Precision Electroweak Data and the Mixed Radion-Higgs Sector

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Outline

- **Review of Randall Sundrum and Radion-Higgs Mixing**
- **Some Details on Precision Electroweak Computations in this context**
- **Phenomenological Implications**

Basic references for Radion-Higgs Mixing

[1] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B 595, 250 (2001) [arXiv:hep-ph/0002178].

[2] C. Csaki, M. L. Graesser and G. D. Kribs, Phys. Rev. D 63, 065002 (2001) [arXiv:hep-th/0008151].

[3] For $\xi = 0$ discussion of precision EW constraints, see P. Das and U. Mahanta, Phys. Lett. B 528, 253 (2002) [arXiv:hep-ph/0107162].

[4] T. Han, G. D. Kribs and B. McElrath, Phys. Rev. D 64, 076003 (2001) [arXiv:hep-ph/0104074].

[5] M. Chaichian, A. Datta, K. Huitu and Z. h. Yu, Phys. Lett. B 524, 161 (2002) [arXiv:hep-ph/0110035].

[6] J. L. Hewett and T. G. Rizzo, JHEP 0308, 028 (2003) [arXiv:hep-ph/0202155].

[7] D. Dominici, B. Grzadkowski, J. F. Gunion and M. Toharia, Nucl. Phys. B 671, 243 (2003) [arXiv:hep-ph/0206192].

[8] M. Battaglia, S. De Curtis, A. De Roeck, D. Dominici and J. F. Gunion, Phys. Lett. B 568, 92 (2003) [arXiv:hep-ph/0304245].

[9] J. F. Gunion, M. Toharia and J. D. Wells, Phys. Lett. B 585, 295 (2004) [arXiv:hep-ph/0311219].

Randall Sundrum Basics: 0th Order

- There are two branes, separated in the 5th dimension (y) and $y \to -y$ symmetry is imposed. With appropriate boundary conditions, the 5D Einstein equations $\Rightarrow$

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - b_0^2 dy^2, \tag{1}$$

where $\sigma(y) \sim m_0|y| \equiv k|y|$ (both the notations k and m_0 appear in the literature).

- $e^{-2\sigma(y)}$ is the warp factor; scales at $y = 0$ of order $M_{\rm P}$ on the hidden brane are reduced to scales at $y = y_0 = b_0/2$ of order TeV on the visible brane; for example, $m_W \sim M_{\rm P} e^{-\frac{1}{2}kb_0}$ if $\frac{1}{2}kb_0 \sim 36$.
- Fluctuations of $g_{\mu\nu}$ relative to $\eta_{\mu\nu}$ are the KK excitations $h^n_{\mu\nu}$.
- Fluctuations of $b(x)$ relative to b_0 define the radion field.
- In the simplest model, all SM gauge and matter fields are placed on the TeV brane and do not propagate in the bulk.

Among these is the Higgs doublet $\widehat{H}$. After EWSB and various rescalings, the surviving properly normalized quantum fluctuation field is called h_0.

- The radion is stabilized by introducing a radion mass by hand.

  A possible mechanism is to have scalar fields in the bulk (Goldberger and Wise).

- Higgs-radion mixing is allowed for.

$$S_\xi = \xi \int d^4x \sqrt{g_{\rm vis}} R(g_{\rm vis}) \widehat{H}^\dagger \widehat{H} \,, \tag{2}$$

  where $R(g_{\rm vis})$ is the Ricci scalar for the metric induced on the visible brane.

- A crucial parameter is the ratio $\gamma \equiv v_0/\Lambda_\phi$, where Λ_ϕ is the vacuum expectation value of the radion field and $v_0 = 246~{\rm GeV}$.

- Net result

  4 independent parameters to completely fix the mass diagonalization of the scalar sector when $\xi \neq 0$. These are:

$$\xi \,, \quad \gamma \,, \quad m_h \,, \quad m_\phi \,, \tag{3}$$

where we recall that $\gamma \equiv v_0/\Lambda_\phi$ with $v_0 = 246~\mathrm{GeV}$.

$\boxed{\Lambda_\phi}$ is a critical parameter:

– Λ_ϕ^{-1} sets strength of interactions of the canonically normalized ϕ_0;
– Λ_ϕ determines the masses of the KK tower of graviton excitations,

$$m_1 = x_1 \frac{k}{M_\mathrm{P}} \frac{\Lambda_\phi}{\sqrt{6}} \quad (x_1 \sim 3.8)\,. \tag{4}$$

Full reliability of RS approximation of flat brane requires $\frac{k}{M_\mathrm{P}} \lesssim 0.1$, but values up to 1 are often considered.

– For $\Lambda_\phi = 5~\mathrm{TeV}$ and $k/M_\mathrm{P} = 0.1$, $m_1 = 750~\mathrm{GeV}$, well within LHC range, but large enough that KK corrections to precision EW observables are small.
– Measuring m_1 fixes $\frac{k}{M_\mathrm{P}}\Lambda_\phi$. The width of the m_1 resonance will provide a determination of $\frac{k}{M_\mathrm{P}}$ (larger $\frac{k}{M_\mathrm{P}} \Rightarrow$ broader resonance shape).
$\Rightarrow$ we are very likely to have a good determination of Λ_ϕ from LHC data.

- After writing out the full quadratic structure of the Lagrangian, including $\xi \neq 0$ mixing, we obtain a form in which the h_0 and ϕ_0 fields for $\xi = 0$ are mixed ($h_0 = dh + c\phi$, $\phi_0 = a\phi + bh$) and have complicated kinetic energy normalization.

We must diagonalize the kinetic energy and rescale to get canonical normalization.

Given m_h and m_ϕ we must invert the mixing equations. The process of inversion is very critical to the phenomenology and somewhat delicate.

The result found is that the physical mass eigenstates h and ϕ cannot be too close to being degenerate in mass, depending on the precise values of ξ and γ; extreme degeneracy is allowed only for small ξ and/or γ.

The $f\overline{f}$ and VV couplings

$$g_{ZZh} = \frac{g\, m_Z}{c_W}\,(d + \gamma b)\ , \quad g_{ZZ\phi} = \frac{g\, m_Z}{c_W}\,(c + \gamma a)\ . \tag{5}$$

The WW couplings are obtained by replacing gm_Z/c_W by gm_W.

$$g_{f\bar{f}h} = -\frac{g\, m_f}{2\, m_W}(d + \gamma b)\, , \quad g_{f\bar{f}\phi} = -\frac{g\, m_f}{2\, m_W}(c + \gamma a)\, . \tag{6}$$

Note same factors for WW and $f\bar{f}$ couplings. Define

$$g_{fVh} \equiv d + \gamma b\,, \quad g_{fV\phi} \equiv c + \gamma a\,. \qquad (7)$$

The gg and $\gamma\gamma$ couplings

- There are the standard loop contributions, except rescaled by $f\overline{f}/VV$ strength factors g_{fVh} or $g_{fV\phi}$.
- In addition, there are anomalous contributions, which are expressed in terms of the SU(3)×SU(2)×U(1) β function coefficients $b_3 = 7$, $b_2 = 19/6$ and $b_Y = -41/6$.
- The anomalous couplings of h and ϕ enter only through their radion admixtures

Parameter Space Example

- Take $m_h = 120$ GeV and $\Lambda_\phi = 5$ TeV.
- In the figure, note the hourglass shape that defines the theoretically allowed region.

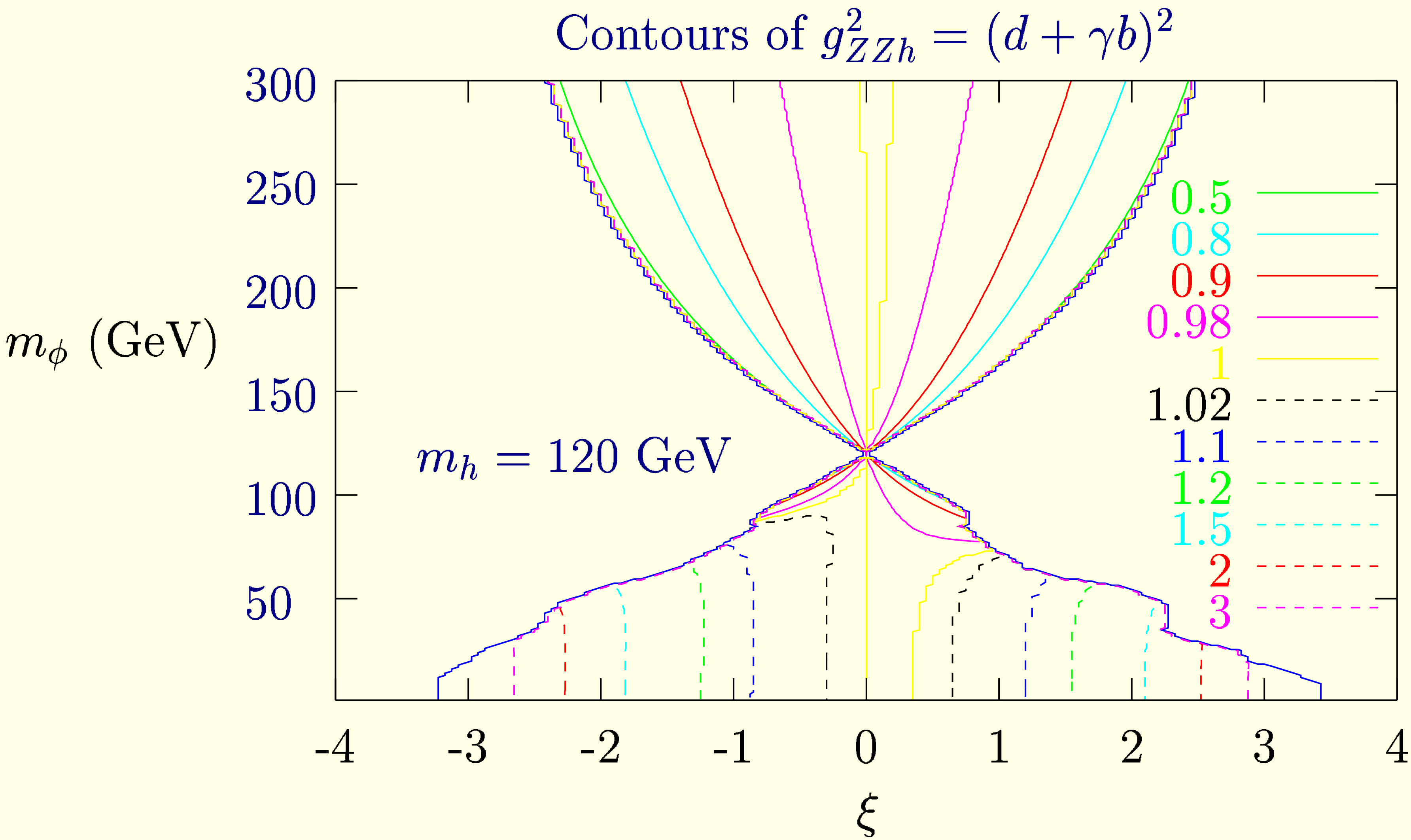


Figure 1: *Contours of* g^2_{fVh} *(relative to SM) for* $\Lambda_\phi = 5$ TeV, $m_h = 120$ GeV.

- **Observe suppression if $m_\phi > m_h$ and vice versa. SM limit is $\xi = 0$.**
- **Large ξ yields big changes in Higgs phenomenology [7, 6], $\Rightarrow$ another talk.**

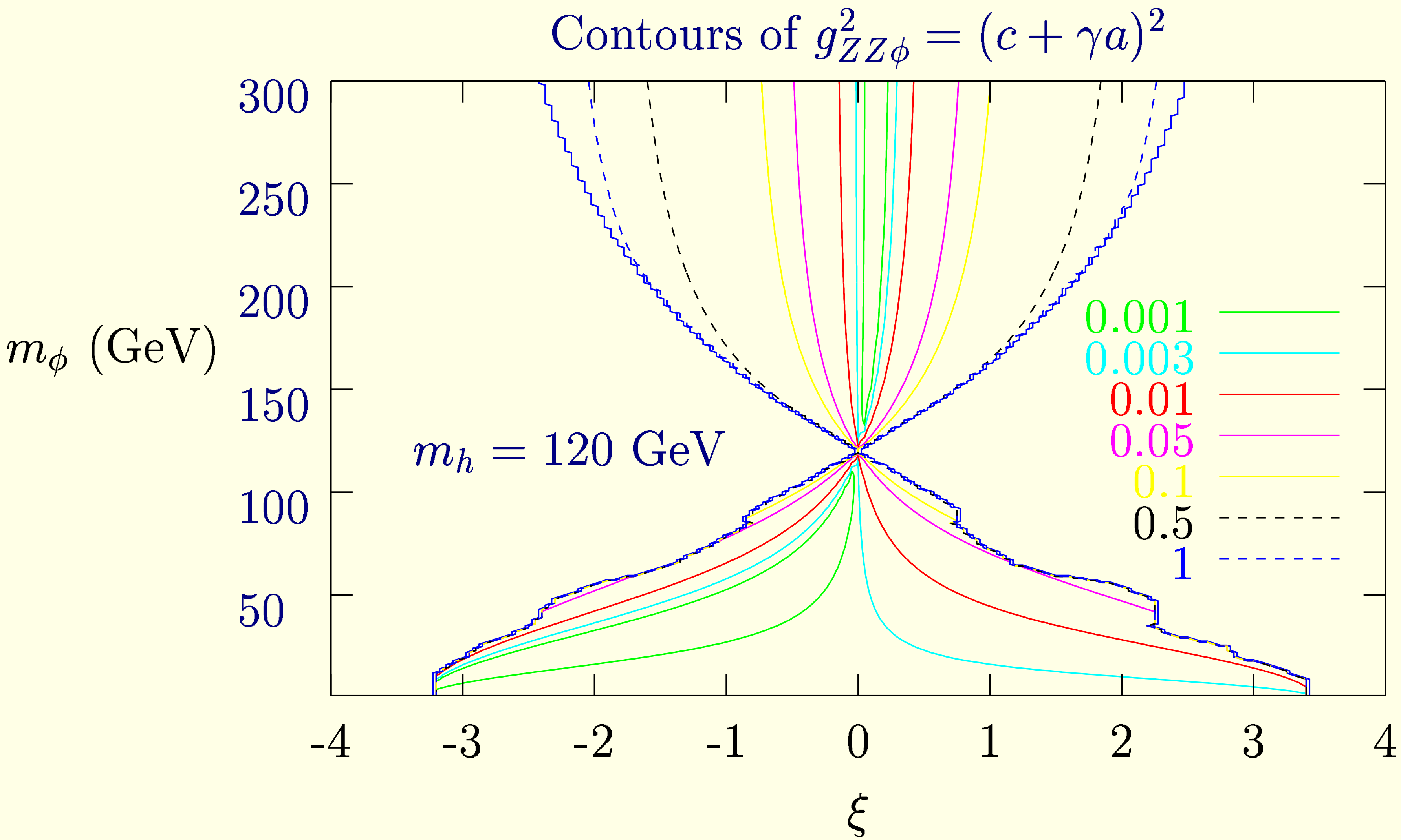


Figure 2: *Contours of* $g^2_{fV\phi}$ *for* $\Lambda_\phi = 5$ TeV, $m_h = 120$ GeV

- **Substantial $g^2_{fV\phi}$ is possible if $m_\phi > m_h$ and ξ is not too small.**
- **$g_{fV\phi} \neq 0$, but is small in SM Higgs limit of $\xi = 0$.**

Randall-Sundrum at 1st and 2nd Order

The metric of the five-dimensional space can be written as

$$ds^2 = \left[e^{-2\sigma}\eta_{\mu\nu} + \hat{\kappa}\left\{e^{-2\sigma}h_{\mu\nu}(x,y) - \eta_{\mu\nu}\ c(y)r(x)\right\} + \hat{\kappa}^2\eta_{\mu\nu}e^{2\sigma}a(y)r^2(x)\right]dx^\mu dx^\nu$$

$$+ \left[1 + \hat{\kappa}2e^{2\sigma}e(y)r(x) + \hat{\kappa}^2e^{4\sigma}f(y)r^2(x)\right]dy^2 \qquad (8)$$

where $\hat{\kappa}^2 = 1/M^3_{\mathrm{Pl}-5}$ and $r(x)$ is the non-canonically normalized radion field. In the unphysical case of no radion stabilization (i.e., no radion mass) $\sigma(y) = ky$ (in the bulk), $c(y) = e(y) = 1$, $f(y) = 2$ **and $a(y) = 1/4$**.

The last choice $\Rightarrow$ no $r\partial_\mu r\partial^\mu r$ trilinear self interactions, $i.e.$ the metric solves the equations of motion up to order $\hat{\kappa}^2$, before introducing stabilization or matter fields.

Once stabilization is introduced, a. c, e, and f become model-dependent. The induced metric on the SM brane up to second-order in the radion will have a form depending on 2 model-dependent parameters:

$$g^{\mathrm{ind}}_{\mu\nu}(x,y_0) = \eta_{\mu\nu}\left[e^{-2ky_0} - \hat{\kappa}\ c_0\ r(x)\ +\ e^{2ky_0}\ a_0\ \hat{\kappa}^2\ r^2(x)\right] \qquad (9)$$

where $a_0 = a(y_0)$ and $c_0 = c(y_0)$ are unknown constants that depend on the stabilization dynamics.

We can then apply this induced metric to the brane Lagrangian, and derive interaction terms between SM states and the radion.

The radion $r(x)$ is proportional to an ordinary 4-dimensional scalar field that is likely to be the lightest state associated with the warped extra dimensions.

The canonically normalized 4-dimensional quantum fluctuation radion field $\phi_0(x)$ is related to $r(x)$ by

$$\frac{c_0}{2}\hat{\kappa}\; e^{2ky_0} r(x) = \left(\frac{1}{\Lambda_\phi}\right)\phi_0(x), \tag{10}$$

where in the no-back-reaction limit of $c(y) = 1$ we would have $\Lambda_\phi = \sqrt{6} M_{\rm Pl} e^{-ky_0}$.

In general, Λ_ϕ is the vacuum expectation value of a 4-dimensional field whose perturbations are related to the radion.

Intuitively, Λ_ϕ can also be thought of as the Planck-warped scale on the SM brane and it should be numerically not dramatically above the weak scale if warped geometry is the origin of the weak scale.

We must compute to $\mathcal{O}(\epsilon = 4 - D)$ (as required to get all finite terms when dimensionally regularizing the loop integrals for precision observables) and $\mathcal{O}(\hat{\kappa}^2)$ (to get quartic interactions required for precision observable loops). For example, the interactions of ϕ_0 with the SM vector fields up to order ϕ_0^2 are

$$\mathcal{L}_{int}(\hat{\kappa}^1) = -\left(\frac{1}{\Lambda_\phi}\right)\ \phi_0(x)\ \left[M_V^2 V^\alpha V_\alpha + \epsilon\ \left(\frac{F^{\alpha\beta}F_{\alpha\beta}}{4} - \frac{M_V^2}{2}V^\alpha V_\alpha\right)\right] \tag{11}$$

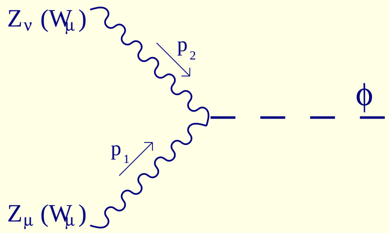
$$\mathcal{L}_{int}(\hat{\kappa}^2) = \frac{4}{\Lambda_\phi^2}\ \phi_0^2(x)\ \left[\frac{\eta}{2}M_V^2 V^\alpha V_\alpha + \epsilon\left(\frac{(2\eta-1)}{16}F^{\gamma\delta}F_{\gamma\delta} - \frac{(2\eta+1)}{8}M_V^2 V^\alpha V_\alpha\right)\right] \tag{12}$$

where $\eta \equiv a_0/c_0^2 = \mathcal{O}(1)$. (We have exchanged the two free parameters a_0 and c_0 of Eq. (9) for the two free parameters Λ_ϕ and η.)

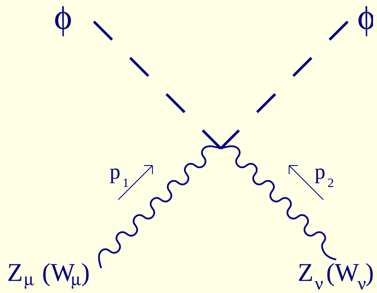
ϵ appears, for example, in the trace of the energy-momentum tensor of a massive vector

$$T^{V\ \mu}_{\ \ \ \mu} = -M_V^2 V^\alpha V_\alpha + \epsilon\mathcal{L}_V\,. \tag{13}$$

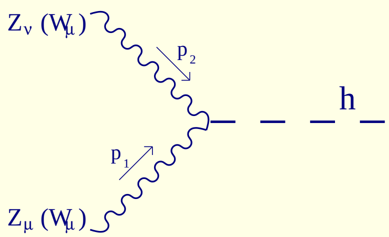
As the radion interacts via the trace of the energy-momentum tensor at one loop, one finds the term $\epsilon\mathcal{L}_V$ in the interaction Lagrangian. The resulting Feynman rules appear on the next page.



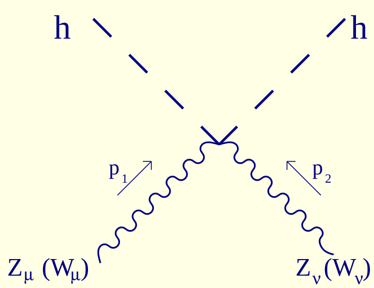
$$i\ g\frac{M_Z}{c_W}\eta_{\mu\nu}\left(c+a\gamma\right)\ -\ i\epsilon\ a\ V^A_{\mu\nu}$$
$$\left(i\ gM_W\eta_{\mu\nu}\left(c+a\gamma\right)-\ i\epsilon\ a\ V^A_{\mu\nu}\right) \tag{1}$$



$$i\ \frac{g^2}{2c_W^2}\eta_{\mu\nu}\left(c^2+4a^2\gamma^2\eta\right)\ +\ i\epsilon\ a^2\ W^A_{\mu\nu}$$
$$\left(i\ \frac{g^2}{2}\eta_{\mu\nu}\left(c^2+4a^2\gamma^2\eta\right)\ +\ i\epsilon\ \ a^2\ W^A_{\mu\nu}\right) \tag{2}$$



$$i\ g\frac{M_Z}{c_W}\eta_{\mu\nu}\left(d+b\gamma\right)-\ i\epsilon\ b\ V^A_{\mu\nu}$$
$$\left(i\ gM_W\eta_{\mu\nu}\left(d+b\gamma\right)-\ i\epsilon\ b\ V^A_{\mu\nu}\right) \tag{3}$$



$$i\ \frac{g^2}{2c_W^2}\eta_{\mu\nu}\left(d^2+4b^2\gamma^2\eta\right)\ +\ i\epsilon\ b^2\ W^A_{\mu\nu}$$
$$\left(i\ \frac{g^2}{2}\eta_{\mu\nu}\left(d^2+4b^2\gamma^2\eta\right)\ +\ i\epsilon\ b^2\ W^A_{\mu\nu}\right) \tag{4}$$

Figure 3: *Feynman rules to necessary order.* $\boldsymbol{V^A_{\mu\nu}}$ *and* $\boldsymbol{W^A_{\mu\nu}}$ *are the anomalous contributions to the trilinear and quartic interactions.* $\boldsymbol{V^A_{\mu\nu} = 2\left(\frac{\gamma}{v}\right)\left[\frac{1}{2}\left(p_1 . p_2\ \eta_{\mu\nu} - p_{1\mu}p_{2\nu}\right) + \frac{1}{2}M_V^2\eta_{\mu\nu}\right]}$, $\boldsymbol{W^A_{\mu\nu} = -2\left(\frac{\gamma}{v}\right)^2\left[(2\eta-1)(p_1 . p_2\ \eta_{\mu\nu} - p_{1\mu}p_{2\nu}) + (2\eta+1)M_V^2\eta_{\mu\nu}\right]}$.

Precision Electroweak Computations

The Feynman rules are employed to compute the S and T parameters:

$$S = \frac{4c_W^2 s_W^2}{\alpha}\left[\frac{\Pi_{ZZ}(M_Z)}{M_Z^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}\right] \tag{14}$$

$$T = \frac{1}{\alpha}\left[\frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}\right]. \tag{15}$$

For both S and T there are several types of contributions.

1. **S_i and T_i are from the sum over direct contributions of each eigenstate of the Higgs-radion system: $i = h, \phi$.**
2. **S^A and T^A are the so-called anomalous terms from radion loop $1/\epsilon$ poles being made finite by linear ϵ terms in the expansion of the radion-matter interaction Lagrangian.**
3. **Non-renormalizable operators arise from integrating out the heavier states that are associated with the SM brane scale Λ_ϕ and Kaluza-Klein excitations above that.**

They act as counter terms for divergences the radion creates from its non-renormalizable interactions with SM particles. Operators such as $\mathcal{O} \sim (H^\dagger D_\mu H)^2$ break isospin symmetry and can have an important effect on the T parameter. Other operators can contribute to the S parameter.

To account for these effects, we define the $\mathcal{O}(1)$ parameters a_M and a_X equivalently to Eq. (12.5) of Ref. [2], and compute the non-renormalizable operator (NRO) contributions to S and T.

The forms of the contributions listed above are:

$$S_i = -\frac{g_i^2}{\pi}\left(B_0(m_Z^2, m_i^2, m_Z^2) - \frac{B_{22}(m_Z^2, m_i^2, m_Z^2)}{m_Z^2} - B_0(0, m_i^2, m_Z^2) + \frac{B_{22}(0, m_i^2, m_Z^2)}{m_Z^2}\right) \quad (16)$$

$$S^A = \frac{v^2}{\pi\Lambda_\phi^2}\frac{1}{Z^2}\left(5\xi - \frac{5}{6} - \left(\eta - \frac{1}{2}\right)\frac{m_\phi^2\cos^2\theta + m_h^2\sin^2\theta}{m_Z^2}\right) \quad (17)$$

$$S^{NRO} = \frac{v^2}{\pi\Lambda_\phi^2}\frac{1}{Z^2}\left(Z^2\, a_X - \frac{(1-6\xi)^2}{12}\ln\frac{\Lambda_\phi^2}{m_Z^2}\right) \quad (18)$$

$$T_i = -\frac{g_i^2}{4\pi s_W^2}\left(B_0(0,m_i^2,m_W^2) - \frac{B_{22}(0,m_i^2,m_W^2)}{m_W^2} - \frac{B_0(0,m_i^2,m_Z^2)}{c_W^2} + \frac{B_{22}(0,m_i^2,m_Z^2)}{m_W^2}\right) \quad (19)$$

$$T^A = \frac{6}{16\pi}\frac{v^2}{c_W^2\Lambda_\phi^2}\frac{6\xi-1}{Z^2} \quad (20)$$

$$T^{NRO} = \frac{3}{16\pi}\frac{v^2}{c_W^2\Lambda_\phi^2}\left(-\frac{a_M}{3} + \frac{(1-6\xi)^2}{Z^2}\ln\frac{\Lambda_\phi^2}{m_Z^2}\right) \quad (21)$$

where the $g_h = g_{fVh} = d + b\gamma$ and $g_\phi = g_{fV\phi} = c + a\gamma$, and we keep only the finite part in the Passarino-Veltman integrals.

Note how $\eta = \frac{1}{2}$, $\eta < \frac{1}{2}$ and $\eta > \frac{1}{2}$ make for very different possibilities since $-(\eta - \frac{1}{2})$ multiplies $m_\phi^2 \cos^2\theta + m_h^2 \sin^2\theta$ in S^A. (No T term.) $\eta = 1/4$ is pre-stabilization .. value noted earlier.

The $\ln\frac{\Lambda_\phi^2}{m_Z^2}$ terms are from evolving from Λ_ϕ down to m_Z. The a_M and a_X defined at scale Λ_ϕ should be $\mathcal{O}(1)$ so long as KK graviton excitations are massive.

Precision Electroweak (PEW) Constraints

- The original work on this was in [2]. We (JFG, Toharia, Wells [9]) claim there were some inaccuracies and, in any case, the model dependence was not fully examined.

- In all our discussions, you should keep in mind that when $\xi \neq 0$, it is mere convention to decide what is the "Higgs" state and what is the "radion" state, especially when they are close in mass.

  We follow the choice of [7] in which the Higgs boson is defined as the state that becomes the Standard Model Higgs in the limit $\xi \to 0$.

- The precision constraints are most interesting when $|\xi|$ is near its upper limits. In this case, and for $\Lambda_\phi > 5~\mathrm{TeV}$ or so, the NRO contributions proportional to a_X and a_M, which implicitly include the KK-exchange contributions are (for $a_X, a_M \sim \mathcal{O}(1)$) quite small compared to the mixing effects, especially those involving $\ln \frac{\Lambda_\phi^2}{m_Z^2}$ and effects of η.

- **Two goals:**

  1. **Determine the portion of the theoretically allowed parameter space for a light Higgs, *e.g.* $m_h = 120$ GeV, that is disfavored by the PEW analysis.**
  2. **Determine the portion of parameter space consistent with PEW when both m_h and m_ϕ are above the SM 95% CL PEW limit of 237 GeV.**

- **In order to determine the allowed regions of S, T parameter space, we have employed the χ^2 ellipse parameterization from the LEPEWWG available before recent increase in m_t. The precise parameterization employed was**

$$\Delta\chi^2 = \frac{(S-S^0)^2}{(0.11)^2} + \frac{(T-T^0)^2}{(0.09)^2} - 2\frac{(S-S^0)(T-T^0)}{(0.11)(0.09)(0.735)(1-[0.735]^2)} \quad (22)$$

where $S^0 = 0.03$ and $T^0 = 0.12$ are the preferred values of S and T relative to those computed in the SM for $m_{H_{SM}} = 150$ GeV. For two parameters, the 68.27% (1σ) and 90% Confidence Levels (CL) correspond to $\Delta\chi^2 = 2.3$ and $\Delta\chi^2 = 4.61$, respectively.

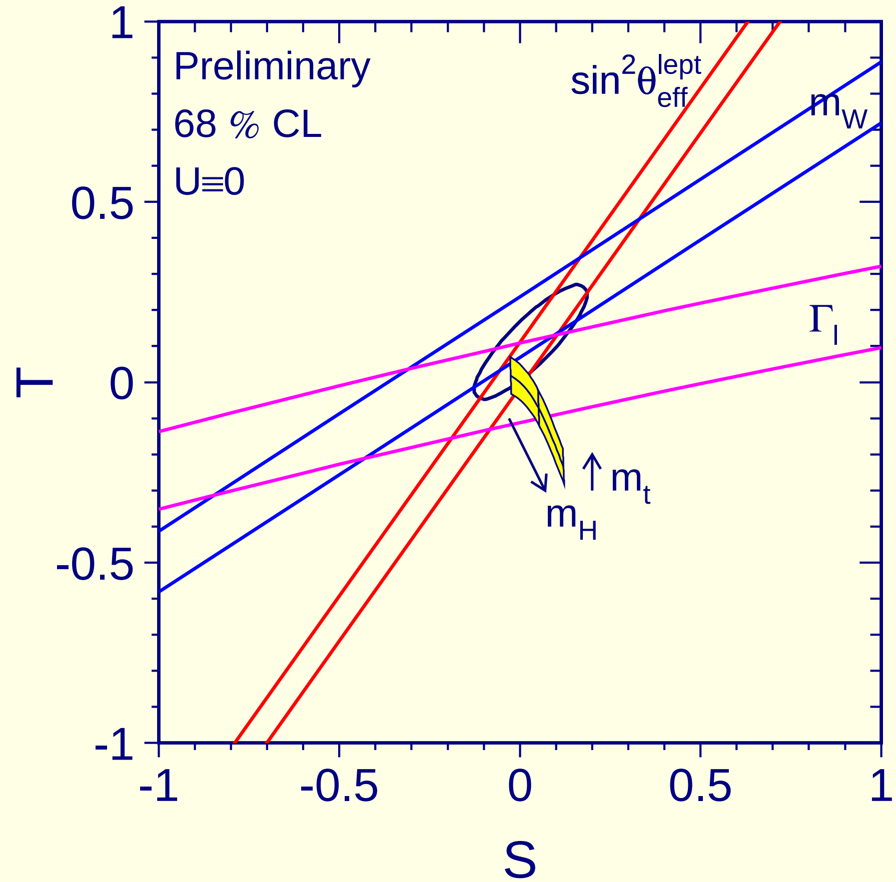


Figure 4: *Most recent LEPEWWG S, T plot for $U = 0$.*

- **The most recent 68% CL plot from LEPEWWG is above.**

Note: **As $m_{h_{SM}}$ increases, T becomes more negative and S becomes more positive.**

For $m_h \gtrsim 500~\mathrm{GeV}$, T is below the 90% CL ellipse and no shift in S (as possible from η term in S^A) will bring you back inside the ellipse.

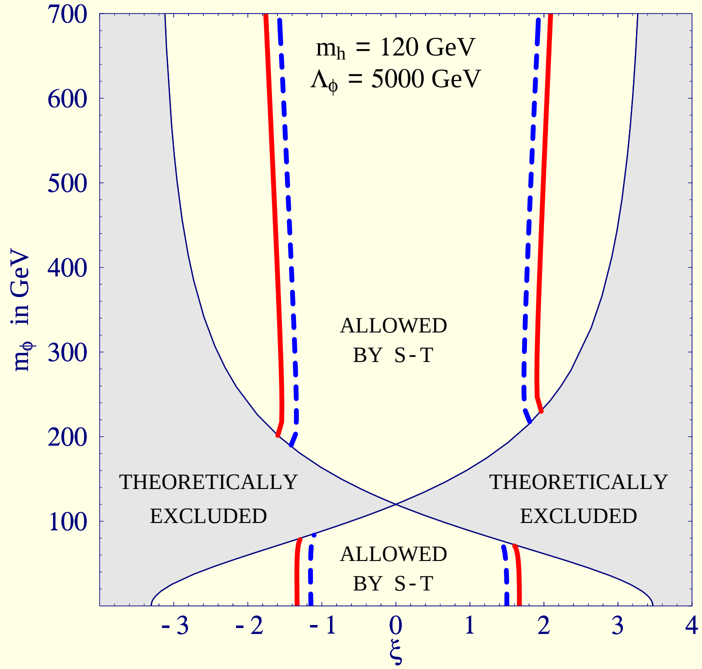


Figure 5: *Typical constraint on m_ϕ, ξ "hourglass" parameter space for $m_h = 120$ GeV. 68% and 90% CL contours are shown. Results not sensitive to η.*

- **For $m_h = 120$, the large $|\xi|$ part of hourglass is excluded at 90% CL.**

  For remaining part of parameter space, detection of the h at the LHC will be possible via usual production/decay modes, except for small (almost negligible for $L = 300\text{fb}^{-1}$) regions along the $m_\phi > m_h$, $\xi < 0$ and $m_\phi < m_h$, $\xi > 0$ hourglass boundaries (where $gg \to h \to \gamma\gamma$ is suppressed). The LC will cover even this region (since $g^2_{ZZh} > 0.1$ everywhere while the LC can probe down to $g^2_{ZZh} \sim 0.01$). The ϕ is not necessarily observable even at the LC when ξ is not large.

- **Another important result, is that m_h and m_ϕ can both be quite large without violating precision electroweak constraints, so long as $|\xi|$ is large enough. Benchmark graphs assume $\eta = 1/4$.**

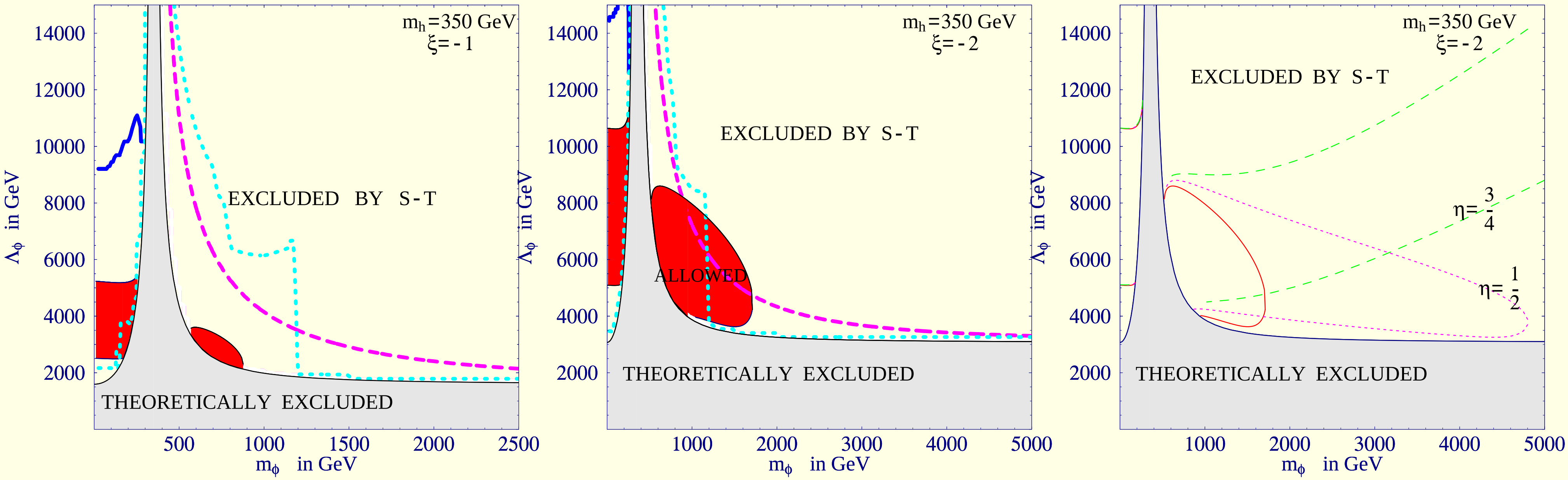


Figure 6: $m_h = 350$ GeV *illustration of how modest* ξ *values allow region for which* m_h *and* m_ϕ *are both relatively large. Contours shown are 90% CL. Between the tower and the thinner dotted (cyan) line,* $N_{SD}^{\phi}(LHC) > 5$ *(*$m_\phi > 1.2$ TeV *not considered).* h *detectable in all allowed regions, but above the blue line and thick dashed mauve line deviations of* h *from* $h_{\rm SM}$ *are* $< 10\%$ *at the LHC. Left and center plots and red contour of right plot are all for* $\eta = 1/4$*.*

- **Comments for $m_h = 350$ GeV:**

1. **Large m_h and m_ϕ are possible without violating precision electroweak constraints because the radion contributions compensate the Higgs contributions in the S, T plane.**
2. **If we increase the magnitude of ξ to -2 from $\xi = -1$, the allowed region increases substantially for the same value of $\eta = 1/4$ — larger negative ξ usually leads to a larger allowed region for heavy Higgs and radion.**
3. **For $\eta = 1/2$ the extra terms in S^A proportional to m_ϕ^2 and m_h^2 are absent.**
   PEW is ok for much larger values of m_ϕ.
4. **For $\eta > 1/2$ and $m_h \leq 500$ GeV the precision constraints can be satisfied for arbitrarily large m_ϕ, as illustrated by the non-closing $\eta = 3/4$ band.**
   One need only choose the value of Λ_ϕ so that the term in S^A of Eq. (17) proportional to m_ϕ^2/Λ_ϕ^2 has the appropriate negative value to compensate the $\Delta S > 0$ contribution from the (somewhat SM-like) h, thereby bringing the net S, T prediction back into the precision electroweak ellipse. This is possible for $m_h \lesssim 500$ GeV, *i.e.* such that the $\Delta T < 0$ contribution from the h is not so negative that the total T lies below the S, T-plane precision electroweak ellipse.
5. **ϕ not observable in all of allowed region. h deviations from $h_{\rm SM}$ can be $< 10\%$ in allowed region.**

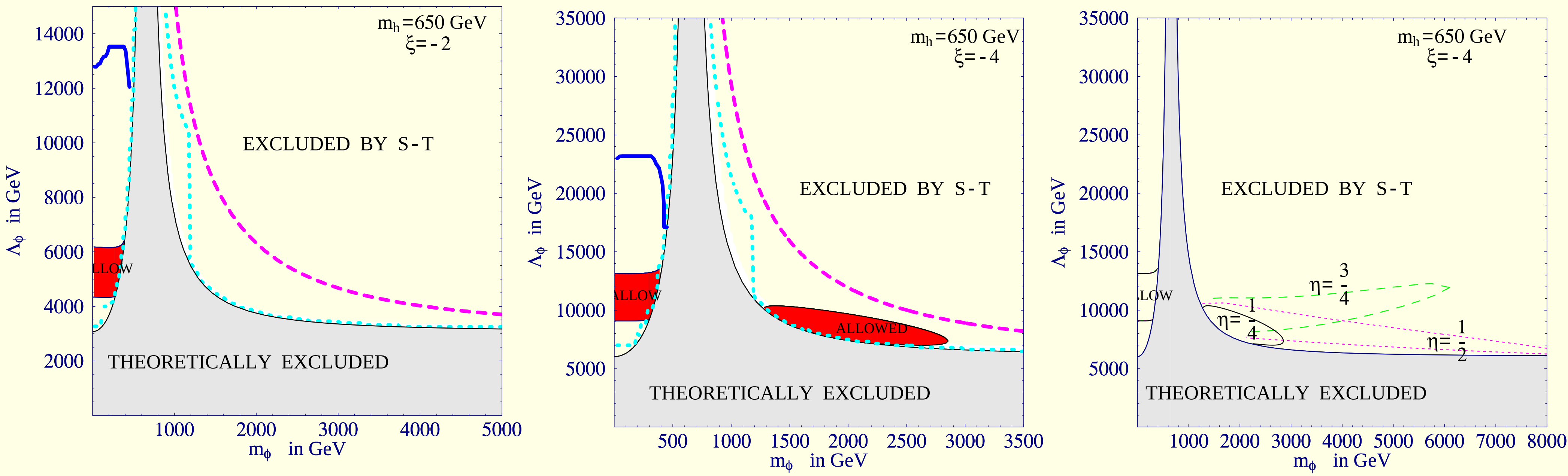


Figure 7: *$m_h = 650$ GeV illustration of how modest ξ values allow region for which m_h and m_ϕ are both relatively large. Contours shown are 90% CL. Between the tower and the thinner dotted (cyan) line, $N_{SD}^{\phi}(LHC) > 5$ ($m_\phi > 1.2$ TeV not considered). Above the blue line and thick dashed mauve line, it is hard to distinguish between the SM Higgs and the RS model h at the LHC — deviations are below 10%. Left and center plots and red contour of right plot are all for $\eta = 1/4$.*

- **Comments for $m_h = 650$ GeV.**

  1. Once $m_h \gtrsim 500$ GeV, we can only reenter PEW ellipse if Λ_ϕ is not too

large (so that the radion does not decouple as it does as $\Lambda_\phi \to \infty$).

2. Values of $|\xi| \gtrsim 3$ might not be easy to generate in the more fundamental theory.
   Also, the very large Λ_ϕ values relative to m_W required by PEW when $|\xi|$ is large might be associated with a significant hierarchy problem.

General Results:

1. A heavy Higgs boson and a heavy radion can both be above the putative Higgs mass upper limit from precision electroweak data.
   Not only is this an example of a case where the Higgs boson limit can be evaded, but it is also an uncommon example of a case for which it becomes *more difficult* to find correlating phenomena as the Higgs boson gets more massive.
   It is more difficult because Λ_ϕ and m_ϕ are also getting larger.
2. We are unlikely to observe the ϕ in much of the precision-electroweak-allowed region (exactly how much depends on the value of η).
   However, for (almost) all of the precision-allowed region, the h will be detectable at the LHC, but not necessarily distinguishable from the $h_{\rm SM}$.
   A future linear collider (with adequate $\sqrt{s}$) would always be able to detect the h. LC discovery of the ϕ is not guaranteed. $\Rightarrow$ Giga-Z to resolve situation.

RS Complications

- Introduction of a mass for the radion either by hand or via Goldberger-Wise approach leads to perturbations of the exact RS metric and/or curvature of the branes.
- However, if one introduces a bulk scalar with carefully tuned brane and bulk potential, it is possible to obtain a mass for the radion while retaining the RS metric as an exact solution. (JFG+Grzadkowski)

  But, then there is mixing between the radion, the Higgs and the KK excitations of the bulk scalar. Phenomenological details have not been worked out.
- Allowing the Higgs field (and others) to propagate in the bulk further complicates the possibilities. See

  1. T. G. Rizzo, JHEP 0206, 056 (2002) [arXiv:hep-ph/0205242].
- In particular, if fermions propagate in the bulk, mixing between the Standard Model top quark and its Kaluza Klein excitations generates

large contributions to the rho parameter and consequently restricts the fundamental RS scale to lie above 100 TeV.

This bound is circumvented in a 'mixed' scenario which localizes the third generation fermions on the TeV brane and allows the lighter generations to propagate in the full five-dimensional bulk. See:

1. J. L. Hewett, F. J. Petriello and T. G. Rizzo, JHEP 0209, 030 (2002) [arXiv:hep-ph/0203091].

- If gauge fields propagate in the bulk, one finds that large negative S and T parameters are induced in the effective theory at tree-level. This is a consequence of the shapes of the W and Z wave functions in the bulk.

  Such effects are generic in extra dimensional theories where the standard model (SM) gauge bosons have non-uniform wave functions along the extra dimension. See:

1. C. Csaki, J. Erlich and J. Terning, Phys. Rev. D 66, 064021 (2002) [arXiv:hep-ph/0203034].

- One might wish to consider multi-doublet models for the Higgs sector on (or off) the TeV brane. Was there a recent paper on this? I could not locate it.

Conclusions

- **While the light Higgs scenario with small mixing remains the most attractive way to satisfy PEW constraints in the RS model, one cannot rule out having m_h and m_ϕ considerably above the putative LEP $m_{h_{\rm SM}}$ limit.**

- **In these cases, although the h is very likely to be observable at the LHC there is no guarantee that other phenomena will become experimentally observable (not even deviations in h properties are guaranteed to be seen) at the LHC that will give us a hint of what is going on.**

- **The same remark applies to an LC with limited energy. In fact, a $1~{\rm TeV}$ LC is needed to guarantee h discovery for RS parameter choices that are not "unreasonable". However, deviations in h properties are much more likely to be seen than at the LHC because of greater measurement precision.**

  If a heavy h is seen, but the ϕ is beyond LC reach (or is weakly produced), Giga-Z measurements will help greatly to clarify the situation, and might tell us where/how to look for the ϕ at the LHC. (an argument for concurrent LHC/LC operation)