Beyond the Standard Model and Collider Physics: the LHC Era and its impact on the ILC

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PPP7
1. Discover and understand in detail the mechanism for EWSB. 
   Higgs bosons, e.g.

2. Determine if the hierarchy problem has been solved or not. 
   SUSY?

3. Discover the dark matter particle(s) and measure the properties of all particles needed for computing the relic density. 
   $R$-parity conserving SUSY for example.

4. Explain electroweak or other baryogenesis using particles seen at the LHC and ILC.
5. Discover a scalar field or other source for dark energy and inflation.

Our prejudices are:

- that the LHC will reveal the EWSB mechanism, but that the ILC will be needed to fully detail it;

- that the LHC will also reveal a mechanism that solves the hierarchy/naturalness problems of the SM;

- that if dark matter is a WIMP of any kind, the LHC is very likely to see it, and the ILC should provide the needed detailed measurements in many cases;

- that it is quite possible that LHC + ILC data will provide the info (e.g. Higgs mass, stop masses, .. in SUSY) needed to assess baryogenesis;

- it is probably wishful thinking to believe that the LHC and/or ILC will see the very weakly coupled scalar(s) that might underly dark energy and inflation.
Given the probable ILC time scales, the big question is how much will the LHC be able to do on its own. **This is very model-dependent.**

Many models have been developed as a result of not having relevant data.

**Indeed, theorists have been adopting the Yogi Berra philosophy:**

*“If you come to a fork in the road, take it!”.*

There are lots of theorists and there have been lots of years, so there have been lots of (admittedly, interesting) roads taken.

**String theory $\Rightarrow 10^{520}$ forks in the road, but most are probably not relevant to our world.**

Our marriage to the SM passed through the **honeymoon stage** long ago, the **reassessment stage** during the 90’s, and has now entered the stage of **questioning the relationship**. Theorists (and a smaller fraction of the experimentalists, experimentalists being more faithful in general) find the many attractive young models far more appealing. We hope to be able to explore their possibilities sooner rather than later. For some us, time is running out in which to enjoy the fruits of our quest.
Bottom Line: WE NEED DATA!
Fortunately, the LHC is at hand.

The CMS Detector

But what will the LHC detectors see?
1. Nothing Interesting?

Unless the Standard Model is itself intrinsically wrong, this is impossible.

Even if there is no Higgs boson, no extra dimension, no ..., we must see strong $WW$ scattering at the TeV scale. The LHC is not exactly the ideal machine for this, but with $L = \text{few} \times 100 \text{ fb}^{-1}$, strong $WW$ scattering should become apparent.

2. Just the Standard Model Higgs boson?

This would be quite surprising to most of us because of the problems with naturalness and hierarchy, but is not totally inconceivable.

One possible mass would be in the $160 - 180$ GeV range for which the SM could be an effective theory to very high mass
scales. A SM-like Higgs boson in this mass range will be seen in the $WW$ final state with just $\sim 1 \, \text{fb}^{-1}$ of $L$.

However, a heavier SM-like Higgs boson cannot be ruled out — the problem with the $S, T$ precision electroweak parameters can be cured with a positive $\Delta T$ from some form of new physics to compensate the negative $\Delta T$ from the SM-like Higgs. This can be made to work for $m_{h_{\text{SM}}} \lesssim 800$ GeV (the outer edge of the PEW $S, T$ space ellipse).

Any SM-like Higgs mass above about $150$ GeV means that low-energy SUSY is probably wrong, but, high $m_{h_{\text{SM}}}$ delays the scale at which new physics is needed to solve the naturalness/hierarchy problem to scales possibly beyond LHC reach.

3. New Gauge Bosons?

This would be exciting and new gauge bosons have some motivation in other models to be discussed below that solve
the naturalness problem, but new gauge bosons per se do not help solve the naturalness / hierarchy problems.

4. New physics that addresses naturalness and hierarchy?

There are many model categories including:

- supersymmetry, including MSSM, NMSSM, E6MSSM, ...
- extra dimensions, including warped or universal or ...
- little Higgs theories
- twin Higgs theories
- ....

I am betting on some type of SUSY with $R$-parity conservation. This class of model has a real shot at explaining naturalness in EWSB, dark matter and baryogenesis all at once.

SUSY also naturally leads to a light SM-like Higgs that is perfect for precision electroweak.
SUSY is also a convenient test case for examining the ILC — LHC connection.

Of course, the MSSM has some problems and so I will give some weight to well-motivated extensions of the MSSM, such as the NMSSM, nMSSM, E6MSSM, ...
Naturalness

The dominant quadratic divergence for the Higgs mass-squared arises from a virtual top quark loop,

\[ \delta m_{h}^{2} = -\frac{3}{4\pi^{2}} \frac{m_{t}^{2}}{v^{2}} \Lambda_{t}^{2}, \]  

where \( \Lambda_{t} \) is the high energy cutoff and \( v = 176 \text{ GeV} \).

This creates the hierarchy/fine-tuning issue in that the SM Higgs mass is very sensitive to the cutoff \( \Lambda_{t} \). A formal definition of fine tuning with respect to \( \Lambda_{t} \) is (for numerics, we take \( m_{t} \sim v \sim 174 \text{ GeV} \))

\[ F_{t}(m_{h}^{SM}) = \left| \frac{\partial \delta m_{h}^{2}}{\partial \Lambda_{t}^{2}} \frac{\Lambda_{t}^{2}}{m_{h}^{2}} \right| = \frac{3}{4\pi^{2}} \frac{\Lambda_{t}^{2}}{m_{h}^{2}_{SM}}. \]
Given a maximum acceptable $F_t$, this equation implies that you must look for new physics at or below the scale

$$\Lambda_t \lesssim \frac{2\pi v}{\sqrt{3}m_t} m_{h_{\text{SM}}} F_t^{1/2} \sim 400 \text{ GeV} \left( \frac{m_{h_{\text{SM}}}}{115 \text{ GeV}} \right) F_t^{1/2},$$  \hspace{1cm} (3)

$F_t > 10$ is deemed problematical, implying (for the precision electroweak preferred SM $m_{h_{\text{SM}}} \sim 100$ GeV mass) new physics well below 1 TeV, in principle well within LHC reach.

**The Alternatives: a partial list**

1. Make $m_{h_{\text{SM}}}$ large while compensating for the associated problems for precision electroweak constraints.
   
   For example, $m_{h_{\text{SM}}} \sim 800$ GeV delays naturalness problems and, therefore, new physics need not enter until $\Lambda_t \gtrsim 5$ TeV.  
   $\Rightarrow$ bad for LHC.

2. Alter the Higgs sector so as to raise $\Lambda_t$, thereby postponing
the need for truly new physics, without raising the SM-like Higgs mass(es).

Instead of looking for SUSY, ..., you would look for the extra Higgs phenomena in the sub- TeV region.

3. Introduce new physics: supersymmetry, little Higgs, .... of a dramatic new kind at $\Lambda_t \lesssim 1$ TeV.

If this new stuff interacts with normal gauge bosons, especially the gluon, this will be great for the LHC which will produce all the new stuff at a high rate.

If this new stuff resides in a sector that only connects to SM gauge bosons and SM fermions via the Higgs (mirror twin Higgs), then LHC signals will be very weak.

If the new stuff has $W, Z$ interactions (but not color, e.g. as in folded SUSY) then LHC signatures will be at measurable level.
An issue for all models other than SUSY: ultraviolet completion above new physics scale $\Lambda_t$ is usually quite messy.
Doublet Models


**Notation:** $h^0$ and $H^0$ for the light and heavy CP-even scalars; $A^0$ for the single CP-odd state; and $H^\pm$ for the charged Higgs pair.

In the limit where the $A^0$ is relatively light, the $h^0$ fairly heavy and the $H^\pm$ and $H^0$ very heavy and almost (but not quite degenerate — need small $m_{H^\pm} - m_{H^0}$ to generate large $\Delta T > 0$) we get the following picture.
Figure 1: Outer ellipses = current 90% CL region for $U = 0$ and $m_{h_{\text{SM}}} = 115$ GeV. Blobs = $S, T$ predictions for 2HDM models with $m_{H^\pm} - m_{H^0}$ for correct $\Delta T > 0$. Innermost (middle) ellipse = 90% (99.9%) CL region for $m_{h_{\text{SM}}} = 115$ GeV after Giga-Z and a $\Delta m_W \lesssim 6$ MeV threshold scan measurement. Stars = SM $S, T$ prediction if $m_{h_{\text{SM}}} = 500$ or 800 GeV.
Add a $Y = 0$ Higgs triplet (with no vev for neutral member for $\rho = 1$ at tree level — use discrete symmetry to guarantee) and you get coupling constant unification at $\text{few} \times 10^{14}$ GeV and the neutral triplet member is a good dark matter candidate.

One-doublet + singlet(s)


We imagine that the singlets mix with the $h_{SM}$ so that the resulting eigenstates, $h_i$ share all the $WW$, $ZZ$, $f\bar{f}$ couplings according to their overlap fraction $f_i$: $h_i = f_i h_{SM} + \ldots$, where $\sum_i f_i^2 = 1$ is required.
\[ T = \sum_i f_i^2 T_{SM}(m_{h_i}), \quad S = \sum_i f_i^2 S_{SM}(m_{h_i}). \quad (4) \]

Recalling that the \( T_{SM} \) and \( S_{SM} \) functions are basically logarithmic, we end up with a requirement for consistency with \( m_{EW} \sim 100 \text{ GeV} \) (central) or \( m_{EW} \sim 200 \text{ GeV} \) (95\% CL) in the SM case of the form \( \log m_{EW} = \sum_i f_i^2 \log m_i \) or

\[ m_{EW} = \prod_i m_i^{f_i^2}. \quad (5) \]

An appropriate \( m_{EW} \) is maintained if all the \( f_i^2 \) are equal and the \( m_i \) are not too widely separated. Or, if they are widely separated, the larger \( m_i \) should have smaller \( f_i^2 \).
Meanwhile, each $h_i$ has top quark loop scaled by $f_i^2$ and thus

$$F_t^i = f_i^2 F_t(m_i) = \frac{3}{4\pi^2} f_i^2 \frac{\Lambda_t^2}{m_i^2}$$

(6)

i.e. significantly reduced. (Note that smaller $f_i$ for larger $m_i$ keeps all $F_t^i$ of similar size.)

Thus, multiple mixed Higgs allow a much larger $\Lambda_t$ for a given maximum acceptable common $F_t^i$. Also, large $\Lambda_t$ implies significant corrections to low-$E$ phenomenology from $\Lambda_t$-scale physics less likely.

Consider for example, one doublet plus 4 complex singlets. This leads to 5 mixed CP-even states $h_i$ and 4 CP-odd states $a_k$.

Using $f_i^2 = 1/5$ and $F_t^i \leq 10$ for each of the $h_i$, $\Lambda_t \sim 5$ TeV is the new requirement if the $m_i$ are spread out in the vicinity.
of 100 GeV.

Meanwhile, the signal for each $h_i$ can be much more difficult than before. There are two sources of difficulty:

– We can spread out the $m_i$ every 10 GeV or thereabouts, so that all but the $4\ell$ signal and $\gamma\gamma$ signal overlap in mass resolution (no peak), and the $4\ell$ and $\gamma\gamma$ signal rates are reduced to $1/5$ of the SM value.

– There can be Higgs to Higgs decays by virtue of the presence of light $a_k$’s leading to $h_i \rightarrow a_k a_l$ and we could also have enough $m_i$ spread for $h_i \rightarrow h_j h_k$.

When the decaying Higgs is light, its would-be decay, $h_i \rightarrow b\bar{b}$, has a very narrow width.

Higgs-to-Higgs decays can easily have much larger width. $\Rightarrow$

Very dangerous for standard LHC Higgs signals.

The importance of such decays was first emphasized by J. R. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D 39, 844 (1989), who found

For example, writing $g_{h_i^0 h_j^0 h_k^0} = \frac{g m_{h_i^0}^2}{2 m_W}$. ignoring phase space suppression and taking $j = k$, we find

\[ \Gamma(h_i^0 \rightarrow h_j^0 h_j^0) = \frac{c^2 g^2 m_{h_i^0}^3}{128 \pi m_W^2} \sim 0.17 \text{ GeV} c^2 \left( \frac{m_{h_i^0}}{100 \text{ GeV}} \right)^3 \text{ vs.} \]  

(7)

\[ \Gamma(h_i^0 \rightarrow b\bar{b}) \sim 0.003 \text{ GeV} \left( \frac{m_{h_i^0}}{100 \text{ GeV}} \right) \text{ and} \]  

(8)

\[ \Gamma(h_i^0 \rightarrow Z Z) = \frac{1}{2} \Gamma(h_i^0 \rightarrow W W) = \frac{g^2 m_{h_i^0}^3}{128 \pi m_W^2}. \]  

(9)
where the latter assumes that $h_i^0$ carries all the vev for giving $W$’s and $Z$’s mass. $c \sim 0.13$ makes the $h_j^0 h_k^0$ or $a_j^0 a_k^0$ mode equal to the $b\bar{b}$ mode. In the Farris et al model, $c = 1$ and there are many models with $c > 0.13$.

Thus, Higgs pair modes will dominate until we pass above the $WW$ threshold.

Scaling $c$ and all other couplings down to a fraction $f_i$ reduces cross sections by $f_i^2$, but does not change the branching ratios.

**One Doublet + Other ’Random’ New Physics with needed $\Delta T$**

This is a very generic possibility. Observation of a high mass $m_{h_{SM}}$ means we have to take this seriously. It will not be very satisfying if the new physics does not fit into a nice framework.

- A very recent example is to introduce a new vector lepton doublet $(L, L^c)$ and a Majorana lepton $N$. Motivation: = dark
matter. (R. Enberg, P. J. Fox, L. J. Hall, A. Y. Papaioannou, M. Papucci) Model is constructed to allow heavy $h_{SM}$ and big enough $\Delta T$ to fit inside the ellipse. The $N$ is crucial to both $\Delta T$ and to having a dark matter candidate.

• Many other models with heavy $h_{SM}$ and sufficient $\Delta T$ from various sources were discussed in M. E. Peskin and J. D. Wells, Phys. Rev. D 64, 093003 (2001) [arXiv:hep-ph/0101342].
The MSSM

- Despite the $\mu$ problem, the MSSM (but not necessarily the cMSSM which is on the verge of being ruled out) is an important benchmark.

- It gives coupling constant unification, electroweak symmetry breaking via renormalization group evolution from the GUT scale, and dark matter (if $R$-parity is conserved).

- To get baryogenesis requires a very light $\tilde{t}_1$ since LEP demands that a SM-like Higgs boson have $m_h \gtrsim 114$ GeV (see talk by C. Wagner) — of course, the $m_h$ lower bound forces $\tilde{t}_2$ to be heavy $\Rightarrow$ large fine-tuning ($i.e.$ sensitivity of $m_Z$ to GUT scale parameters).

- Precision electroweak is not a problem if SUSY is reasonably light and stops are split.
Figure 2: Precision electroweak constraints in the MSSM context prefer light SUSY, but with big stop mass ratio, but that is good for baryogenesis.
The MSSM brings up many interesting LHC — ILC connections. In particular, the LHC inverse problem: Can we use LHC data to determine the fundamental Lagrangian parameters? And, can we do so with sufficient accuracy as to allow a meaningful extrapolation to the GUT scale and an accurate calculation of DM density?

The general picture:

This picture presumes that the LHC will have a hard time determining the absolute mass scale. For example, the SPS1a’ point gives a spectrum of the following type:

![Mass spectra of an SPS1a’-like point.](image)

\[ m_0 = 57, \ m_{1/2} = 250, \ A_0 = 0, \ \tan\beta = 10, \ \text{sgn}(\mu) = +1 \]

**Figure 4:** Mass spectra of an SPS1a’-like point.

Using lepton spectrum edges and the like, one gets quite a bit
of information about the spectrum, but a good determination of the overall mass scale is elusive. $m_{\tilde{\chi}_1^0}$ sets the overall scale.

\[ m_{e\chi_0} \]

Figure 5: Accuracy for $m_{\tilde{\chi}_1^0}$ determination. Dots=LHC alone ⇒ $\Delta m_{\tilde{\chi}_1^0} \sim \pm 20$ GeV. Vertical band=ILC. G. Weiglein et al. [LHC/LC Study Group], Phys. Rept. 426, 47 (2006) [arXiv:hep-ph/0410364]. Note: Errors assume you are at the SPS1a’ dot.
How does the LHC accuracy compare to what we need?

a) Dark matter calculation:

![Diagram showing the accuracy of WMAP, LHC, and ILC in determining the mass of the lightest neutralino, $m_{\chi^0_1}$, and its relic density $\Omega_{\chi}h^2$. LHC is assumed to get 10% accuracy on absolute $m_{\chi^0_1}, m_{\chi^0_2}, m_{\chi^\pm_1}$ masses = very optimistic using usual techniques. The yellow dot denotes the actual values of $m_{\chi^0_1}$ and $\Omega_{\chi}h^2$ for point B'. A. Birkedal, et al. hep-ph/0507214]

**Figure 6:** Accuracy of WMAP (horizontal green shaded region), LHC (outer red rectangle) and ILC (inner blue rectangle) in determining $m_{\chi^0_1}$, the mass of the lightest neutralino, and its relic density $\Omega_{\chi}h^2$. LHC is assumed to get 10% accuracy on absolute $m_{\chi^0_1}, m_{\chi^0_2}, m_{\chi^\pm_1}$ masses = very optimistic using usual techniques. The yellow dot denotes the actual values of $m_{\chi^0_1}$ and $\Omega_{\chi}h^2$ for point B’. A. Birkedal, et al. hep-ph/0507214

A precision calculation of the primordial density primarily

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needs accurate masses (couplings being fixed by supersymmetry).

The ILC measures $m_{\tilde{\chi}_1^0}$ and other masses to within $\Delta m_{\tilde{\chi}_1^0} \sim \pm 3$ GeV. Could we possibly reach this level at the LHC?

b) Precision mass measurement are needed to meaningfully assess GUT scale boundary conditions.

**Figure 7:** Evolution to the GUT scale using LHC + ILC1000 measurements. On the left, $1/M_i [\text{GeV}^{-1}]$ is plotted vs. $Q(\text{GeV})$. On the right, $M_j^2 [10^3 \text{ GeV}^2]$ for 3rd soft masses squared are plotted vs. $Q(\text{GeV})$.

Differences between colored sparticle masses and the weakly
interacting sparticle masses are determined at the LHC, and absolute scale for the latter is possible at the ILC (esp. threshold scans).

We must work to get all (or at least most) of the required accuracy at the LHC.

Since the LHC/ILC report, there have been many further efforts. A few important ones are

1. Gjeltsen, Miller, Osland: hep-ph/0410303

These all make use of one chain decay, such as \( \tilde{g} \rightarrow q\tilde{q} \) with \( \tilde{q} \rightarrow q\tilde{\chi}_2^0 \rightarrow q\ell\ell \rightarrow q\ell\ell\tilde{\chi}_1^0 \), with at least three visible particles.

These have not been successful in obtaining the needed accuracy from LHC data alone.
We claim to have an innovative and very promising technique. (JG, McElrath, Cheng, Marandella, Han, in preparation). It makes use of the full event properties.

Consider the chain decay sequence:

Figure 8: A typical chain decay topology to be decided on after cuts using OSET techniques e.g.
This topology can be applied to many processes with 4 visible and 2 invisible particles.

Suppose $M_Y = M_{Y'}$, $M_X = M'_{X}$, and $M_N = M'_{N}$.

Examples that fit this:

\[
\begin{align*}
\bar{t}t & \rightarrow bW^+ bW^- \rightarrow bl^+ \nu bl^- \bar{\nu} \\
\tilde{\chi}_2^0 \tilde{\chi}_2^0 & \rightarrow l\tilde{l}l\tilde{l} \rightarrow ll\tilde{\chi}_1^0 ll\tilde{\chi}_1^0 \\
\tilde{q}\tilde{q} & \rightarrow q\tilde{\chi}_2^0 q\tilde{\chi}_2^0 \rightarrow ql\tilde{l}ql\tilde{l} \rightarrow qll\tilde{\chi}_1^0 qll\tilde{\chi}_1^0 \\
\tilde{t}\tilde{t} & \rightarrow b\tilde{\chi}^+ b\tilde{\chi}^- \rightarrow bW^+ \tilde{\chi}_1^0 bW^- \tilde{\chi}_1^0
\end{align*}
\]

The third entry above is the SPS1a’ case of interest.

We take each event and determine from available constraints and the visible energies of the leptons the values of $M_X$, $M_Y$ and $M_N$ that are allowed. (There are no additional degrees of freedom.)
After overlapping the allowed regions for many events, but neglecting backgrounds, combinatorics and resolutions, one finds a picture like

![Diagram](image)

Figure 9: Mass region (in GeV) that can solve all events. 500 generated events for $m_Y = 246.6$ GeV, $m_X = 128.4$ GeV and $m_N = 85.3$ GeV, using correct chain assignments and perfect resolution.
It is easy to show that the correct input masses are at the tip of the allowed region.

But, after including resolution and combinatorics the correct mass solution actually typically lies outside the mass region consistent with all events (obviously there should be some events inconsistent with values at the tip).

The solution is to look for the values of the masses that maximize the number of consistent events.

At a certain point near the correct solution, as we vary $M_N$ we get the following plot.
Remarkably, the point at which the turnover occurs gives $M_N$ (and $M_X$ and $M_Y$) to good accuracy.

Figure 10: Number of events consistent with $M_N$ choice as a function of $M_N$. 
The final values for the masses are determined as

\{252.2, 130.4, 85.0\} \text{ GeV} \hspace{1cm} \text{vs.} \hspace{1cm} \{246.6, 128.4, 85.3\} \text{ GeV} \hspace{1cm} (10)

Remarkably, the $N$ mass is extremely accurate and the $Y$ mass quite close as well.

- **Error evaluation:**

Must adopt an ‘experimental’ approach for such an empirical procedure:

Generate 10 different $50 \ \text{fb}^{-1}$ data samples and apply the procedure to each sample.

Estimate the errors of our method by examining the statistical variations of the 10 samples, which yields
\[ m_Y = 252.2 \pm 4.3 \text{ GeV}, \]
\[ m_X = 130.4 \pm 4.3 \text{ GeV}, \]
\[ m_N = 86.2 \pm 4.3 \text{ GeV}. \]

The statistical variations for the mass differences are much smaller:

\[ \Delta m_{YX} = 119.8 \pm 1.0 \text{ GeV}, \quad \Delta m_{XN} = 46.4 \pm 0.7 \text{ GeV}. \quad (11) \]

Compared with the correct values \( \mathcal{M}_A = \{246.6, 128.4, 85.3\} \), we observe small biases in the mass determination, especially for the mass differences, which means that our method has some “systematic errors”.

However, these systematic errors are determined once we fix the experimental resolutions, the kinematic cuts and the fit.
Therefore, they can be easily corrected for, which leaves us errors for the absolute mass scale of $\sim few$ GeV and for the mass differences of $\sim 1$ GeV.

**Backgrounds**

In the above example, the background is negligible with the applied cuts.

However, if in some other case the backgrounds turned out to be substantial, they could decrease the accuracy of the mass determination.

We observe increases in both the biases and variations.
Figure 11: $m_N$ determination with different background-signal ratio. The dashed horizontal line corresponds to the correct $m_N$. 
We are confident that the experimental groups will actually end up doing even better in the end.

In particular, if there are backgrounds, but they understand them, then they can separately apply our procedure to them and subtract the backgrounds in the event number plots, returning us to a situation close to the zero-background case.

Higgs detection in the MSSM

There is certainly a no-lose theorem in the CP-conserving case.

Figure 12: Coverage of the $M_{H^\pm}$–$\tan\beta$ plane in the CPX scenario with $M_{\text{SUSY}} = 0.5$ TeV and for the $A_t = A_b$ and $m_{\tilde{g}}$ phases shown at top.

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In these non-discovery regions, one of two phenomena occurs. 

(1) The neutral Higgs boson with dominant couplings to the $W$ and $Z$ bosons is not the lightest. As a result, it can decay predominantly into channels which contain either two neutral Higgs bosons, or a neutral Higgs boson and a $Z$ boson.

The lightest Higgs boson has only feeble couplings to the $W$ and $Z$ bosons and top quarks, and escapes detection both at LEP and the hadron colliders.

(2) All three neutral Higgs bosons can share the coupling to $W$ and $Z$ bosons and the top quark, resulting in three marginal signal excesses.

- Fine-tuning in the MSSM

The latter should not be ignored since coupling unification
suggests we should consider the model all the way up to the GUT scale.

Here, the important fine-tuning is how precisely GUT-scale parameters must be tuned in order to get the correct $m_Z$ after RGE evolution:

$$F \equiv \text{Max}_p \left| \frac{\partial \log m_Z}{\partial \log p} \right|, \quad (12)$$

where $p$ are the GUT scale parameters (e.g. $\mu, M_3, m_{H_u}^2, A_t$, to name the usually critical ones).

In the CP-conserving MSSM, the lowest $F$ ($\sim 25$) with $m_{h^0}$ above the 114 GeV LEP limit (assuming $m_{A^0} > 100$ GeV) is achieved in the maximal mixing scenario when $A_t \sim -500$ GeV (rather precisely).

This is relaxed in the CP-violating case.
The Next-to-Minimal Supersymmetric Model

Some electroweak priors

Figure 13: Perhaps we really should believe in a light SM-like Higgs!
Figure 14: There is an observed vs. expected 2.6σ discrepancy that is fit perfectly if there is a Higgs with $m_h \sim 100$ GeV having close to SM $ZZh$ coupling, but $B(h \rightarrow b\bar{b}) \sim 1/10$ the SM value.

The NMSSM can fit all these priors, solves the hierarchy problem and has many very attractive features. R. Dermisek and J. F. Gunion,
• The MSSM \( \mu \) parameter problem is solved using \( \mathcal{L} \supset \lambda \hat{S} \hat{H}_u \hat{H}_d \) with \( \mu_{\text{eff}} = \lambda \langle S \rangle \).

• Gauge coupling unification and RGE EWSB is just as in the MSSM.

• Fine-tuning \( (F) \) is absent if the light \( h_1 \) has SM-like \( VV \) and \( f \bar{f} \) couplings and if \( m_{h_1} \sim 100 \) GeV \( (\tan \beta \gtrsim 5) \) — SUSY sparticles, especially the stops and gluino, should be close to being observable at the Tevatron.
Figure 15: $F$ vs. $m_{h_1}$ in the NMSSM for $\tan \beta = 10$, $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV. Large yellow crosses are fully consistent with LEP constraints. See earlier Dermisek + JFG refs.
• This fits precision EWSB very nicely.

• It is natural for the lightest CP-odd Higgs, \( a_1 \), to be such that 
  \[ B(h_1 \rightarrow a_1a_1) \sim 0.8 \] 
  and \( m_{a_1} < 2m_b \) so that \( a_1 \rightarrow 2\tau \) or 
  2 jets (which evades LEP limits on \( e^+e^- \rightarrow Z + b's \)).\(^1\)

• LEP analyzes have not constrained the \( Z + 4\tau \) mode for 
  \( m_{h_1} \sim 100 \text{ GeV} \). It is now being actively pursued.

  The \( Z + h_1 \rightarrow Z + b\bar{b} \) enhancement at 2.3\( \sigma \) level is well fit 
  for \( B(h_1 \rightarrow a_1a_1) \sim 0.8 \).

What will the LHC see?

• Light SUSY (needed for low fine-tuning).

\(^1\)Note: Since the \( h_1b\bar{b} \) coupling is so small, it does not take much \( h_1a_1a_1 \) coupling 
  for the \( h_1 \rightarrow a_1a_1 \) to be the dominant decay since \( m_{h_1} < 2m_W \). There are many 
  papers that make use of the small \( \Gamma(h_1 \rightarrow b\bar{b}) \) to allow dominance of other decays.
• But it will not see the $h_1$ in the usual channels (e.g. $h_1 \rightarrow \gamma\gamma$, ...).

• LHC analyzes must be sensitive to $h_1 \rightarrow a_1a_1 \rightarrow 4\tau$ (if $m_{a_1} > 2m_\tau$ for which there is some fine-tuning preference) or $h_1 \rightarrow a_1a_1 \rightarrow 4$ jets (if $m_{a_1} < 2m_\tau$).

Possibilities:

- $WW \rightarrow h_1 \rightarrow a_1a_1 \rightarrow 4\tau$, $WW$ fusion production.
- $t\bar{t}h_1 \rightarrow t\bar{t}4\tau$.
- $pp \rightarrow pp + h_1 \rightarrow pp + 4\tau$ diffractive channel.
- While there is real hope for the above $4\tau$ channels, the corresponding cases with $4\tau$ replaced by $4$ jets are probably impossible at the LHC.

Even if the LHC sees the Higgs signal, the ILC will be crucial to really detail the $h_1$. 

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• The Higgs-power of the ILC resides in the fact that the $e^+e^- \rightarrow ZX$ missing-mass $M_X$ spectrum will have strong Higgs peak regardless of how the $h_1$ decays.

![Graph showing the number of events vs. recoil mass for $m_H = 120$ GeV.]

**Figure 16:** Decay-mode-independent Higgs $M_X$ peak in the $Zh \rightarrow \mu^+\mu^-X$ mode for $L = 500 \text{ fb}^{-1}$ at $\sqrt{s} = 350$ GeV, taking $m_h = 120$ GeV.

There are lots of events in just the $\mu^+\mu^-$ channel (which you may want to restrict to since it has the best mass resolution).
• Can then check $h_1 \rightarrow b \bar{b}$ and $h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$ branching ratios for consistency with the model.

• Can study the $a_1$ decays as well, and probably get a good determination of many $a_1$ properties (e.g. how much singlet).

Other benefits:

• Light SM-like $h_1$ as well as the $\lambda A_\lambda S H_u H_d$ coupling makes baryogenesis highly viable, especially if the stop squarks are not particularly heavy (as preferred for low fine-tuning).

• There is a new dark matter scenario in which the $\tilde{\chi}^0_1$ is very light but annihilates sufficiently via the $a_1$ resonance — can we measure the $a_1$ and $\tilde{\chi}^0_1$ properties sufficiently accurately to check? J. F. Gunion, D. Hooper and B. McElrath, Phys. Rev. D 73, 015011 (2006) [arXiv:hep-ph/0509024].
There is a new task for the $B$ factories: look for $\Upsilon \rightarrow \gamma a_1 \rightarrow \gamma + 2\tau, \gamma + 2j$. R. Dermisek, J. F. Gunion and B. McElrath, arXiv:hep-ph/0612031.

Figure 17: $B(\Upsilon \rightarrow \gamma a_1)$ for NMSSM scenarios with various ranges for $m_{a_1}$. (blue= $< 2m_\tau$, red= $[2m_\tau, 7.5]$, green= $[7.5, 8.8]$, black= $[8.8, 9.2]$) The lower bound on $B(\Upsilon \rightarrow \gamma a_1)$ arises basically from the LEP requirement of $B(h_1 \rightarrow a_1 a_1) > 0.7$. 
The LHC will yield SUSY spectrum and perhaps initial SM-like $h_1$ discovery.

If $\tan \beta \gtrsim 20$, then $h_2, h_3, a_2$ will be detectable in $b\bar{b}h$ with $h \rightarrow \tau^+\tau^-$, since low $F$ predicts $m_{h_2}, m_{h_3}, m_{a_2} \lesssim 400$ GeV.

The ILC will be needed to really verify $h_1$ properties and to check whether dark matter works, i.e. the $\tilde{\chi}_1^0$ and possibly (depending upon $m_{\tilde{\chi}_1^0}$) the $a_1$ properties.

Must continue LEP data reanalysis.

$B$ factory data on $\Upsilon$ resonances should be used to constrain or observe $\Upsilon \rightarrow \gamma a_1$.

Many of these same conclusions apply to other MSSM extensions such as ....
• Strings like lots of singlets.

• In the SUSY context, you could have many singlets, which leads to many $h_1 \to aa$ type decays, not to mention $h_1 \to \tilde{\chi}\tilde{\chi}$ decays.


  – This model has an extra $U(1)'$ gauge group added to the MSSM along with a singlet $S$ as well as 3 other $S_{1,2,3}$; all are charged under the $U(1)'$, but not under the SM groups. $S$ gives the $\mu$ parameter as in the NMSSM. The model has some attractive features, but also a lot of complexity. Some problems and features are:
* The lightest Higgs with $WW$ couplings can be heavy because of extra $D$-term contributions to its mass.
* The lightest Higgs need not have $WW$ couplings. If it doesn’t, then it is usually somewhat singlet in nature.
* Gauge coupling unification would appear to require significant extra matter at high scales.
* A more complete model would be required to assess fine-tuning with respect to GUT-scale parameters.
* There are 4 light $a_k^0$’s and these are definitely important in Higgs decays, especially for a light singlet-like Higgs with suppressed couplings to SM particles, but also for the heavier SM-like Higgs if it has mass below $2m_W$.
* There are many neutralinos, some of which are singlet-like and very light, but coupled to the Higgs so that $h_i \rightarrow \tilde{\chi}_j^0 \tilde{\chi}_k^0$ is often a dominant or at least important channel, again especially for the lighter singlet-like Higgs boson.
The decays of the lightest $a_1$ can be dominated by neutralino pairs.

Figure 18: Branching ratios for the somewhat heavy lightest Higgs with substantial $WW$ coupling.
- A lot more work is needed on this kind of model with regard to baryogenesis, dark matter, gauge coupling unification (possibly problematical), ... to fully assess.

**Bottom line:**

- The LHC is very likely to miss the Higgs because of the many channels it would appear in.

  High $\tan \beta$ could come to the rescue and allow $b\bar{b}h_2 + b\bar{b}a_2 \rightarrow b\bar{b}\tau^+\tau^-$ detection. (There are Tevatron hints from CDF and anti-hints from D0.)

- The LHC would probably see lots of SUSY, unless all colored sparticles are heavy (not preferred by fine-tuning).

- The ILC would absolutely be required to detail the SUSY spectrum and generally sort things out.
This possibility is quite independent of the EWSB scenario, but does have some impact on dark matter.

- Many soft-SUSY-breaking boundary conditions can lead to near degeneracy of the $\tilde{\chi}_1^0$ and $\tilde{\chi}_1^\pm$.


- AMSB $\Rightarrow M_1 : M_2 : M_3 \sim 2 : 1 : 7$.

This implies that $m_{\tilde{\chi}_1^\pm} \sim m_{\tilde{\chi}_1^0}$ and that the gluino is about 7 times heavier than the $\tilde{\chi}_1^0$.

Since $\Delta m_{\tilde{\chi}} \equiv m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$ will be very small, $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 + ...$ will have soft ... In particular, the leptons in the $\ell\nu$ decay mode are very soft.

- For a moderate mass scenarios, $\tilde{g}$'s and $\tilde{q}$'s have high production rate and discovery will be possible in the leptons plus jets plus...
$\mathcal{E}_T$ channel where the leptons come from the $\tilde{\chi}^0_2$ decays that appear in $\tilde{g}$ chain decays.

- The really dangerous situation for a hadron collider, including the LHC, would be as follows.

1. The colored sparticles are very heavy. This is quite ‘easy’ for AMSB compared to mSUGRA. This is because $M_3 = 7m_{\tilde{\chi}^0_1}$ and because the squark mass soft term $m_0$ is an independent parameter that could be very large.
   Roughly, to make life difficult we would need $m_{\tilde{g}} \gtrsim 2$ TeV which translates to $m_{\tilde{\chi}^0_1} \sim m_{\tilde{\chi}^\pm_1} \sim 300$ GeV and $m_{\tilde{\chi}^0_2} \sim 600$ GeV.
   Of course, we would expect large fine-tuning measure $F$ for such a large $m_{\tilde{g}}$, and so perhaps we need not worry.

2. $\mu$ is large, so that $\tilde{\chi}^0_3$ and $\tilde{\chi}^0_4$ are heavy.
This would also make $F$ large.

3. Combining the above, one must rely almost entirely on light gaugino $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_1^{0}$ production.

4. Maximally bad mass difference:

$$\Delta m_{\tilde{\chi}} \equiv m_{\tilde{\chi}^{\pm}_1} - m_{\tilde{\chi}^{0}_1} \sim \text{few GeV}. \quad (13)$$

This is the natural result if one starts with AMSB-like b.c. at tree-level and then inputs the one-loop corrections. In this case,

$$\tilde{\chi}_1^{\pm} \rightarrow \tilde{\chi}_1^{0} + \pi' \text{ s or } \tilde{\chi}_1^{\pm} \rightarrow \tilde{\chi}_1^{0} \ell \nu \quad (14)$$

promptly (so can’t use vertex detection or stable track or ...) and the $\pi$’s or $\ell$ are soft. ($B(\tilde{\chi}_1^{0} \ell \nu) \sim 0.1$.)

In this scenario,

1. $\tilde{\chi}_1^{\pm} \tilde{\chi}_1^{0}$ and $\tilde{\chi}_1^{+} \tilde{\chi}_1^{-}$ production rates are not large enough to give a dramatic excess of anything.
2. The like-sign di-lepton and tri-lepton signals are very difficult since the leptons are very soft.

3. Without a $\gamma$ or $jet$ tag, missing energy will tend to cancel between the two sides of the event. The tag will reduce the event rate further. ⇒ Not clear that SUSY will be discovered.

4. Once $\Delta m_{\tilde{\chi}} \gtrsim 5 - 7 \text{ GeV}$, there may be enough energy in the soft pion 'jets' that progress can be made (Wang, Han et al).

5. Generally speaking, the ILC may be necessary for SUSY discovery if $m_{\tilde{\chi}_1^\pm} \sim m_{\tilde{\tau}_1^0} > 300 \text{ GeV}$ (for $m_{\tilde{g}} > 2 \text{ TeV}$ so that LHC signals are weak) and this will require ILC1000.

**Bottom line:**

The LHC could fail to see SUSY because of heavy AMSB-like b.c. AND fail to see the Higgs because of unexpected decays coming from an extended scalar etc. sector like the NMSSM with $m_{\sigma_1} < 2m_\tau$ (so that Higgs detection will probably not be
This would be a scenario with no LHC signal for which the ILC1000 (but not the ILC500) would see a lot of spectacular things ($h_1, a_1, \tilde{\chi}_1^\pm, \tilde{\chi}_1^0$ at the least).

A maximally bad scenario

- If we do not demand that SUSY gives us dark matter, then we could introduce baryonic $R$-parity violation (proton still stable at required level if no leptonic $R$-parity violation).

- The Higgs boson could decay via $h^0 \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0$ and the $\tilde{\chi}_1^0$ decays via $\tilde{\chi}_1^0 \rightarrow 3j$.

  Makes $h^0$ discovery extremely difficult.

- Meanwhile, we have no missing energy signal for SUSY discovery and only very weak like-sign dilepton signal (recall that the leptons in $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0\ell\nu$ are very soft).
Not clear how to discover SUSY in this case.
Conclusions

1. In most reasonable SUSY models with $R$-parity conservation (so as to address dark matter), the LHC will see both Higgs boson signals and missing energy that can be associated with dark matter.

However,

2. Beware of an NMSSM-like or CPX-like Higgs sector.

In the NMSSM case, work on the $4\tau$ modes at the very least. LEP analyzes might have impact.

Is there anything to be done for the $4\ jet$ modes?

In the CPX case, there are no known improvements to be made at the LHC, but one should try to close the hole using extended LEP analyzes.
3. AMSB-like b.c. with large $m_{\tilde{g}}$ and/or $R$-parity violation are worrisome.

Further work by the ATLAS and CMS collaborations may increase the robustness of the marginal signals that would be present for $m_{\tilde{\chi}^0_1} \gtrsim 300$ GeV (implying $m_{\tilde{g}} \gtrsim 2$ TeV) and heavy $\tilde{q}$’s in the $R$-parity conserving case.

If the LHC fails in either Higgs or SUSY discovery, the ILC would be likely to provide coverage.