m_{h_{eff}} < 140~\mathrm{GeV}$ and in the WW/ZZ final states at higher $m_{h_{eff}}$.

- $\boxed{\gamma + \not{E}_T \text{ at the LC}}$ unlike LHC, subprocess energy fixed $=\sqrt{s}$.

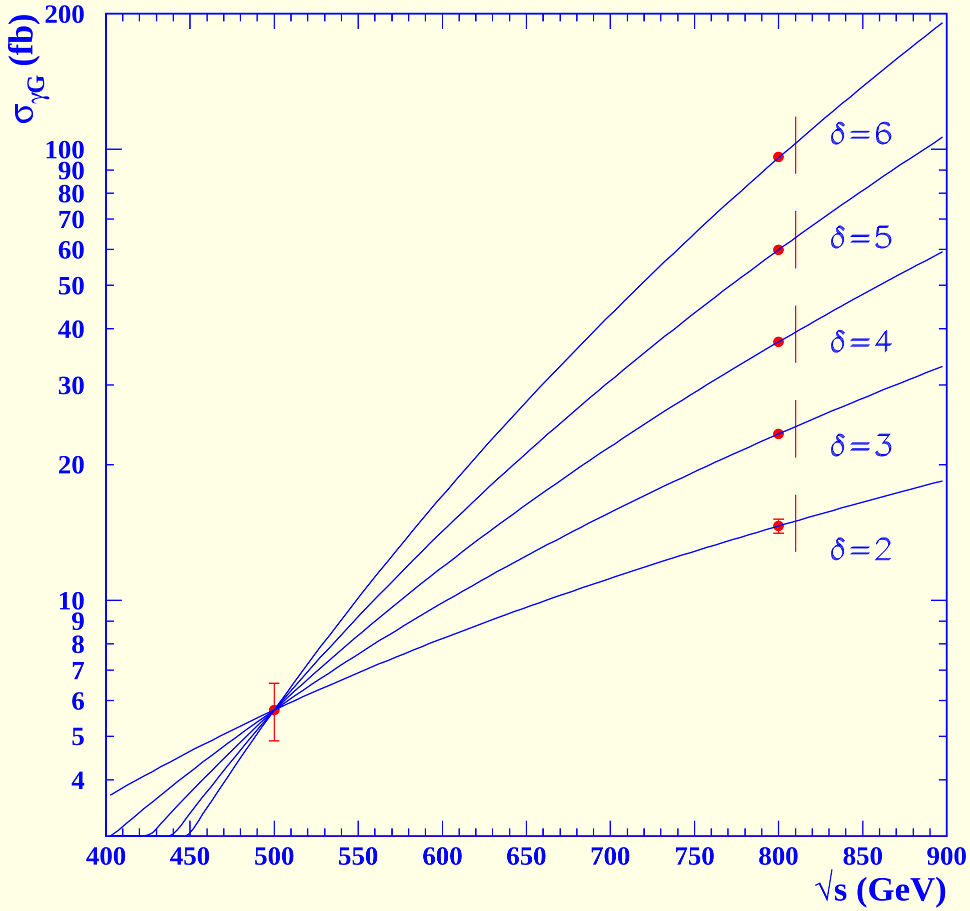

Figure 4: $\gamma + \not{E}_T$ cross sections vs. $\sqrt{s}$, normalized to common value at $\sqrt{s} = 500$ GeV. Thus, energy dependence gives δ via ratio of cross sections. Absolute normalization then gives M_D.

In the following figures (which assume $L = 100\mathrm{fb}^{-1}$ at LHC):

- Green region = visible channels $< 5\sigma$ at LHC.

- The regions above the blue line = LHC invisible Higgs signal in the WW-fusion channel exceeds 5 σ.

- The solid vertical line = maximum M_D which can be probed at the $5\ \sigma$ level by the analysis of $jets/\gamma$ with missing energy at the LHC.

- The middle dotted vertical line = M_D below which the theoretical computation at the LHC is ambiguous — a signal could still be present there, but its magnitude is uncertain.

  (For $\delta = 5$, there is no value of M_D for which the LHC computation is reliable.)

- The dashed vertical line at the lowest M_D value is the 95% CL lower limit coming from combining Tevatron and LEP/LEP2 limits.

- Region above the yellow line = LC invisible Higgs signal exceeds $5\ \sigma$ assuming $\sqrt{s} = 350\ \mathrm{GeV}$ and $L = 500\mathrm{fb}^{-1}$.

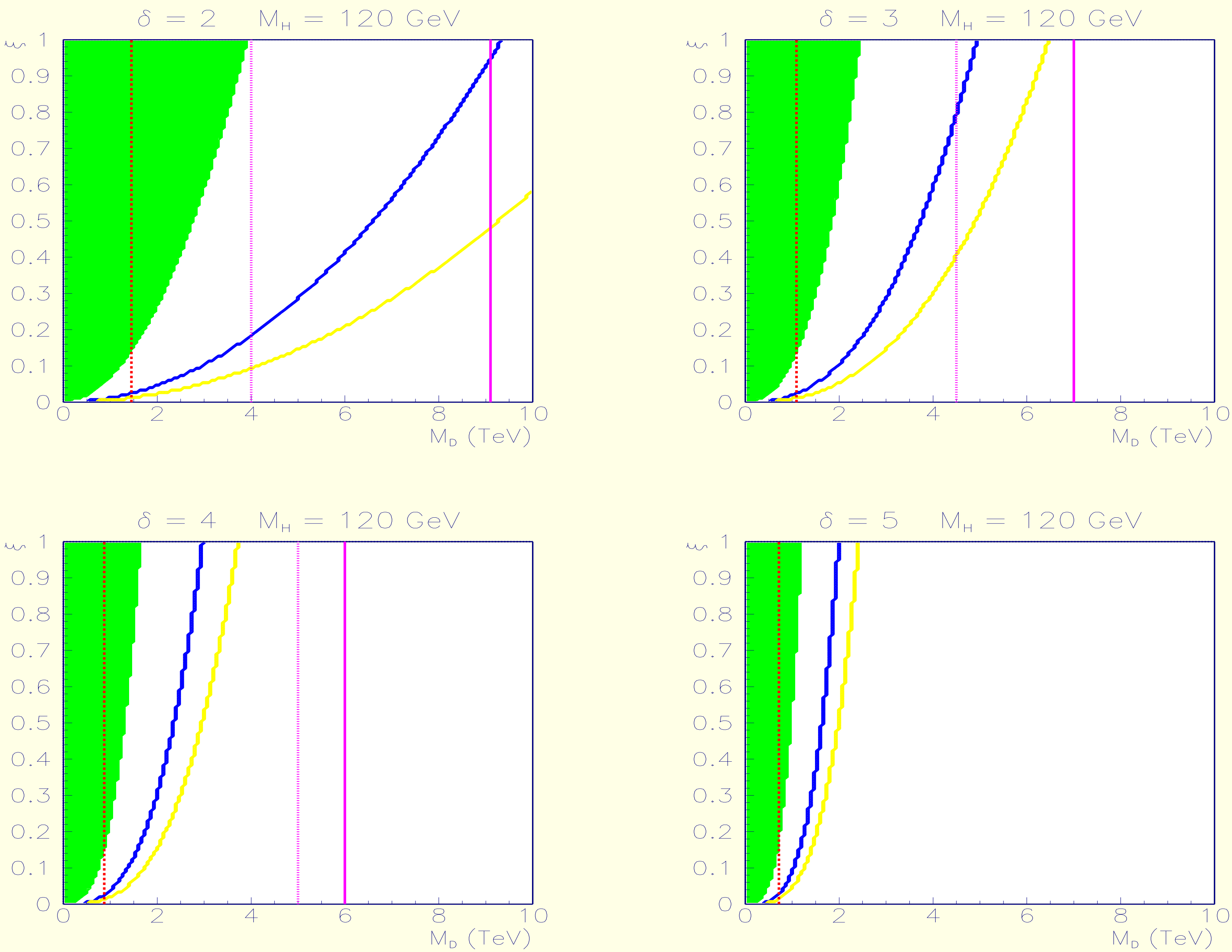

Figure 5: Results for $m_h = 120$.

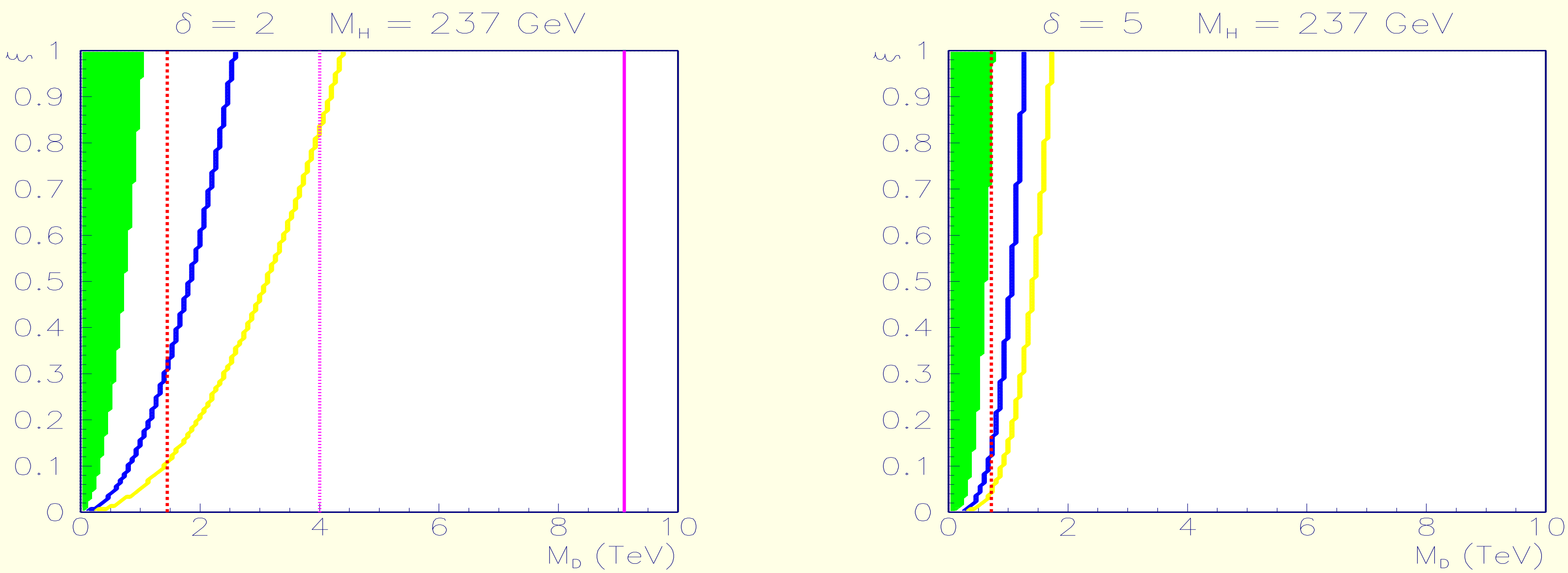

Figure 6: As in Fig. 5, for $m_h = 237$ GeV and $\delta = 2$ and $\delta = 5$.

- $\boxed{\textbf{Summary}}$

Whenever the LHC Higgs boson sensitivity in standard visible decays is lost due to their suppression, the invisible rate is large enough to still ensure detection through a dedicated analysis.

For $m_h = 237$ GeV, Fig. 6 shows that regions where visible signal $< 5\sigma$ appear to be fully excluded by LEP and Tevatron.

Determining ADD parameters from LHC and LC data

- **If the LC is operating, the very first test of the model will be to determine if the $e^+e^- \to ZX$ events do indeed exhibit a resonance structure with the predicted rate for a SM Higgs with the observed peak mass.**

  This can be done at about the 3% level.

  If this test works, then one can proceed with the parameter determination.

- **If the LC is not operating, there will be no decay-mode-independent means for checking that the Higgs is produced with SM-like rate.**

  At the LHC, this can only be done by looking for consistency of the collection of visible and invisible final state rates in various production modes with the assumption of a SM $production$ rate combined with the ADD prediction that the standard visible final state BR's are reduced in rate by the uniform factor of $1 - BR(h_{eff} \to invisible)$.

- **We will determine the error with which the LHC can determine the parameters under the assumption that the production cross section for the**

Higgs signal in each of the many production modes studied by ATLAS and CMS is SM-like.

The errors on the parameter determinations will be somewhat increased if we allow for the possibility of non-SM production rates. Thus, the results presented for LHC operation alone are somewhat optimistic.

- Our LHC procedures are as follows.
  - For the LHC, we have not made use of the jets/$\gamma + \not{E}_T$ signal for determining M_D and δ because of theoretical uncertainties described earlier.
  - For the LHC Higgs signal in visible channels, we compute the $\Delta\chi^2$ for a model relative to expectations as

$$\Delta\chi^2 = \frac{(S - S_0)^2}{\Delta S_0^2} \tag{13}$$

where $\Delta S_0^2 = S_0 + B$ and S and S_0 are computed from the SM rates by multiplying by $1 - BR_{h_{eff}\to invisible}$ and $1 - BR^0_{h_{eff}\to invisible}$.
Analogous procedures for $\Delta\chi^2$ contributions are followed in other channels.

– For the LHC Higgs signal in the invisible final state, we employed the detailed results of [8] (used in [6]), in which the Higgs signal and background event rates are given for the $WW \to Higgs \to invisible$ channel assuming SM production rate and 100% invisible branching ratio.

- A TeV-class e^+e^- linear collider will be able to improve the determination of the ADD model parameters very considerably with respect to the LHC alone. Here, we make use of the Higgs signals in both visible and invisible final states and also of the $\gamma + \not{E}_T$ signal.

– For the $\gamma + \not{E}_T$ signal, we have employed the TESLA study results of [9] for the signal.

We will present results obtained assuming measurements performed at the two energies of $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1000$ GeV assuming integrated luminosities of either $500\mathrm{fb}^{-1}$ and $1000\mathrm{fb}^{-1}$, respectively, or $1000\mathrm{fb}^{-1}$ and $2000\mathrm{fb}^{-1}$, respectively.

The reason for considering two energies is that the ratio of the cross sections at the two energies gives a strong constraint on δ, independent of cross section normalization. The value of M_D can then be thought of as being determined by the absolute value of the cross sections.

– For the invisible Higgs signal, we employ the $\sqrt{s} = 350$ GeV results of [7].

– For the visible Higgs signal, we employ a simple summary of the best available LC errors on the various SM Higgs signals, especially the $b\overline{b}$ and $WW^{(*)}$ final states, assuming running at energies of $\sqrt{s} = 500~\mathrm{GeV}$ and $\sqrt{s} = 1000~\mathrm{GeV}$ with luminosities of at least $500\mathrm{fb}^{-1}$ and $1000\mathrm{fb}^{-1}$, respectively, and with polarization.
We do not consider $m_h > 500~\mathrm{GeV}$.

- The $BR_{h_{eff}\to visible}$ measurement turns out to be quite important in discriminating between different models when the invisible branching fraction is large (the latter requiring small to moderate m_h, small M_D, $\delta = 2$ or 3, depending on M_D, and substantial ξ).

  In such a case, the visible branching fraction can be quite small and typically varies rapidly as a function of the ADD parameters (in particular, ξ), whereas the invisible branching fraction, although large, will be relatively more slowly varying and will not provide as good a discrimination between different parameter choices.

  Of course, if $BR_{heff\to visible}$ is so small that the background is dominant, the error in the measurement deteriorates and our ability to determine ξ, M_D and δ from this measurement deteriorates.

- Complementary statements apply to the case when $BR_{heff\to invisible}$ is small and $BR_{h_{eff}\to visible}$ is slowly varying.

  In particular, $BR_{heff\to invisible}$ varies very rapidly with ξ at small ξ in the case of $m_h = 120~\mathrm{GeV}$, $\delta = 2$, even for quite large M_D.

  As δ increases, the branching ratio contours become more vertical, and it becomes more difficult to determine ξ accurately.

- In the best cases, the visible and invisible branching fractions are comparable and both are rapidly varying as a function of ξ and the other ADD parameters. In such a case, measurements of these branching fractions combine to yield an excellent determination of all the ADD parameters.

- Given the (currently) five different $\Delta\chi^2$ outlined above, which we denote by $\Delta\chi^2(LHC~Hvis)$, $\Delta\chi^2(LHC~Hinv)$, $\Delta\chi^2(LC~\gamma \not{E}_T)$, $\Delta\chi^2(LC~Hinv)$, and $\Delta\chi^2(LC~Hvis)$, respectively, the net discrimination between models can be characterized using

$$
\begin{aligned}
\Delta\chi^2(LHC) &= \Delta\chi^2(LHC~Hvis) + \Delta\chi^2(LHC~Hinv) \\
\Delta\chi^2(LC) &= \Delta\chi^2(LC~\gamma \not{E}_T) + \Delta\chi^2(LC~Hinv) + \Delta\chi^2(LC~Hvis) \\
\Delta\chi^2(LHC+LC) &= \Delta\chi^2(LHC) + \Delta\chi^2(LC)\,. \qquad (14)
\end{aligned}
$$

- Since we assume that m_h will be very precisely measured, we concentrate on our ability to determine the parameters M_D, δ and ξ.

  We will present regions of parameter space corresponding to 95% CL determination, which for three parameters corresponds to $\Delta\chi^2 = 7.82$.

  Some sample results appear in Figs. 7 and 8, where we continue to focus on the light Higgs mass case of $m_h = 120~\mathrm{GeV}$.

- In the first figure, we present 95% CL contours for determination of the ADD parameters, M_D, ξ and δ assuming $m_{h_{eff}} = 120~\mathrm{GeV}$. The plots are all for input values if $\delta^0 = 2$ and $\xi^0 = 0.5$.

  The upper two plots and lower left plot are obtaining assuming $L = 100\mathrm{fb}^{-1}$ at the LHC, $\sqrt{s} = 350~\mathrm{GeV}$ Higgs measurements at the LC, and $\sqrt{s} = 500~\mathrm{GeV}$ and $\sqrt{s} = 1000~\mathrm{GeV}$ $\gamma + \not{E}_T$ measurements at the LC with $L = 1000\mathrm{fb}^{-1}$ and $L = 2000\mathrm{fb}^{-1}$ at the two respective energies. They are for different M_D^0 values: upper left — $M_D^0 = 2~\mathrm{TeV}$; upper right — $M_D^0 = 5~\mathrm{TeV}$; lower left — $M_D^0 = 8~\mathrm{TeV}$.

  The lower right plot is a repeat of the $M_D^0 = 5~\mathrm{TeV}$ case, but assuming

lower integrated luminosities: $L = 30\mathrm{fb}^{-1}$ **at the LHC and** $L = 500\mathrm{fb}^{-1}$ **and** $L = 1000\mathrm{fb}^{-1}$ **at** $\sqrt{s} = 500~\mathrm{GeV}$ **and** $\sqrt{s} = 1000~\mathrm{GeV}$ **at the LC.**

The larger light grey (yellow) regions are the 95% CL regions in the ξ, M_D **and** δ, M_D **planes using only** $\Delta\chi^2(LHC)$.

The smaller dark grey (blue) regions or points are the 95% CL regions in the ξ, M_D **and** δ, M_D **planes using** $\Delta\chi^2(LHC + LC)$.

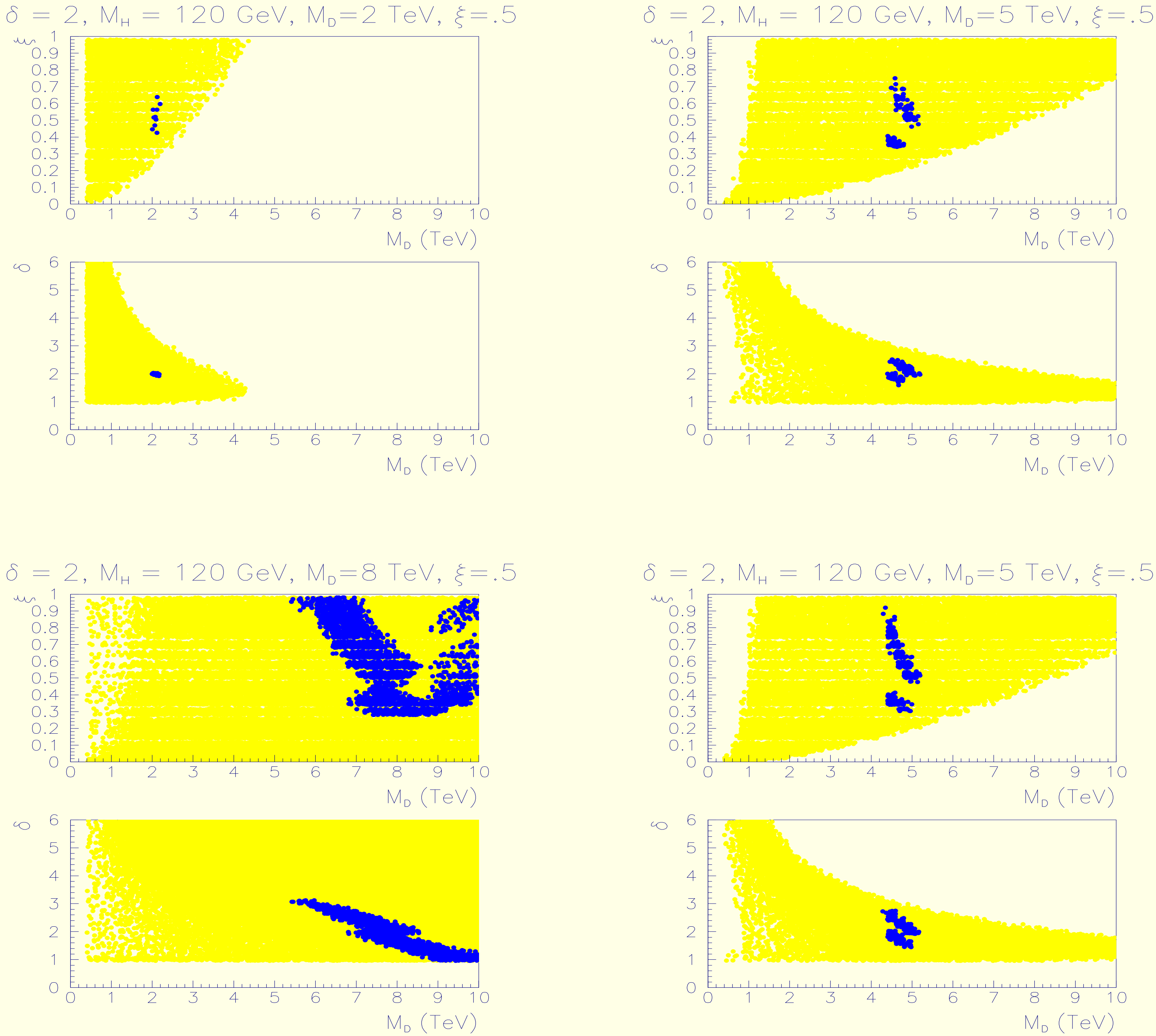
δ = 2, M_H = 120 GeV, M_D=2 TeV, ξ=.5
δ = 2, M_H = 120 GeV, M_D=5 TeV, ξ=.5
δ = 2, M_H = 120 GeV, M_D=8 TeV, ξ=.5
δ = 2, M_H = 120 GeV, M_D=5 TeV, ξ=.5
ξ
δ
M_D (TeV)

Figure 7:

Figure 8 considers the higher δ values of 4 and 5. The first three subfigures show again the decrease of precision with increasing M_D. (Adequate precision is lost at a lower M_D value than for $\delta = 2$.) Comparing the lower right to lower left figure, we see that at fixed M_D and ξ the precision of parameter determination increases as δ is lowered.

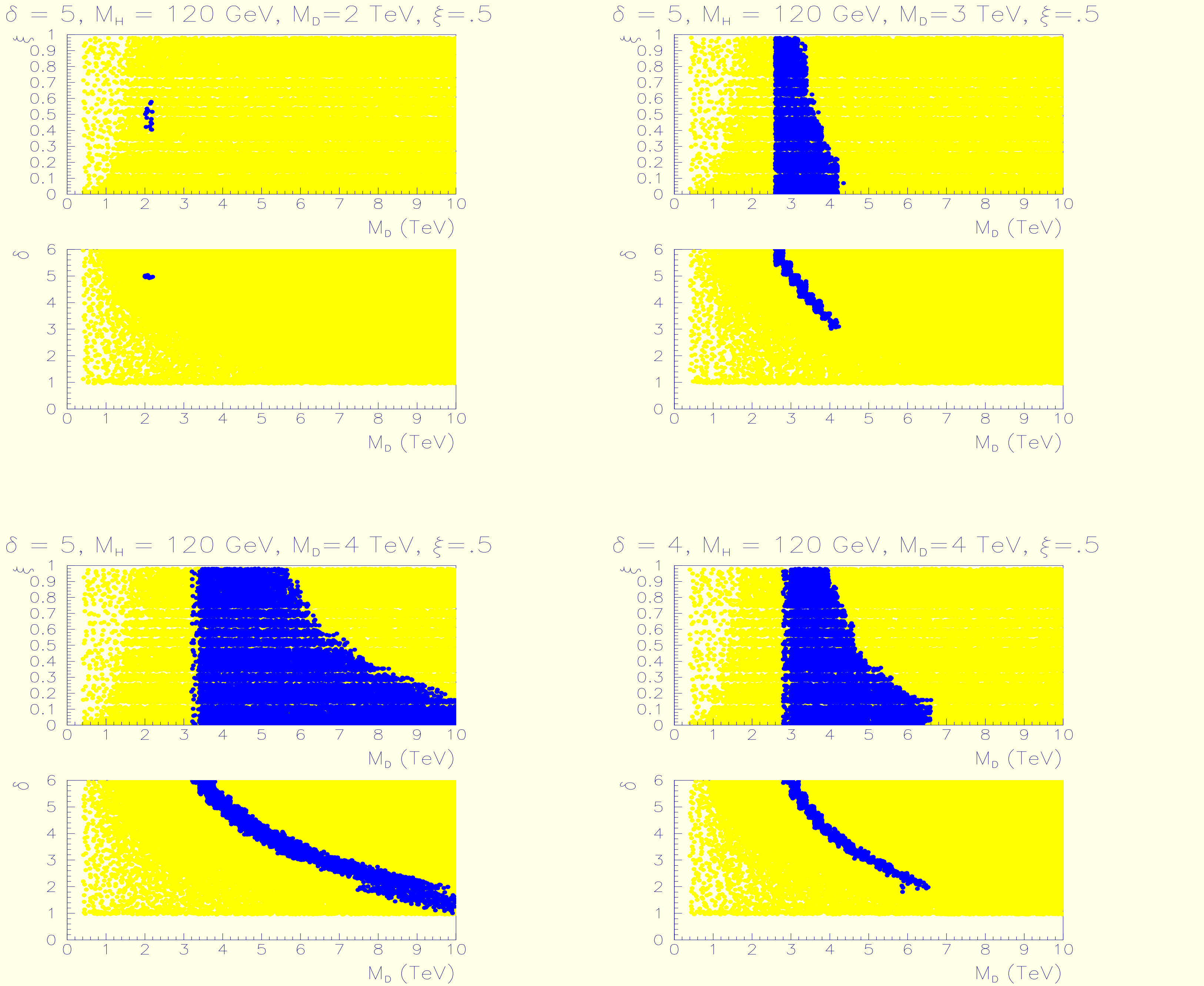
δ = 5, M_H = 120 GeV, M_D=2 TeV, ξ=.5
δ = 5, M_H = 120 GeV, M_D=3 TeV, ξ=.5
δ = 5, M_H = 120 GeV, M_D=4 TeV, ξ=.5
δ = 4, M_H = 120 GeV, M_D=4 TeV, ξ=.5
ξ
δ
M_D (TeV)

Figure 8:

ADD Conclusions

- If the Higgs boson is light (we have focused on $m_h = 120$ GeV and the current 95% CL upper limit from precision electroweak data of 237 GeV), then the invisible final state Higgs signal at the LHC could provide the most definitive evidence for the existence of extra dimensions before LC operation unless the mixing parameter ξ is much smaller than its expected $\mathcal{O}(1)$ magnitude.
- However, although the LHC has a good chance of seeing a signal, it will not be able to determine M_D, δ and ξ with any real precision.

  In particular, $jets/\gamma + \not{E}_T$ predictions as a function of M_D and δ are ambiguous in such a way that a given signal rate cannot be reliably interpreted.
- A variety of measurements at the LC will be required:
  - $\gamma + \not{E}_T$,
  - Higgs production/decay in the usual visible SM-like final states,
  - and Higgs production/decay in the invisible final state.

  Once these measurement have been made with the high precision expected at the LC, the M_D, δ and ξ will be determined with good to reasonable accuracy so long as not both δ and M_D are large. Then go back to LHC and look for consistency.