Higgs Bosons in the NMSSM and its $U(1)$ Extensions

Jack Gunion
U.C. Davis

SUSY08, Seoul, Korea, June 16, 2008
1. The “ideal” Higgs boson.
2. Why we must go beyond the SM and MSSM to have an ideal Higgs boson.
3. Extensions of the MSSM.
4. The NMSSM ideal Higgs boson.
5. Implications for Higgs detection at the LHC.
6. Other experimental consequences.
7. Impact of $U(1)$ extensions/variations of the NMSSM.
The NMSSM is an attractive possibility!

**Warning:** To understand why the NMSSM path is particularly attractive, one will have to worry about many types of “fine-tuning”: i) quadratic-divergence; ii) electroweak; iii) $\mu$; iv) dark matter; v) electroweak baryogenesis; and vi) light-pseudoscalar.
Criteria for an ideal Higgs theory

• The theory should allow for a light Higgs boson without fine-tuned cancellation of the quadratic divergence. This I term the “quadratic-divergence” fine-tuning issue.

• Whatever theory is employed to remove the quadratic divergence should also predict $m_Z^2$ or equivalently $v^2$ without having to fine-tune the high scale (e.g. GUT-scale) parameters of the theory. I term this the “electroweak symmetry breaking” (“EWSB”) fine-tuning issue.

We will return to discuss these two issues more thoroughly. For now, we continue with purely phenomenological criteria.

• The theory should predict a Higgs (or collection of Higgses) with SM coupling-squared (or summed coupling-squared) to $WW$, $ZZ$ and with mass (or weighted average mass) in the range preferred by precision electroweak data. The latest plot is:
At 95% CL, $m_{h_{\text{SM}}} < 160$ GeV and the $\Delta \chi^2$ minimum is near 85 GeV when all data are included.

The latest $m_W$ and $m_t$ measurements also prefer $m_{h_{\text{SM}}} \sim 100$ GeV.

The blue-band plot may be misleading due to the discrepancy between the "leptonic" and "hadronic" measurements of $\sin^2 \theta_W^{\text{eff}}$, which yield
\[ \sin^2 \theta_W^{\text{eff}} = 0.23113(21) \] and \[ \sin^2 \theta_W^{\text{eff}} = 0.23222(27), \] respectively. The SM has a CL of only 0.14 when all data are included.

If only the leptonic \( \sin^2 \theta_W^{\text{eff}} \) measurements are included, the SM gives a fit with CL near 0.78. However, the central value of \( m_{h_{\text{SM}}} \) is then near 50 GeV with a 95\% CL upper limit of \( \sim 105 \) GeV (Chanowitz, xarXiv:0806.0890 e.g.).

A \( Z' \) can affect these conclusions, but also need not; more later.

- Thus, in an ideal model, a Higgs with SM-like \( ZZ \) coupling should have mass no larger than 105 GeV. Our generic notation will be \( H \).

But, at the same time, it should avoid the LEP limits on such a light Higgs. One generic possibility is for its decays to be non-SM-like.
Table 1: LEP $m_H$ Limits for a $H$ with SM-like $ZZ$ coupling, but varying decays.

<table>
<thead>
<tr>
<th>Mode Limit (GeV)</th>
<th>SM modes 114.4</th>
<th>$2\tau$ or $2b$ only 115</th>
<th>$2j$ 113</th>
<th>$WW^* + ZZ^*$ 100.7</th>
<th>$\gamma\gamma$ 117</th>
<th>$\not{E}$ 114</th>
<th>$4e, 4\mu, 4\gamma$ 114?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4b$ 110</td>
<td>$4\tau$ 86</td>
<td>any (e.g. $4j$) 82</td>
<td>$2f + \not{E}$ 90?</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that to have $m_H \leq 105$ GeV requires one of the final three modes or something even more exotic. We also note that the mode-independent limit of 82 GeV still makes some assumptions about the nature of the final state and for some final states is probably (no explicit statements from LEP collaborations are available) lower.

- Perhaps the ideal Higgs should be such as to predict the $2.3\sigma$ excess at $M_{b\bar{b}} \sim 98$ GeV seen in the $Z + b\bar{b}$ final state.
Figure 1: Plots for the $Zb\bar{b}$ final state.

Possibilities for explaining the excess are:

1. Very roughly, to give the excess seen $B(H \rightarrow b\bar{b}) \sim 0.1 B(H \rightarrow b\bar{b})_{SM}$ is required if $H$ has SM $ZZ$ coupling.
2. Or, you could have SM-like decay pattern but $g_{ZZH}^2 \sim 0.1 g_{ZZh_{SM}}^2$. However, in this latter case there must be other Higgs bosons with “average” mass near 100 GeV such that $\sum_i g_{ZZh_i}^2 = g_{ZZh_{SM}}^2$. 
• Number 1 is the simplest possibility, and is easily achieved.

Indeed, almost any additional decay channel will severely suppress the $b\bar{b}$ branching ratio.

A Higgs of mass, *e.g.*, 100 GeV has a decay width into Standard Model particles that is only 2.6 MeV, or about $10^{-5}$ of its mass.

It doesn’t take a large Higgs coupling to some new particles for the decay width to these new particles to dominate over the decay width to SM particles (early references = JFG+Haber, 1984; Li, 1985; JFG+Gunion, 1986; NMSSM context: JFG+Haber+Moroi, hep-ph/9610337; Dobrescu+Matchev, hep-ph/0008192—full review Chang+Dermisek+JFG+Weiner, arXiv:0801.4554).

For example, compare the decay width for $h \to b\bar{b}$ to that for $h \to aa$, where $a$ is a light pseudoscalar Higgs boson. Writing $\mathcal{L} \ni g_{haa}h_{aa}$ with $g_{haa} = c \frac{g m_h^2}{2 m_W}$ and ignoring phase space suppression, we find

$$\frac{\Gamma(h \to aa)}{\Gamma(h \to b\bar{b})} \sim 310 c^2 \left( \frac{m_h}{100 \text{ GeV}} \right)^2. \quad (1)$$
This expression includes QCD corrections to the $b\bar{b}$ width as given in HDECAY which decrease the leading order $\Gamma(h \to b\bar{b})$ by about 50%.

The decay widths are comparable for $c \sim 0.057$ when $m_h = 100$ GeV. Values of $c$ at this level or substantially higher (even $c = 1$ is possible) are generic in BSM models containing an extended Higgs sector.

- Regarding possibility #2 (many light Higgs bosons), one easily arrange to satisfy LEP limits and fit precision electroweak data (Espinosa+JFG, hep-ph/9807275) and delay quadratic fine-tuning to large $\Lambda$.

- Finally, perhaps the Higgs should be such as to allow for a strong 1st-order phase transition in the early universe for electroweak baryogenesis. Easiest if $m_H \lesssim 100$ GeV for $H$ with SM $WW/ZZ$ coupling.
Why we must go beyond the SM and MSSM to have an ideal Higgs

Why not the SM

- $m_{h_{SM}} < 114.4$ GeV is inconsistent with direct LEP limit.

- Quantum corrections to the Higgs mass-squared lead to severe quadratic-divergence fine-tuning unless new physics enters at a low scale. After including the one loop corrections we have

$$m_{h_{SM}}^2 = \mu^2 + \frac{3\Lambda^2}{32\pi^2 v^2}(2m_W^2 + m_Z^2 + m_{h_{SM}}^2 - 4m_t^2)$$ (2)

where $\mu^2 = 2\lambda v^2_{SM}$, and $\lambda$ is the quartic coupling in the Higgs potential.

The $\mu^2$ and $\Lambda^2$ terms have entirely different sources, and so a value of $m_{h_{SM}} \sim m_Z$ should not arise by fine-tuned cancellation between the two terms. $\Rightarrow \Lambda \lesssim 1$ TeV.
Go to SUSY or extra-dimensions, or both, to solve completely (as opposed to delaying the solution to some uncertain ”ultraviolet completion”).

Why Supersymmetry

- SUSY is mathematically intriguing.
- SUSY is naturally incorporated in string theory.
- Elementary scalar fields have a natural place in SUSY, and so there are candidates for the spin-0 fields needed for electroweak symmetry breaking and Higgs bosons.
- Dark matter = Lightest Supersymmetric Particle (LSP) is natural.
- SUSY cures the quadratic-divergence fine-tuning problem, and it does so without EWSB fine-tuning (see definition below) provided the SUSY breaking scale is $\lesssim 500$ GeV.
In particular, the top quark loop (which comes with a minus sign) is canceled by the loop of the spin-0 partner "stop" (which loop comes with a plus sign). Thus, \( \Lambda^2 \) is effectively replaced by \( \overline{m}_t^2 \equiv \frac{1}{2} (m_{tL}^2 + m_{tR}^2) \).

- If we assume that all sparticles reside at the \( \mathcal{O}(1 \ \text{TeV}) \) scale and that \( \mu \) is also \( \mathcal{O}(1 \ \text{TeV}) \), then, the MSSM has two particularly wonderful properties.

1. **Gauge Coupling Unification**
   
   Couplings unify very precisely at a high scale of order \( f_{ew} \times 10^{16} \ \text{GeV} \).

2. **RGE EWSB**
   
   Starting with soft-SUSY-breaking masses-squared at \( M_U \), the RGE’s predict that the top quark Yukawa coupling will drive one of the soft-SUSY-breaking Higgs masses squared \( (m_{H_u}^2) \) negative at a scale of order \( Q \sim m_Z \), thereby automatically generating electroweak symmetry breaking \( (\langle H_u \rangle = h_u, \langle H_d \rangle = h_d) \).

**Why not the MSSM**

1. **The \( \mu \) parameter** in \( W \ni \mu \hat{H}_u \hat{H}_d \),\(^1\) is dimensionful, unlike all other

\(^1\)Hatted (unhatted) capital letters denote superfields (scalar superfield components).
superpotential parameters. A big question is why is it $\mathcal{O}(1 \ TeV)$ (as required for EWSB and $m_{\tilde{\chi}_1^\pm}$ lower bound), rather than $\mathcal{O}(M_U, M_P)$ or 0.

Getting the appropriate $\mu$ value is a severe fine-tuning problem for the MSSM. There are many suggested approaches, but none are really compelling.

2. $m_Z$ IS FINE-TUNED.

So long as $m^2_t$ is not too far above $m^2_Z$, getting $m^2_Z$ correct does not involve any highly precise cancellations of the different contributions to $m^2_Z$ (really the Higgs field vev-squared $v^2_{SM}$) as determined by evolving the SUSY breaking parameters from $M_U$ to $m_Z$.

However, such a choice for $m^2_t$ creates a problem!!!!

3. The Higgs Mass

In the presence of soft-SUSY-breaking, the light Higgs has $(\tan \beta \equiv$
$h_u/h_d$:

$$m_h^2 \sim m_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} v^2 y_t^4 \sin^4 \beta \log \left( \frac{m_t^2}{m_t^2} + \frac{X_t^2}{m_t^2} \left[ 1 - \frac{X_t^2}{12m_t^2} \right] \right)$$

large $\tan \beta$ \begin{align*}
\sim &(91 \text{ GeV})^2 + (38 \text{ GeV})^2 \log \left( \frac{m_t^2}{m_t^2} + \ldots \right). 
\end{align*}

Here, $X_t = A_t - \frac{\mu}{\tan \beta}$ determines the amount of stop-squark mixing.

For stop masses $\sim 2m_t$, $m_h \sim 100$ GeV, in perfect accord with precision electroweak data and EWSB fine-tuning is minimal, i.e. $m_Z$ is not sensitive to GUT scale parameters.

The Problem: In such cases, $h$ is rather SM-like and LEP rules out the $h$.

The only escape is to have strong Higgs mixing, for which the $h$ lies below 114 GeV but does not have SM-like $ZZ$ coupling. In this case, most of the SM $ZZ$ coupling resides in the heavier $H$ with mass above 114 GeV so that precision electroweak fits are not good. (Also for the strong-mixing cases there is significant EWSB fine-tuning problem, i.e. the precise value of $m_Z$ is very sensitive to the GUT scale parameters, see below.)
4. **EWSB Fine-tuning** (different from quadratic-divergence fine-tuning)

\[ F = \max_p \left| \frac{p}{m_Z} \frac{\partial m_Z}{\partial p} \right|, \]  

(4)

where \( p \in \{ M_{1,2,3}, m_Q^2, m_U^2, m_D^2, m_{H_u}^2, m_{H_d}^2, \mu, A_t, B\mu, \ldots \} \) (all at \( M_U \)).

These \( p \)'s are the GUT-scale parameters that determine all the \( m_Z \)-scale SUSY parameters, and these (via RGEs) determine \( m_Z^2 \propto v_{SM}^2 \).

\( F > 20 \) means worse than 5% fine-tuning of the GUT-scale parameters is required to get the right value of \( m_Z \). This would be bad.

5. So, what is the smallest \( F \) that can be achieved in the MSSM?

(a) For most of parameter space, \( m_h > 114 \text{ GeV} \) is required. Then, \( F > 100 \) or so unless there is large stop mixing, in which case \( F > 30 \) at best.

(b) For special cases characterized by large Higgs mixing, \( F \) can be reduced to 16 at best (6% fine-tuning), but this part of parameter space

Absence of EWSB fine-tuning corresponds to $F \sim 5$, i.e. $\lesssim 20\%$ tuning of GUT-scale parameters.

6. For the part of MSSM parameter space allowed after Higgs mass constraints are imposed, electroweak baryogenesis, and to some extent correct relic LSP abundance, require fine-tuning of soft-SUSY-breaking parameters.
The Extensions of the MSSM

Depending on the symmetry, the MSSM can be extended in various ways:


2. the Minimal Next-to-minimal Supersymmetric SM (MNSSM) (or the nearly Minimal Supersymmetric SM (nMSSM)) (Panagiotakopoulos:1999ah, Panagiotakopoulos:2000wp, Dedes:2000jp, Menon:2004wv), and


4. The Exceptional Supersymmetric SM (ESSM) (King:2005jy) is, to a large extent, similar to the UMSSM.

5. The secluded $U(1)'$ model (sMSSM) (Erler:2002pr) has multiple Higgs singlets and, in a decoupling limit of the extra singlets, the low energy spectrum is similar to the nMSSM.
Table 2 (from Barger, Langacker, Lee, Shaughnessy, CPNSH and hep-ph/0603247) shows the symmetry, superpotential and the Higgs spectrum of several models.

**Table 2: Higgs bosons of the MSSM and several of its extensions**

<table>
<thead>
<tr>
<th>Model</th>
<th>Symmetry</th>
<th>Superpotential</th>
<th>CP-even</th>
<th>CP-odd</th>
<th>Charged</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSSM</td>
<td>–</td>
<td>$\mu \hat{H}_u \cdot \hat{H}_d$</td>
<td>$H_1, H_2$</td>
<td>$A_1$</td>
<td>$H^\pm$</td>
</tr>
<tr>
<td>NMSSM</td>
<td>$Z_3$</td>
<td>$\lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{\kappa}{3} \hat{S}^3$</td>
<td>$H_1, H_2, H_3$</td>
<td>$A_1, A_2$</td>
<td>$H^\pm$</td>
</tr>
<tr>
<td>nMSSM</td>
<td>$Z_5^R, Z_7^R$</td>
<td>$\lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + t_F \hat{S}$</td>
<td>$H_1, H_2, H_3$</td>
<td>$A_1, A_2$</td>
<td>$H^\pm$</td>
</tr>
<tr>
<td>UMSSM</td>
<td>$U(1)'$</td>
<td>$\lambda \hat{S} \hat{H}_u \cdot \hat{H}_d$</td>
<td>$H_1, H_2, H_3$</td>
<td>$A_1$</td>
<td>$H^\pm$</td>
</tr>
<tr>
<td>sMSSM</td>
<td>$U(1)'$</td>
<td>$\lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \lambda_s \hat{S}_1 \hat{S}_2 \hat{S}_3$</td>
<td>$H_1, \ldots, H_6$</td>
<td>$A_1, \ldots, A_4$</td>
<td>$H^\pm$</td>
</tr>
</tbody>
</table>

With apologies, I won’t consider the nMSSM (or MNSSM) since they introduce a dimensionful parameter in $W$.

We will be adopting a rather different motivation for looking at these models than the original one.

Previously, these extensions were used to raise the Higgs mass above 114 GeV for lower $\overline{m}_t$ (thereby reducing EWSB fine-tuning) and lower $\tan \beta$.

The precision electroweak data now imply that this is not the most motivated route. It is best to use the NMSSM or $U(1)$ MSSM to allow a light Higgs with very SM-like $ZZ, WW$ coupling to evade LEP limits via extra decays.
The NMSSM ideal Higgs boson

- The NMSSM is defined by adding a single SM-singlet superfield $\hat{S}$ to the MSSM and imposing a $Z_3$ symmetry on the superpotential, implying

$$W = \lambda \hat{S}\hat{H}_u\hat{H}_d + \frac{\kappa}{3}\hat{S}^3$$  \hspace{1cm} (5)

The reason for imposing the $Z_3$ symmetry is that then only dimensionless couplings $\lambda$, $\kappa$ enter. There can in particular be no $\mu\hat{H}_u\hat{H}_d$ term — the effective value of $\mu$ and, indeed, all dimensionful parameters will then be determined by the soft-SUSY-breaking parameters.

- The presence of the $\hat{S}^3$ term is crucial to explicitly break a Peccei Quinn symmetry that would lead to a massless axion problem when SUSY is spontaneously broken.

- The $Z_3$ symmetry must be slightly broken to avoid a cosmological domain wall problem, but this can be arranged without affecting any low scale

- The soft-SUSY-breaking terms corresponding to the terms in $W$ are:

$$\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3. \quad (6)$$

- The tree level Higgs potential is given by

$$V = \lambda^2(|H_u|^2|S|^2 + |H_d|^2|S|^2 + |H_u \cdot H_d|^2) + \kappa^2|S|^2$$

$$+ \lambda\kappa(H_u \cdot H_d S^{*2} + \text{h.c.}) + \frac{1}{4}g^2(|H_u|^2 - |H_d|^2)^2$$

$$+ \frac{1}{2}g_2^2|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 + m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_S^2|S|^2$$

$$+ (\lambda A_\lambda H_u \cdot H_d S + \frac{1}{3}\kappa A_\kappa S^3 + \text{h.c.}) \quad (7)$$

where $g^2 = \frac{1}{2}(g_1^2 + g_2^2)$. Note: quartic $|S|^4$ terms come from $\kappa$ part of $W$.

There is a long list of good features of the NMSSM.

1. The NMSSM maintains all the attractive features (GUT unification, RGE EWSB) of the MSSM while avoiding all its problems.
2. The NMSSM solves the $\mu$ problem. The $\mu$ parameter is automatically generated by $\langle S \rangle$ leading to $\mu_{\text{eff}} \hat{H}_u \hat{H}_d$ with $\mu_{\text{eff}} = \lambda \langle S \rangle$. $\mu$ is automatically of order a TeV (as required) since $\langle S \rangle$ is of order the SUSY-breaking scale, which will be below a TeV.

3. Further, there are very attractive scenarios in the NMSSM with no EWSB fine-tuning. To avoid EWSB fine-tuning, sparticles must be light, especially the stops; the optimal is $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \sim 350$ GeV, somewhat above Tevatron limits but accessible at the LHC. Also, the gluino should be light.

For such stop masses, $m_{h_1} \sim 100$ GeV is predicted. This is perfect for precision electroweak, but what about LEP?
Figure 2: $F$ vs. $m_{h_1}$ for $M_{1,2,3} = 100, 200, 300$ GeV and $\tan\beta = 10$. Small $\times$ = no constraints other than global and local minimum, no Landau pole before $M_U$ and neutralino LSP. The O’s = stop and chargino limits imposed, but NO Higgs limits. The □’s = all LEP single channel, in particular $Z + 2b$, Higgs limits imposed. The large FANCY CROSSES are after requiring $m_{a_1} < 2m_b$, so that LEP limits on $Z + b’$s, where $b’$s = $2b + 4b$, are not violated. Taken from Dermisek+JFG, arXiv:0705.4387.
4. The points with smallest $F$ are such that $m_{h_1} \sim 100$ GeV, $g_{ZZ h_1} \sim g_{ZZ h_{SM}}$ but $B(h_1 \rightarrow a_1 a_1) > 0.75$, with $m_{a_1} < 2m_b$ to avoid LEP limits on $Z + b's$ ($b's = 2b + 4b$).

In the $h_1 \rightarrow a_1 a_1 \rightarrow \tau^+ \tau^- \tau^+ \tau^-$ channel, the LEP lower limit is $m_{h_1} > 87$ GeV.

In the $h_1 \rightarrow a_1 a_1 \rightarrow 4j$ channel, the LEP lower limit is $m_{h_1} > 82$ GeV.

5. If $B(h_1 \rightarrow a_1 a_1) > 0.75$ to avoid LEP limits then $B(h_1 \rightarrow b \bar{b}) \sim 0.1$ is common and the $2.3\sigma$ LEP excess near $m_{b \bar{b}} \sim 98$ GeV in $e^+e^- \rightarrow Z + b's$ is perfectly explained.

6. GUT-scale boundary conditions are generic 'no-scale'. That is, for the lowest $F$ points we are talking about, almost all the soft-SUSY-breaking parameters are small at the GUT scale. This is a particularly attractive possibility in the string theory context.
One possible issue for the proposed scenario.

Is a light $a_1$ with the right properties natural, or does this require fine-tuning of the GUT-scale parameters?

- The naturalness of a light-$a_1$ scenario is the topic of Dermisek +JFG, hep-ph/0611142. I only state some results.

- The NMSSM has a natural $U(1)_R$ symmetry when the soft-SUSY-breaking $A_\lambda$ and $A_\kappa$ in $V \ni \lambda A_\lambda S H_u H_d + \frac{1}{3} \kappa A_\kappa S^3$ are set to zero.

If this limit is applied at scale $m_Z$, then, $m_{a_1} = 0$.

But, it turns out that then $B(h_1 \rightarrow a_1 a_1) \lesssim 0.3$ which does not allow escape from the LEP limit.

However, the much more natural idea would be to impose the $U(1)_R$ symmetry at the GUT scale.

Then, the renormalization group often generates exactly the values for the parameters needed to obtain a light $a_1$ with large $B(h_1 \rightarrow a_1 a_1)$. 
We measure the tuning needed to get small $m_{a_1}$ and large $B(h_1 \to a_1 a_1)$ using $G$ (the "light-$a_1$ tuning measure"). We want small $G$. 

Figure 3: $G$ vs. $F$ for $M_{1,2,3} = 100, 200, 300$ GeV and $\tan \beta = 10$ for points with $F < 15$ having $m_{a_1} < 2m_b$ and large enough $B(h_1 \to a_1 a_1)$ to escape LEP limits. The color coding is: blue = $m_{a_1} < 2m_\tau$; red = $2m_\tau < m_{a_1} < 7.5$ GeV; green = $7.5$ GeV < $m_{a_1} < 8.8$ GeV; and black = $8.8$ GeV < $m_{a_1} < 9.2$ GeV. Really small $G$ requires $m_{a_1} > 7.5$ GeV.

A phenomenologically important quantity is $\cos \theta_A$, the coefficient of the
MSSM-like doublet Higgs component of the $a_1$:

$$a_1 = \cos \theta_A A_{MSSM} + \sin \theta_A A_S.$$  \hspace{1cm} (8)

Figure 4: $G$ vs. $\cos \theta_A$ for $M_{1,2,3} = 100, 200, 300$ GeV and $\tan \beta = 10$ from $\mu_{\text{eff}} = 150$ GeV scan (left) and for points with $F < 15$ (right) having $m_{a_1} < 2m_b$ and large enough $B(h_1 \rightarrow a_1 a_1)$ to escape LEP limits. The color coding is: blue = $m_{a_1} < 2m_\tau$; red = $2m_\tau < m_{a_1} < 7.5$ GeV; green = $7.5$ GeV $< m_{a_1} < 8.8$ GeV; and black = $8.8$ GeV $< m_{a_1} < 9.2$ GeV.
We observe:

1) The blue ‘+’s, which are the points with $m_{a_1} < 2m_\tau$, have rather large $G$ and tend to require precise tuning of $A_\lambda$ and $A_\kappa$ (the relevant soft parameters) at scale $M_U$.

2) Really small $G$ occurs for $m_{a_1} > 7.5$ GeV and $\cos \theta_A \sim -0.1$.

3) A lower bound on $|\cos \theta_A|$ is apparent. It arises because $B(h_1 \rightarrow a_1a_1)$ falls below 0.75 for too small $|\cos \theta_A|$.

4) The preferred small $\cos \theta_A \sim -0.1$ implies that the $a_1$ is mainly singlet and its coupling to $b\bar{b}$, being proportional to $\cos \theta_A \tan \beta$ is not enhanced. However, it is also not that suppressed, which has important implications.
Detecting the $h_1$ and/or the $a_1$.

**LHC**

All standard LHC channels fail: e.g. $B(h_1 \rightarrow \gamma\gamma)$ is much too small because of large $B(h_1 \rightarrow a_1a_1)$.

The possible new LHC channels include:

1. $WW \rightarrow h_1 \rightarrow a_1a_1 \rightarrow 4\tau$.

   Looks moderately promising (see, e.g., A. Belyaev, S. Hesselbach, S. Lehti, S. Moretti, A. Nikitenko and C. H. Shepherd-Themistocleous, arXiv:0805.3505 [hep-ph].) but far from definitive results at this time.

2. $t\bar{t}h_1 \rightarrow t\bar{t}a_1a_1 \rightarrow t\bar{t}\tau^+\tau^-\tau^+\tau^-$.

   Study begun.

3. $\tilde{\chi}_2^0 \rightarrow h_1\tilde{\chi}_1^0$ with $h_1 \rightarrow a_1a_1 \rightarrow 4\tau$. 

(Recall that the $\tilde{\chi}_2^0 \rightarrow h_1\tilde{\chi}_1^0$ channel provides a signal in the MSSM when $h_1 \rightarrow b\bar{b}$ decays are dominant.)

4. Last, but definitely not least: diffractive production $pp \rightarrow pph_1 \rightarrow ppX$.

The mass $M_X$ can be reconstructed with roughly a $1 - 2$ GeV resolution, potentially revealing a Higgs peak, independent of the decay of the Higgs.

The event is quiet so that the tracks from the $\tau$’s appear in a relatively clean environment, allowing track counting and associated cuts.

Our (JFG, Forshaw, Pilkington, Hodgkinson, Papaefstathiou: arXiv:0712.3510) results are that one expects about 3 clean, i.e. reconstructed and tagged, events with very small background ($\sim 0.1$ event) per $90 \text{ fb}^{-1}$ of luminosity.

$\Rightarrow$ clearly a high luminosity game.

We estimate the significance, $S$, of the observation by equating the probability of $s + b$ events given a Poisson distribution with mean $b$ to the probability of $S$ standard deviations in a Gaussian distribution.
Signal significances are plotted in Fig. 5 for a variety of luminosity and triggering assumptions.

Figure 5: (a) The significance for three years of data acquisition at each luminosity. (b) Same as (a) but with twice the data. Different lines represent different $\mu$ trigger thresholds and different forward detector timing. Some experimentalists say more efficient triggering is possible, doubling the number of events at given luminosity.

CMS folk claim we can increase our rates by about a factor of 2 to 3 using additional triggering techniques.
The Collinearity Trick

- Since $m_a \ll m_h$, the $a$’s in $h \rightarrow aa$ are highly boosted.  
  $\Rightarrow$ the $a$ decay products will travel along the direction of the originating $a$.  
  $\Rightarrow$ $p_a \propto \sum \text{visible 4-momentum}$ of the charged tracks in its decay.  
  Labeling the two $a$’s with indices 1 and 2 we have

$$p_{\text{vis}}^i = f_i p_{a,i}$$  \hspace{1cm} (9)

where $1 - f_i$ is the fraction of the $a$ momentum carried away by neutrals.

- The accuracy of this has now been tested in the $pp \rightarrow pph$ case, but after other cuts it is almost not needed.

- This reconstruction procedure will most likely be quite crucial in the $WW \rightarrow h$ case.

$$pp \rightarrow pph \text{ with } h \rightarrow aa$$
• The two unknowns, $f_1$ and $f_2$ can be determined using information from the forward proton detectors:

\[ p_{a,1} + p_{a,2} = p_h \]  \hspace{1cm} (10)

and $p_h$ is measured.

• In fact, the situation is over constrained. Although the transverse momentum of the Higgs can be measured using the forward detectors it will typically be rather small. Assuming it to be zero leaves us with the three equations:

\[ \left( p^{vis}_{1} \right)_{x,y} f_1 + \left( p^{vis}_{2} \right)_{x,y} f_2 = 0 \]  \hspace{1cm} (11)

and

\[ \left( p^{vis}_{1} \right)_{z} f_1 + \left( p^{vis}_{2} \right)_{z} f_2 = (\xi_1 - \xi_2) \frac{\sqrt{s}}{2} \]  \hspace{1cm} (12)

where $x$ and $y$ label the directions transverse to the beam axis and the $1 - \xi_i$ are the longitudinal momenta of the outgoing protons expressed as fractions of the incoming momenta.
Solving (11) and (12) gives

\[
f_1 = \frac{2}{(\xi_1 - \xi_2)\sqrt{s}} \left[ (p_{\text{vis}}^1)_z - \frac{(p_{\text{vis}}^2)_z(p_{\text{vis}}^1)_{x,y}}{(p_{\text{vis}}^2)_{x,y}} \right], \tag{13}
\]

\[
f_2 = -\frac{(p_{\text{vis}}^2)_{x,y}}{(p_{\text{vis}}^1)_{x,y}} f_1. \tag{14}
\]

Equations (13) and (14), provide two solutions depending on whether we solved using the \((x, z)\) or \((y, z)\) pair of equations.

Note that we are able to make \(4 = 2 \times 2\) a mass measurements per event.

Figure 6 shows the distribution of masses obtained for 180 fb\(^{-1}\) of data collected at \(3 \times 10^{33}\) cm\(^{-2}\)s\(^{-1}\), corresponding to about 6 Higgs events and therefore 24 \(m_\alpha\) entries.

In the right-hand figure the integer in each box labels one of the 6 signal events.

By considering many pseudo-data sets, we conclude that a typical experiment would yield \(m_\alpha = 9.3 \pm 2.3\) GeV, which is in re-assuringly
good agreement with the expected value of 9.7 GeV.

Figure 6: (a) A typical $a$ mass measurement. (b) The same content as (a) but with the breakdown showing the 4 Higgs mass measurements for each of the 6 events, labeled 1 − 6 in the histogram.

\[ WW \rightarrow h \]

- For $m_h = 100$ GeV and SM-like $WWh$ coupling, $\sigma(WW \rightarrow h) \sim 7$ pb, implying $7 \times 10^5$ events before cuts for $L = 100$ fb$^{-1}$. 
• In this case, we do not know the longitudinal momentum of the $h$, but we should have a good measurement of its transverse momentum from the tagging jets and other recoil jets. In fact, in this case, $p_T^h$ must be large enough that the $a$’s are not back to back; this is the case for almost all events even before cuts.

• We then have the two equations:

$$p_h^x = \frac{(p_{1\text{vis}}^x)}{f_1} + \frac{(p_{2\text{vis}}^x)}{f_2} \quad p_h^y = \frac{(p_{1\text{vis}}^y)}{f_1} + \frac{(p_{2\text{vis}}^y)}{f_2}$$

(15)

with solution

$$f_1 = \frac{(p_{1\text{vis}}^y)(p_{2\text{vis}}^x) - (p_{1\text{vis}}^x)(p_{2\text{vis}}^y)}{p_h^y(p_{2\text{vis}}^x) - p_h^x(p_{2\text{vis}}^y)} \quad f_2 = \frac{(p_{1\text{vis}}^y)(p_{2\text{vis}}^x) - (p_{1\text{vis}}^x)(p_{2\text{vis}}^y)}{-p_h^y(p_{1\text{vis}}^x) + p_h^x(p_{1\text{vis}}^y)}$$

(16)

• Of course, this follows very much the same pattern as in $WW \rightarrow h_{\text{SM}}$ with $h_{\text{SM}} \rightarrow \tau^+ \tau^-$ decays. Use of the collinear $\tau$ decay approximation and using the same equations for the visible $\tau$ decay products yields a pretty good $h_{\text{SM}}$ mass peak in the LHC studies done of this mode.

• A signal only Monte-Carlo run without lepton or tag jet momentum
smearing yields encouraging results

Figure 7: (a) A typical $h$ mass distribution. (b) A typical $a$ mass distribution. No cuts imposed; signal only

- The main issue is that the techniques for and ability to isolate a di-tau system as opposed to a single tau have not yet been established at the LHC so backgrounds are yet to be determined.
At the ILC, there is no problem since $e^+e^- \rightarrow ZX$ will reveal the $M_X \sim m_{h_1} \sim 100$ GeV peak no matter how the $h_1$ decays.

But the ILC is decades away.

As it turns out, $\Upsilon \rightarrow \gamma a_1$ decays hold great promise for $a_1$ discovery (or exclusion) as I now outline.

This kind of search should be pushed to the limit.

This idea has gained some traction with the $B$ factory managers.

In particular, CLEO has started looking at their existing data and placed some useful, but not (yet) terribly constraining, new limits.
Figure 8: PRELIMINARY New Limits from CLEO III from $\Upsilon(1S) \to \gamma \tau^+ \tau^-$. Total of 22 Million $\Upsilon(1S)$ events. Left plot: fixed $\mu = 150$ GeV scan without constraint on $F$; right plot: all $F < 15$ points in general scan. Both scans are for $\tan \beta = 10$, $M_{1,2,3} = 100, 200, 300$ GeV. Code: $2m_\tau < m_{a_1} < 7.5$ GeV; green $= 7.5$ GeV $< m_{a_1} < 8.8$ GeV; and black $= 8.8$ GeV $< m_{a_1} < 9.2$ GeV.
• Of course, we cannot exclude the possibility that \( 9.2 \text{ GeV} < m_{a_1} < 2m_b \).

Phase space for the decay causes increasingly severe suppression.

And, there is the small region of \( M_{\Upsilon} < m_{a_1} < 2m_b \) that cannot be covered by \( \Upsilon \) decays.

• However, if \( B(\Upsilon \rightarrow \gamma a_1) \) sensitivity can be pushed down to the \( 10^{-7} \) level, one might discover the \( a_1 \).

This would be very important input to the LHC program.

• Note: For preferred \( B(\Upsilon \rightarrow \gamma a_1) \) levels, the \( a_1 \) contribution to \( a_\mu \) (which contribution is \( < 0 \), i.e. in the wrong direction) is negligible.

Other scenarios/cautions

• If you ignore "ideal" criteria and relax \( F \) a bit, then possibilities expand tremendously. I mention just a few.
1. If $m_{h_1} > 110$ GeV, then $m_{a_1} > 2m_b$ is ok and $h_1 \rightarrow a_1 a_1 \rightarrow 4b, 2b2\tau$ is open. Many explorations: Ellwanger+JFG+Hugonie+Moretti: hep-ph/0305109, hep-ph/0401228, hep-ph/0503203, for the $2b2\tau$ channel, but tough; Cheung, K et al., hep-ph/0703149 and Carena, M. et al., arXiv:0712.2466 claim the $4b$ channel is observable at the LHC.

2. If the $a_1$ is very-very singlet (requires extreme tuning), then $a_1 \rightarrow \gamma\gamma$ and $h_1 \rightarrow 4\gamma$ is open (probably ruled out by LEP for $m_{h_1} < 114$ GeV). Studies include direct detection of the $a_1$ in production with charginos. A. Arhrib, K. Cheung, T. J. Hou and K. W. Song, JHEP 0703, 073 (2007) [arXiv:hep-ph/0606114].


4. The list is very long and I can only apologize to those I should mention but have no space for.
Implications for the "ideal" Higgs

- $Z - Z'$ mixing causes a downwards shift in $m_Z^2$ (level repulsion) which in turn implies an upwards shift in the $T$ parameter: $\delta T = -\frac{\delta m_Z^2}{m_Z^2}$.

In principle, this allows a large Higgs mass from precision fits. Chanowitz (hep-ph/0806.0890) finds that the CL for the fit including both leptonic and hadronic determinations of $\sin^2 \theta^\ell_{W}^{\text{eff}}$ is still very low: $CL = 0.09$, which is actually worse than for pure SM.

Dropping the hadronic $\sin^2 \theta^\ell_{W}^{\text{eff}}$ information leads to $CL \gtrsim 0.7$ (only slightly worse than SM), raises the central $m_{h_{\text{SM}}}$ value to perhaps as high as 70 GeV and allows for $m_{h_{\text{SM}}}$ up to 260 GeV at 95% CL vs. 105 GeV without $Z - Z'$ mixing.

Thus, the range of "ideal" possibilities is increased, but a Higgs with SM-like $ZZ, WW$ coupling and mass below 105 GeV is still preferred.
More generally, the mass of an extra $Z'$ and its possible mixing with the $Z$ are strongly constrained by experiment (see, for example, Carena:2004xs).

**The Motivations**

Some key points are the following.

- One main motivation for the $U(1)$ extension vs. NMSSM is that it provides an alternative way to remove the PQ symmetry; namely, promote it to a local symmetry.

  This requires the introduction of a new gauge boson, $Z'$, mediating a new force, which will gain a mass when the PQ symmetry is broken.

  As usual, the Goldstone boson (the axion) will be “eaten” by the gauge boson to provide the extra degree of freedom needed for its longitudinal polarization, and consequently there would be no axion to be found in low energy experiments.

- The existence of additional $U(1)$ gauge groups at TeV energies is well motivated by GUT and string models (Cvetic:1997ky, Demir:2003ke, Han:2004yd,
In particular, compactification of the extra dimensions in string theories often leads to large gauge groups such as $E_6$ or $E_8$. These gauge groups can then break down to the gauge groups of the SM with extra (local) $U(1)'$s. For example, one possible breaking would be $E_6 \to SO(10) \times U(1)_\phi$ followed by $SO(10) \to SU(5) \times U(1)_\chi$.

In general, the gauge bosons of these two new $U(1)$ symmetries mix, and one can arrange the symmetry breaking such that one combination maintains a GUT scale mass, while the other is manifest at (just above) the electroweak scale and becomes the $Z'$. (A detailed study of such a model was presented in King:2005jy.)

- However, from a low-energy perspective, there is a price to pay.

The field content of the model needs to be extended by adding new chiral quark and lepton states in order to ensure anomaly cancellation related to the gauged $U(1)'$ symmetry.
• A $U(1)'$ transformation for $\hat{S}$ avoids the possible domain wall problems of the NMSSM.

But also it forbids the $\hat{S}^3$ superpotential term for which some indirect substitute is needed for reasonable phenomenology.

The role of the $\hat{S}^3$ term with regard to generating quartic terms in the scalar potential is played by $D$ terms involving the various SM singlets.

**The Models**

There is a huge literature that I cannot do justice to — see recent review by Langacker (xarXiv:0801.1345) and the CPNSH report. I discuss only a couple.


• This model has an extra $U(1)'$ gauge group added to the MSSM along with a singlet $\hat{S}$ as well as 3 other $\hat{S}_{1,2,3}$; all are charged under the $U(1)'$, but not under the SM groups.
• This model will illustrate the potentialities of $U(1)'$ MSSM extensions and especially the challenges associated with the Higgs sector.
• The superpotential is

$$W = \lambda \hat{S} \hat{H}_u \hat{H}_d + \kappa \hat{S}_1 \hat{S}_2 \hat{S}_3.$$  (17)

• The $\lambda$ terms gives the $\mu$ parameter as in the NMSSM.
• $U(1)'$ charges are chosen to be non-trivial for all $\hat{S}, \hat{S}_{1,2,3}$ but such as to allow only these terms with dimensionless parameters.
• The reason for having a different $\hat{S}$ for the $\mu$ parameter as opposed to the $\hat{S}_{1,2,3}$ is that without such separation the $Z'$ mass tends to be comparable to $m_Z$ if SUSY-breaking parameters are below a TeV (as desirable from EWSB fine-tuning perspective).

In this model, a large $Z'$ mass can be generated by large vevs for the $S_{1,2,3}$ (which of course have $U(1)'$ charges. Since these communicate to $S$ only via the $U(1)'$, the phenomenology of the MSSM-like sector of the model is not much modified except for Higgs decays.
The scalar potential contains $F$ and $D$ term components:

$$
V_F = \chi^2 (|H_u|^2 |H_d|^2 + |S|^2 |H_d|^2 + |S|^2 |H_u|^2) + \kappa^2 (|S_1|^2 |S_2|^2 + |S_2|^2 |S_3|^2 + |S_3|^2 |S_1|^2)
$$

$$
V_D = \frac{G^2}{8} (|H_u|^2 - |H_d|^2)^2 + \frac{1}{2} g_{Z'}^2 \left( Q_S |S|^2 + Q_{H_d} |H_d|^2 + Q_{H_u} |H_u|^2 + \sum_{i=1}^{3} Q_{S_i} |S_i|^2 \right)
$$

where the $Q$s are the $U(1)'$ charges of the various fields, $G^2 = g_1^2 + g_2^2$ and $g_1$, $g_2$ and $g_{Z'}$ are the coupling constants for the $U(1)$, $SU(2)$ and $U(1)'$ groups. Gauge unification suggests taking $g_{Z'} = \sqrt{5/3} g_1$.

The model has 6 CP-even Higgs states, $h_{1,2,3,4,5,6}$ and 4 CP-odd states $a_{1,2,3,4}$, as well as 9 neutralinos $\tilde{\chi}_{1,2,3,4,5,6,7,9}$ where the 1st indices go with $\tilde{B}$, $\tilde{Z}$, $\tilde{H}_u$ and $\tilde{H}_d$ of the MSSM subcomponent.

The model has some attractive features, but also a lot of complexity. Some problems and features are:

- Anomaly cancellation in the model quires the introduction of additional chiral supermultiplets with exotic SM quantum numbers. These can be consistent with gauge unification, but do introduce additional model dependence.

The exotics are given mass by the same scalars that give rise to the $Z'$
mass.
- The lightest Higgs with $WW$ couplings can be heavy because of extra $D$-term contributions to its mass.
- Gauge coupling unification would appear to require significant extra matter at high scales.
- A more complete model would be required to assess fine-tuning with respect to GUT-scale parameters.
- Light $a_k^0$'s are definitely important in Higgs decays if the Higgs mass is below 100 GeV.
- There are many neutralinos, some of which are singlet-like and very light, but coupled to the Higgs so that $h_i \rightarrow \tilde{\chi}_j^0\tilde{\chi}_k^0$ is often a dominant or at least important channel, again especially for the lighter Higgs boson.
Figure 9: Branching ratios for the lightest Higgs.

- In the figure, solid symbols near $\text{BR} \sim 1$ show cases where $H_1$ has SM-like $WW, ZZ$ coupling but decays to $a$ s or $\chi$ s and therefore escapes LEP limit.
The decays of the lightest $a_1$ can be dominated by neutralino pairs. Look for the solid symbols with $BR \sim 1$ starting above $m_{A_1} \gtrsim 20 \text{ GeV}$.

Figure 10: Branching ratios for the lightest CP-odd $A_1$. 

Lightest CP-Odd Higgs Branching Ratios

![Branching Ratios Graph]

$M_{A_1}$ (GeV) vs. Branching Ratio

- $\chi^0_1 \chi^0_1$
- $\chi^0_1 \chi^{0}_{i>1}$
- $\chi^{0}_{i>1} \chi^{0}_{i>1}$
- $\chi^+ \chi^-$
- $H^+ H^-$
- $H_1 Z$
- $t \bar{t}$
- $b \bar{b}$
- $\tau^+ \tau^-$
- $c \bar{c}$
- $s \bar{s}$
Decay Channels Examples only.

\[ H_1 \to A_1 A_1 \to 4\tilde{\chi}'s \to \text{visible} + \tilde{\chi}_1^0 \tilde{\chi}_1^0 \]

\[ H_1 \to 2\tilde{\chi}'s \to \text{visible} + \tilde{\chi}_1^0 \tilde{\chi}_1^0 \]

The above will contain a mixture of visible and invisible energy and not have a reconstructable mass peak.

\[ H_1 \to A_1 A_1 \to \text{all the NMSSM channels} \]

Probably, the most likely result is a mixture of all possibilities.

I hope we will not have to contend with such a complex model, but one should keep in mind that string theory can easily produce models of this type.

- A lot more work is needed on this kind of model with regard to baryogenesis, dark matter, gauge coupling unification (possibly problematical), ...

... to fully assess.

2. One can also consider cases in which there is significant \( Z - Z' \) mixing.

In S. W. Ham, E. J. Yoo, S. K. Oh and D. Son, arXiv:0801.4640 [hep-ph]. It is shown that mixing can be significant enough to reduce the \( ZZ \) coupling to an extent observable at an ILC.
3. In S. W. Ham, T. Hur, P. Ko and S. K. Oh, arXiv:0801.2361 [hep-ph]. it is shown that loops contributing to $gg$ fusion production containing the colored exotics of the UMSSM can lead to a significant enhancement of Higgs cross sections at the LHC.

4. The list of possible models and their implications is vast and most will have strongly modified Higgs phenomenology at the LHC and other colliders.

5. In general, it will be valuable to place the strongest possible limits on $BR(\Upsilon \rightarrow \gamma A)$ since most $U(1)$ MSSM extensions will have one or more light pseudoscalars.
Conclusions

- The NMSSM naturally has small fine-tuning of all types, i.e. for:

  1) Quadratic divergence fine-tuning is erased ab initio.

  2) EWSB, i.e. $m_Z^2$, fine-tuning can be avoided for $m_{h_1} \lesssim 100$ GeV, which is consistent with LEP limits when $m_{a_1} < 2m_b$ and $BR(h_1 \rightarrow a_1a_1) > 0.75$.

  3) Light-$a_1$ fine-tuning to achieve $m_{a_1} < 2m_b$ and (simultaneously) large $B(h_1 \rightarrow a_1a_1)$ can be avoided.

  $m_{a_1} > 2m_{\tau}$ is preferred to minimize light-$a_1$ fine-tuning.

  4) Electroweak baryogenesis becomes entirely viable, not just because $m_{h_1} < 100$ GeV, but also because of extra terms in NMSSM potential.

  5) There is much more freedom in obtaining correct relic LSP density (e.g. LSP can have singlet component).

- If low fine-tuning is imposed for an acceptable SUSY model, the NMSSM example suggests we should expect:
– a \( h_1 \) with \( m_{h_1} \sim 100 \text{ GeV} \) and SM-like couplings to SM particles but with primary decays \( h_1 \rightarrow a_1 a_1 \) with \( m_{a_1} < 2m_b \), where the \( a_1 \) is mainly singlet.

**Consequences**

Higgs detection will be quite challenging at a hadron collider. Higgs detection at the ILC is easy using the missing mass \( e^+e^- \rightarrow ZX \) method of looking for a peak in \( M_X \).

Higgs detection in \( \gamma\gamma \rightarrow h_1 \rightarrow a_1 a_1 \) will be easy.

Detection of the \( a_1 \) could easily result from pushing on \( \Upsilon \rightarrow \gamma a_1 \).

– the stops and other squarks are light;
– the gluino, and, by implication assuming conventional mass orderings, the wino and bino all have modest mass;

• In short, SUSY will be easily seen at the LHC, but Higgs detection requires hard work. Still, it now appears possible with high luminosity using doubly-diffractive \( pp \rightarrow pph_1 \rightarrow pp4\tau \) events.

• Even if the LHC sees the Higgs \( h_1 \rightarrow a_1 a_1 \) directly, it will not be able to get much detail. Only the ILC and possibly \( B \)-factory results for \( \Upsilon \rightarrow \gamma a_1 \)
can provide the details needed to verify the model.

- It is likely that other models in which the MSSM $\mu$ parameter is generated using additional scalar fields can achieve small fine-tuning in a manner similar to the NMSSM.

Low fine-tuning typically requires low SUSY masses which in turn typically imply $m_{h_1} \sim 100$ GeV.

And, to escape LEP limits large $B(h_1 \to a_1 a_1 + \ldots)$ with most final states not decaying to $b$'s ($e.g.$ $m_{a_1} < 2m_b$) would be needed. In general models, there would be many channels in \ldots and detection of any one channel would be a huge challenge.

In general, the $a_1$ might not need to be so singlet as in the NMSSM and would then have larger $B(\Upsilon \to \gamma a_1)$.

- If the LHC Higgs signal is really marginal in the end, and even if not, the ability to check perturbativity of $WW \to WW$ at the LHC might prove to be very crucial to make sure that there really is a light Higgs accompanying light SUSY and that it carries most of the SM coupling strength to $WW$. 

J. Gunion

SUSY08, Seoul, Korea, June 16, 2008 54
• A light $a_1$ allows for a light $\tilde{\chi}_1^0$ to be responsible for dark matter of correct relic density: annihilation would be via $\tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow a_1$. To check the details, properties of the $a_1$ will need to be known fairly precisely.

The ILC might (but might not) be able to measure the properties of the very light $\tilde{\chi}_1^0$ and of the $a_1$ in sufficient detail to verify that it all fits together.

But, also $\Upsilon \rightarrow \gamma a_1$ decay information would help tremendously.