Dark Matter from Light Neutralinos and CP-odd Higgs Bosons in the NMSSM and the ILC

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Motivations for the NMSSM

SM problems:

- No explanation for the huge hierarchy of $m_{h_{\rm SM}} \ll M_{\rm P}$, as required for perturbativity of $W_L W_L \to W_L W_L$, ... If the scale of new physics is Λ, then

$$\delta m_h^2\big|_{top} \sim -\frac{N_c|\lambda_t|^2}{8\pi^2}\Lambda^2 \tag{1}$$

  and in the absence of new physics communicating to the Higgs sector before $M_{\rm P}$, $\Lambda \sim M_{\rm P}$ leads to huge fine-tuning.

- No explanation for negative m^2 in Higgs potential needed for EWSB.

- Gauge coupling unification does not take place.

MSSM successes:

- Gauge coupling unification works very well (though not perfectly).

- Evolution from GUT scale to m_Z can naturally produce $m_{H_u}^2 < 0$ and, hence, EWSB.

- Dark matter.

- Low-Scale ($\lesssim$ TeV) Supersymmetry could in principle solve the naturalness / hierarchy problem.

$\boxed{\text{BUT}}$ there are significant problems for the MSSM

$\boxed{\text{MSSM problems:}}$

- The CP-conserving MSSM is being pushed into parameter regions characterized by substantial fine tuning and a "little" hierarchy problem (i.e. large stop masses) in order to have a heavy enough Higgs boson for consistency with LEP limits.

- A strong phase transition for baryogenesis is hard to arrange when the Higgs is heavy enough to evade LEP. It requires that one stop is very light and the other stop very heavy, the latter leading to a very high level of fine tuning.

- **No really attractive explanation for the μ parameter has emerged.**

- **One can marginally escape all but the last of these problems if significant Higgs sector CP violation is introduced through SUSY loops.**

What are the alternatives to the MSSM?:

- **"Split Supersymmetry" (Arkani-Hamed etal).**

- **"Little Higgs" models (Arkani-Hamed etal).**

- **"Large Extra Dimensions" (Dimopoulos, ...)**

- **"Higgsless" models (Terning etal)**

Compared to the NMSSM, all are complicated, incomplete, ... Why not something simple?

- **The NMSSM**

  – **The CP-conserving NMSSM can solve all these problems.**

Indeed, the NMSSM can have a very low-level of fine-tuning, small little hierarchy, good electroweak baryogenesis,...

Is it not time to adopt the NMSSM as the baseline supersymmetric model?

– The NMSSM is the simplest of a class of models that emerge from string theory with extra singlet super fields. For example, there are models with extra superfields that are singlets under the SM groups, but charged under a new $U(1)'$. One such model was studied by McElrath, Han, Langacker, ...

– The focus here is on Dark Matter.

The NMSSM phenomenology for dark matter is considerably richer than that of the MSSM in many important ways.

The NMSSM is a big step up in the complexity of the possibilities and analyzes that will be required to confirm that the LSP of the model is the (or at least a) source of dark matter.

If string theory is any guide, nature is likely to be overly generous when it comes to the number of neutralinos and Higgs bosons.

The NMSSM: Brief Review and Relevant Experimental Constraints

- The Next to Minimal Supersymmetric Standard Model (NMSSM [1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13]) provides a very elegant solution to the μ problem of the MSSM via the introduction of a singlet superfield $\widehat{S}$.

  For the simplest possible scale invariant form of the superpotential, the scalar component of $\widehat{S}$ acquires naturally a vacuum expectation value of the order of the SUSY breaking scale, giving rise to a value of μ of order the electroweak scale.

- The NMSSM is actually the simplest supersymmetric extension of the standard model in which the electroweak scale originates from the SUSY breaking scale only.

- The NMSSM preserves all the successes of the MSSM (gauge coupling unification, RGE EWSB, dark matter, . . .).

Hence, the phenomenology of the NMSSM deserves to be studied at least as fully and precisely as that of the MSSM.

Its particle content differs from the MSSM by the addition of one CP-even and one CP-odd state in the neutral Higgs sector (assuming CP conservation), and one additional neutralino. Thus, the physics of the Higgs bosons – masses, couplings and branching ratios [1, 7, 8, 9, 10, 11, 12, 13] and of the neutralinos (too many references to list) can differ significantly from the MSSM.

I will be following the conventions of Ellwanger, Hugonie, JFG [14]. The NMSSM parameters are as follows.

a) Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

$$\lambda\ \widehat{S}\widehat{H}_u\widehat{H}_d + \frac{\kappa}{3}\ \widehat{S}^3 \tag{2}$$

depending on two dimensionless couplings λ, κ beyond the MSSM. (Hatted capital letters denote superfields, and unhatted capital letters will denote their scalar components).

The μ term of the MSSM arises from

$$\lambda\widehat{S}\widehat{H}_u\widehat{H}_d \to \lambda\langle S\rangle\widehat{H}_u\widehat{H}_d \equiv \mu_{\rm eff}\widehat{H}_u\widehat{H}_d\,. \tag{3}$$

b) The associated trilinear soft terms are

$$\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3 \,. \tag{4}$$

In the MSSM language,

$$\lambda A_\lambda S H_u H_d \to \lambda A_\lambda \langle S \rangle H_u H_d \equiv B_\mu \mu_{\rm eff} H_u H_d \,. \tag{5}$$

c) In addition to λ, κ, A_λ and A_κ, other crucial input parameters are

$$\tan\beta = \langle H_u \rangle / \langle H_d \rangle \;,\; \mu_{\rm eff} = \lambda \langle S \rangle \equiv \lambda x \;. \tag{6}$$

These, along with M_Z, can be viewed as determining the three SUSY breaking masses squared for H_u, H_d and S through the three minimization equations of the scalar potential as defined in the soft-SUSY-breaking potential components

$$m_{H_u}^2 H_u^2 + m_{H_d}^2 H_d^2 + m_S^2 S^2 \,. \tag{7}$$

Thus, to specify the Higgs sector at tree level the NMSSM needs six parameters

$$\lambda\ ,\ \kappa\ ,\ A_\lambda\ ,\ A_\kappa,\ \tan\beta\ ,\ \mu_{\rm eff}\ . \tag{8}$$

We will choose sign conventions for the fields such that λ and $\tan\beta$ are positive, while κ, A_λ, A_κ and $\mu_{\rm eff}$ should be allowed to have either sign.

In addition, values for the gaugino masses and of the soft terms related to the squarks and sleptons must be input. These determine the radiative corrections to the Higgs sector and the properties of the sfermions and gauginos.

We will be focusing on the lightest CP-odd Higgs boson, the a_1, and on the lightest neutralino, the $\widetilde\chi_1^0$, which will be stable (assuming conventional R-parity conservation).

An important issue will be the composition of these states.

- The eigenvector of the lightest neutralino, $\widetilde\chi_1^0$, in terms of gauge eigenstates is:

$$\widetilde\chi_1^0 = \epsilon_u \tilde H_u^0 + \epsilon_d \tilde H_d^0 + \epsilon_W \tilde W^0 + \epsilon_B \tilde B + \epsilon_s \tilde S, \tag{9}$$

where ϵ_u, ϵ_d are the up-type and down-type higgsino components, ϵ_W, ϵ_B are the wino and bino components and ϵ_s is the singlet component of the lightest neutralino.

- We write the lightest CP-odd Higgs as:

$$a_1 = \cos\theta_A A_{\rm MSSM} + \sin\theta_A A_s, \tag{10}$$

where A_s is the CP-odd piece of the singlet and $A_{\rm MSSM} \equiv A$ is the state that would be the MSSM pseudoscalar Higgs if the singlet were not present. θ_A is the mixing angle between these two states.

There is also a third imaginary linear combination of H_u^0, H_d^0 and S that we have removed by a rotation in β. This field becomes the longitudinal component of the Z after electroweak symmetry is broken.

- In the basis $\widetilde\chi^0 = (-i\widetilde\lambda_1, -i\widetilde\lambda_2, \psi_u^0, \psi_d^0, \psi_s)$, the tree-level neutralino mass matrix takes the form

$$\mathcal{M}_{\widetilde\chi^0} = \begin{pmatrix} M_1 & 0 & \frac{g_1 v_u}{\sqrt2} & -\frac{g_1 v_d}{\sqrt2} & 0 \\ 0 & M_2 & -\frac{g_2 v_u}{\sqrt2} & \frac{g_2 v_d}{\sqrt2} & 0 \\ \frac{g_1 v_u}{\sqrt2} & -\frac{g_2 v_u}{\sqrt2} & 0 & -\mu & -\lambda v_d \\ -\frac{g_1 v_d}{\sqrt2} & \frac{g_2 v_d}{\sqrt2} & -\mu & 0 & -\lambda v_u \\ 0 & 0 & -\lambda v_d & -\lambda v_u & 2\kappa x \end{pmatrix}. \tag{11}$$

In the above, the upper 4×4 matrix corresponds to $\mathcal{M}^{\rm MSSM}_{\widetilde{\chi}^0}$.

From the lower 3×3 matrix, we find that if $\lambda v_{u,d} = (\mu/x) v_{u,d}$ are small compared to $|\mu|$ and/or $2|\kappa x|$ then the singlino decouples from the MSSM and has mass (as found from $[M_{\widetilde{\chi}^0}]^2_{55}$)

$$m_{\rm singlino} \simeq \sqrt{\lambda^2 v^2 + 4\kappa^2 x^2} = \sqrt{\mu^2 v^2/x^2 + 4\kappa^2 x^2}\,. \qquad (12)$$

If $2|\kappa x|$ and λv are both $< M_1, M_2, |\mu|$, then the $\widetilde{\chi}^0_1$ will tend to be singlino-like. If λv is small and $2|\kappa x|$ and M_1 are similar in size and $< M_2, |mu|$, then the $\widetilde{\chi}^0_1$ will be a bino – singlino mixture.

- After removing the CP-odd degree of freedom that is absorbed in giving the Z its mass, the remaining CP-odd states have the squared-mass matrix

$$\mathcal{M}^2_A = \begin{pmatrix} \frac{2\lambda x}{\sin 2\beta}(A_\lambda + \kappa x) & \lambda v (A_\lambda - 2\kappa x) \\ \lambda v (A_\lambda - 2\kappa x) & (2\lambda\kappa + \frac{\lambda A_\lambda}{2x}) v^2 \sin 2\beta - 3x\kappa A_\kappa \end{pmatrix} \qquad (13)$$

where $v^2 = v_u^2 + v_d^2$.

The a_1 becomes very singlet-like if A_κ or κ is small. (These limits are actually associated with additional symmetries of the model.)

If we take $A_\kappa \to 0$, one finds the results

$$\tan\theta_A \sim \frac{x}{v\sin 2\beta}\left[\frac{1+A_\lambda/(\kappa x)}{1-A_\lambda/(2\kappa x)}\right], \tag{14}$$

$$m_{a_1}^2 \sim \frac{\frac{9}{2}\lambda A_\lambda v^2 x \sin 2\beta}{x^2+v^2\sin^2 2\beta + A_\lambda x/\kappa}, \tag{15}$$

valid whenever the numerator of the preceding equation is much smaller than the square of the denominator, as for example if $\tan\beta \to \infty$ or $|x|$ is large (as required for finite $|\mu| = \lambda|x|$ when λ is small). Note that $\cos\theta_A$ will be quite small typically and the a_1 relatively singlet like in this limit. Note: $\cos\theta_A$ small is bad for dark mark annihilation.

If $\kappa \to 0$ one finds

$$\tan\theta_A \sim -\frac{2x}{v\sin 2\beta}, \quad \cos^2\theta_A \sim \frac{v^2\sin^2 2\beta}{v^2\sin^2 2\beta + 4x^2} \tag{16}$$

and

$$m_{a_1}^2 \sim \frac{6\kappa x^2(3\lambda v^2 \sin 2\beta - 2A_\kappa x)}{4x^2+v^2\sin^2 2\beta}. \tag{17}$$

For $|x| > v$, $|\cos\theta_A|$ is small for moderate $\tan\beta$ and approaches 0 at large $\tan\beta$.

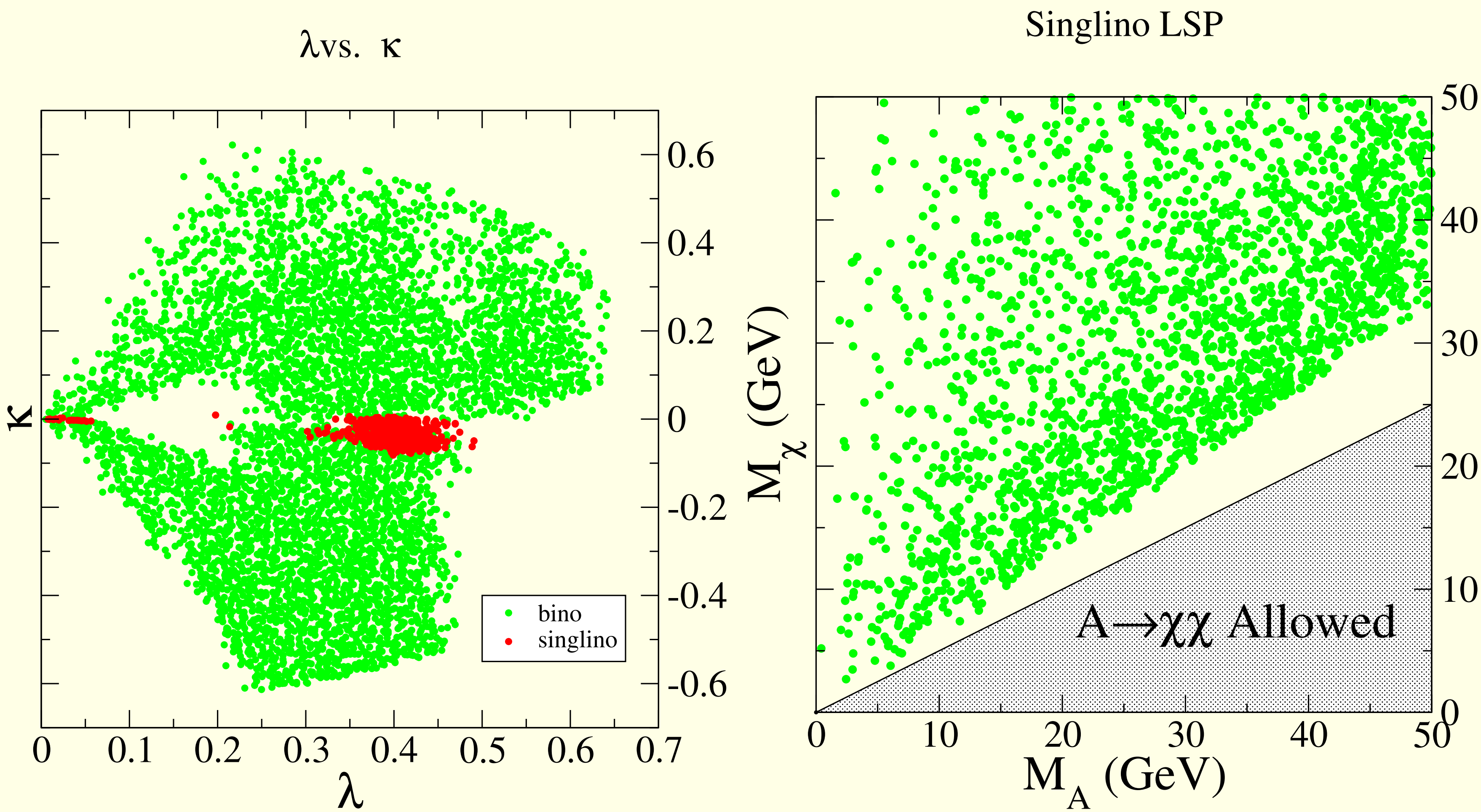

Figure 1: On the left, we show regions of λ–κ parameter space for which the $\widetilde\chi_1^0$ is singlino-like (defined by $\epsilon_s^2 > 0.5$) and bino-like (defined by $\epsilon_s^2 \leq 0.5$). On the right, we plot m_A vs. $m_{\widetilde\chi_1^0}$ for singlino-like neutralinos with $\epsilon_s^2 > 0.9$. Each point shown is consistent with all LEP constraints as contained in NMHDECAY. The left plot shows that unless κ is very small, the $\widetilde\chi_1^0$ is bino-like. The right plot shows that $m_{a_1} \simeq 2m_{\widetilde\chi_1^0}$ is impossible for a singlino $\widetilde\chi_1^0 \Rightarrow$ too weak $\widetilde\chi_1^0\widetilde\chi_1^0 \to a_1 \to q\bar q$ given weak $q\bar q a_1$ coupling.

To explain dark matter we do not want the a_1 to be highly singlet-like. Otherwise, the $\widetilde{\chi}_1^0\widetilde{\chi}_1^0 \to a_1 \to q\overline{q}$ (dominant) annihilation mode will be too weak to avoid overclosing the universe.

Since the a_1 is singlet-like whenever κ is so small that the $\widetilde{\chi}_1^0$ is highly singlet-like, this means that to explain dark matter a $\widetilde{\chi}_1^0$ with substantial bino component is preferred.

The only escape would have been if $m_{a_1} \simeq 2m_{\widetilde{\chi}_1^0}$ were possible, but the figure shows and one can see analytically that $m_{a_1} < 2m_{\widetilde{\chi}_1^0}$ by a significant amount when the $\widetilde{\chi}_1^0$ is mostly singlino.

- We (Ellwanger, Hugonie, JFG [14]) have developed the NMSSM analogue of HDECAY: NMHDECAY. NMHDECAY performs the following tasks:

  1. It checks for theoretical consistency of any given parameter set.
  2. It determines the properties of all the Higgs bosons, sfermions and gauginos.
  3. It checks that the parameter set is consistent with basic experimental limits on Higgs bosons and on neutralinos and charginos.

  All scenarios presented are processed through NMHDECAY.

- There are additional constraints on scenarios with a light $\widetilde{\chi}_1^0$ and a_1 coming from

  1. δa_μ – a positive value of order 7×10^{-10} ($\tau^+\tau^-$ data) or 25×10^{-10} (direct e^+e^- data) is desirable..;
  2. rare K decays;
  3. rare B decays;
  4. Υ and J/Ψ decays.
     Of these, the latter are the most constraining, especially $\Upsilon \to \gamma\widetilde{\chi}_1^0\widetilde{\chi}_1^0$.
     CLEO limits are $BR(\Upsilon \to \gamma\widetilde{\chi}_1^0\widetilde{\chi}_1^0) \simeq 3 \times 10^{-5}$ for $m_{\widetilde{\chi}_1^0} < 1.5$ GeV.
     CLEO used only $48\mathrm{pb}^{-1}$ of data (about 1M $\Upsilon(1S)$). They have 20 times this recorded. BaBar and Belle have produced about 5M $\Upsilon(1S)$ each with ISR.
     This measurement can be drastically improved with existing data!

  Nonetheless, it is not difficult to find models with a light $\widetilde{\chi}_1^0$ and correct Ωh^2 relic density that avoid these limits, as shown in the following figure.

  Observe that even if the limits on $BR(\Upsilon \to \gamma\widetilde{\chi}_1^0\widetilde{\chi}_1^0)$ are improved by a factor of 10, there are still solutions with good Ωh^2, even for this limited scan.

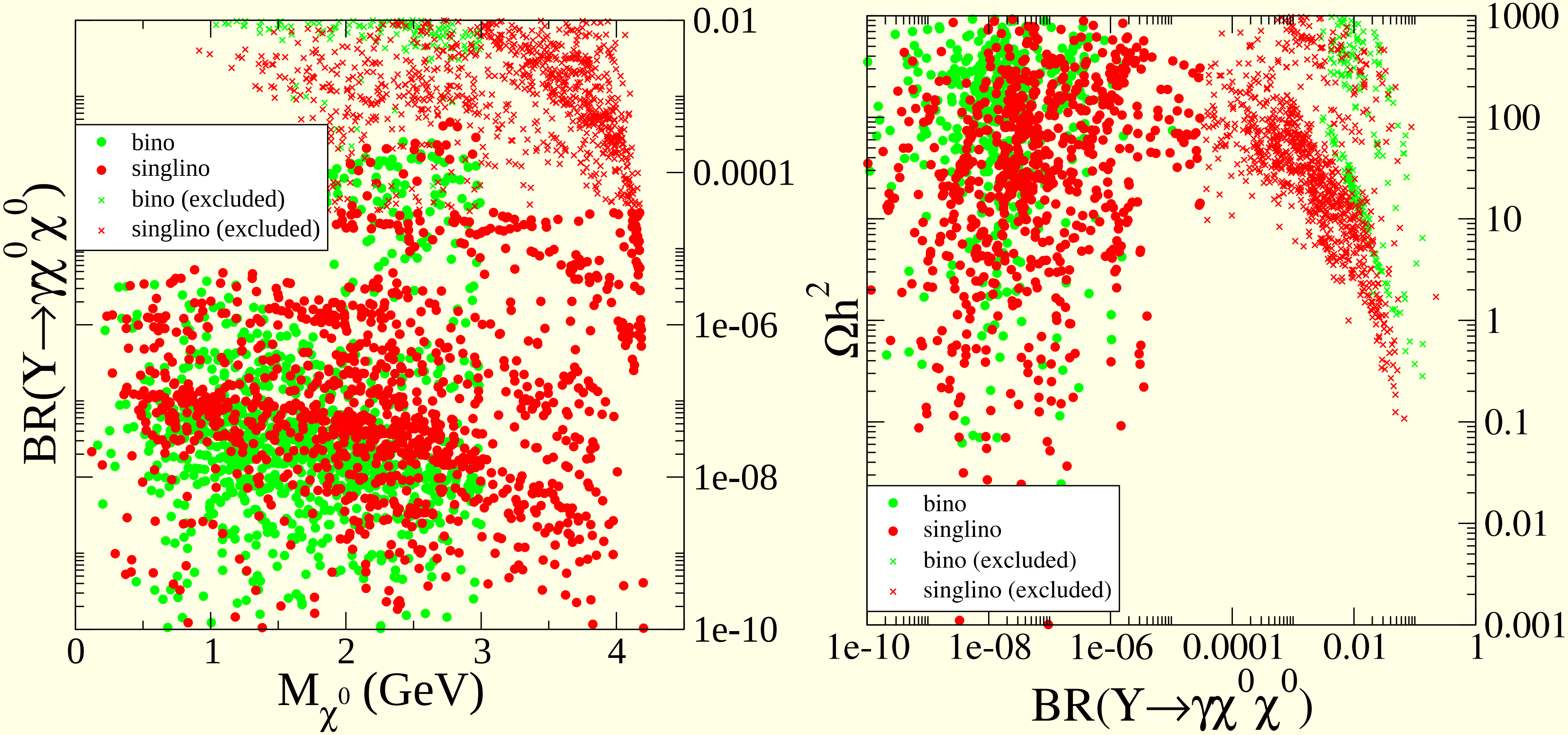

Figure 2: Constraints from Υ decays and the correlation with Ωh^2.

NB: the points plotted in the figures are the same. There are still some scan artifacts in these plots.

Dark Matter

- The squared amplitude for the processes, $\widetilde{\chi}_1^0\widetilde{\chi}_1^0 \to A \to f\bar{f}$, averaged over the final state angle is given by:

$$\omega_{f\bar{f}}^A = \frac{C_{f\bar{f}A}^2 C_{\widetilde{\chi}_1^0\widetilde{\chi}_1^0 A}^2}{(s-m_A^2)^2 + m_A^2\Gamma_A^2} \frac{s^2}{16\pi}\sqrt{1-\frac{4m_f^2}{s}}, \tag{18}$$

$$\tag{19}$$

where the label A denotes a CP-odd Higgs.

Here, $C_{f\bar{f}A}^2$ and $C_{\widetilde{\chi}_1^0\widetilde{\chi}_1^0 A}^2$ are the fermion-fermion-Higgs coupling and the neutralino-neutralino-Higgs coupling, and m_A and Γ_A are the A mass and width. In the NMSSM case, we will be considering only $A = a_1$. The relevant couplings are then given by:

$$\begin{aligned} C_{\widetilde{\chi}_1^0\widetilde{\chi}_1^0 A} &= \cos\theta_A\Big[(g_2\epsilon_W - g_1\epsilon_B)(\epsilon_d\cos\beta - \epsilon_u\sin\beta) \\ &+ \sqrt{2}\lambda\epsilon_s(\epsilon_u\sin\beta + \epsilon_d\cos\beta)\Big] + \sin\theta_A\sqrt{2}\left[\lambda\epsilon_u\epsilon_d - \kappa\epsilon_s^2\right] \end{aligned}$$

$$C_{f\bar{f}A} \;=\; \begin{cases} \frac{m_f}{\sqrt{2}v}\cos\theta_A\tan\beta, & f=d,s,b,l \\ \frac{m_f}{\sqrt{2}v}\cos\theta_A\cot\beta, & f=u,c \end{cases}$$

Note all the $\cos\theta_A$'s.

- **We expect $\Gamma_A \approx$ eV-MeV if $A = a_1$ is mostly singlet and $\Gamma_A \approx$ 1-10 MeV otherwise.**

  The width is strongly affected by the many kinematic thresholds due to hadronic resonances with masses less than 10 GeV.

  Therefore, any computation of the relic density is inherently limited by our ability to compute hadronic form factors and sum over hadronic decays which may be on-shell and may enhance the annihilation.

  We require only that the relic density is $\mathcal{O}(0.1)$. There is sufficient parameter space to make the relic density precisely the value measured by WMAP when all hadronic corrections are taken into account.

  In our computations, we neglect the widths since they are very small compared to the masses considered. Of course, one could always tune $2m_{\widetilde{\chi}_1^0}$ to some hadronic resonance or threshold in order to drastically

increase the cross section and thus reduce the thermal relic density, but we do not employ such precision tuning.

- The squared amplitude of Eq. (18) can be used to obtain the thermally averaged annihilation cross section. Using the notation $s_0 = 4m^2_{\widetilde{\chi}^0_1}$, we have

$$\begin{aligned} \langle \sigma v \rangle &= \frac{\omega(s_0)}{m^2_{\widetilde{\chi}^0_1}} - \frac{3}{m_{\widetilde{\chi}^0_1}}\left[\frac{\omega(s_0)}{m^2_{\widetilde{\chi}^0_1}} - 2\omega'(s_0)\right]T + \mathcal{O}(T^2) \\ &= \frac{1}{m^2_{\widetilde{\chi}^0_1}}\left[1 - \frac{3T}{m_{\widetilde{\chi}^0_1}}\right]\omega(s)\Big|_{s\to 4m^2_{\widetilde{\chi}^0_1}+6m_{\widetilde{\chi}^0_1}T} + \mathcal{O}(T^2), \end{aligned} \tag{20}$$

where T is the temperature. Keeping terms to zeroth and first order in T should be sufficient for the relic abundance calculation. Writing this as an expansion in $x = T/m_{\widetilde{\chi}_1^0}$, $\langle \sigma v \rangle = a + bx + \mathcal{O}(x^2)$, we arrive at:

$$
\begin{aligned}
a_{\chi\chi\to A\to f\bar{f}} &= \frac{g_2^4 c_f m_f^2 \cos^4\theta_A \tan^2\beta}{8\pi m_W^2} \frac{m_{\widetilde{\chi}_1^0}^2 \sqrt{1 - m_f^2/m_{\widetilde{\chi}_1^0}^2}}{(4m_{\widetilde{\chi}_1^0}^2 - m_A^2)^2 + m_A^2\Gamma_A^2} \qquad (21) \\
\times\Big[-\epsilon_u(\epsilon_W &- \epsilon_B \tan\theta_W)\sin\beta + \epsilon_d(\epsilon_W - \epsilon_B\tan\theta_W)\cos\beta \\
&+ \sqrt{2}\frac{\lambda}{g_2}\epsilon_s(\epsilon_u\sin\beta + \epsilon_d\cos\beta) + \frac{\tan\theta_A}{g_2}\sqrt{2}(\lambda\epsilon_u\epsilon_d - \kappa\epsilon_s^2)\Big]^2, \\
b_{\chi\chi\to A\to f\bar{f}} &\simeq 0, \qquad (22)
\end{aligned}
$$

where c_f is a color factor, equal to 3 for quarks and 1 otherwise. For this result, we have assumed that the final state fermions are down-type. If they are instead up-type fermions, the couplings used must be modified as described above.

- The annihilation cross section can now be used to calculate the thermal relic abundance present today.

$$\Omega_{\widetilde{\chi}_1^0} h^2 \approx \frac{10^9}{M_{\rm Pl}} \frac{x_{\rm FO}}{\sqrt{g_\star}} \frac{1}{(a + 3b/x_{\rm FO})}, \tag{23}$$

where $g_\star$ is the number of relativistic degrees of freedom available at freeze-out and $x_{\rm FO}$ is given by:

$$x_{\rm FO} \approx \ln\left(\sqrt{\frac{45}{8}} \frac{m_{\widetilde{\chi}_1^0} M_{\rm Pl}(a + 6b/x_{\rm FO})}{\pi^3 \sqrt{g_\star x_{\rm FO}}}\right). \tag{24}$$

For the range of cross sections and masses we are interested in, $x_{\rm FO} \approx 20$.

- **MSSM benchmark**

  To benchmark the NMSSM, we first consider a light bino which annihilates through the exchange of an MSSM-like CP-odd Higgs ($\cos\theta_A = 1$). The results for this case are shown in Fig. 3.

  In this figure, the thermal relic density of LSP neutralinos exceeds the measured value for CP-odd Higgses above the solid and dashed curves, for values of $\tan\beta$ of 50 and 10, respectively.

MSSM Case

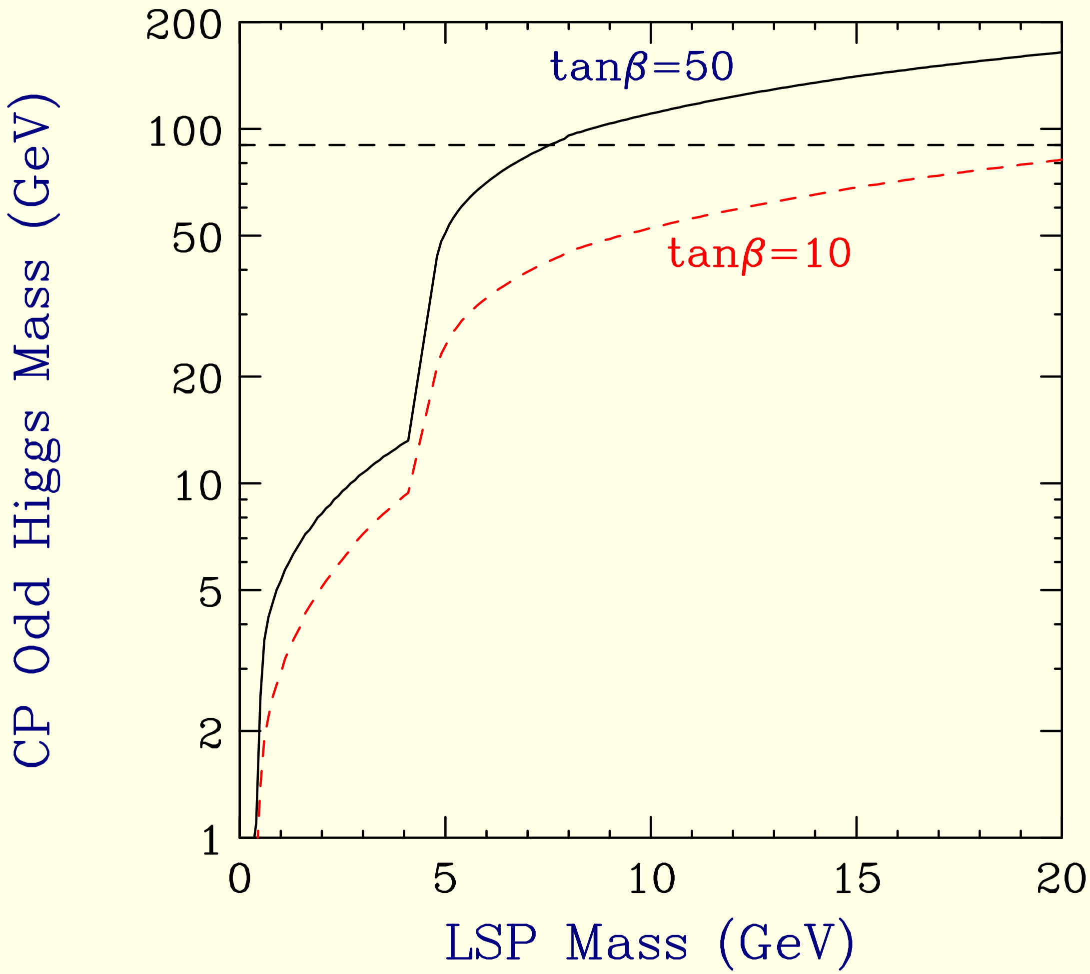

Figure 3: The CP-odd Higgs mass required to obtain the measured relic density for a light neutralino in the MSSM. Models above the curves produce more dark matter than in observed. These results are for the case of a bino-like neutralino with a small higgsino admixture ($\epsilon_B^2 = 0.94$, $\epsilon_u^2 = 0.06$).

Shown as a horizontal dashed line is the lower limit on the the MSSM CP-odd Higgs mass from collider constraints.

This figure demonstrates that even in the case of very large $\tan\beta$, the lightest neutralino must be heavier than about 7 GeV. For moderate values of $\tan\beta$, the neutralino must be heavier than about 20 GeV.

- $\boxed{\textbf{NMSSM sample points}}$

In the NMSSM framework, there is much more freedom.

One can construct a huge number of points that satisfy all experimental constraints and give good Ωh^2.

This is true even restricting to small $m_{\widetilde\chi_1^0}$ and associated small m_{a_1}.

These points can have a range of characteristics.

Below, I present two of the points that satisfy all constraints and give good Ωh^2

Table 1: Sample point 1: note singlet-like h_1.

λ	κ	$\tan\beta$	μ	A_λ	A_κ	M_1	M_2
0.436736	-0.049955	1.79644	-187.931	-458.302	-40.4478	1.92375	390.053

m_{a_1}	$\cos\theta_A$
7.17307	-0.193618

m_{h_1}	ξ_u	ξ_d	ξ_S
73.8217	0.1127	-0.0277	0.9932

$m_{\widetilde\chi_1^0}$	$\epsilon_{\tilde B}$	$\epsilon_{\tilde W}$	ϵ_u	ϵ_d	$\epsilon_{\tilde S}$
3.49603	-0.781466	-0.00594669	0.11476	0.26493	0.553099

δa_μ	BR($b \longrightarrow s\mu^+\mu^-$)	BR($\Upsilon \longrightarrow \gamma + A_1$)
1.24968e-10	3.1597e-09	8.12331e-06

$\langle\sigma v\rangle$	Ωh^2
4.55841e-26 cm^3/s	0.107689

The above point has:

1. a light $\widetilde\chi_1^0$, that is mainly bino, but with significant singlino component;
2. a singlet-like h_1;
3. a quite singlet-like a_1;
4. a small δa_μ that neither hurts nor helps;
5. excellent Ωh^2.

Table 2: Sample point 2: note MSSM-like h_1.

λ	κ	$\tan\beta$	μ	A_λ	A_κ	M_1	M_2
0.224982	-0.47912	7.58731	-174.624	-421.908	-30.6106	21.0909	984.116

m_{a_1}	$\cos\theta_A$
46.6325	-0.570716

m_{h_1}	$\sqrt{\xi_u^2+\xi_d^2}$
117.72	0.999

$m_{\widetilde\chi_1^0}$	$\epsilon_{\tilde B}$	$\epsilon_{\tilde W}$	ϵ_u	ϵ_d	$\epsilon_{\tilde S}$
22.3693	-0.971512	-0.00241597	0.00204445	0.236626	0.0127527

δa_μ	BR$(b\longrightarrow s\mu^+\mu^-)$
-1.37801e-10	3.16178e-09

$\langle\sigma v\rangle$	Ωh^2
2.17478e-26 cm^3/s	0.108649

The above point has:

1. a modest mass $\widetilde\chi_1^0$, that is almost purely bino;
2. a SM-like h_1;
3. an a_1 with substantial non-singlet component;
4. a small δa_μ that neither hurts nor helps;
5. excellent Ωh^2.

- **Given that our scans show that we can freely adjust the masses and nature of the a_1 and $\widetilde{\chi}_1^0$, while still satisfying all constraints, we find it appropriate to simply fix the compositions of the a_1 and $\widetilde{\chi}_1^0$ and vary m_{a_1} and $m_{\widetilde{\chi}_1^0}$ so as to illustrate what mass ranges can give appropriate Ωh^2 for a sample set of composition choices and several $\tan\beta$ values.**

  This kind of plot is presented in Fig. 4. The results shown are for a CP-odd Higgs which is a mixture of MSSM-like and singlet components specified by $\cos^2\theta_A = 0.6$. The $\widetilde{\chi}_1^0$ is bino-like with a small higgsino admixture as specified by $\epsilon_B^2 = 0.94$, $\epsilon_u^2 = 0.06$.

  For each pair of contours (solid black, dashed red, and dot-dashed blue), the region between the lines is the space in which the neutralino's relic density does not exceed the measured density.

  The solid black, dashed red, and dot-dashed blue lines correspond to $\tan\beta = 50$, 15 and 3, respectively. Also shown as a dotted line is the contour corresponding to the resonance condition, $2m_{\widetilde{\chi}_1^0} = m_{a_1}$.

  For the $\tan\beta = 50$ or 15 cases, neutralino dark matter can avoid being overproduced for any a_1 mass below $\sim 20-60$ GeV, as long as $m_{\widetilde{\chi}_1^0} > m_b$. For smaller values of $\tan\beta$, a lower limit on m_{a_1} can apply as well.

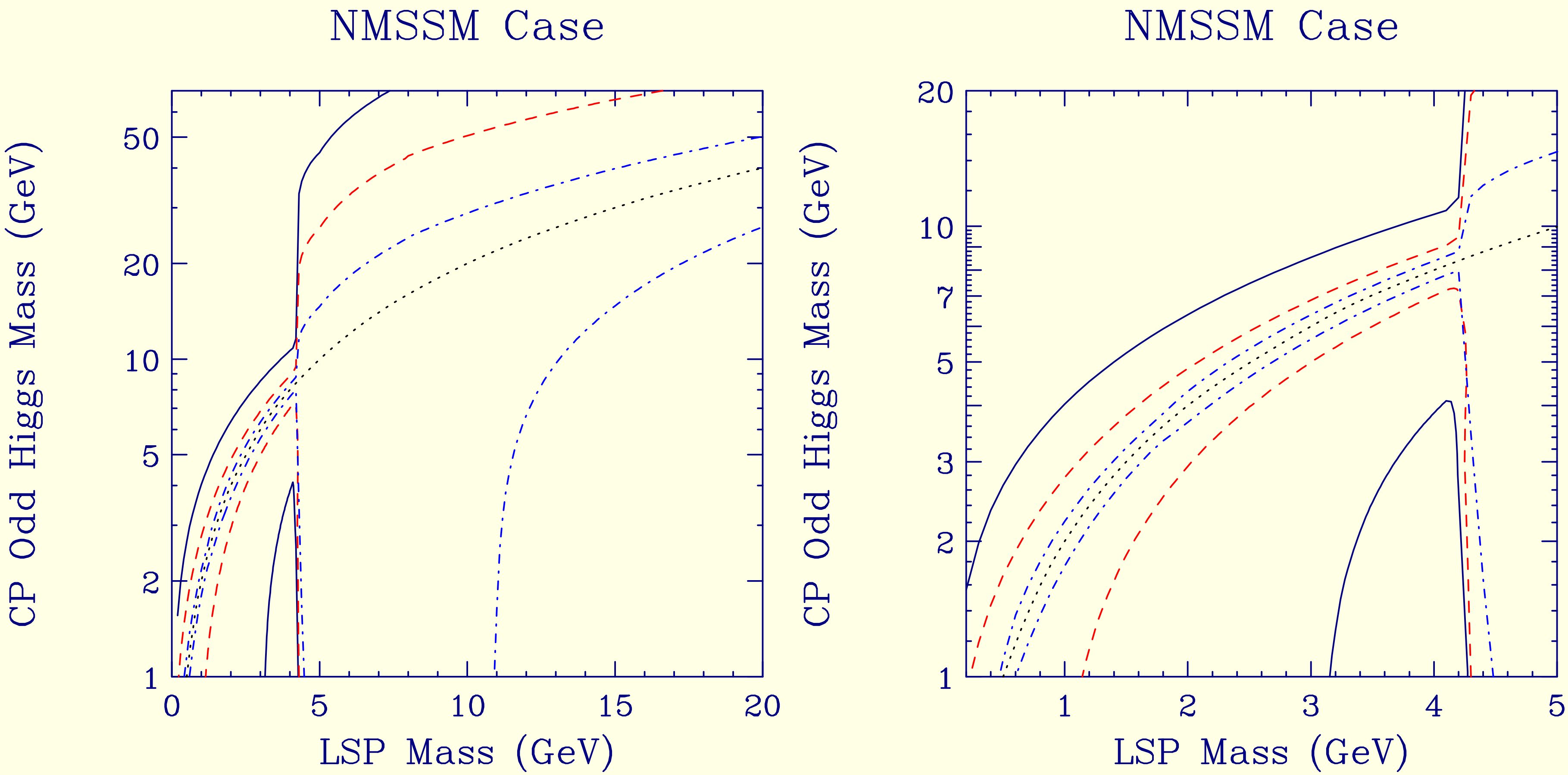

Figure 4: The CP-odd Higgs mass required to obtain the measured relic density for a light neutralino in the NMSSM. The $\tan\beta = 50$ case is highly constrained for very light neutralinos, and is primarily shown for comparison with the MSSM case.

For neutralinos lighter than the mass of the b-quark, annihilation is generally less efficient. This region is shown in detail in the right frame of

Fig. 4.

In this funnel region, annihilations to $c\bar{c}$, $\tau^+\tau^-$ and $s\bar{s}$ all contribute significantly. Despite the much smaller mass of the strange quark, its couplings are enhanced by a factor proportional to $\tan\beta$ (as with bottom quarks) and thus can play an important role in this mass range.

In this mass range, constraints from Upsilon and J/ψ decays can be very important, often requiring fairly small values of $\cos\theta_A$.

For annihilations to light quarks, $c\bar{c}$, $s\bar{s}$, etc., the Higgs couplings to various meson final states should be considered, which include effective Higgs-gluon couplings induced through quark loops.

In our calculations here, we have used the conservative approximation of the Higgs-quark-quark couplings alone, even for these light quarks, but with kinematic thresholds set by the mass of the lightest meson containing a given type of quark, rather than the quark mass itself. This corresponds to thresholds of 9.4 GeV, 1.87 GeV, 498 MeV and 135 MeV for bottom, charm, strange and down quarks, respectively.

A more detailed treatment, which we will not undertake here, would include the proper meson form factors as well as allowing for the possibility of virtual meson states.

- **In the figure, we focused on the case of a bino-like LSP. If the LSP is mostly, but not purely, singlino, it is also possible to generate the observed relic abundance in the NMSSM.**

  A number of features differ for the singlino-like case in contrast to a bino-like LSP, however.

  1. **First, the ratio $m_{\widetilde\chi_1^0}/m_{a_1}$ cannot be arbitrarily small. The relationship between these two masses was shown for singlino-like LSPs in Fig. 1. As discussed earlier, and shown in this figure, an LSP mass that is chosen to be precisely at the Higgs resonance, $m_{a_1} \simeq 2m_{\widetilde\chi_1^0}$, is not possible for this case: m_{a_1} is always less than $2m_{\widetilde\chi_1^0}$ by a significant amount.**
  2. **Second, in models with a singlino-like LSP, the a_1 is generally also singlet-like and the product of $\tan^2\beta$ and $\cos^4\theta_A$ is typically very small. This limits the ability of a singlino-like LSP to generate the observed relic abundance. However, the last two terms in Eq. (21) can be important.**
     Overall, the inability to compensate the smallness of the coefficients in Eq. (21) by being nearly on-pole implies that annihilation is too inefficient for an LSP that is more than 80% singlino.

  A sample point is presented in the table below.

Table 3: Sample point 3: singlino-like $\widetilde{\chi}_1^0$.

λ	κ	$\tan\beta$	μ	A_λ	A_κ	M_1	M_2
0.415867	-0.029989	1.78874	-175.622	-455.387	-39.671	7.1098	289.115

m_{a_1}	$\cos\theta_A$
8.35008	-0.187349

m_{h_1}	$\sqrt{\xi_u^2+\xi_d^2}$
63.3851	0.229555

$m_{\widetilde{\chi}_1^0}$	$\epsilon_{\tilde{B}}$	$\epsilon_{\tilde{W}}$	ϵ_u	ϵ_d	$\epsilon_{\tilde{S}}$
-3.97588	-0.369729	0.0261634	0.252368	0.256015	0.856377

δa_μ	BR($b\to s\mu^+\mu^-$)
-1.17325e-10	3.16148e-09

$\langle\sigma v\rangle$	Ωh^2
4.0846e-26 cm^3/s	0.120289

The above point has:

1. a light $\widetilde{\chi}_1^0$, that is mainly singlino;
2. a singlet-like h_1;
3. an a_1 with small non-singlet component;
4. a small δa_μ that neither hurts nor helps;
5. acceptable Ωh^2.

Direct DM Detection Possibilities

Here, the CP-odd Higgs bosons play no role. We must deal with a CP-even H. We will employ the approximate LEP limits on a Higgs boson with $m_H < 120~\mathrm{GeV}$

$$\xi_{u,d} \lesssim \left(\frac{m_H}{120\mathrm{GeV}}\right)^{3/2} + 0.1, \tag{25}$$

Recall also that LEP limits on a light $\widetilde{\chi}_1^0$ from invisible Z decays roughly imply $\epsilon_{u,d} < 0.06$. Here $\xi_{u,d}$ denote the H composition in terms of H_u and H_d and $\epsilon_{u,d}$, as earlier, denote $\widetilde{\chi}_1^0$ Higgsino composition.

- **DAMA and Elastic Scattering**

  The claim of a positive WIMP detection made by the DAMA collaboration is not consistent with the limits placed by CDMS and others for a WIMP in the mass range normally considered (above a few tens of GeV).

  Very light WIMPs, however, scatter more efficiently with light target nuclei than with heavier nuclei, which can complicate this picture.

For a WIMP with a mass between about 6 and 9 GeV, it has been shown that the DAMA results can be reconciled with the limits of CDMS and other experiments.[1] This is made possible by the relatively light sodium (A=23.0) component of the DAMA experiment compared to germanium (A=72.6) and silicon (A=28.1) of CDMS.

To produce the rate observed by DAMA, a light WIMP would need an elastic scattering cross section of 7×10^{-40} cm^2 to 2×10^{-39} cm^2 ($0.7 - 2$ fb). For the case of a bino-like or singlino-like neutralino capable of resolving the DAMA discrepancy, the predicted cross section is roughly:

$$\sigma_{\rm elastic} \lesssim 1.4 \times 10^{-42} cm^2 \left(\frac{120~\mathrm{GeV}}{m_H}\right)^4 \left(\left(\frac{m_H}{120~\mathrm{GeV}}\right)^{3/2} + 0.1\right)^2 \left(\frac{\tan\beta}{50}\right)^2 F_\lambda \tag{26}$$

assuming $m_{\widetilde\chi_1^0} > m_p$ and $\tan\beta > 1$, using the $\xi_{u,d}$ limit of Eq. (25) and adopting $\epsilon_{u,d} \sim 0.06$.

One has $F_\lambda = 1$ for the bino-like case and $F_\lambda = 2\lambda^2/(g_2^2 \tan^2\theta_W) \approx 0.67 \times (\lambda/0.2)^2$ for the singlino-like case.

For $\tan\beta = 50$, $\lambda = 0.2$ and a Higgs mass of 120 GeV, we estimate a

[1] If a tidal stream of dark matter is present in the local halo, WIMP masses over a somewhat wider range can reconcile DAMA with CDMS as well.

neutralino-proton elastic scattering cross section on the order of 4×10^{-42} cm^2 (4×10^{-3} fb) for either a bino-like or a singlino-like LSP.

This value may be of interest to direct detection searches such as CDMS, DAMA, Edelweiss, ZEPLIN and CRESST. To account for the DAMA data, the cross section would have to be enhanced by a local over-density of dark matter .

The cross section in Eq. (26) is small unless $\tan\beta$ is quite large, in which case the scenario will run into difficulty with LEP limits unless $\cos\theta_A$ is quite small.

To explain the DAMA result, we can instead require m_H to be small, implying of course small $\xi_{u,d}$ (the limits being already incorporated in the above equation). For instance, with $m_{\widetilde{\chi}_1^0} = 6$ GeV, $m_H = 3$ GeV, and $\tan\beta = 10$, the DAMA result can be reproduced with $\sigma_{\rm elastic} \sim 4 \times 10^{-39} {\rm cm}^2$ (~ 4 fb), *without* requiring a dark matter wind through our solar system.

It is consistent within in the NMSSM for a mostly-singlet H_1 to be this light if λ is small. In this case the singlet decouples from the MSSM and the whole singlet supermultiplet is light.

- **The 511 keV Emission Line**

If the LSP's mass is even smaller, below ~ 1 GeV, it may still be possible to generate the observed relic density. In this mass range, in addition to annihilations to strange quarks ($K^{\pm}$, K^0), final state fermions can include muons and even lighter quarks ($\pi^{\pm}$, π^0).

There is a $\widetilde{\chi}_1^0$ mass range in which neutralinos will annihilate mostly to muon pairs. This range is $m_\mu < m_{\widetilde{\chi}_1^0} < m_{\pi^+} + m_{\pi^0}/2$, or 106 MeV $< m_{\widetilde{\chi}_1^0} < 207$ MeV. (The upper limit will be explained shortly.) This range of parameter space is of special interest within the context of the 511 keV emission observed from the galactic bulge by the INTEGRAL/SPI experiment. Muons produced in neutralino annihilations will quickly decay, generating electrons with energies of $\sim m_{\widetilde{\chi}_1^0}/3$, which may be sufficiently small for them to come to rest in the galactic bulge before annihilating.

The upper limit above derives from the fact that the $\widetilde{\chi}_1^0\widetilde{\chi}_1^0$ annihilations should not create many π^0's. In this way, we avoid gamma ray constraints from EGRET. If we assume that the annihilation mediator is the CP-odd a_1, $\widetilde{\chi}_1^0\widetilde{\chi}_1^0 \to a_1 \to$ pions is only possible if $2m_{\widetilde{\chi}_1^0} \gtrsim 2m_{\pi^+} + m_{\pi^0}$ since the lowest threshold channel is to three pions: $\widetilde{\chi}_1^0\widetilde{\chi}_1^0 \to a_1 \to \pi^+\pi^-\pi^0$.

It has been shown that a ∼100 MeV dark matter particle annihilating through an a-term (low velocity) cross section can simultaneously yield the measured relic density and generate the number of positrons needed to accommodate the INTEGRAL/SPI data. These are precisely the features of a $106 - 207$ MeV neutralino combined with the presence of a 100 MeV $- 1$ GeV CP-odd Higgs.

The main difficulty with this scenario comes from the constraints on Upsilon decays, discussed earlier. To evade the CLEO limitof $BR(\Upsilon \to \gamma a_1) < 2 \times 10^{-5}$ in this mass region, we must require $\cos^2\theta_A \tan^2\beta < 0.13$.

Given these constraints, and considering a bino-like neutralino with a 6% higgsino admixture and $m_{\widetilde{\chi}_1^0} = 150$ MeV, the annihilation cross section needed to avoid overproducing dark matter can only be attained for a fairly narrow range of $m_{a_1} \approx 2m_{\widetilde{\chi}_1^0} \pm 10$ MeV.

This scenario, although not particularly attractive due to this requirement, does demonstrate that it is possible to generate the INTEGRAL signal with neutralinos in the NMSSM.

This can be confirmed or ruled out by improving the limit on $BR(\Upsilon \to \gamma a_1)$ where the a_1 is not observed or where the a_1 decays to a muon pair.

In the latter case, the a_1 may have a significant displaced vertex of a few cm, especially for small $\tan\beta$ and $m_{a_1} < 2m_{\widetilde{\chi}_1^0}$.

An a_1 this light ($300~\mathrm{MeV}$) is too light to be technically natural, however. Radiative corrections pull up its mass and a cancellation between different orders in perturbation theory is required for a_1 to be this light. While we have found parameter points capable of yielding the INTEGRAL signal, we find that they are not stable in the sense that if any of the Higgs-sector parameters are adjusted by a very small amount, the a_1 is pulled up in mass to $\mathcal{O}(10~\mathrm{GeV})$. From our numeric analysis, m_{a_1} as small as a few GeV is technically natural.

Implications for the LHC and ILC

We will assume the more natural bino-like $\widetilde{\chi}_1^0$ case for this discussion.

- **The LHC must be sensitive to a very light LSP.**

  Presumably no problem since missing momentum is just as good as missing mass.

  However, it seems likely that the LHC will only set an upper limit on $m_{\widetilde{\chi}_1^0}$.

  There are the standard SPS1a"" decay chains that can be used to do this.

- **At the ILC, we will want to get a direct handle on $m_{\widetilde{\chi}_1^0}$. It seems that this will be straightforward using the usual approaches.**

  One needs to study how well the composition of the $\widetilde{\chi}_1^0$ can be determined at the ILC. We need to get all 5 components.

- **A bigger difficulty, I believe, in checking whether or not the $\widetilde{\chi}_1^0$ explains the dark matter will be the necessity to observe and measure the composition of the a_1.**

Probably the best ways to probe the a_1 are:

1. $t\bar{t}a_1$ production, which has a substantial cross section for a non-singlet light a_1 so long as $\tan\beta$ is not too large.
2. $b\bar{b}a_1$ production at large $\tan\beta$. The cross section is big for light a_1, but backgrounds for a light a_1 have not been studied.
3. $WW \to a_1 a_1$ which relies only on the $SU(2)$ quantum numbers of the a_1. Again, what are the backgrounds for a light a_1?

Of course, all these processes are suppressed as $\cos\theta_A \to 0$, so we could have trouble for those points where the a_1 is singlet-like.

At the ILC, the same processes as listed above are applicable. There is also $e^+e^- \to Za_1a_1$ via the ZZa_1a_1 coupling.

The ILC environment will be much cleaner and one could hope to more easily see a very light a_1 in the relevant final states (that depend up m_{a_1}). Again, singlet suppression will take place.

A sample plot for the $WW \to a_1a_1$ fusion and Za_1a_1 rates is below.

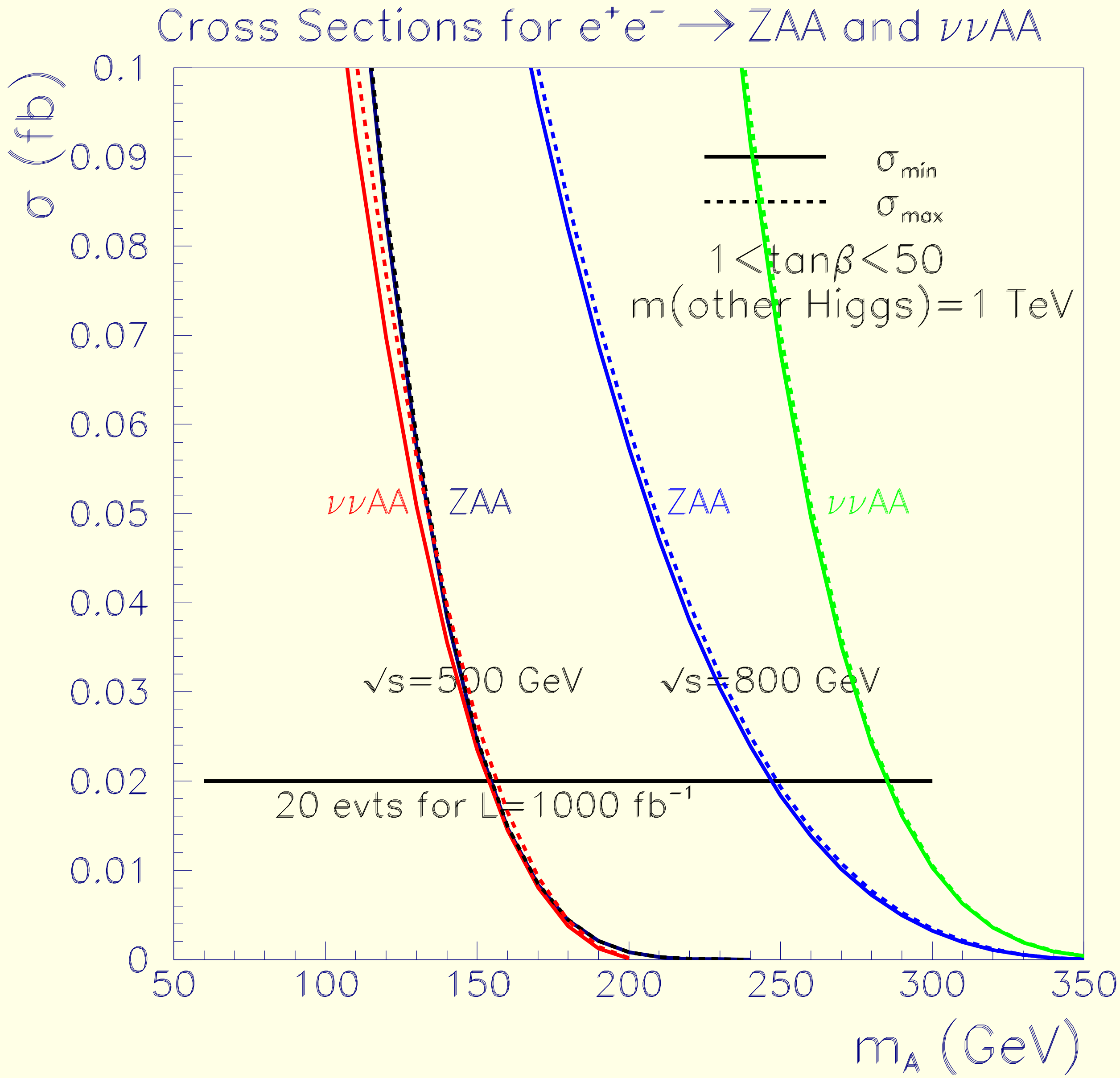

Figure 5: WW fusion rates at the ILC.

(I did not have quickly available the rates for very low m_{a_1}, but obviously they are large.)

At the ILC, there may also be some sensitivity to a_1h_1, a_1h_2, a_1h_3 production, depending upon parameters. However, these rates will also depend on the CP-even Higgs compositions.

More useful may be the one-loop processes $e^+e^- \to \gamma a_1$, $e^+e^- \to Z a_1$ and $e^+e^- \to \nu\overline{\nu} a_1$. Theses were studied by JFG, T. Farris, H. Logan and S. Su. Some figures appear on the next page.

As shown in previous studies, $\gamma\gamma \to a_1$ production has a substantial rate, although the backgrounds and such have not been examined for very low a_1. In principle, as shown by JFG and B. Grzadkowksi, various γ polarization asymmetries can be employed to determine that the a_1 observed is precisely CP-odd (implying a CP conserving NMSSM Higgs sector).

All of these have sensitivity to the detailed composition of the a_1 and extraction of its properties may be possible. **A dedicated study of this is needed.**

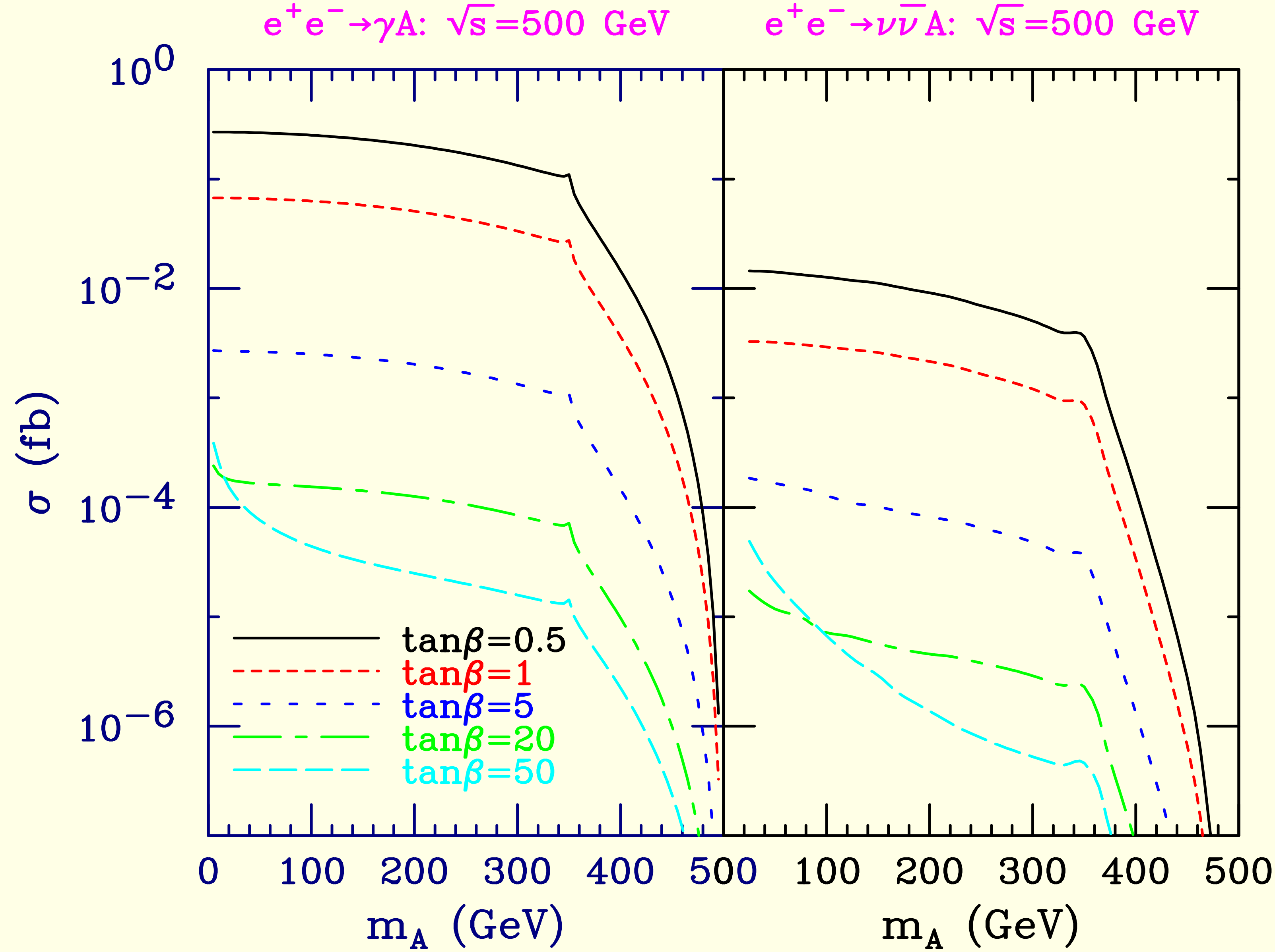

Figure 6: Sample one-loop rates for a CP-odd Higgs boson.

Conclusions

- We should avoid getting trapped in the MSSM Dark Matter scenarios.

  After all the MSSM has significant problems.

- Nature (string theory?) may well yield something like the NMSSM.

  Certainly, the NMSSM provides a good baseline in which to explore how much more flexibility there is for DM predictions and scenarios.

- If the NMSSM is any guide, we need to pay more attention to the possibility of a quite light $\widetilde{\chi}_1^0$ associated with a a_1 with about twice the mass.

  Such scenarios generate many possible signals in Υ decays and direct detection that could provide first hints. Maybe the DAMA observation or the $511~\mathrm{keV}$ photon are such a hint (but not both).

- Studies are needed to determine if the ILC can determine the $\widetilde{\chi}_1^0$ and a_1 properties to the precision needed to confirm that a light $\widetilde{\chi}_1^0$ is the source of DM (at least partially). IT MAY NOT BE EASY.