NMSSM Tevatron/LHC Scenarios: Motivation and Phenomenology

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News from the NMSSM and Beyond

- NMSSM Naturalness Issues
- NMSSM Baryogenesis
- NMHDECAY
- NMSSM LHC and Tevatron Phenomenology

MSSM problems:

- The MSSM is being pushed into parameter regions characterized by substantial fine tuning and a “little” hierarchy problem (i.e. large stop masses) in order to have a heavy enough Higgs boson for consistency with LEP limits.

- A strong phase transition for baryogenesis is hard to arrange when the Higgs is heavy and the stops are heavy.

- No really attractive explanation for the $\mu$ parameter has emerged.
One can marginally escape all but the last of these problems if significant Higgs sector CP violation is introduced through SUSY loops. However, I will propose that it is time to adopt the NMSSM as the baseline supersymmetric model.

The NMSSM phenomenology is considerably richer than that of the MSSM in many important ways. The focus here is on Higgs physics.

References


The NMSSM

- The Next to Minimal Supersymmetric Standard Model (NMSSM \[1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13]\)) provides a very elegant solution to the \(\mu\) problem of the MSSM via the introduction of a singlet superfield \(\hat{S}\).

For the simplest possible scale invariant form of the superpotential, the scalar component of \(\hat{S}\) acquires naturally a vacuum expectation value of the order of the SUSY breaking scale, giving rise to a value of \(\mu\) of order the electroweak scale.

The NMSSM is actually the simplest supersymmetric extension of the standard model in which the electroweak scale originates from the SUSY breaking scale only.

- In addition, the NMSSM renders the “little fine tuning problem” of the MSSM, originating from the non-observation of a neutral CP-even Higgs boson at LEP II, less severe \([2]\). Fine-tuning was also studied earlier in \([3]\). Our discussion here comes to rather different conclusions as compared to either reference.
A possible cosmological domain wall problem \[4\] can be avoided by introducing suitable non-renormalizable operators \[5\] that do not generate dangerously large singlet tadpole diagrams \[6\].

Hence, the phenomenology of the NMSSM deserves to be studied at least as fully and precisely as that of the MSSM.

Its particle content differs from the MSSM by the addition of one CP-even and one CP-odd state in the neutral Higgs sector (assuming CP conservation), and one additional neutralino. Thus, the physics of the Higgs bosons – masses, couplings and branching ratios \[1, 7, 8, 9, 10, 11, 12, 13\] can differ significantly from the MSSM.

I will be following the conventions of Ellwanger, Hugonie, JFG \[14\]. The NMSSM parameters are as follows.

a) Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

\[ \lambda \, \hat{S} \hat{H}_u \hat{H}_d + \frac{\kappa}{3} \, \hat{S}^3 \]  

(1)

depending on two dimensionless couplings \(\lambda, \kappa\) beyond the MSSM. (Hatted capital letters denote superfields, and unhatted capital letters will denote their scalar components).
b) The associated trilinear soft terms are

\[ \lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3. \]  \hspace{1cm} (2)

c) The final two input parameters are

\[ \tan \beta = \langle H_u \rangle / \langle H_d \rangle, \ \mu_{\text{eff}} = \lambda \langle S \rangle. \]  \hspace{1cm} (3)

These, along with \( M_Z \), can be viewed as determining the three SUSY breaking masses squared for \( H_u, H_d \) and \( S \) through the three minimization equations of the scalar potential.

Thus, as compared to two independent parameters in the Higgs sector of the MSSM (often chosen as \( \tan \beta \) and \( M_A \)), the Higgs sector of the NMSSM is described by the six parameters

\[ \lambda, \ \kappa, \ A_\lambda, \ A_\kappa, \ \tan \beta, \ \mu_{\text{eff}}. \]  \hspace{1cm} (4)

We will choose sign conventions for the fields such that \( \lambda \) and \( \tan \beta \) are positive, while \( \kappa, A_\lambda, A_\kappa \) and \( \mu_{\text{eff}} \) should be allowed to have either sign.
In addition, values for the gaugino masses and of the soft terms related to the squarks and sleptons that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths must be input.
Fine Tuning

w. Radovan Dermisek

The MSSM

Sample discussions of the issues appear in the papers cited in [16].

A typical and useful discussion for the MSSM is that given by Kane and King. They find that even at high $\tan\beta$ it is difficult to reduce fine tuning

$$F = \text{Max}_a F_a \equiv \text{Max}_a \left| \frac{d \log m_Z^2}{d \log a} \right|, \quad (5)$$

where the parameters $a$ are the GUT scale soft-SUSY-breaking parameters and the $\mu$ parameter, below the level of about 50 for $M_3 = 200$ GeV. A typical graph was that presented for $m_0 = 100$ GeV and $M_2 = M_1 = 200$, and $M_3 = 200, 150$ and 100 GeV. (All parameters given are GUT scale
values.)

Figure 1: Higgs mass $m_h$ and $F_\mu$ as functions of $\tan \beta$ for $m_0 = 100$ GeV, $M_{1,2} = 200$ GeV and $M_3 = 200, 150$ and $100$ GeV.
One can write down formulae for $m_Z^2$ and $F_\mu$. The procedure is to evolve GUT-scale parameters down to $m_Z$ and then insert the evolution results into

$$\frac{1}{2} m_Z^2 = -\mu^2 + \frac{m_{H_d}^2 - t_\beta m_{H_u}^2}{t_\beta^2 - 1}.$$  \hspace{1cm} (6)

For example, at $\tan\beta = 2.5$ they find (GUT parameters again):

$$\frac{1}{2} m_Z^2 = -0.87\mu^2 + 3.6M_3^2 - 0.12M_2^2 + 0.007M_1^2 - 0.71m_{H_u}^2 + 0.10m_{H_d}^2$$
$$+ 0.48(m_Q^2 + m_U^2) - 0.34A_t M_3 + 0.25M_2 M_3 + \text{small}.$$ \hspace{1cm} (7)

From this you already see the problem with large $M_3^2$. You must have carefully tuned cancellation to get $m_Z^2$ right. Of course, one cannot rule out the possibility that such cancellation is natural in particular models that have a built in correlation between $\mu$ and $M_3$, for example.

The NMSSM

We now contrast this to the NMSSM situation. Here, the computation of $m_Z^2$ is much more complicated. Some results on this have appeared in refs. [2] and [3], but I will claim they missed the most interesting part of parameter
space with the smallest finetuning. We start with

\[ V = \lambda^2 (h_u^2 s^2 + h_d^2 s^2 + h_u^2 h_d^2) + \kappa^2 s^4 - 2\lambda\kappa h_u h_d s^2 - 2\lambda A \lambda h_u h_d s \]
\[ + \frac{2}{3} \kappa A \kappa s^3 + m_{H_u}^2 h_u^2 + m_{H_d}^2 h_d^2 + m_S^2 s^2 + \frac{1}{4} g^2 (h_u^2 - h_d^2)^2. \] (8)

In the above, \( h_u \) and \( h_d \) are the vevs of the up and down type Higgs fields (without any \( \sqrt{2} \)) and \( s \) is the vev of the singlet Higgs field in the normalizations of NMHDECAY. (What I call \( g^2 \) is \( g^2 \equiv \frac{1}{2} (g_2^2 + g'^2) \) so that \( m_Z^2 = g^2 (h_u^2 + h_d^2) \).)

One must then solve the minimization equations

\[ \frac{\partial V}{\partial h_u} = 0, \quad \frac{\partial V}{\partial h_d} = 0, \quad \frac{\partial V}{\partial s} = 0 \] (9)

for the soft masses squared and explore combinations thereof for reexpressing
the minimization conditions. One finds

\[ m^2_{H_u} = \frac{1}{2h_u} \left( g^2 h_u^2 - g^2 h_d^3 - 2h_d h_u \lambda^2 + 2A_\lambda h_d \lambda s + 2h_d \kappa \lambda s^2 - 2h_u \lambda^2 s^2 \right) \]  \hspace{0.5cm} (10)

\[ m^2_{H_d} = \frac{1}{2h_d} \left( g^2 h_d^2 h_u^2 - g^2 h_u^3 - 2h_d h_u \lambda^2 + 2A_\lambda h_u \lambda s + 2h_u \kappa \lambda s^2 - 2h_d \lambda^2 s^2 \right) \]  \hspace{0.5cm} (11)

\[ m^2_S = \frac{1}{s} \left( \lambda A_\lambda h_d h_u + 2h_d h_u \kappa \lambda s - h_d^2 \lambda^2 s - h_u^2 \lambda^2 s - \kappa A_\kappa s^2 - 2\kappa^2 s^3 \right) \]  \hspace{0.5cm} (12)

One then defines

\[ \mu_{\text{eff}} = \lambda s, \quad \tan \beta \equiv \frac{h_u}{h_d}. \]  \hspace{0.5cm} (13)

It is then easy to eliminate terms linear in \( s \) to find that

\[ \frac{1}{2} m^2_Z = -\mu_{\text{eff}}^2 + \frac{m^2_{H_d} - \tan^2 \beta m^2_{H_u}}{\tan^2 \beta - 1}. \]  \hspace{0.5cm} (14)

However, \( \mu_{\text{eff}} \) is not a fundamental parameter in this case. Taking \( \frac{\kappa \lambda}{\tan \beta} - \)
\( \lambda^2 (11) - (\kappa \lambda \tan \beta - \lambda^2) \) (10), we obtain a second equation

\[
\kappa \lambda \left( \frac{1}{\tan \beta} m^2_{Hd} - m^2_{Hu} \tan \beta \right) - \lambda^2 \left( m^2_{Hd} - m^2_{Hu} \right) 
= \frac{1}{2} m_Z^2 \tan^2 \beta - 1 \left[ \kappa \lambda \left( \frac{1}{\tan \beta} + \tan \beta \right) - 2 \lambda^2 + \frac{2}{g^2} \lambda^4 \right] 
+ \mu_{eff} A \lambda \lambda^2 \left( \frac{1}{\tan \beta} - \tan \beta \right)
\]

(15)

Let’s make it simpler by defining

\[
a = - \frac{1}{2} \tan^2 \beta - 1 \left[ \kappa \lambda \left( \frac{1}{\tan \beta} + \tan \beta \right) - 2 \lambda^2 + \frac{2}{g^2} \lambda^4 \right] \quad (16)
\]

\[
b = \frac{1}{\tan \beta} k \lambda \left( m^2_{Hd} - m^2_{Hu} \tan^2 \beta \right) - \lambda^2 \left( m^2_{Hd} - m^2_{Hu} \right) \quad (17)
\]

\[
c = A \lambda \lambda^2 \left( \frac{1}{\tan \beta} - \tan \beta \right) \quad (18)
\]

so that it is simply

\[
a M_Z^2 + b = c \mu_{eff}. \quad (19)
\]
Squaring this equation and plugging in $\mu_{eff}$ from Eq. (14) we can eliminate $\mu_{eff}$ completely, and we obtain a quadratic equation for $M_Z^2$ with coefficients given in terms of soft susy breaking parameters:

$$AM_Z^4 + BM_Z^2 + C = 0,$$

where

$$A = a^2$$

(21)

$$B = 2ab + c^2/2$$

(22)

$$C = b^2 + c^2 \frac{m_{Hd}^2 - m_{Hu}^2 \tan^2 \beta}{1 - \tan^2 \beta}.$$  

(23)

This is the equivalent formula to that in the case of the MSSM. $A$, $B$, and $C$ can be expressed in terms of SSB parameters at the GUT scale; the only difference is that it is a quadratic equation. Therefore there are two solutions:

$$m_Z^2 = \frac{1}{2A} \left( -B \pm \sqrt{B^2 - 4AC} \right).$$

(24)

Only one applies for any given set of parameter choices.

To explore fine tuning, we begin at scale $m_Z$. 

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• We fix \( \lambda \) and \( \kappa \), choose values for \( \tan \beta \) and \( \tan \gamma \equiv s/v \), and of course fix \( h_u^2 + h_d^2 = v^2 \). In the NMHDECAY conventions employed, \( \lambda > 0 \) and \( \tan \beta > 0 \), but \( \kappa \) can have either sign.

We also find it easiest to fix the soft-SUSY-breaking parameters \( A_\lambda, A_\kappa \), and \( A_t = A_b \) at scale \( m_Z \).

• We also wish consider given GUT scale values for

\[
M_1, \quad M_2, \quad M_3, \quad m_Q^2, \quad m_U^2, \quad m_D^2, \quad m_L^2, \quad \text{and} \quad m_E^2. \quad (25)
\]

These we will take to have respective universal values.

• We use the usual back and forth RGE iteration approach to determine the values of \( m_Q^2, \quad m_U^2, \quad m_D^2, \quad m_L^2, \quad \text{and} \quad m_E^2 \) at scale \( m_Z \) that are consistent with these GUT scale values and the scale-\( m_Z \) values for \( \lambda, \kappa, \tan \beta, \tan \gamma, A_t, A_\lambda, A_\kappa \). These are then input into the Higgs multi-loop mass and analysis program.

At the same time, we obtain \( m_{H_u}^2, \quad m_{H_d}^2, \quad \text{and} \quad m_S^2 \) GUT scale values that are consistent with the choices determined by our \( m_Z \) scale inputs.
(which immediately fix the above quantities at scale $m_Z$ via minimization conditions).

- We can compute the $F_a$ by perturbing the GUT scale input $a$ a bit, recomputing the resulting $m_{H_u}^2(m_Z)$, $m_{H_d}^2(m_Z)$, $m_S^2(m_Z)$, $A_\lambda(m_Z)$ and $A_\kappa(m_Z)$ and then reminimizing the potential, which will yield new values of $m_Z$ (and $\tan \beta$ and $\tan \gamma$).

We use the shifted $m_Z$ computed as above to compute $F_a$.

Resulting observations

- One finds, depending upon input GUT scale parameters, that the largest of the $F_a$ ($F$) can be quite modest in size even if the GUT scale parameters are quite large.

In fact, we can always find parameter choices such that $F \sim 9 \div 10$ can be achieved, far below the MSSM values.

(Of course, there are other choices that give large $F$.)

An example of small $F$
• We consider Kane-King like choices: $\tan \beta = 3$, $M_1 = M_2 = M_3 = 300$ GeV (higher than their 200 GeV) and a universal value for $m_0^2 = m_Q^2 = m_U^2 = m_D^2 = m_L^2 = m_E^2 = (400 \text{ GeV})^2$.

• We scan over various possible $A_t(m_Z)$, $A_\lambda(m_Z)$ and $A_\kappa(m_Z)$ values.

• We require $\mu_{\text{eff}} > 100$ GeV and $M_2(m_Z) > 100$ GeV (to avoid a light chargino).

**A typical small $F$ case**

F=9.3

H masses={361, 283, 72}  P masses= {355, 17}  H+ mass= 351

At low scale
l= 0.363 k= 0.214 tanb=3.  s=665  M1= -126 M2= -248 M3= -888
mHu2u= -49708.3  mHd2u= 52699.7  mS2u= 043036.8
mQ2= 448390.  mu2= 106338.  md2= 758901.  mL2= 241725.  me2= 102443.
Au= 218.3  Al= 11.53  Ak= 0.462

At GUT scale
l= 0.4796 k= -0.2915  Ak= -180.5  Al= -1008.8  Au= -2495.4
mHu2= 1065740  mHd2=  4936.6  mS2= 19770.1
Our scanning statistics are still low, but it can certainly be said that a very efficient means for selecting scenarios with small $F$ is to focus on small $A_{\kappa}$, which generically leads to $h_1 \to a_1 a_1$ decays as being possible and not infrequently dominant, with the $h_1$ being quite SM-like. Such scenarios can evade current LEP constraints (as we shall come to).

Typical expressions for the things that enter into the calculation of $a$, $b$, and $c$ and thence $A$, $B$ and $C$ are:

\[
A_{\text{MZ}} = 0.760 \ A_{\text{G}} - 0.353 \ A_{\text{G}} + 0.314 \ M_{2\text{G}} + 0.699 \ M_{3\text{G}} + \text{small}
\]

\[
A_{\text{kMZ}} = 0.867 \ A_{\text{G}} - 0.375 \ A_{\text{G}} + \text{small}
\]

\[
m_{\text{Hu2MZ}} = 0.151 \ M_{2\text{G}}^2 + 0.281 \ A_{\text{G}} M_{3\text{G}} - 0.189 \ M_{2\text{G}} M_{3\text{G}} - 3.1 \ M_{3\text{G}}^2 + 0.523 \ m_{\text{Hu2G}} - 0.43 \ m_{\text{Q2G}} - 0.33 \ m_{\text{mu2G}}
\]

\[
m_{\text{Hd2MZ}} = 0.446 \ M_{2\text{G}}^2 + 0.899 \ m_{\text{Hd2G}} + \text{small}
\]

\[
m_{\text{S2MZ}} = 0.742 \ m_{\text{S2G}} + \text{small}
\]

Clearly, the analysis of exactly why there is cancellation in the computation of $F$ is somewhat complex, but we are working on it. What is clear is the general fact that there is a cancellation going on for all the small fine-tuning solutions. For example, in the above case, $B = 13210$ while $\sqrt{B^2 - 4AC} = 21556$ and $m_Z^2 = (-B + \sqrt{B^2 - 4AC})/(2A)$. Thus, $B$
is fairly dominant (often it is very dominant) and whatever dependence on some GUT parameter is present in $B$, it is also present with similar strength in $\sqrt{B^2 - 4AC}$, implying (for the sign shown) that the change in $-B$ is compensated fairly well by the change in $\sqrt{B^2 - 4AC}$.
Baryogenesis in the NMSSM

w. K. Kelley

The only work on this in the literature is that of ref. [2]. Others have focused on models with different or specialized superpotentials such as $W = \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{m^2_{12}}{\chi} \hat{S}$ [17] or $W = \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{\kappa}{3} \hat{S}^3 + \mu \hat{H}_u \hat{H}_d + r \hat{S}$ [18]. We are revisiting this to see to what extent the parameter regions with $h \rightarrow aa$ decays might be preferred over other regions.

We stick to the NMSSM as already defined. We employ the usual types of machinery to evaluate the strength of the phase transition prior to introducing CP violation into the Higgs sector (either through loops or explicitly). As usual, we employ the criterion of $\frac{v}{T_c} > 1$ as being required for a strong enough phase transition (as needed for the out-of-equilibrium condition for adequate baryogenesis). We have so far only looked at top and stop loop contributions. We are in the process of putting in contributions from the neutralino and chargino sectors, etc. The results are thus quite PRELIMINARY.

As we expected, electroweak baryogenesis is more easily accommodated in the NMSSM than in the MSSM. The reasons are:

- The SM-like Higgs can be lighter and still escape detection via $h_1 \rightarrow a_1 a_1$
dominance. (Recall that a light SM-like Higgs strengthens the phase transition.)

- If you require $m_{h_1}$ to be up near the LEP limit because $h_1 \rightarrow a_1 a_1$ decays are absent, you can succeed with a lighter $\tilde{t}$ than in the MSSM. This is because the $h_1$ mass gets an extra contribution at tree level:

$$m_{h_1}^2 \leq m_Z^2 \left( \cos^2 2\beta + \frac{2\lambda^2}{g_1^2 + g_2^2} \sin^2 2\beta \right). \quad (26)$$

which can give substantial $m_{h_1}^2$ even at tree-level for moderate $\tan\beta$. For example, for small $\kappa$ and $\tilde{X}_t = \sqrt{6}$ (maximal mixing), $m_{h_1}^2$ is maximum for $\tan\beta \sim 3$ where, depending upon $\kappa$, $\lambda$ can be big enough to give $m_{h_1} \sim 130$ GeV.

So far, we have kept $m_{\tilde{t}_{1,2}} \sim 1$ TeV and explored parameter space in the region defined by:

$$\lambda \in [0.1, 0.65], \quad \kappa \in [0.1, 0.65], \quad \tan\beta \in [1.6, 3.0], \quad \mu_{\text{eff}} = \lambda s \in [17.5, 350],$$

$$A_\lambda \in [-1000, 1000] \text{ GeV}, \quad A_\kappa \in [-1000, 1000] \text{ GeV}, \quad A_t = 1.5 \text{ TeV} \quad (27)$$

the latter being for roughly maximal mixing.
Below is a plot showing the $v/T_c > 1$ cases.

Figure 2: Scatter plot of $BR(h_1 \rightarrow a_1 a_1)$ vs. $m_{h_1}$ for points with $v/T_c > 1$. Baryogenesis favors $h_1 \rightarrow a_1 a_1$ scenarios!
We (Ellwanger, Hugonie, JFG [14]) have developed the NMSSM analogue of HDECAY. We provide two forms of the NMHDECAY program:

- **NMHDECAY_SLHA.f** — for study of one parameter point in the SLHA conventions for particle labeling etc. familiar to experimentalists;

- **NMHDECAY_SCAN.f** — designed for general phenomenological work including scanning over ranges of NMSSM parameters.

The programs, and associated data files, can be downloaded from the two web pages:

http://www.th.u-psud.fr/NMHDECAY/nmhdecay.html

http://higgs.ucdavis.edu/nmhdecay/nmhdecay.html

The web pages provide simplified descriptions of the programs and instructions on how to use them. The programs will be updated to include additional features and refinements in subsequent versions. We welcome comments with regard to improvements that users would find helpful.

**Input files are slhainp.dat and scaninp.dat, respectively.** They are simple!
# Total number of points scanned
1000

# Output format 0=short 1=long (not recommended for big scannings)
0

# lambda
0.5
0.5

# kappa
-0.15
-0.15

# tan(beta)
3.5
3.5

# mu
200.
200.

# A_lambda
780.
780.

# A_kappa
150.0
250.0

Table 1: Sample scaninp.dat file — 1st half for sample case #2.
# Remaining soft terms (no scan)

mQ3= 1.D3
mU3= 1.D3
mD3= 1.D3
mL3= 1.D3
mE3= 1.D3
AU3= 1.5D3
AD3= 1.5D3
AE3= 1.5D3
mQ= 1.D3
mU= 1.D3
mD= 1.D3
mL= 1.D3
mE= 1.D3
M1= 5.D2
M2= 1.D3
M3= 3.D3

**Table 2:** The 2nd half of `scaninp.dat` file for sample case #2.
NMHDECAY performs the following tasks:

1. It computes the masses and couplings of all physical states in the Higgs, chargino and neutralino sectors.\(^1\)

   Error messages are produced if a Higgs or squark mass squared is negative.

2. It computes the branching ratios into two particle final states (including charginos and neutralinos — decays to squarks and sleptons will be implemented in a later release) of all Higgs particles.

3. It checks whether the Higgs masses and couplings violate any bounds from negative Higgs searches at LEP, including many quite unconventional channels that are relevant for the NMSSM Higgs sector.

   It also checks the bound on the invisible $Z$ width (possibly violated for light neutralinos).

\(^1\) For the Higgses, we have included the leading two-loop effects, but neglected subleading two-loop contributions and subleading one-loop purely electroweak contributions. In MSSM limit, our Higgs masses agree to within a few GeV with HDECAY.
In addition, NMHDECAY checks the bounds on the lightest chargino and on neutralino pair production.

Corresponding warnings are produced in case any of these phenomenological constraints are violated.

4. It checks whether the running Yukawa couplings encounter a Landau singularity below the GUT scale.

A warning is produced if this happens.

5. Finally, NMHDECAY checks whether the physical minimum (with all vevs non-zero) of the scalar potential is deeper than the local unphysical minima with vanishing $\langle H_u \rangle$ or $\langle H_d \rangle$.

If this is not the case, a warning is produced.

- Below, I will discuss an example we employ to illustrate the use of these programs.

It represents a scenario in which Higgs to Higgs decays make LHC Higgs detection very difficult.

Other cases will be discussed.
Scenarios where LHC Higgs detection is hard

• First, recall that normal MSSM Higgs detection at the LHC relies on:

1) \( gg \rightarrow h/a \rightarrow \gamma\gamma \);
2) associated \( Wh/a \) or \( t\bar{t}h/a \) production with \( \gamma\gamma\ell^{\pm} \) in the final state;
3) associated \( t\bar{t}h/a \) production with \( h/a \rightarrow b\bar{b} \);
4) associated \( b\bar{b}h/a \) production with \( h/a \rightarrow \tau^+\tau^- \);
5) \( gg \rightarrow h \rightarrow ZZ^{(*)} \rightarrow 4 \) leptons;
6) \( gg \rightarrow h \rightarrow WW^{(*)} \rightarrow \ell^+\ell^-\nu\bar{\nu} \);
7) \( WW \rightarrow h \rightarrow \tau^+\tau^- \);
8) \( WW \rightarrow h \rightarrow WW^{(*)} \).

In supersymmetric models, it is also useful to include the mode

9) \( WW \rightarrow h \rightarrow invisible \).

which, however, plays little role in the following. We also assume that \( t \rightarrow H^{\pm}b \) will be observable for \( m_{H^{\pm}} < 155 \) GeV (could be raised).

• We estimate the expected statistical significances at the LHC in all Higgs
boson detection modes 1) – 9) by rescaling results for the SM Higgs boson and/or the the MSSM \( h, H \) and/or \( A \).

- **Scenarios for which LHC Higgs detection is “easy”, for \( L = 300 \text{fb}^{-1} \)**

If Higgs decays to Higgs and/or SUSY are forbidden, then [26]: We can always detect at least one of the NMSSM Higgs bosons.

This was not the case [19] until the \( t\bar{t}h \to t\bar{t}b\bar{b} \) mode [20, 21] (We have had the experimentalists extrapolate this beyond the usual SM mass range of interest.) and the \( WW \) fusion modes [22, 23, 24] were brought into play.

The point yielding the very lowest LHC statistical significance in an extensive scan over \( 10^9 \) points in parameter space had the following parameters:

\[
\lambda = 0.0535; \quad \kappa = 0.0259; \quad \tan \beta = 5.42; \quad \mu_{\text{eff}} = 145; \quad A_\lambda = -46 \text{ GeV}; \quad A_\kappa = -141 \text{ GeV}.
\]  

Properties of the Higgs bosons for this point are listed in table 3.

Other points with relatively weak LHC signals are similar in that:

1. the Higgs masses are closely spaced and below or at least not far above the \( WW/ZZ \) decay thresholds,
2. the CP-even Higgs bosons tend to share the $WW/ZZ$ coupling strength (indicated by $R_i$ in the table),

3. couplings to $b\bar{b}$ of all Higgs bosons (the $b_i$ or $b'_i$ in the table) are not very enhanced,

4. and couplings to $gg$ (the $g_i$ or $g'_i$ in the table) are suppressed relative to the SM Higgs comparison.

The most visible process for this point was the $WW \to h_3 \to \tau^+\tau^-$ channel, but many other (notably $t\bar{t}h \to t\bar{t}b\bar{b}$) channels are also visible.

Overall, we have a quite robust LHC no-lose theorem for NMSSM parameters such that LEP constraints are passed and Higgs-to-Higgs decays are not allowed once full LHC luminosity is achieved.

It would be a good idea for the LHC experimentalists to check that one really can see the Higgs signals at our estimated levels for this worst case no-Higgs-to-Higgs point.

It would be a good idea to see what can the Tevatron do with such a point!
Table 3: Properties of the neutral NMSSM Higgs bosons for the most difficult no-Higgs-to-Higgs-decays LHC point. In the table, $R_i = \frac{g_{hi}VV}{g_{h_SM}VV}$, $t_i = \frac{g_{hi}t\bar{t}}{g_{h_SM}t\bar{t}}$, $b_i = \frac{g_{hi}b\bar{b}}{g_{h_SM}b\bar{b}}$ and $g_i = \frac{g_{hi}gg}{g_{h_SM}gg}$ for $m_{h_SM} = m_{h_i}$. Similarly, $t'_i$ and $b'_i$ are the $i\gamma_5$ couplings of $a_i$ to $t\bar{t}$ and $b\bar{b}$ normalized relative to the scalar $t\bar{t}$ and $b\bar{b}$ SM Higgs couplings and $g'_i$ is the $a_i gg \epsilon \times \epsilon'$ coupling relative to the $\epsilon \cdot \epsilon'$ coupling of the SM Higgs.

<table>
<thead>
<tr>
<th>Higgs</th>
<th>$h_1$</th>
<th>$h_2$</th>
<th>$h_3$</th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (GeV)</td>
<td>94</td>
<td>113</td>
<td>147</td>
<td>133</td>
<td>173</td>
</tr>
<tr>
<td>$R_i$</td>
<td>$-0.440$</td>
<td>$-0.743$</td>
<td>$-0.505$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$t_i$ or $t'_i$</td>
<td>$-0.421$</td>
<td>$-0.647$</td>
<td>$-0.662$</td>
<td>$-0.183$</td>
<td>0.026</td>
</tr>
<tr>
<td>$b_i$ or $b'_i$</td>
<td>$-0.993$</td>
<td>$-3.55$</td>
<td>4.10</td>
<td>$-5.37$</td>
<td>0.757</td>
</tr>
<tr>
<td>$g_i$ or $g'_i$</td>
<td>0.470</td>
<td>0.554</td>
<td>0.435</td>
<td>0.139</td>
<td>0.021</td>
</tr>
<tr>
<td>$B(h_i or a_i \rightarrow b\bar{b})$</td>
<td>0.902</td>
<td>0.908</td>
<td>0.870</td>
<td>0.911</td>
<td>0.903</td>
</tr>
<tr>
<td>$B(h_i or a_i \rightarrow \tau^+\tau^-)$</td>
<td>0.081</td>
<td>0.085</td>
<td>0.086</td>
<td>0.088</td>
<td>0.095</td>
</tr>
<tr>
<td>Chan. 1) $S/\sqrt{B}$</td>
<td>0.00</td>
<td>0.20</td>
<td>0.26</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Chan. 2) $S/\sqrt{B}$</td>
<td>0.83</td>
<td>0.76</td>
<td>0.22</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>Chan. 3) $S/\sqrt{B}$</td>
<td>3.03</td>
<td>6.28</td>
<td>5.64</td>
<td>5.64</td>
<td>0.00</td>
</tr>
<tr>
<td>Chan. 4) $S/\sqrt{B}$</td>
<td>0.00</td>
<td>0.88</td>
<td>3.24</td>
<td>3.24</td>
<td>0.04</td>
</tr>
<tr>
<td>Chan. 5) $S/\sqrt{B}$</td>
<td>0.00</td>
<td>0.12</td>
<td>1.59</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Chan. 6) $S/\sqrt{B}$</td>
<td>0.00</td>
<td>0.00</td>
<td>1.26</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Chan. 7) $S/\sqrt{B}$</td>
<td>0.00</td>
<td>6.88</td>
<td>6.96</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Chan. 8) $S/\sqrt{B}$</td>
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<td>0.44</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>All-channel $S/\sqrt{B}$</td>
<td>3.14</td>
<td>9.39</td>
<td>9.75</td>
<td>6.50</td>
<td>0.04</td>
</tr>
</tbody>
</table>
The difficult scenarios: Higgs to Higgs (or SUSY) decays

The importance of Higgs to Higgs decays was first realized at Snowmass 1996 (JFG, Haber, Moroi [19]) and was later elaborated on in [25]. Detailed NMSSM scenarios were first studied in [26, 27].

We have shown that (for relatively heavy squarks and gauginos) all scenarios of this type for which discovery is not possible in modes 1) – 9) are such that there is a SM-like Higgs $h_H$ which decays to a pair of lighter Higgs, $h_L h_L$.

In general, the $h_L$ decays to $b\bar{b}$ and $\tau^+\tau^-$ (if $m_{h_L} > 2m_b$) or to $jj$ and $\tau^+\tau^-$ (if $2m_\tau < m_{h_L} < 2m_b$) or, as unfortunately still still possible, to $jj$ if $m_{h_L} < 2m_\tau$.

In the first two cases, a possibly viable LHC signal then comes [26, 27] from $WW \rightarrow h_H \rightarrow h_L h_L \rightarrow jj \tau^+\tau^-$ in the form of a bump in the $M_{jj\tau^+\tau^-}$ reconstructed mass distribution. It is not a wonderful signal, but it is a signal.

For most such cases, $h_L$ is actually the lightest CP-odd scalar $a_1$ and $h_H$ is the lightest or 2nd lightest CP-even scalar, $h_1$ or $h_2$. 
Experimentalists should work hard to see if our crude estimates that there would be an observable signal at the LHC will survive reality.

- As regards the cases where \( m_{a_1} < 2m_\tau \Rightarrow a_1 \rightarrow c\bar{c}, s\bar{s}, gg \), these can often evade LEP limits (but we are pushing the LEP people for improvements).

It will be very difficult extract a signal in these cases where neither \( b \) nor \( \tau \) tagging is relevant. The only hope would be jet counting, but QCD backgrounds are probably enormous.

Since the \( b\bar{b} \) coupling of these very light \( a_1 \)'s is not enhanced significantly (typically), there are no reliable exclusions coming from \( \Upsilon \) or \( B_{s,d} \) decays. We believe there is simply too much model dependence in the theory for such decays, although we would be happy to be persuaded otherwise.

- There are also cases in which \( h_H = h_2 \) and \( h_L = h_1 \), \( m_{h_1} > 2m_b \), but yet \( h_1 \rightarrow c\bar{c}, gg \) decays are completely dominant — parameters are chosen near a special region where the \( h_1 \) decouples from leptons and down-type quarks.

Again, it is very hard to imagine a technique for extracting a signal at the LHC.
The basic question: Can the Tevatron be sensitive to the Higgs-to-Higgs decay scenarios?

To assess this, we go through some benchmark points that will appear in a forthcoming paper (JFG, Ellwanger, Hugonie, Moretti).
Some Benchmark Points

Generically speaking, the NMSSM is capable of producing a large variety of Higgs phenomenologies. Here, we wish to delineate the kinds of scenarios that have not been excluded by LEP but might have a significant chance of allowing Higgs discovery at the Tevatron. Primary among these appear to be the scenarios in which there is a somewhat light SM-like Higgs boson that decays in unconventional fashion to two still lighter Higgs bosons, most commonly in the CP-conserving framework that we focus on here, a pair of the lightest CP-odd states. Thus, we focus on the $h \rightarrow aa$ situations that can arise in the NMSSM. These are also those favored by finetuning and electroweak baryogenesis.

The $h \rightarrow aa$ decays lead to the following final states:

(i) 4 $b$'s, 2$b$'s + 2$\tau$'s, or 4$\tau$'s, when $m_a > 2m_b$;
(ii) 4 $c$'s, 2$c$'s + 2$\tau$'s, or 4$\tau$'s, when $2m_\tau < m_a < 2m_b$;
(iii) 4$c$'s, 2$c$'s + 2 jets, or 4 jets, when $2m_c < m_a < 2m_\tau$;
(iv) or 4 jets, when $m_a < 2m_c$. Here, jet = $s$ or $g$. 
There are only a limited number of LEP limits that can be applied in these cases.

- In case (i) above, the best limits are certainly those recently extracted specifically for the $Zh \rightarrow Zaa \rightarrow Zb\bar{b}b\bar{b}$ final state. The 95% CL contours for $C^2 = \left[\frac{g^2_{Zh}}{g^2_{Zh}}\right]_{SM} \times Br(h \rightarrow aa) \times [BR(a \rightarrow b\bar{b})]^2$, for various values of $m_a$, are shown in Fig. 3.

We are not aware of any limits yet extracted for the $Zh \rightarrow Zaa \rightarrow Zb\bar{b}\tau^+\tau^-$ or $Zh \rightarrow Zaa \rightarrow Z\tau^+\tau^-\tau^+\tau^-$ final states. However, we understand that some are forthcoming.

- The $Z + 4b$ final state limit is only superior to the $Zh \rightarrow Z + \text{hadrons}$ final state independent limit, plotted in Fig. 3 and, in more detail, in Fig. 4 for $m_h \gtrsim 45$ GeV. The Fig. 4 limit is for $C^2 = \left[\frac{g^2_{Zh}}{g^2_{Zh}}\right]_{SM} \times Br(h \rightarrow aa) \times [BR(a \rightarrow \text{hadrons})]^2$, where the hadrons refers to a state with any number of jets.
Figure 3: Plot of the 95% CL limits on $C^2 = \left(\frac{g_{Zh}^2}{g_{Zh}^{SM}}\right) \times BR(h \rightarrow aa) \times [BR(a \rightarrow b\bar{b})]^2$. The different curves are for different $m_a$ values: solid lines are for 12, 13, 14, 15, 16 and 17 GeV in order of red, blue, green, yellow, magenta, black; dotted lines are for 18, 19, 20, 21, 22 and 23 in same color order; dotted lines are for 24, 25, 26, 27, 28 and 29 in same color order; dotdash lines are for 30, 31, 32, 33, 34, and 35 in same color order; long-dash lines are for 36, 37, 38, 39, 40 and 41 in same color order; and dot-dot-dash lines are for 42, 43, 44, 45, 46 and 47 in same color order. The thick solid red line is the line of Fig. 4 for this same mass region.
Figure 4: Plot of the 95% CL limit on $C^2 = \left[ \frac{g_{Zh}^2}{g_{Zh}^2}_{SM} \right] \times BR(h \rightarrow \text{hadrons})$, where $h$ is only assumed to decay to hadrons, not any specific number of jets.
• In cases (ii), (iii), and (iv), only the $Zh \rightarrow Z + \text{hadrons}$ limits remain potentially useful.

• However, some specific limits have been obtained for $m_h \in [40, 90]$ GeV with $m_a < 2m_b$.

So far these are only available as upper bound plots in the $m_h, m_a$ parameter space of the regions in which the $Zh \rightarrow Zaa \rightarrow Z + F$ signals are excluded at 95% CL for values of $C^2 = 0.2, 0.4, 0.5, 0.6, 0.8$ and 1, where $C^2 = [g^2_{Zh}/g^2_{Zh}^{SM}] \times BR(aa \rightarrow F)$, where $F$ stands for any of the relevant final states, such as $F = \tau^+\tau^-\tau^+\tau^-, c\bar{c}\tau^+\tau^-, c\bar{c}c\bar{c}$, ... .

• Typically, for $2m_\tau < m_a < 2m_b$ one finds $BR(a \rightarrow \tau^+\tau^-) \sim 0.8$ and $BR(a \rightarrow c\bar{c}) \sim 0.2$, implying dominance of the $\tau^+\tau^-\tau^+\tau^-, \tau^+\tau^-c\bar{c}$ and $c\bar{c}c\bar{c}$ final states, with other final states being negligible.

• For $2m_c < m_a < 2m_\tau$, the $a \rightarrow c\bar{c}$ decay is dominant and the $Z + c\bar{c}c\bar{c}$ final state will be of primary interest.

• The relevant plots appear in Figs. 5-10. For $m_h < 40$ GeV, one must turn to Fig. 4 as the only available limit.

A plot like Fig. 4 and ones like Figs. 5 and 6 appear on page 10 and page 12, respectively, of Boonekamp’s talk at CPNSH.
Figure 5: Contours of limits on $C^2 = \left[ \frac{g_{Zh}^2}{[g_{Zh}^2]_{SM}} \right] \times BR(h \rightarrow aa) \times [BR(a \rightarrow \tau^+\tau^-)]^2$ at $C^2 = 0.2, 0.4, 0.5, 0.6, 0.8$ and 1 (red, blue, green, yellow, magenta, and black, respectively). For example, if $C^2 > 0.2$, then the region below the $C^2 = 0.2$ contour is excluded at 95% CL.
Figure 6: Contours of limits on $C^2 = [g_{Zh}/g_{Zh}^{SM}] \times BR(h \to aa) \times [BR(a \to c\bar{c})]^2$ at $C^2 = 0.2, 0.4, 0.5, 0.6, 0.8$ and 1 (red, blue, green, yellow, magenta, and black, respectively). For example, if $C^2 > 0.2$, then the region below the $C^2 = 0.2$ contour is excluded at 95% CL.
Figure 7: Contours of limits on
\( C^2 = \frac{g_{Zh}^2}{[g_{Zh}^2]_{SM}} \times BR(h \rightarrow aa) \times 2BR(a \rightarrow c\bar{c}) \times BR(a \rightarrow s\bar{s} + gg) \)
at \( C^2 = 0.2, 0.4, 0.5, 0.6, 0.8 \) and 1 (red, blue, green, yellow, magenta, and black, respectively). For example, if \( C^2 > 0.2 \), then the region below the \( C^2 = 0.2 \) contour is excluded at 95\% CL.
Figure 8: Contours of limits on
\[ C^2 = \frac{g_{Zh}^2}{[g_{Zh}^2]_SM} \times BR(h \rightarrow aa) \times 2BR(a \rightarrow c\bar{c}) \times BR(a \rightarrow \tau^+\tau^-) \]
at \( C^2 = 0.2, 0.4, 0.5, 0.6, 0.8 \) and 1 (red, blue, green, yellow, magenta, and black, respectively). For example, if \( C^2 > 0.2 \), then the region below the \( C^2 = 0.2 \) contour is excluded at 95\% CL.
Figure 9: Contours of limits on 
\[ C^2 = \left[ \frac{g^2_{Zh}}{g^2_{Zh}}_{SM} \right] \times BR(h \rightarrow aa) \times 2 \times BR(a \rightarrow \tau^+\tau^-) \times BR(a \rightarrow s\bar{s} + gg) \]
at \[ C^2 = 0.2, 0.4, 0.5, 0.6, 0.8 \] and 1 (red, blue, green, yellow, magenta, and black, respectively). For example, if \[ C^2 > 0.2 \], then the region below the \[ C^2 = 0.2 \] contour is excluded at 95% CL.
Figure 10: Contours of limits on
\[ C^2 = \left[ \frac{g^{2}_{Zh}}{g^{2}_{Zh}} \right]_{SM} \times BR(h \rightarrow aa) \times [BR(a \rightarrow s\bar{s} + gg)]^2 \]
at \[ C^2 = 0.2, 0.4, 0.5, 0.6, 0.8 \] and 1 (red, blue, green, yellow, magenta, and black, respectively). For example, if \[ C^2 > 0.2 \], then the region below the \[ C^2 = 0.2 \] contour is excluded at 95% CL.
Figure 11: 95% CL upper limit on $C^2 = \left[ \frac{g_{Zh}^2}{g_{Zh}^{SM}} \right] \times BR(h \rightarrow jj)$ from LEP analyzes.
Tevatron Implications of LHC scenarios

All the parameter space points listed below escape the LEP limits for one reason or another, as sketched below.

In the tables,

- \( R_i = \frac{g_{h_iVV}}{g_{h_{SM}VV}} \),
- \( t_i = \frac{g_{h_i\bar{t}t}}{g_{h_{SM}\bar{t}t}} \),
- \( b_i = \frac{g_{h_i\bar{b}b}}{g_{h_{SM}\bar{b}b}} \),
- and \( g_i = \frac{g_{h_igg}}{g_{h_{SM}gg}} \)

for \( m_{h_{SM}} = m_{h_i} \).

Similarly,

- \( t'_i \) and \( b'_i \) are the \( i\gamma_5 \) couplings of \( a_i \) to \( \bar{t}t \) and \( \bar{b}b \) normalized relative to the scalar \( \bar{t}t \) and \( \bar{b}b \) SM Higgs couplings
- and \( g'_i \) is the \( a_igg \epsilon \times \epsilon' \) coupling relative to the \( \epsilon \cdot \epsilon' \) coupling of the SM Higgs.
<table>
<thead>
<tr>
<th>Bare Parameters</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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<td>λ</td>
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<td>0.220</td>
<td>0.400</td>
<td>0.340</td>
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<tr>
<td>κ</td>
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<td>−580.0</td>
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<tr>
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</table>

<table>
<thead>
<tr>
<th>CP-even Higgs Boson Masses and Couplings</th>
<th>m_{h_1} (GeV)</th>
<th>R_1</th>
<th>t_1</th>
<th>b_1</th>
<th>g_1</th>
<th>B(h_1 \rightarrow b \bar{b})</th>
<th>B(h_1 \rightarrow \tau^+ \tau^-)</th>
<th>B(h_1 \rightarrow a_1 a_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>m_{h_1} (GeV)</td>
<td>79.2</td>
<td>90.4</td>
<td>99.7</td>
<td>109.8</td>
<td>119.5</td>
<td></td>
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</tr>
<tr>
<td>R_1</td>
<td>0.893</td>
<td>0.986</td>
<td>0.966</td>
<td>−0.994</td>
<td>−1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t_1</td>
<td>0.893</td>
<td>0.986</td>
<td>0.966</td>
<td>−0.979</td>
<td>−1.000</td>
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</tr>
<tr>
<td>b_1</td>
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<td>0.901</td>
<td>−1.117</td>
<td>−1.010</td>
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</tr>
<tr>
<td>g_1</td>
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<td>0.985</td>
<td>0.969</td>
<td>0.974</td>
<td>0.999</td>
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<td></td>
</tr>
<tr>
<td>B(h_1 \rightarrow b \bar{b})</td>
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<td>0.076</td>
<td>0.024</td>
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<td>0.009</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B(h_1 \rightarrow \tau^+ \tau^-)</td>
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<td>0.007</td>
<td>0.002</td>
<td>0.000</td>
<td>0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B(h_1 \rightarrow a_1 a_1)</td>
<td>0.958</td>
<td>0.910</td>
<td>0.970</td>
<td>0.997</td>
<td>0.988</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| m_{h_2} (GeV)                            | 221.1 | 478.9 | 287.9 | 304.1 | 1430.6 |
| R_2                                      | −0.451 | −0.165 | −0.260 | 0.084 | −0.001 |
| t_2                                      | −0.450 | −0.164 | −0.259 | −0.016 | −0.129 |
| b_2                                      | −0.731 | −0.193 | −0.570 | 0.922 | 7.798 |
| g_2                                      | 0.446 | 0.164 | 0.256 | 0.022 | 0.121 |
| B(h_2 \rightarrow b \bar{b})             | 0.002 | 0.000 | 0.001 | 0.004 | 0.154 |
| B(h_2 \rightarrow \tau^+ \tau^-)         | 0.000 | 0.000 | 0.000 | 0.000 | 0.022 |
| B(h_2 \rightarrow W^+ W^- + ZZ)           | 0.417 | 0.569 | 0.338 | 0.051 | 0.000 |
| B(h_2 \rightarrow a_1 a_1)               | 0.256 | 0.041 | 0.438 | 0.590 | 0.003 |
| B(h_2 \rightarrow h_1 h_1)               | 0.324 | 0.247 | 0.214 | 0.013 | 0.001 |</p>
<table>
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<tr>
<th>Point Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tr>
<td><strong>CP-odd Higgs Boson Masses and Couplings</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_{a_1}$ (GeV)</td>
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<td>9.8</td>
<td>20.4</td>
<td>40.5</td>
<td>31.5</td>
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<td>$-0.009$</td>
<td>$-0.004$</td>
<td>$0.128$</td>
<td>$-0.009$</td>
</tr>
<tr>
<td>$b_1'$</td>
<td>$-1.067$</td>
<td>$-0.217$</td>
<td>$-0.846$</td>
<td>$1.074$</td>
<td>$-0.533$</td>
</tr>
<tr>
<td>$g_1'$</td>
<td>$0.757$</td>
<td>$0.153$</td>
<td>$0.481$</td>
<td>$0.242$</td>
<td>$0.190$</td>
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<tr>
<td>$B(a_1 \to bar{b})$</td>
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<td>$0.000$</td>
<td>$0.938$</td>
<td>$0.928$</td>
<td>$0.932$</td>
</tr>
<tr>
<td>$B(a_1 \to \tau^+\tau^-)$</td>
<td>$0.800$</td>
<td>$0.830$</td>
<td>$0.058$</td>
<td>$0.069$</td>
<td>$0.065$</td>
</tr>
<tr>
<td>$B(a_1 \to c\bar{c} + s\bar{s} + gg)$</td>
<td>$0.197$</td>
<td>$0.167$</td>
<td>$0.004$</td>
<td>$0.002$</td>
<td>$0.003$</td>
</tr>
</tbody>
</table>

**Table 4:** Properties of selected scenarios for which Higgs detection at a hadron collider must rely on the $h_{1,2} \to a_1a_1 \to jj\tau^+\tau^-$ or $h_2 \to h_1h_1 \to jj\tau^+\tau^-$ modes, where $jj = b\bar{b}$ in many cases. The quantities $R_i, t_i, b_i, g_i, t'_i, b'_i$ and $g'_i$ are discussed in the text. Important absolute branching ratios are also displayed.
<table>
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<td>$\lambda$</td>
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<td>$m_{h_1}$ (GeV)</td>
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<td>0.007</td>
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<td>0.000</td>
<td>0.001</td>
<td>0.071</td>
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<tr>
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<td>0.001</td>
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<td>0.000</td>
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<td><strong>CP-odd Higgs Boson Masses and Couplings</strong></td>
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<tr>
<td>$m_{a_1}$ (GeV)</td>
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<td>32.9</td>
<td>45.0</td>
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<td>0.007</td>
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<td>$g'_1$</td>
<td>0.018</td>
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<td>$B(a_1 \to bb)$</td>
<td>0.924</td>
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<td>0.927</td>
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<td>0.073</td>
<td>0.066</td>
<td>0.071</td>
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<td>1200.0</td>
<td>904.4</td>
<td>750.2</td>
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<td>$m_{h^\pm}$ (GeV)</td>
<td>1196.9</td>
<td>901.8</td>
<td>742.3</td>
<td>486.8</td>
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</table>

**Table 5:** Four additional scenarios for which Higgs discovery would need sensitivity to the $h \to a_1a_1 \to jj\tau^+\tau^-$ ($h = h_1$ or $h_2$) or $h_2 \to h_1h_1 \to jj\tau^+\tau^-$ modes, with $jj = b\bar{b}$ in many cases. Notations as in table 4.
Discussion of points 1 to 9: $a_1 \rightarrow b\bar{b}$ decays present

- **Point 1** provides a useful first example.

Since $m_{h_1^0} \sim 79$ GeV is rather low, and since the $gg$ coupling to $h_1$ is about 0.9 times SM strength, the Tevatron should have a reasonable production rate in the $gg$ fusion channel.

In this case, $h_1 \rightarrow a_1a_1$ decay completely dominates and since $m_{a_1} \sim 8.4$ GeV, $a_1a_1 \rightarrow \tau^+\tau^-$ is the relevant channel.

This point escapes the LEP limits by virtue of the unusual $h_1 \rightarrow a_1a_1 \rightarrow \tau^+\tau^-\tau^+\tau^-$ decay which avoids the constraints of both Fig. 3 and 4 by virtue of the fact that the final state is dominated by leptons or unexpectedly soft jets.

Meanwhile $C^2(\tau^+\tau^-\tau^+\tau^-) = \left[\frac{g_{Zh_1}^2}{g_{Zh_{SM}}^2}\right] \times BR(h_1 \rightarrow a_1a_1) \times [BR(a_1 \rightarrow \tau^+\tau^-)]^2 \sim 0.49$ just barely avoids being eliminated by the $C^2 = 0.4$ contour which excludes only $m_{a_1} \lesssim 8.3$ GeV at $m_{h_1^0} = 79$ GeV, and is not subject to elimination by the $C^2 = 0.5$ contour (which would exclude $C^2 \geq 0.5$ for $m_{a_1} \leq 9.2$ for $m_{h_1^0} = 79$ GeV).
Better analysis of this specific configuration by the LEP experimentalists might well exclude this point, but we could easily shift parameters slightly.

At the Tevatron, the $gg$ production rate for the $h_1$ will be $g_1^2 = 0.9^2 = 0.81$ of the SM rate.

Is it possible to extract the $4\tau$ signal for the $h_1$ in the $gg$ fusion mode?

Of course, we should also note that the $WW$ fusion coupling squared ($R_1^2 \sim 0.893^2 \sim 0.8$) is close to SM-like and one can tag forward and backward jets.

$Wh$ associated production could also be considered. It provides a trigger and some handle on backgrounds as compared to $gg$ fusion.

However, the lower masses of the Higgs boson being considered here and at subsequent parameter space points only gives a factor of $\sim 2 \div 4$ increase relative to $m_h \gtrsim 115$ GeV.

The main question is whether backgrounds are lower for this kind of scenario as compared to the LHC situation with much higher Higgs cross section, but also much higher backgrounds.
Cross Section Reality Check

Figure 12: Various cross sections at the Tevatron for a SM Higgs boson. Note the small size of $WW$ fusion at low $m_h$. Better is $Wh$ associated production,
Point 2 provides a similar example at somewhat higher $m_{h_1^0} = 90.4$ GeV and similar $m_{a_1} \sim 9.8$ GeV.

In particular, even though $C^2(\tau^+\tau^-\tau^+\tau^-) \sim 0.61$ in this case, the $m_{a_1}$ value is sufficiently large that even an extension of the 0.6 LEP contour of Fig. 5 to $m_{h_1^0} \sim 90.4$ GeV would be unlikely to exclude this point.

Meanwhile, the $gg$ fusion and $WW$ fusion production rates are $0.97 \times$ the SM rate.

Point 3 has $m_{h_1^0} = 99.7$ GeV and $m_{a_1} = 20.4$ GeV.

The dominant final state is $h_1 \to a_1a_1 \to b\bar{b}b\bar{b}$.

From the table, we extract $C^2(b\bar{b}b\bar{b}) = \left[\frac{g_{Zh_1}^2}{g_{Zh_{SM}}^2}\right] \times BR(h_1 \to a_1a_1) \times [BR(a_1 \to b\bar{b})]^2 \sim 0.8$.

This barely sneaks below the relevant contour of Fig. 3.

This is a canonical example of a point for which one could look for the above final state in $gg$ or $WW$ fusion, or $Wh$ associated production, using $b$ tagging.
• The $m_{h_1^0}$ values for points 4 and 5 are somewhat higher (and certainly beyond all LEP limits) and production rates for the $h_1$ (even though essentially of SM strength) would be somewhat lower.

Still, these kinds of points with dominant $a_1a_1 \rightarrow b\bar{b}b\bar{b}$ final state might be interesting because of the possibility of using $b$-tagging.

• Moving on, the next really interesting point in the tables is point 7.

Here we have a rather low $m_{h_1^0} \sim 69.5$ GeV matched with a fairly high $m_{a_1} \sim 32.9$ GeV.

The dominant decay mode is $h_1 \rightarrow a_1a_1 \rightarrow b\bar{b}b\bar{b}$ with $C^2(b\bar{b}b\bar{b}) \sim 0.22$.

This again falls slightly below the relevant contour of Fig. 3 and well below the limit of Fig. 4.

Despite the 0.32 and 0.38 suppression factors for $WW$ and $gg$ fusion respectively (relative to the SM rates), the low mass implies substantial SM rates to begin with and $b$ tagging could yield a signal.
• Now let us examine point 9 with $m_{h_1^0} \sim 39.8$ GeV and $m_{a_1} \gg m_{h_1^0}$.

This $h_1$ is mainly singlet and has very tiny $ZZ$ coupling. However, its $b\bar{b}$ coupling is 0.352 of SM strength and so production and detection in the $gg \rightarrow b\bar{b}h_1 \rightarrow b\bar{b}b\bar{b}$ mode might be worth examining.

The $h_2$ is the (very) SM-like guy in this case, and decays primarily via $h_2 \rightarrow h_1h_1 \rightarrow b\bar{b}b\bar{b}$.

Since $m_{h_2} \sim 125$ GeV, there is no problem with LEP limits, but the Tevatron cross sections would not be large.
### Table 6: Properties of points for which the $WW \rightarrow h_H \rightarrow h_L h_L \rightarrow jj\tau^+\tau^-$ modes don’t work.
Table 7: Properties (continued) of selected scenarios for which LHC Higgs detection would not even be possible in the $WW \rightarrow h_H \rightarrow h_L h_L \rightarrow jj\tau^+\tau^-$ modes.

Discussion of points 10 to 14: no $a_1 \rightarrow b\bar{b}$ decays

- **Point 10** provides something quite unique.

  The $h_1$ with $m_{h_1^0} \sim 94.1$ GeV is quite SM-like, but it decays via $h_1 \rightarrow a_1a_1 \rightarrow \gamma\gamma\gamma\gamma$ as the dominant final state, with an effective $C^2 \sim 0.82$.

  The possibility of reconstructing each $a_1$ in the $\gamma\gamma$ channel at $m_{a_1} \sim 40$ GeV should give a really clean signature.

  Searches for this kind of unusual signal should not be ignored!
• The next point that presents something quite new and might be accessible at the Tevatron is **point 11**, with \( m_{h_1^0} = 57.3 \text{ GeV} \) and \( m_{h_2} = 124.7 \text{ GeV} \).

Because of an accidental suppression of the \( h_1b\bar{b} \) coupling, the dominant \( h_1 \) decay is to \( c\bar{c} + s\bar{s} + gg \) (mainly \( c\bar{c} \)).

This escapes LEP limits in the \( Zjj \) final state because the effective \( C^2(jj) = [g_{Zh_1}^2/g_{Zh_{SM}}^2] \times BR(h_1 \rightarrow jj) \sim 0.07 \) falls below (but, as usual, not by much) the 95% CL limit for this quantity at \( m_{h_1^0} \sim 57.3 \text{ GeV} \) from the \( Zh \rightarrow Zjj \) LEP searches, as plotted in Fig. 11.

(This dedicated search provides the strongest limits for this kind of case.)

This same suppression factor would apply to \( WW \rightarrow h_1 \rightarrow jj \) searches and triggering on the \( c \)'s in the final state would be inefficient.

Thus, to us, it looks hard.

Meanwhile, there is also the \( h_2 \rightarrow h_1h_1 \rightarrow jjjj \) possibility, but the high \( m_{h_2} \sim 124.7 \text{ GeV} \) coupled with no \( b \)'s probably makes this one hard too.

• **Points 12, 13 and 14** are all characterized by a SM-like \( h_1 \) with masses
of \( m_{h_1^0} \sim 95.4 \) GeV, 88 GeV and 113.8 GeV, respectively, decaying via \( h_1 \rightarrow a_1a_1 \) with each \( a_1 \) decaying to \( c\bar{c} + s\bar{s} + gg \).

Because the \( m_{h_1^0} \) values are beyond the specialized LEP limits, these are all allowed scenarios. One would need to look for \( WW \) or \( gg \) fusion to \( h_1 \) followed by \( h_1 \) decays as specified above.

Again, this is likely to be difficult because of the lack of \( b \) or \( \tau \) tagging.
Conclusions

We believe that these scenarios represent test cases of real importance at the Tevatron.

- The NMSSM is an attractive model, and the $h \rightarrow aa$ decay modes have significantly nice features with regard to finetuning and electroweak baryogenesis.

- The sometimes modest Higgs masses involved and the typically fairly SM-like couplings of the primary Higgs mean that the Tevatron production rates are significant. Further, the smaller backgrounds at the Tevatron might make it possible to see these Higgs-to-Higgs pair signals before the LHC turns on if efficient background reduction techniques can be found.

- There are lots of NMSSM Higgs scenarios, and it will probably take years to place limits or detect a weak signal, but hopefully not more years than we have left before LHC turn-on.