MUON COLLIDER HIGGS FACTORY

J. F Gunion, UCLA Higgs Factory Workshop (Feb. 28 – March 1, 2001)

OUTLINE

Selected items in Higgs Theory

- The decoupling limit, and escapes therefrom.
- The heavy SM-like Higgs scenario consistent with precision EW.

The Machine

- The key parameters.
- The key machine features.

Higgs Physics at the Higgs Factory

- Basic reminders
- The alternative to scanning
- Review of expected errors for SM-like Higgs
- Comparison to NLC/TESLA
- Special capabilities
- Special Higgs scenarios
- The PNGB possibility

1 Selected items from Higgs Theory

The decoupling limit and escapes therefrom.

- Many models of new physics related to technicolor, string theory, . . . have a two-doublet (or more complicated) Higgs sector + SM as the effective theory below some relatively low scale $\Lambda \sim 10$ TeV.
- In such a model, the decoupling limit can be defined in 2 ways.
  1. One neutral Higgs boson has full strength WW/ZZ coupling, and all others decouple.
  2. Only one Higgs boson is light and all the others are heavy.

  Except in special cases, one can show that these are equivalent.

E.G., consider the general CP-conserving 2HDM, with $h^0, H^0, A^0, H^\pm$, $\tan\beta = v_2/v_1$, α =CP-even mixing angle and Higgs potential

$$\begin{aligned}\mathcal{V} = {} & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}] \\ & +\tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ & + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \Big[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\Big]\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right\} . \qquad (1.1)\end{aligned}$$

Define $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$ and

$$\begin{aligned}\lambda & \equiv \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2\lambda_6 c_\beta^2 s_{2\beta} + 2\lambda_7 s_\beta^2 s_{2\beta} \,, \\ \hat{\lambda} & \equiv \tfrac{1}{2} s_{2\beta}\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta}\right] + \lambda_6 c_\beta^2 (3 s_\beta^2 - c_\beta^2) + \lambda_7 s_\beta^2 (s_\beta^2 - 3 c_\beta^2) \,.\end{aligned}$$

Recall:

$$\boldsymbol{h^0 \to WW, ZZ \propto \sin(\beta-\alpha)\,, \quad H^0 \to WW, ZZ \propto \cos(\beta-\alpha)\,.}$$

Then, there are two possible decoupling limits in sense 1.:

- (a) $c_{\beta-\alpha} \to 0$, implying H^0 decouples from WW/ZZ, and (b) $s_{\beta-\alpha} \to 0$, implying h^0 decouples.

  However, unless $\hat{\lambda} = 0$, only the first is consistent with other constraints (e.g. positive masses squared); for $\hat{\lambda} \neq 0$ we have

$$m_{h^0}^2 \simeq v^2(\lambda \mp \hat{\lambda} c_{\beta-\alpha})\,, \tag{1.2}$$

$$m_K^2 \simeq v^2 \left[\pm \frac{\hat{\lambda}}{c_{\beta-\alpha}} + \lambda_K \right]\,, \qquad K = A^0, H^0, H^\pm\,, \tag{1.3}$$

  where the λ_K are functions of β and the λ_i of $\mathcal{V}$. Then, as $c_{\beta-\alpha} \to 0$ all other Higgs masses $\to \infty$, implying decoupling in sense 2.

  Thus, for example, if the LEP2 Higgs signal is real, which requires near maximal coupling for a light h^0, unless $\hat{\lambda} = 0$ we infer that all other Higgs bosons are quite heavy.

- There are two $\hat{\lambda} = 0$ possibilities: (a) $\kappa = 1$ case — a special choice of the λ_i such as $\lambda_1 = \lambda_2 = \lambda_3$, $\lambda_4 = -\lambda_5$ (and $\lambda_6 = \lambda_7 = 0$ as generally chosen anyway); (b) $\kappa = 2$ case — $\tan\beta \to \infty$.

  In either case we have: two possible WW/ZZ 'decoupling' sub-cases:

  1. $m_{A^0}^2 + \lambda_5 v^2 > \lambda_\kappa v^2$: $\cos(\beta - \alpha) \to 0$, $m_{h^0}^2 = \lambda_\kappa v^2$ and

$$m_{H^0}^2 = \frac{m_{12}^2}{s_\beta c_\beta} = m_{A^0}^2 + \lambda_5 v^2 = m_{H^\pm}^2\,.$$

     - The h^0 is SM-like and its mass is limited if $|\lambda_\kappa| < 1$ is required.
     - For $m_{A^0}^2 \gg |\lambda_5| v^2$ all the other Higgs bosons are heavy.
     - $m_{A^0}^2 \ll m_{h^0}^2$ OK if $\lambda_5 > \lambda_\kappa$ (so that $m_{H^0} > m_{h^0}$).

       $\Rightarrow m_{A^0} \ll m_{h^0} < m_{H^0} \sim m_{H^\pm}$ with h^0 SM-like.

2. $m_{A^0}^2 + \lambda_5 v^2 < \lambda_\kappa v^2$: $\sin(\beta - \alpha) \to 0$, $m_{H^0}^2 = \lambda_\kappa v^2$ and

$$m_{h^0}^2 = \frac{m_{12}^2}{s_\beta c_\beta} = m_{A^0}^2 + \lambda_5 v^2 = m_{H^\pm}^2 \,.$$

- **This a peculiar case in which the *heavier* CP-even Higgs boson can have exactly SM couplings.**
- Note: m_{H^0} is limited if perturbativity for λ_κ is required.
- $m_{A^0}^2$ is limited by the requirement $m_{h^0}^2 < m_{H^0}^2$ and perturbativity for $|\lambda_5|$ and $|\lambda_\kappa|$. In particular, if $m_{A^0}^2 \gg m_{h^0}^2$ then $\lambda_5 \to -\frac{m_{A^0}^2}{v^2}$, which limits the size of $m_{A^0}^2$ if $|\lambda_5| < 1$ is required.
- $m_{A^0} \ll m_{h^0} = m_{H^\pm} \lesssim m_{H^0}$ is possible if $\lambda_5 < \lambda_\kappa$.

Note: the special $\hat{\lambda} = 0$ scenarios correspond to special symmetries of the potential or the large-$\tan\beta$ limit required for b, t, τ Yukawa coupling unification. Thus, they are not unmotivated.

So, an interesting question is whether we could detect the light A^0 predicted in these unusual cases?

Also, could such an unconventional Higgs scenario be consistent with precision EW measurements?

Scenarios with a heavy SM-like Higgs that are consistent with precision electroweak data. Ingredients (an example).

- Effective theory below $\Lambda = 1 - 10$ TeV = a two-Higgs doublet model (more complicated also OK) with a light (or maybe not so light) A^0 or h^0 with no WW/ZZ coupling.

- A moderate value of $\tan\beta$.

- A heavy (but $\lesssim 1$ TeV) SM-like Higgs; e.g. the h^0 when A^0 is light.

- A still heavier neutral Higgs that is almost, but not quite, degenerate with the $H^\pm$ charged Higgs bosons.

- We can get any value of $\Delta\rho$ we want: e.g. for light A^0 case, one finds

$$\begin{aligned}\Delta\rho = \frac{\alpha}{16\pi m_W^2 c_W^2}\Bigg\{ & \frac{c_W^2}{s_W^2}\frac{m_{H^\pm}^2 - m_{H^0}^2}{2} \\ & -3m_W^2\left[\log\frac{m_{h^0}^2}{m_W^2} + \frac{1}{6} + \frac{1}{s_W^2}\log\frac{m_W^2}{m_Z^2}\right]\Bigg\}\end{aligned} \tag{1.4}$$

- ΔS worsens logarithmically with the increase in the SM-like Higgs mass: e.g.

$$S(0) \sim \frac{1}{12\pi}\left(-\frac{5}{3} + \log\frac{m_{h^0}^2}{m_W^2}\right), \tag{1.5}$$

  in the light A^0 case.

- For any given NLC/TESLA $\sqrt{s}$, choose (light A^0 case) h^0 to be SM-like and with $m_{h^0} \gtrsim \sqrt{s}$. $\Rightarrow$ perfectly consistent with current precision constraints.

- Giga-Z could distinguish light SM-like h^0 from this alternative.

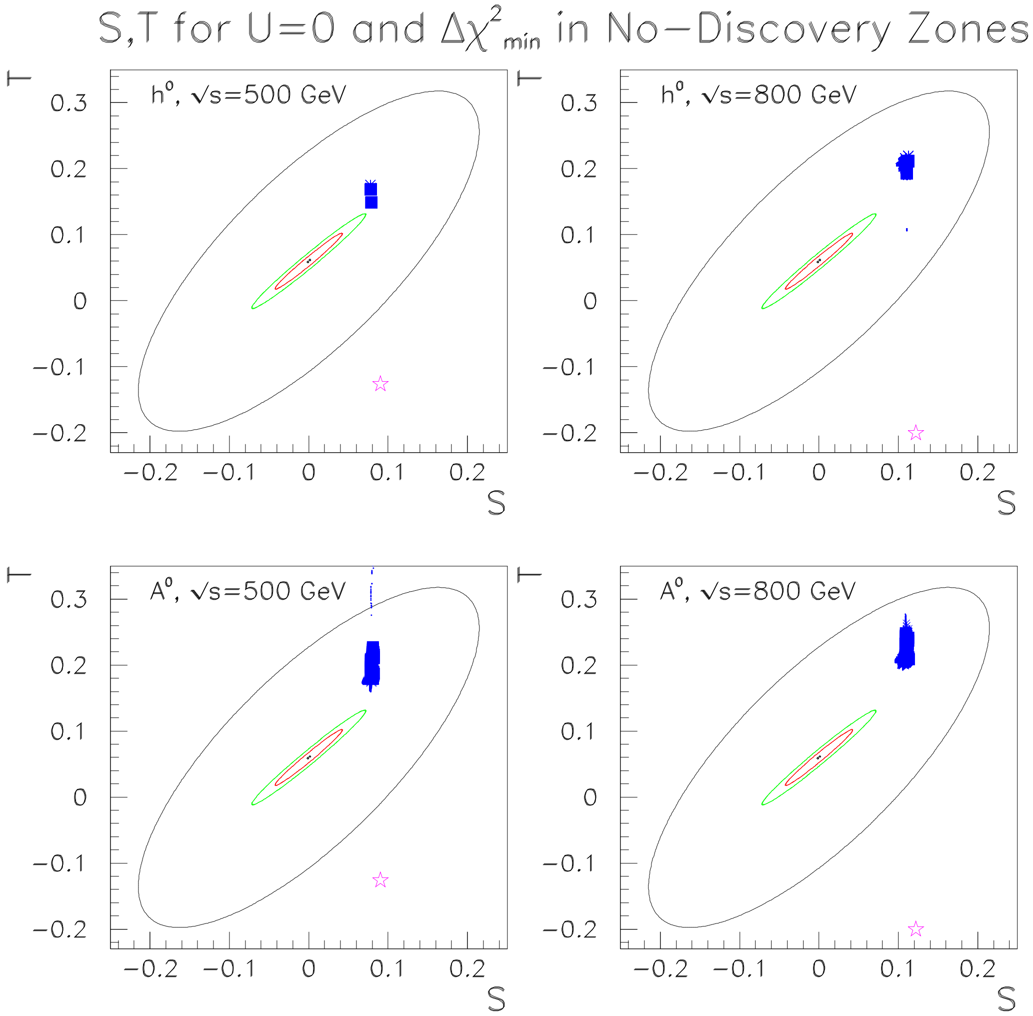


Figure 1: The outer ellipse gives the current 90% CL region for $U = 0$ and SM Higgs mass of 115 GeV. S, T predictions for the 2HDM $\Delta\chi^2_{\rm min}$ models with $[m_h, \tan\beta]$ values in the 'no-discovery' wedges of Fig. 2 are shown by the blobs of overlapping points. The innermost (middle) ellipse gives the 90% (99.9%) CL region for $m_{h_{SM}} = 115$ GeV obtained after Giga-Z precision measurements *and* a $\Delta m_W \lesssim 6$ MeV threshold scan measurement of m_W. The stars indicate the SM S, T prediction if $m_{h_{SM}} = \sqrt{s} - m_Z$.

- Would see the SM-like Higgs at LHC but not at NLC/TESLA.
- Could not see the possibly lighter decoupled Higgs at LHC/NLC/TESLA unless quite light (via, e.g. $e^+e^- \to A^0A^0Z$) or unless $\tan\beta$ either small or very large (via $t\bar{t}A^0$ or $b\bar{b}A^0$, respectively).

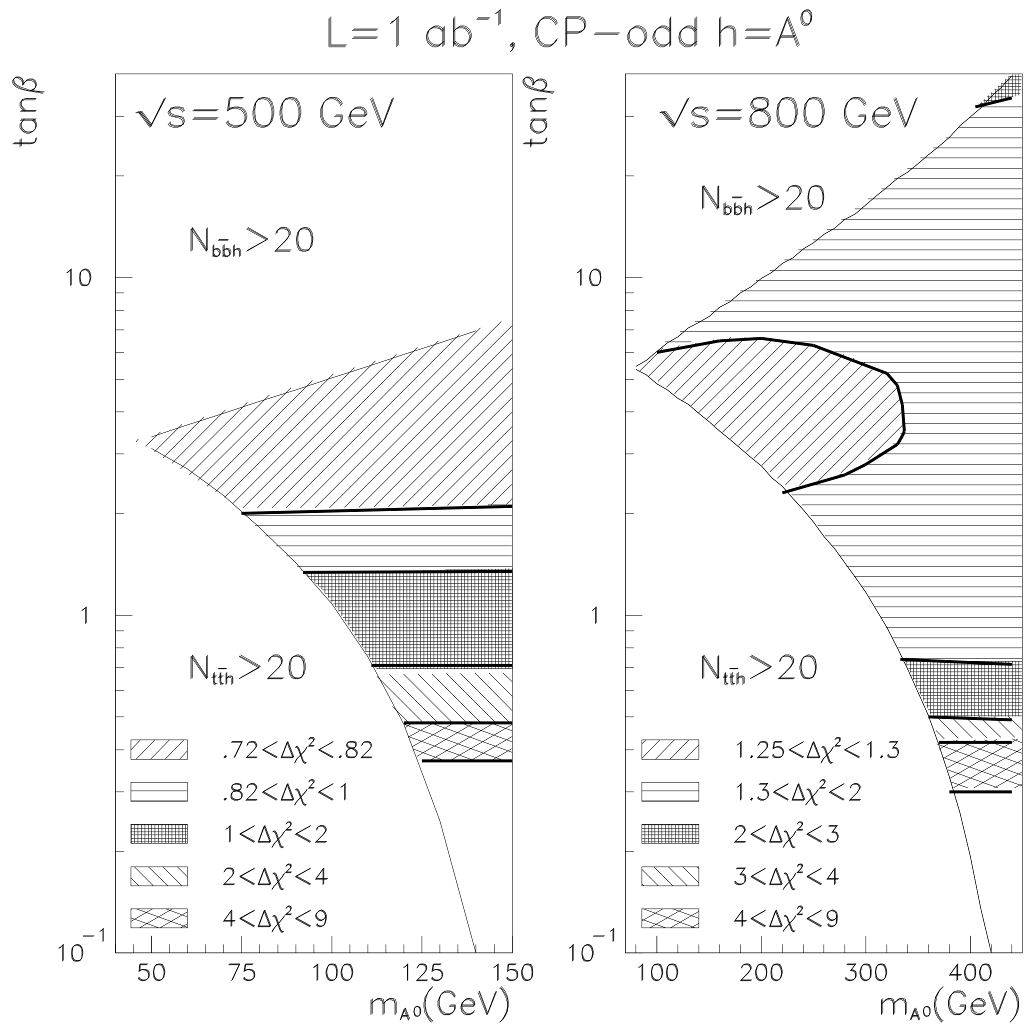


Figure 2: For $\sqrt{s} = 500$ GeV and $\sqrt{s} = 800$ GeV, the solid lines show as a function of m_{A^0} the maximum and minimum $\tan\beta$ values between which $t\bar{t}A^0$, $b\bar{b}A^0$ final states will both have fewer than 20 events assuming $L = 1~\mathrm{ab}^{-1}$. The different regions indicate the best $\Delta\chi^2$ values (relative to the best SM χ^2) obtained for fits to precision electroweak data after scanning: a) over the masses of the remaining Higgs bosons subject to the constraint they are too heavy to be directly produced; and b) over the mixing angle in the CP-even sector. Results are shown only for $m_h < \sqrt{s} - 2m_t$, but extrapolate to higher m_h in obvious fashion.

- In the Yukawa modes, 20-event discovery = optimistic, $\Rightarrow$ difficult region for an e^+e^- collider (the region inside the wedge) is probably larger.

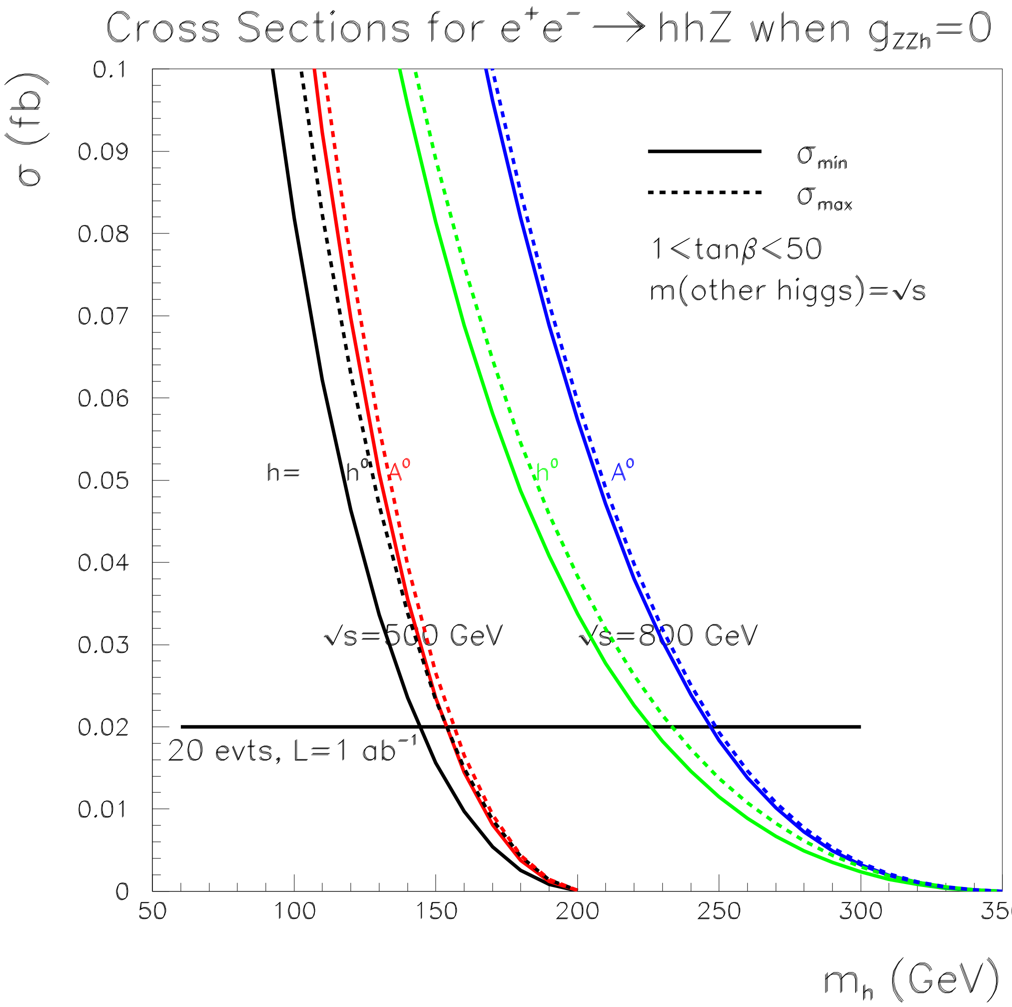


Figure 3: For $\sqrt{s}=500$ GeV and $\sqrt{s}=800$ GeV and for $h=h^0$ and $h=A^0$, we plot as a function of m_h the maximum and minimum values of $\sigma(e^+e^-\to hhZ)$ found after scanning $1<\tan\beta<50$ taking all other higgs masses equal to $\sqrt{s}$. For $h=h^0$, we require $\sin(\beta-\alpha)=0$ during the scan. The 20 event level for $L=1~\mathrm{ab}^{-1}$ is indicated.

- 20 events for A^0A^0Z final state could probe up to $m_{A^0}\sim 150$ GeV for $\sqrt{s}\sim 500$ GeV and up to $m_{A^0}\sim 250$ GeV for $\sqrt{s}\sim 800$ GeV. Realistic background estimate might $\Rightarrow$ less mass reach.

Net result: Could see a heavy SM-like Higgs at LHC and have no idea there is a lighter Higgs without a muon collider capable of scanning the appropriate region.

2 Key muon collider parameters and features

Yearly integrated luminosities. Benchmarks are (using $\mathcal{L} = 1\times 10^{32}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ $\Rightarrow L = 1~\mathrm{fb}^{-1}/\mathrm{yr}$):

- $L_{\rm year} \gtrsim 0.1, 0.22, 1~\mathrm{fb}^{-1}$ at $\sqrt{s} \sim 100$ GeV for beam energy resolutions of $R = 0.003\%, 0.01\%, 0.1\%$, respectively;
- $L_{\rm year} \sim 2, 6, 10~\mathrm{fb}^{-1}$ at $\sqrt{s} \sim 200, 350, 400$ GeV, respectively, for $R \sim 0.1\%$;

Key features of the muon collider:

- Beam energy spread of $\Delta E_{\rm beam}/E_{\rm beam} = 0.01R(\%)$ with $R = 0.003\%$ can be achieved at the low-energy Higgs factory so that the Gaussian spread in $\sqrt{s}$, given by

$$\sigma \sim 2~\mathrm{MeV}\left(\frac{R}{0.003\%}\right)\left(\frac{\sqrt{s}}{100~\mathrm{GeV}}\right), \tag{2.1}$$

  can be as small as the natural width of a light SM-like Higgs boson.

- For very narrow resonance scan:
  - Need to know $E_{\rm beam}$ to few parts in 10^{-6}.
  - Need stability in E at this level.
  - Need to know σ

  Measurements for each muon spill (10^8 spills per year) will be performed using muon spin precession and spectra of e's from the decays. Defining $\nu = (g_\mu - 2)E_{\rm beam}/[2m_\mu]$, $N(T) \propto \int e^{i2\pi\nu T} S(\nu)d\nu$, where $S(\nu)$ is the distribution of μ's as a function of ν.

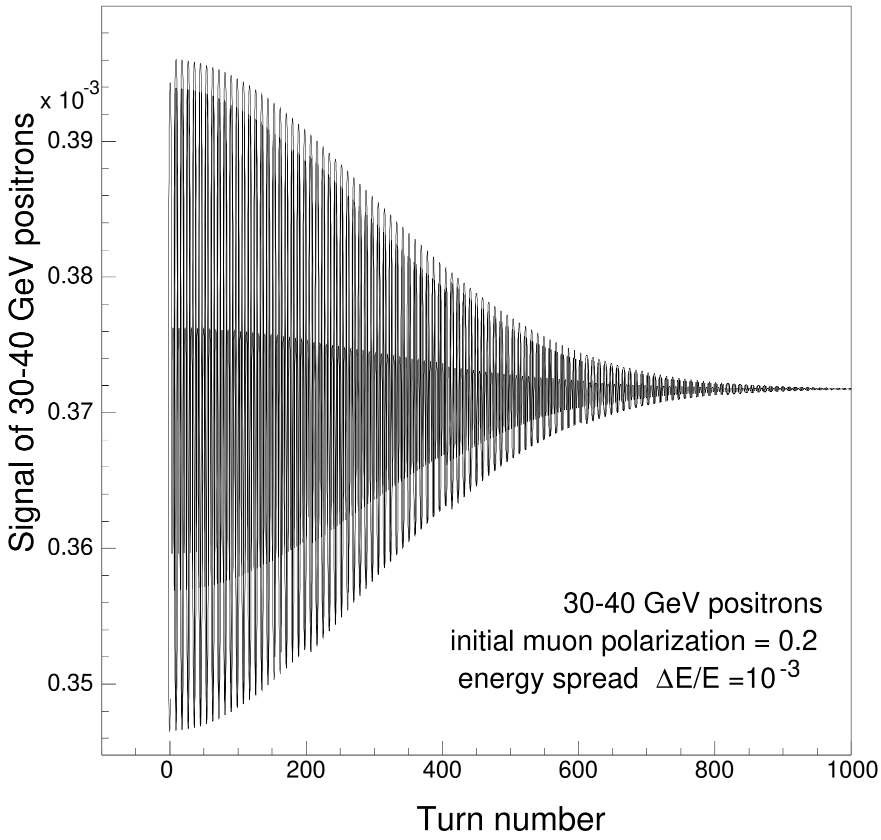


Figure 4: Oscillation with turn number for number of e's in $30-40$ GeV range from 50 GeV muons.

– Oscillation frequency $\to E_{\rm beam}$.

– Amplitude of envelope $\propto e^{-\frac{1}{2}(T2\pi\nu\frac{\Delta E_{\rm beam}}{E_{\rm beam}})^2}$.

$\Rightarrow$ For $R = 0.003\%$, $\Delta E_{\rm beam}/E_{\rm beam} \sim 10^{-7}$ ($\Delta E_{\rm beam} \sim 5$ keV), $\Delta\sigma/\sigma \sim 0.01$ *per spill.*

$\Rightarrow$ For each spill these errors are tiny compared to statistical errors for the channel rates. For 10^8 spills, only $1/N_{\rm channel}^{1/2}$ matters.

$\Rightarrow$ Main concern is systematic $\Delta\sigma/\sigma$ error.

How important might a systematic $\Delta\sigma$ systematic error be?

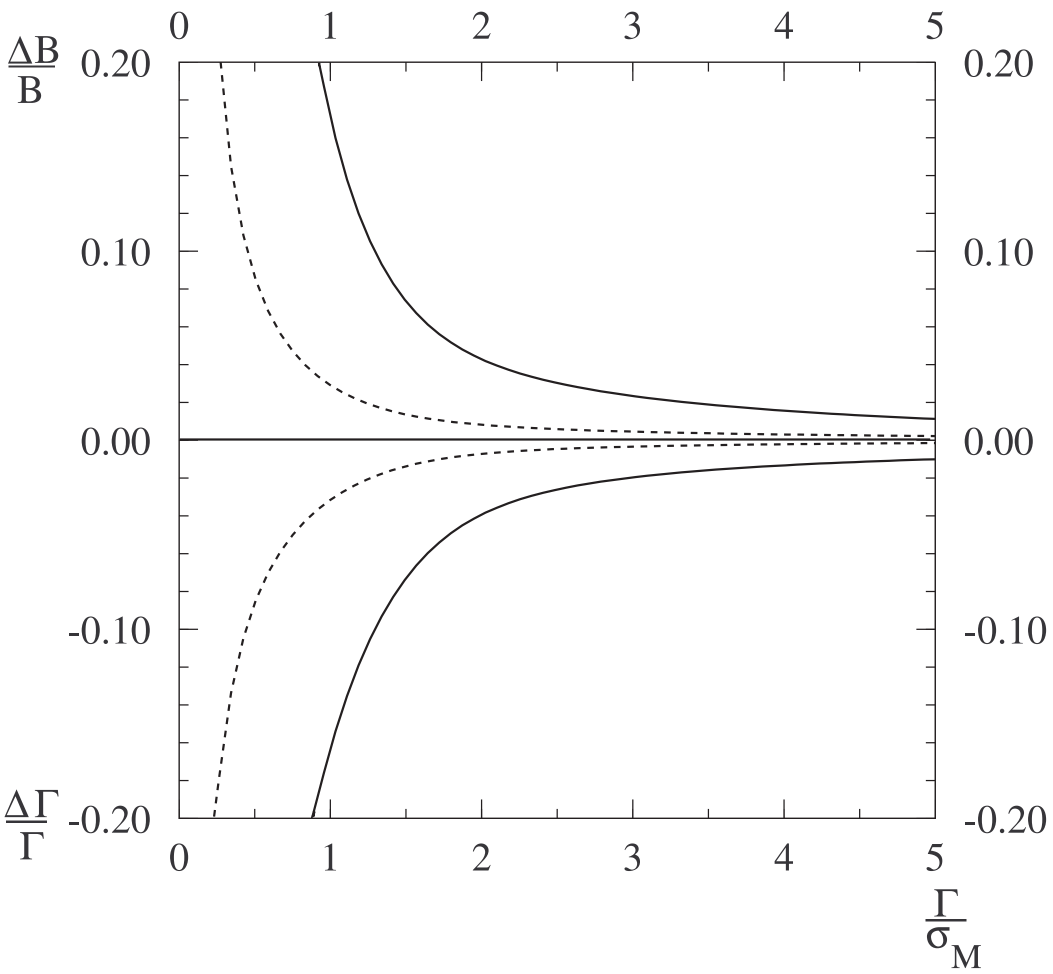


Figure 5: The fractional errors induced in B and Γ by $\Delta\sigma_M$ as functions of Γ/σ_M when employing the three-point scan measurement ($E = M$, $E = M \pm 2\Gamma$); results are shown for $\Delta\sigma_M/\sigma_M = 0.05$ (solid line) and $\Delta\sigma_M/\sigma_M = 0.01$ (dashed line).

Fig. 5 $\Rightarrow \Delta\Gamma/\Gamma, \Delta B/B \sim 3\Delta\sigma/\sigma$ at $\Gamma/\sigma \sim 1$, for scan.

3 Narrow Resonance Generalities

The totally unique feature of a muon collider is the possibility of s-channel $\mu^+\mu^- \to X$ with $\sigma < \Gamma_T \sim$ few MeV, assuming X has enough $\mu^+\mu^-$ coupling.

Compute $\overline{\sigma}$ by convoluting a Gaussian $\sqrt{s}$ distribution of width σ with the standard s-channel Breit Wigner Higgs resonance cross section. For $\sqrt{s} = M$, one obtains

$$\overline{\sigma} \simeq \frac{\pi\sqrt{2\pi}\Gamma(X \to \mu\mu)\, B(X \to F)}{M^2} \times \begin{cases} \frac{1}{\sigma}\,; & \sigma \gg \Gamma_T \\ \frac{2\sqrt{2}}{\Gamma_T\sqrt{\pi}}\,; & \sigma \ll \Gamma_T \end{cases} . \quad (3.1)$$

Eq. (3.1) $\Rightarrow$ for big $\overline{\sigma}$, need small Γ_T and $\sigma \lesssim \Gamma_T$.

Locate X and center on resonance by rough scan.

Determine width and rates. Two procedures for the latter.

- High precision scan. Three-point scan = optimal – total $L_{\rm total}$ distributed as $L_{\rm total}/5$ at M, and $2L_{\rm total}/5$ a $M \pm 2\sigma$ each.

  Although smaller R implies smaller $\mathcal{L}$, the correlation is such that for small Γ_T (*e.g.* a SM-like Higgs boson with $m_h \lesssim 2m_W$) it is best to use the smallest R that can be achieved.

  Statistics for Γ_T limited by small S/B on wings.

- New technique: two measurements at $\sqrt{s} = M$ — one with $\sigma^{\rm min} \ll \Gamma_T$ and one with $\sigma^{\rm max} \gg \Gamma_T$: $\quad \overline{\sigma}(\sigma^{\rm min})/\overline{\sigma}(\sigma^{\rm max}) \propto \sigma^{\rm max}/\Gamma_T$.

  Statistics for Γ_T limited by small S/B for larger $\sigma^{\rm max}$.

Critical issues:

– Sensitivity of Γ_T to $\overline{\sigma}(\sigma^{\rm min})/\overline{\sigma}(\sigma^{\rm max})$.

– Statistical error in $\overline{r} \equiv \overline{\sigma}(\sigma^{\rm min})/\overline{\sigma}(\sigma^{\rm max})$.

Need to know how $\mathcal{L}$ varies with R. Benchmark values given earlier $\Rightarrow$ $\mathcal{L}(R = 0.03\%) = 4.7\mathcal{L}(R = 0.003\%)$, for example.

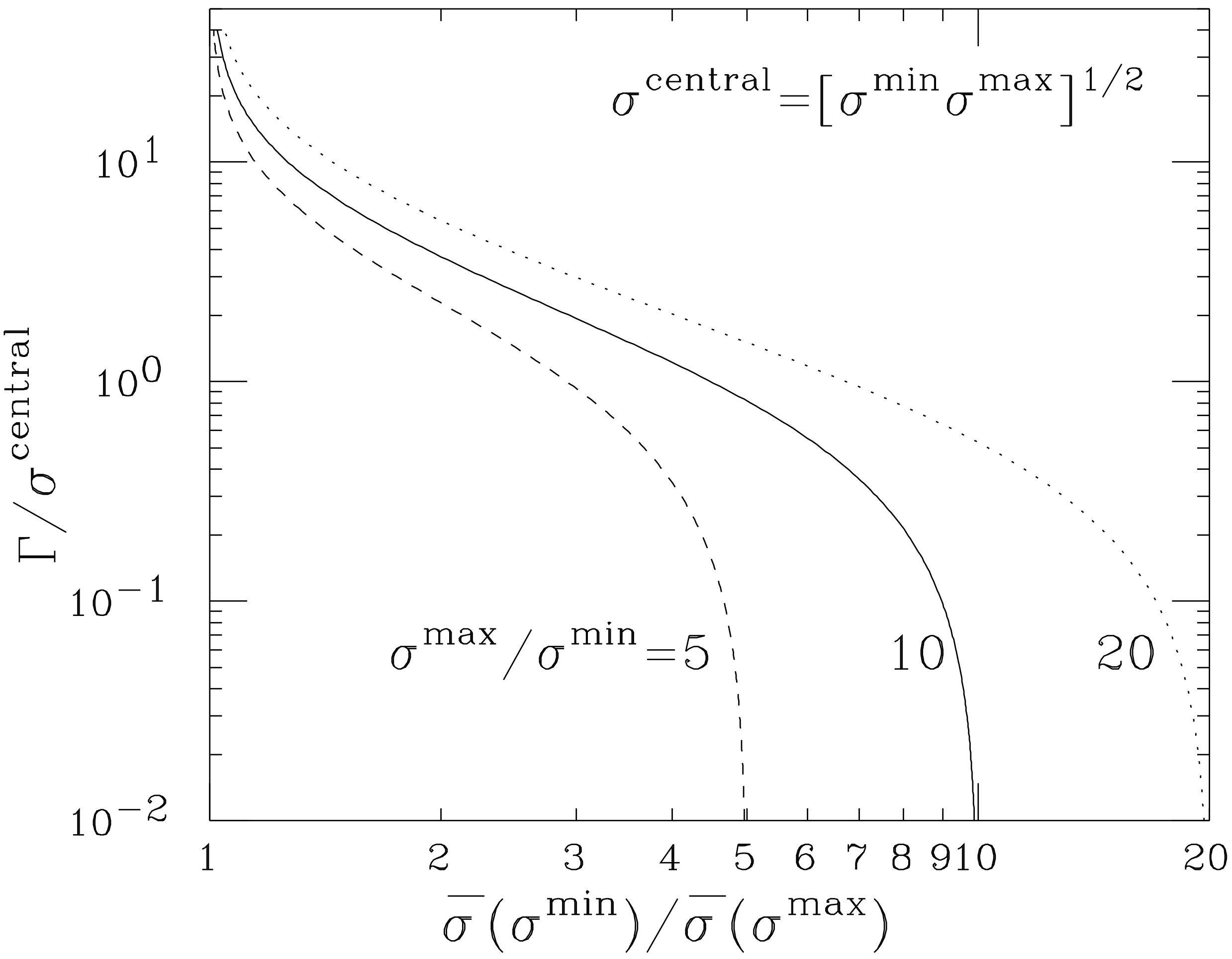


Figure 6: $\Gamma_T/\sigma^{\rm central}$ plotted as a function of the cross section ratio $\overline{\sigma}(\sigma^{\rm min})/\overline{\sigma}(\sigma^{\rm max})$ for the indicated values of $\sigma^{\rm max}/\sigma^{\rm min}$ keeping $\sigma^{\rm central} \equiv \sqrt{\sigma^{\rm max}\sigma^{\rm min}}$ fixed.

Note: $\Delta\Gamma_T/\Gamma_T = |s|\Delta\overline{r}/\overline{r}$, where $|s|$ is the slope in the log – log plot.

$|s| \sim 1.5$ (~ 2) for $\sigma^{\rm max}/\sigma^{\rm min} = 10$ and $\Gamma_T/\sigma^{\rm central} \sim 2 - 3$ (~ 1).

Note: Systematic errors from ratio technique are $\Delta\Gamma_T/\Gamma_T = \Delta\sigma/\sigma$ and $\Delta B = 0$.

4 Higgs physics

Standard Model-Like Higgs

$m_h \lesssim 2m_W$ is required for good s-channel cross section. Above that $\Gamma_h^{\rm tot}$ becomes big and $\overline{\sigma}_h \propto BF(h \to \mu^+\mu^-)$ is too small.

Strategy: $\Delta m_h \sim 100$ MeV from LHC if $m_h \lesssim 2m_W$ (2γ mode) $\Rightarrow$ First center on $\sqrt{s} \sim m_h$ via rough scan and then measure Higgs properties.

$m_h > 100$ GeV means no Z-pole problem; centering might take $L = 0.1$ fb^{-1} or about 1 yr.

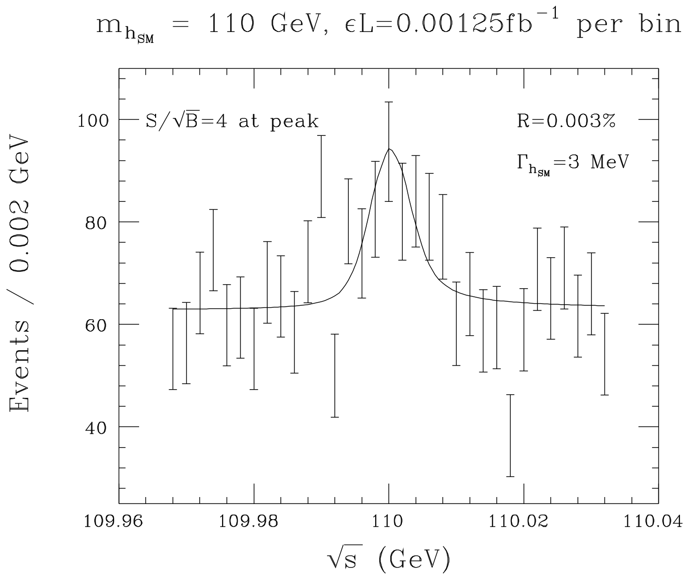


Figure 7: Initial scan for centering on $m_{h_{SM}} = 110$ GeV Higgs boson.

- The crucial measurements are then:
  - The very tiny Higgs width: $\Gamma_h^{\rm tot} = 1 - 10$ MeV for a SM-like Higgs with $m_h \lesssim 140$ GeV (i.e. mass as predicted for h^0 of SUSY).
  - $\sigma(\mu^+\mu^- \to h \to X)$ for $X = b\overline{b}, WW^\star, ZZ^\star, c\overline{c}, \tau^+\tau^-$.

Accuracy achievable? First consider optimized 3-point scan.

At $L_{\rm total} = 0.4~{\rm fb}^{-1}$ (≥ 4 years at $R = 0.003\%$) the errors for $\sigma B(h_{SM} \to b\overline{b})$ are small, but those for $\Gamma^{\rm tot}_{h_{SM}}$ are bad if m_h large.

Table 1: Percentage errors (1σ) for $\Gamma_h^{\rm tot}$ and $\overline{\sigma}B(h \to b\overline{b}, WW^\star, ZZ^\star)$ for $\mu^+\mu^- \to h$. For the scan procedure we use $R = 0.003\%$ and 4-year luminosity of $L = 0.4~{\rm fb}^{-1}$, using $L/5$ at $E = m_h$, $2L/5$ at $E = m_h + 2\sigma$ and $2L/5$ at $E = m_h - 2\sigma$. For the $\overline{r}$-ratio procedure, we assume $E = m_h$ and accumulate $L = 0.2~{\rm fb}^{-1}$ at $R = 0.003\%$ and $L = 0.94~{\rm fb}^{-1}$ at $R = 0.03\%$, corresponding to two years of running at each R. Efficiencies, cuts and bremsstrahlung included.

Quantity	Errors for the scan procedure					
Mass (GeV)	**100**	**110**	**120**	**130**	**140**	**150**
$\overline{\sigma}B(b\overline{b})$	4%	3%	3%	5%	9%	28%
$\overline{\sigma}B(WW^\star)$	32%	15%	10%	8%	7%	9%
$\overline{\sigma}B(ZZ^\star)$	–	190%	50%	30%	26%	34%
$\Gamma_h^{\rm tot}$	30%	16%	16%	18%	29%	105%
Quantity	Errors for the $\overline{r}$-ratio procedure					
Mass (GeV)	**100**	**110**	**120**	**130**	**140**	**150**
$\overline{\sigma}B(b\overline{b})$	3.8%	2.8%	2.8%	4.4%	7.6%	21%
$\overline{\sigma}B(WW^\star)$	26%	12%	7.7%	5.7%	5.0%	5.6%
$\overline{\sigma}B(ZZ^\star)$	–	190%	46%	25%	20%	22%
$\Gamma_h^{\rm tot}$	45%	25%	20%	19%	17%	18%

What about the new ratio technique?

– $L_{\rm year}\overline{\sigma}_h$ ~constant in going from $R = 0.003\%$ to $R = 0.03\%$, but S/B decreases.

– For same $L_{\rm total}$, $\overline{r}$ statistical Γ_T errors are smaller (larger) for larger (smaller) m_h. For $\overline{\sigma}_h B$, $\overline{r}$ errors smaller for all m_h.

– For $\Gamma_h^{\rm tot} \sim \sigma$ (e.g. $m_h \leq 120$ GeV), systematic errors for scan are $>$ than for $\overline{r}$-technique ($\Delta\Gamma_h^{\rm tot}/\Gamma_h^{\rm tot} = (3~vs.~1) \times \Delta\sigma/\sigma$).

  * $\Rightarrow$ if $\Delta\sigma/\sigma \sim 0.01$, $\overline{r}$ technique better for rates for all m_h.
  * $\Rightarrow$ if $\Delta\sigma/\sigma \sim 0.05$, $\overline{r}$ technique better for $\Gamma_h^{\rm tot}$ for all m_h.

- How does this compare to what we get from NLC/TESLA?

  The NLC/TESLA is now more than competitive given the now accepted $L = 500~{\rm fb}^{-1}/{\rm yr}$ combined with its ability to measure the $h_{SM} \to ZZ$ coupling (inclusive missing mass technique in $e^+e^- \to Zh_{SM}$).

Table 2: Percentage errors (1σ) for 1 year of e^+e^- collisions assuming $L = 500~{\rm fb}^{-1}$. From TESLA orange book report.

Quantity	Errors		
Mass (GeV)	**120**	**140**	**150**
$\Gamma(ZZ)$	0.025	0.027	0.030
$B(b\overline{b})$	0.024	0.026	0.065
$B(c\overline{c})$	0.083	0.19	-
$B(\tau^+\tau^-)$	0.05	0.08	-
$B(WW^*)$	0.051	0.025	0.021
$B(ZZ^*)$	-	-	0.169
$B(\gamma\gamma)$	0.19	-	-
$\Gamma_h^{\rm tot} = \frac{\Gamma(WW)_{WW\to h}}{B(WW^*)}$	0.061	0.045	0.134
$\Gamma_h^{\rm tot} = \frac{\Gamma(WW)=\Gamma(ZZ)}{B(WW^*)}$	0.056	0.037	0.036
$\Gamma_h^{\rm tot} = \frac{\Gamma(\gamma\gamma)_{\gamma\gamma\to h}}{B(\gamma\gamma)}$	0.19	-	-

NLC/TESLA in 1 year does better than muon collider in 4 years, **unless we can increase the $R = 0.003\%$ luminosity**.

- Use these measurements to distinguish between h_{SM} and h^0 of some other model; e.g. in the MSSM $\Rightarrow$ constraints on H^0 and A^0.

  – σB ratios from s-channel Higgs factory μC data are not competitive with NLC/TESLA; **ignoring systematics in interpretation** $\Rightarrow$ latter probe $m_{A^0} \lesssim 700$ GeV.

  – What about absolute $\mu^+\mu^- \to h \to b\bar{b}$ rate? This is the most accurately measured quantity at the μC, **but systematics?**

  Can we disentangle effects from $\Gamma_h^{\rm tot}$, m_b, ...?

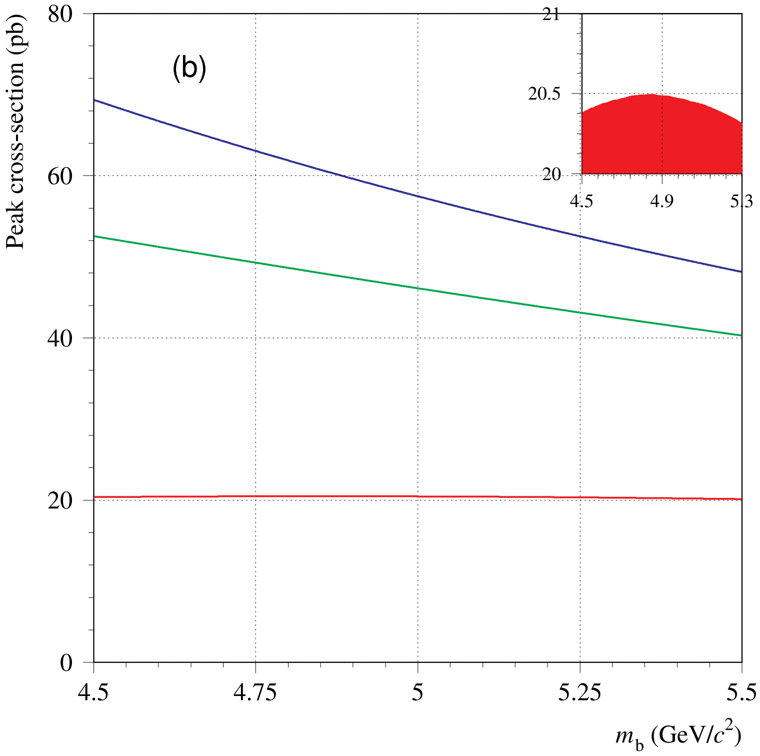


Figure 8: We plot as a function of m_b, $\mu^+\mu^- \to h$ for $\sigma = 0$ (top), $\mu^+\mu^- \to h \to b\bar{b}$ for $\sigma = 0$ (middle), $\mu^+\mu^- \to h \to b\bar{b}$ for $\sigma(R = 0.003\%)$. From Janot.

$B(h \to b\bar{b}) = \frac{\Gamma(h \to b\bar{b})}{\Gamma_h^{\rm tot}}$ varies slowly with m_b; $\sigma \neq 0$ damps small $\Gamma_h^{\rm tot}$ more than large $\Gamma_h^{\rm tot}$ $\Rightarrow$ little m_b dependence.

Now look at χ^2 of deviation from SM expectation. Claim:

* Muon Collider with $L_{\rm tot} = 0.2~{\rm fb}^{-1}$ probes $m_{A^0} \lesssim 600$ GeV at 3 sigma level. **Beware: SUSY decays, large $\tan\beta$ one-loop couplings to $b\overline{b}$, . . .**

– Using NLC $\sqrt{s} = 500$ GeV ($L_{\rm tot} = 200~{\rm fb}^{-1}$) e^+e^- data there is a cleaner alternative.

There are 4 ways to determine $\Gamma(h \to \mu^+\mu^-)$:

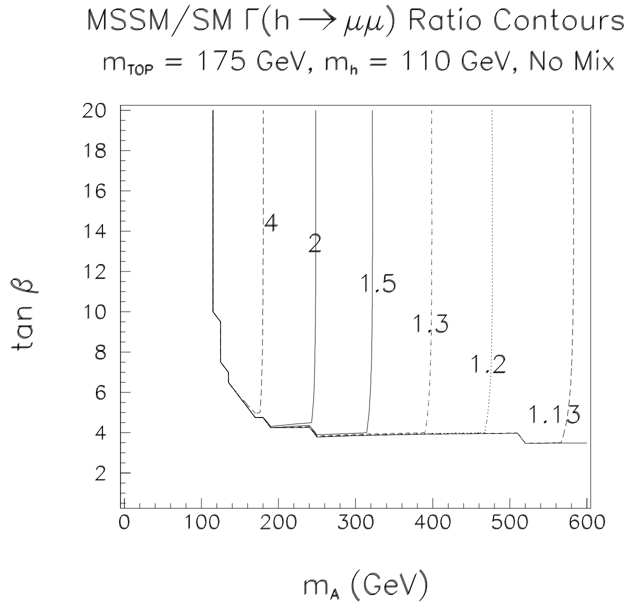
1) $\Gamma(h_{SM} \to \mu^+\mu^-) = \frac{[\Gamma(h_{SM}\to\mu^+\mu^-)B(h_{SM}\to b\overline{b})]_{\mu C}}{B(h_{SM}\to b\overline{b})_{\rm NLC}}$;

2) $\Gamma(h_{SM} \to \mu^+\mu^-) = \frac{[\Gamma(h_{SM}\to\mu^+\mu^-)B(h_{SM}\to WW^\star)]_{\mu C}}{B(h_{SM}\to WW^\star)_{\rm NLC}}$;

3) $\Gamma(h_{SM} \to \mu^+\mu^-) = \frac{[\Gamma(h_{SM}\to\mu^+\mu^-)B(h_{SM}\to ZZ^\star)]_{\mu C}\Gamma^{\rm tot}_{h_{SM}}}{\Gamma(h_{SM}\to ZZ^\star)_{\rm NLC}}$;

4) $\Gamma(h_{SM} \to \mu^+\mu^-) = \frac{[\Gamma(h_{SM}\to\mu^+\mu^-)B(h_{SM}\to WW^\star)\Gamma^{\rm tot}_{h_{SM}}]_{\mu C}}{\Gamma(h_{SM}\to WW^\star)_{\rm NLC}}$.

* Determine to $\pm 4\%$ with $L = 0.4~{\rm fb}^{-1}$ scan and NLC ($200~{\rm fb}^{-1}$) data $\Rightarrow$ 3σ sensitivity for m_{A^0} up to 600 GeV ($m_h \lesssim 2m_W$).
* **No systematics like those for $b\overline{b}$ rate procedure.**
* μC provides only probe of this 2nd generation lepton coupling.



5 Using muon spin precision to determine the CP nature of a Higgs boson

For a light Higgs, the NLC/TESLA techniques are via $\gamma\gamma$ collisions (generally quite feasible) or $t\bar{t}h$ production final state angular distributions (requires $\sqrt{s} \gtrsim 800$ GeV). **But μC has cleanest technique of all.** For resonance, R, production at a μC with $\overline{\mu}(a + ib\gamma_5)\mu$ coupling to the muon the cross section takes the form

$$\begin{aligned}\overline{\sigma}_S(\zeta) &= \overline{\sigma}_S^0 \left(1 + P_L^+ P_L^- + P_T^+ P_T^- \left[\frac{a^2-b^2}{a^2+b^2}\cos\zeta - \frac{2ab}{a^2+b^2}\sin\zeta\right]\right) \\ &= \overline{\sigma}_S^0 \left[1 + P_L^+ P_L^- + P_T^+ P_T^- \cos(2\delta + \zeta)\right] ,\end{aligned} \tag{5.1}$$

- $\delta \equiv \tan^{-1}\frac{b}{a}$,
- P_T (P_L) is the degree of transverse (longitudinal) polarization: no P_T $\Rightarrow$ sensitivity to $\overline{\sigma}_S^0 \propto a^2 + b^2$ only.
- ζ = angle of the μ^+ transverse polarization relative to that of the μ^- as measured using the the direction of the μ^-'s momentum as the $\hat{z}$ axis.
- Only the $\sin\zeta$ term is truly CP-violating, but $\cos\zeta$ also $\Rightarrow$ significant sensitivity to a/b.

Ideal = isolate $\frac{a^2-b^2}{a^2+b^2}$ and $\frac{-2ab}{a^2+b^2}$ via the asymmetries (take $P_T^+ = P_T^- \equiv P_T$ and $P_L^\pm = 0$)

$$\begin{aligned}\mathcal{A}_I &\equiv \frac{\overline{\sigma}_S(\zeta=0) - \overline{\sigma}_S(\zeta=\pi)}{\overline{\sigma}_S(\zeta=0) + \overline{\sigma}_S(\zeta=\pi)} = P_T^2 \frac{a^2-b^2}{a^2+b^2} = P_T^2 \cos 2\delta\,, \\ \mathcal{A}_{II} &\equiv \frac{\overline{\sigma}_S(\zeta=\pi/2) - \overline{\sigma}_S(\zeta=-\pi/2)}{\overline{\sigma}_S(\zeta=\pi/2) + \overline{\sigma}_S(\zeta=-\pi/2)} = -P_T^2 \frac{2ab}{a^2+b^2} = -P_T^2 \sin 2\delta\,.\end{aligned}$$

If a^2+b^2 already well determined, and background is known,

$$[\delta\mathcal{A}]^2 = \frac{\overline{\sigma}_S^0 + \overline{\sigma}_B^0 - \mathcal{A}^2[\overline{\sigma}_S^0]^2(\overline{\sigma}_S^0+\overline{\sigma}_B^0)^{-1}}{L\,[\overline{\sigma}_S^0]^2} \tag{5.2}$$

Since $\mathcal{A}^2 \leq P_T^4$, $P_T < 0.5 \Rightarrow$ neglect $\mathcal{A}^2$ in numerator $\Rightarrow \frac{\mathcal{A}^2}{[\delta\mathcal{A}]^2} \propto P_T^4 L$.

- Ideal would be be run at fixed $\zeta = 0, \pi/4, \pi/2, 3\pi/2$.
- Precession makes this impossible. Take $\vec{B} = -B\hat{y}$, and write

$$s_{\mu^-} = P_H^-\left[\gamma(\beta,\hat{z})\cos\theta^- - (0,\hat{x})\sin\theta^-\right] + P_V^-(0,\hat{y})\,. \tag{5.3}$$

$$s_{\mu^+} = P_H^+\left[\gamma(\beta,-\hat{z})\cos\theta^+ - (0,\hat{x})\sin\theta^+\right] + P_V^+(0,\hat{y})\,. \tag{5.4}$$

  - $\hat{z}$ is direction of μ^- instantaneous momentum;
  - P_H (P_V) is horizontal (vertical, i.e. $\hat{y}$) degree of polarization; $P_H^\pm\cos\theta^\pm = P_L^\pm$; $\sqrt{[P_H^\pm\sin\theta^\pm]^2 + [P_V^\pm]^2} = P_T^\pm$
  - If μ^- beam enters the storage ring with $\widehat{P_H^-} = \hat{p}_{\mu^-}$, $\theta^-(N_T) = \omega(N_T - 1/2)$, where N_T is the number of turns during storage, counted starting with $N_T = 1$ the first time the bunch passes the IP, and $\omega = 2\pi\gamma\frac{g_\mu - 2}{2}$, with $\gamma = E/m_\mu$.

As a function of θ^- and θ^+, defining $c_- \equiv \cos\theta^-$ etc.,

$$\frac{\overline{\sigma}_S(\theta^+,\theta^-)}{\overline{\sigma}_S^0} = (1 + P_H^+P_H^-c_+c_-) + \cos 2\delta(P_V^+P_V^- + P_H^+P_H^-s_+s_-) + \sin 2\delta(P_H^-P_V^+s_- - P_H^+P_V^-s_+)\,. \tag{5.5}$$

I: To approximate the $\zeta = 0$ configuration, we choose $P_H^+ = P_H^- = P_H = 0.05$, $\theta^- = \theta^+$, $P_V^+ = P_V^- = \sqrt{P^2 - P_H^2}$.

II: To approximate the $\zeta = \pi$ configuration, we choose $P_H^+ = P_H^- = P_H = 0.05$, $\theta^- = \theta^+ + \pi$, $P_V^- = -P_V^+ = -\sqrt{P^2 - P_H^2}$.

III: To emphasize the $\zeta = \pi/2$ and $\zeta = 3\pi/2$ configurations over many turns of the bunches, we choose $P_H^- = P$ ($P_V^- = 0$), $P_H^+ = P_H = 0.05$ and $P_V^+ = \sqrt{P^2 - P_H^2}$.

To determine how well one can perform the measurements, must develop strategy for maximizing $\langle P_T^4 \rangle L$ by selecting only energetic muons to accelerate and combining bunches. No time to describe in detail.

Our (Grzadkowski, Gunion, Pliszka) conclusions are encouraging if you have a proton source that is 3 times as intense as needed for studies not requiring transverse polarization, the 3 compensating for selecting against slow muons.

- Define $(\hat{a}, \hat{b}) = (a, b)/\left(g m_\mu / 2 m_W\right)$.
- Give contours at $\Delta\chi^2 = 1, 4, 6.635, 9$ in the $\delta = \tan^{-1}\frac{\hat{b}}{\hat{a}}$, $r = \sqrt{\hat{a}^2 + \hat{b}^2}$ parameter space.
- define $I =$ proton source intensity enhancement.
- Compare 4 cases
  - (i) $P = 0.2$, $L = 0.15~\mathrm{fb}^{-1}$ (which corresponds to $I = 1$);
  - (ii) maintaining same proton intensity, $I = 1$, but merge the bunches, $\Rightarrow P^m(I = 1) \sim 0.39$, for which $L = 0.075~\mathrm{fb}^{-1}$;
  - (iii) increasing the proton source intensity by a factor of two, $I = 2$, while merging the bunches, corresponding to $P^m(I = 2) \sim 0.48$, for which $L = 0.075~\mathrm{fb}^{-1}$;
  - (iv) $I = 3$, using just-full bunches, corresponding to $P^f(I = 3) \sim 0.45$, for which $L = 0.15~\mathrm{fb}^{-1}$.

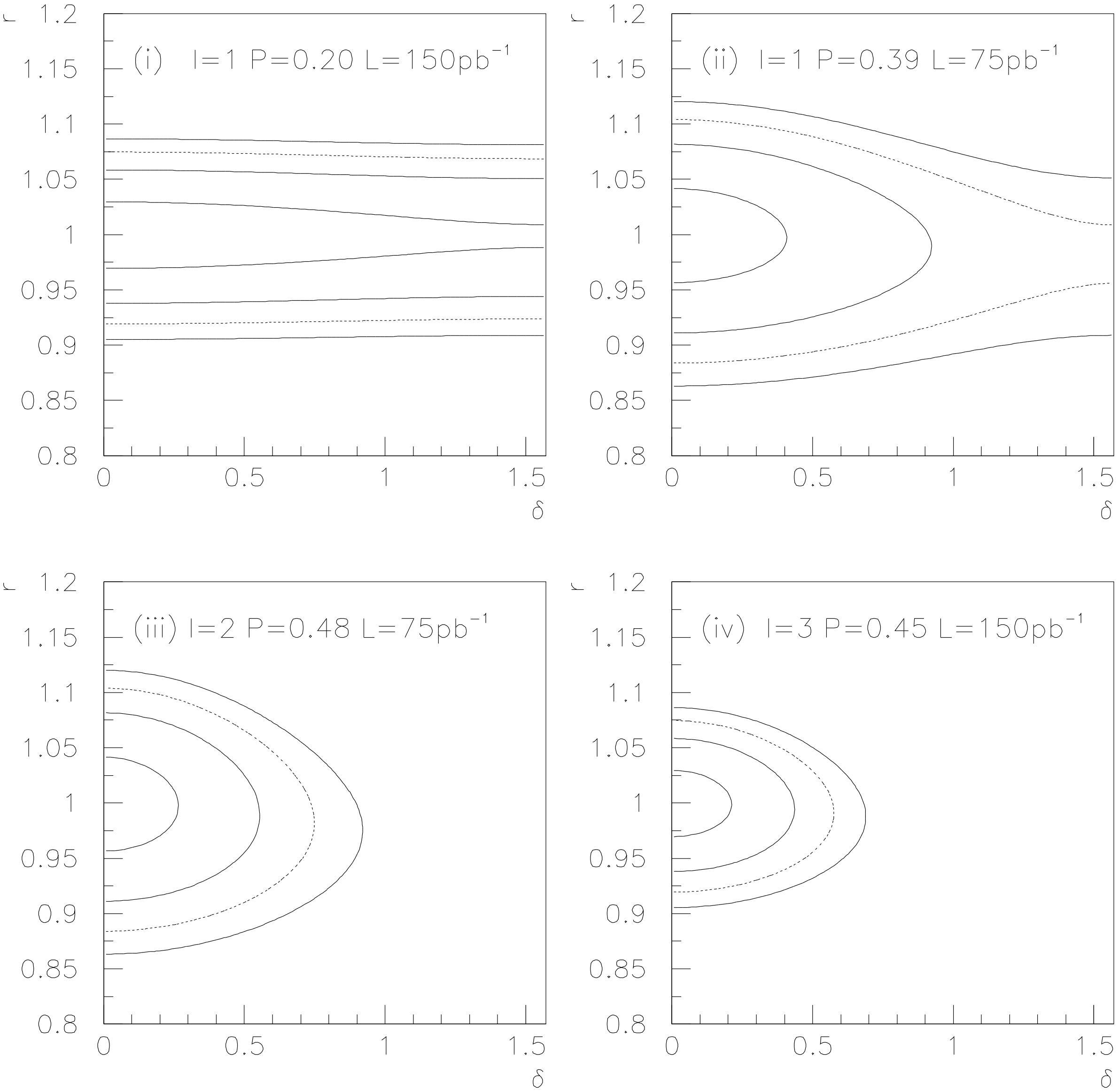


Figure 9: SM ($\hat{a} = 1, \hat{b} = 0$) with $m_h = 130$.

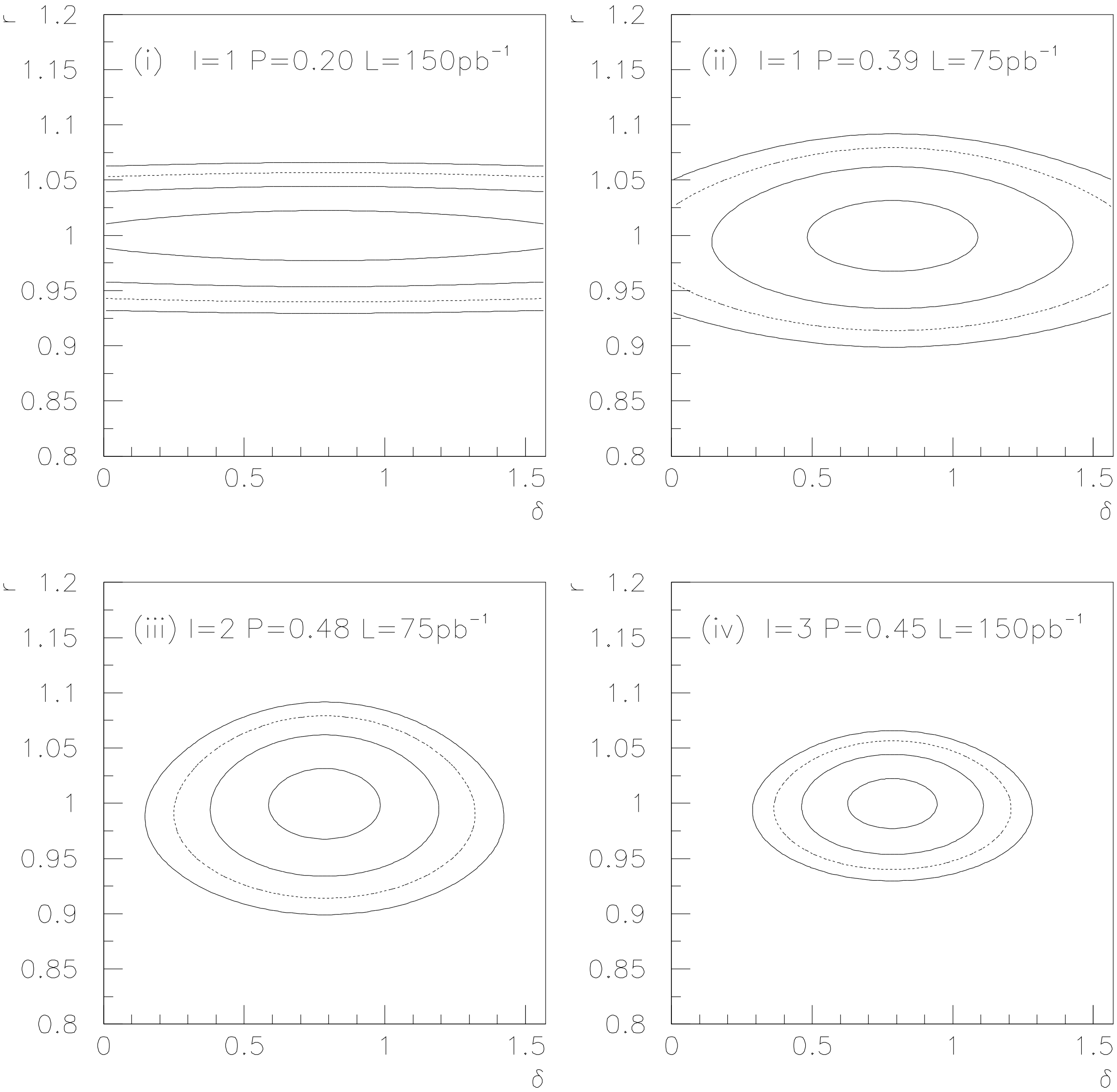


Figure 10: Mixed-CP higgs boson with $\hat{a} = \hat{b} = 1/\sqrt{2}$ and $m_h = 110$ GeV.

6 Scanning for a decoupled h^0 or an A^0

Use notation h for the decoupled h^0 or the A^0.

- Earlier discussion $\Rightarrow \sqrt{s}=500$ GeV NLC/TESLA will find such an h if $m_h < 150$ GeV or so. If found, μC with $\sqrt{s}=m_h$ is unique machine for performing a detailed study when other Higgs are heavy enough such that relevant Higgs pair h^0A^0 process is kinematically forbidden at available NLC/TESLA $\sqrt{s}$.

- If not found, begin scan at $\sqrt{s}\sim 150$ GeV.

  Scan procedure depends upon $\Gamma_h^{\rm tot}(m_h)$ vs. luminosity for required R such that $\sigma \lesssim \Gamma_h^{\rm tot}$.

  Must make steps of size $\Gamma_h^{\rm tot}\sim\sigma$.

- For $2m_t > m_h > 150$ GeV, $h\to b\overline{b}$ and $\Gamma_h^{\rm tot}\sim 0.05-0.1$ GeV unless $\tan\beta<1$.

  For $m_h > 2m_t$, $\Gamma_h^{\rm tot}$ rises to at least 1 GeV or so.

  $\Rightarrow$ only need $R=0.05-0.1\%$ or so for $m_h<2m_t$ rising to $R=0.5-1\%$ for $m_h>2m_t$.

- In a $3-4$ year program, using earlier quoted nominal yearly L's for such R's as function of $\sqrt{s}$, we could imagine devoting:
  - $L = 0.003~\mathrm{fb}^{-1}$ to 2000 points separated by 0.1 GeV in $\sqrt{s} = 150-350$ GeV range — total $L = 4~\mathrm{fb}^{-1}$, requires 3 years;

    Find h in $b\bar{b}$ state if $\tan\beta \gtrsim 4-5$.
  - $L = 0.03~\mathrm{fb}^{-1}$ to each of 100 points separated by 0.5 GeV in $\sqrt{s} = 350-400$ GeV range — total $L = 3~\mathrm{fb}^{-1}$, $= 1/2$ year;

    For $\tan\beta > 6$, find h in $b\bar{b}$ final state.

    For $\tan\beta < 6$, find h in $t\bar{t}$ final state.
  - $L = 0.01~\mathrm{fb}^{-1}$ to each of 100 points separated by 1 GeV in $\sqrt{s} = 400-500$ GeV range — total $L = 1~\mathrm{fb}^{-1}$, $= 1/10$ year;

    For $\tan\beta > 7$, find h in $b\bar{b}$ final state.

    For $\tan\beta < 8$, find h in $t\bar{t}$ final state.

  In this way, would find h (whether decoupled h^0 or A^0) if $m_h < 2m_t$ and $\tan\beta \gtrsim 5$ or if $m_h > 2m_t$ for any $\tan\beta$.

  $\Rightarrow$ Need more luminosity to guarantee discovery if $m_h < 2m_t$.

  $\sqrt{s} = 800$ GeV TESLA/NLC $m_h > 250$ GeV limit on such an h would help a lot to limit $m_h < 2m_t$ scan region.

- If $\tan\beta > 5$, then could run at $\sqrt{s} =$ maximal and see h in bremsstrahlung tail of $m_{b\bar{b}}$ distribution after 3-4 years of running.

7 Pseudo-Goldstone boson physics

- All dynamical symmetry breaking (DSB) theories (including technicolor) with symmetry in the DSB sector larger than $SU(2)_L \times SU(2)_R$ will contain PGB's.
- In large class of models, the lightest PGB is the colorless, neutral, $T_3 = -1/2$ mass eigenstate, P^0. It is much lighter than any other DSB state, since its mass is proportional to m_b.
- Because the P^0 is $T_3 = -1/2$ it couples to $\mu^+\mu^-$ with $\lambda_\mu \sim m_\mu/v$, similar to Higgs.

  Further, it is almost as narrow as Higgs: for 'typical' model parameter choices $\Gamma^{\rm tot}_{P^0} = 2$ MeV $-$ 20 MeV for $m_{P^0} = 50 - 200$ GeV.
- No current constraints on P^0 from LEP, LEP2, Tevatron RunI.

  Some hope at RunII.

  LHC will detect $gg \to P^0 \to \gamma\gamma$ Higgs like mode, with very high event rate, from $?? \leq m_{P^0} \leq 200$ GeV. ?? might be of order 30 GeV (uncertainty due to $\gamma - j$ discrimination level being uncertain); detailed study required.
- Because the P^0 has no tree-level coupling to ZZ, $\Rightarrow$ not easily studied at e^+e^- collider (best mode: $e^+e^- \to \gamma P^0$): only $\gamma\gamma$ collider provides some potential: $\gamma\gamma \to P^0 \to b\bar{b}$ robust; $\tau^+\tau^-$ and $\gamma\gamma$ final states uncertain.
- Only the muon collider could really study the P^0 in detail and have the ability to scan for its presence at low mass.

$\lesssim 1$ year would be required to scan $m_{P^0} < 80$ GeV.

LHC discovery certain above this and $\Rightarrow \sim 100$ MeV mass uncertainty; for m_{P^0} not close to m_Z μC can scan this in $\lesssim 1$ year and center on $\sqrt{s} \simeq m_{P^0}$.

- Once centered on $\sqrt{s} \sim m_{P^0}$, good measurements of rates in the $b\overline{b}$, $\tau^+\tau^-$, $c\overline{c}$ and gg s-channel production final states will be possible:

  An accurate measurement of the total width $\Gamma^{\rm tot}_{P^0}$ will also result.

Table 3: Fractional statistical errors (1σ) for $\overline{\sigma}B(P^0 \to {\rm all})$ (combining $b\overline{b}$, $\tau^+\tau^-$, $c\overline{c}$ and gg tagged-channel rates) and $\Gamma^{\rm tot}_{P^0}$ for $\mu^+\mu^- \to P^0$. We compare results for an $R = 0.003\%$ three-point scan with total integrated luminosity of $L = 0.4~{\rm fb}^{-1}$ to results obtained using the $\overline{\tau}$-ratio technique assuming accumulated luminosities at $E = m_{P^0}$ of $L = 0.2f~{\rm fb}^{-1}$ at $R = 0.003\%$ and $L = 0.94(2-f)~{\rm fb}^{-1}$ at $R = 0.03\%$ (corresponding to roughly $2f$ years of running at $R = 0.003\%$ and $(4-2f)$ years of running at $R = 0.03\%$ for pessimistic instantaneous luminosity assumptions). f (tabulated below) is chosen to minimize the error in $\Gamma^{\rm tot}_{P^0}$. Efficiencies, cuts, tagging and bremsstrahlung included.

Quantity	Errors for the scan procedure					
Mass (GeV)	**60**	**80**	m_Z	**110**	**150**	**200**
$\overline{\sigma}B$	0.0029	0.0054	0.043	0.0093	0.012	0.018
$\Gamma^{\rm tot}_{P^0}$	0.014	0.029	0.25	0.042	0.052	0.10
Quantity	Errors for the $\overline{\tau}$-ratio procedure					
Mass (GeV)	**60**	**80**	m_Z	**110**	**150**	**200**
f	0.8	0.7	0.6	0.8	0.9	1.0
$\overline{\sigma}B$	0.0029	0.0062	0.055	0.010	0.011	0.016
$\Gamma^{\rm tot}_{P^0}$	0.014	0.028	0.24	0.041	0.039	0.053

- This μC info (+ LHC measurement) $\Rightarrow$ measure many interesting parameters of the effective DSB Lagrangian describing the P^0.

Conclusion If DSB is nature's choice and PGB's exist, there is no substitute for the muon collider.

8 Conclusions

The muon collider Higgs factory could play an important role in delineating a SM-like Higgs boson. Even if $L = 500~\mathrm{fb}^{-1}$ is possible at NLC/TESLA there is no substitute for a direct probe of $\Gamma_h^{\rm tot}$ and no other access to $\mu^+\mu^-$ coupling.

If the Higgs sector is not SUSY-like, and there are light Higgs bosons with no WW/ZZ coupling, it might be that only the muon collider could detect them.

And certainly only a μC could study such a 'decoupled' Higgs in a thorough manner.

A technicolor scenario with a light pseudo-Nambu Goldstone boson demands a muon collider.

It remains highly desirable to exceed the luminosities now used as benchmarks for a Higgs factory.