The NMSSM $h \rightarrow aa$ Scenario

A Supersymmetric model with minimal fine-tuning, coupling constant unification, RGE generated EWSB and a SM-like Higgs with mass

$\sim 100$ GeV.

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Based largely on:
R. Dermisek and J. Gunion, hep-ph/0510322
R. Dermisek and J. Gunion, hep-ph/0502105
J. Gunion, D. Hooper and B. McElrath, hep-ph/0509024

See also: J. Gunion, D. Miller, A. Pilaftsis, forthcoming CPNSH (CP-violating and Non-Standard Higgses) CERN Yellowbook Report.
1. The LHC is coming, but is it guaranteed to find the Higgs of any SUSY model? Or might the ILC be even more critical than imagined?

2. Why focus on SUSY? It remains the most attractive way to solve the hierarchy problem.

3. The Minimal SUSY Model (MSSM) is very attractive, but LEP limits on the lightest Higgs and the gluino imply that it is in a fine-tuned part of parameter space.

4. The Next to Minimal Supersymmetric Model (NMSSM) maintains all the attractive features of the MSSM while avoiding fine tuning, especially if $m_{h_1} \sim 100$ GeV, as preferred by LEP data (precision and direct search).

5. Collider Implications Low-fine-tuning NMSSM models change how to search for the Higgs at the LHC and imply that one should look again at the LEP data for $h \rightarrow aa$ Higgs signals. The ILC will be essential.
The LHC is at hand
The Tunnel
The Magnets
The ATLAS Detector
The CMS Detector

But will the LHC detectors detect the Higgs boson. The very attractive NMSSM SUSY scenario suggests we may have to work hard.
The Beauty of Supersymmetry

- SUSY is mathematically intriguing.

- SUSY is naturally incorporated in string theory.

- Scalar fields have a natural place in SUSY, and so there are candidates for the spin-0 fields needed for electroweak symmetry breaking and Higgs bosons.

- SUSY cures the naturalness / hierarchy problem provided the SUSY breaking scale is of order $\sim 1 \text{ TeV}$.

- The MSSM comes close to being very nice.

  If we assume that all sparticles reside at the $O(1 \text{ TeV})$ scale and that $\mu$ is also $O(1 \text{ TeV})$, then, the MSSM has two particularly wonderful properties.
1. **Gauge Coupling Unification**

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**Standard Model**

![Graph showing Standard Model](image)

**MSSM**

![Graph showing MSSM](image)

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**Figure 1:** Unification of couplings constants ($\alpha_i = g_i^2/(4\pi)$) in the minimal supersymmetric model (MSSM) as compared to failure without supersymmetry.

The MSSM sparticle content + two-doublet Higgs sector $\Rightarrow$ gauge coupling unification at $M_U \sim \text{few} \times 10^{16} \text{ GeV}$, close to $M_P$. High-scale unification correlates well with the attractive idea of gravity-mediated SUSY breaking.
Figure 2: Evolution of SUSY-breaking masses or masses-squared, showing how $m_{Hu}^2$ is driven $< 0$ at low $Q \sim \mathcal{O}(m_Z)$.

Starting with universal soft-SUSY-breaking masses-squared at $M_U$, the RGE’s predict that the top quark Yukawa coupling will drive one of the soft-SUSY-breaking Higgs masses squared ($m_{Hu}^2$) negative at a scale of order $Q \sim m_Z$, thereby automatically generating electroweak symmetry breaking ($\langle H_u \rangle = h_u, \langle H_d \rangle = h_d$), BUT MAYBE $m_Z$ IS FINE-TUNED.
The Higgs Mass

In the presence of soft-SUSY-breaking, the light Higgs has (\( \tan \beta = h_u/h_d \))

\[
m_h^2 \sim m_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} v^2 y_t^4 \sin^4 \beta \log \left( \frac{m_{t_1} m_{t_2}}{m_t^2} \right) + \ldots
\]

\[\text{large } \tan \beta \sim (91 \text{ GeV})^2 + (38 \text{ GeV})^2 \log \left( \frac{m_{t_1} m_{t_2}}{m_t^2} \right). \quad (1)\]

A Higgs mass of order 100 GeV, as predicted for stop masses \( \sim 2m_t \), is in wonderful accord with precision electroweak data.

So, why haven’t we seen the Higgs? Is SUSY wrong, are stops heavy, or is the MSSM too simple?
MSSM Problems

- The \( \mu \) parameter in \( W \supset \mu \widehat{H}_u \widehat{H}_d \),\(^1\) is dimensionful, unlike all other superpotential parameters. A big question is why is it \( \mathcal{O}(1 \text{ TeV}) \) (as required for EWSB and \( m_{\tilde{\chi}^\pm_1} \) lower bound), rather than \( \mathcal{O}(M_U, M_P) \) or 0.

- LEP limits:

**Figure 3:** Maximal-mixing (\( X_t = A_t - \mu \cot \beta = -2m_{\text{SUSY}} = -2 \text{ TeV, } \mu > 0 \)) and no-mixing (with \( \mu > 0 \)) LEP exclusions at 90\% CL. From CERN-PH-EP/2006-001.

\(^1\) Hatted (unhatted) capital letters denote superfields (scalar superfield components).
The LEP limits on Higgs bosons have pushed the CP-conserving MSSM into an awkward corner of parameter space characterized by large $\tan\beta$ and large $\sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}}$. For $m_{\tilde{t}_L} = m_{\tilde{t}_R} = 1$ TeV $\equiv m_{\text{SUSY}}$, we have the MSSM exclusion plots shown.

There is still room, but we need $\sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}} \gtrsim 900$ GeV.

**Fine-tuning**

Minimization of the Higgs potential gives (at scale $m_Z$)

$$\frac{1}{2} m_Z^2 = -\mu^2 + \frac{m_{H_d}^2 - \tan^2 \beta m_{H_u}^2}{\tan^2 \beta - 1}$$

and the $m_Z$-scale $\mu, m_{H_u}^2, m_{H_d}^2$ parameters are sensitive to their GUT scale values yielding at $\tan\beta = 10$ (similar to $\tan\beta = 2.5$ results in Kane and King hep-ph/9810374 and Bastero-Gil, Kane, and King hep-ph/9910506)

$$m_Z^2 = -2.0 \mu^2(M_U) + 5.9 M_3^2(M_U) + 0.8 m_Q^2(M_U) + 0.6 m_U^2(M_U) - 1.2 m_{H_u}^2(M_U) - 0.7 M_3(M_U) A_t(M_U) + 0.2 A_t^2(M_U) + \ldots$$
One would expect that $m_Z \sim 2M_3(M_U), m_Q(M_U), m_u(M_U) \sim m_{\tilde{g}}, m_{\tilde{t}}$, we need a very light gluino and a rather light stop to avoid fine-tuning OR we need highly correlated cancellations and large $A_t$ (Nomura’s talk).

A rigorous measure is $F$ plotted below.

\[ F = \text{Max}_p \left| \frac{p}{m_Z} \frac{\partial m_Z}{\partial p} \right|, \]

where $p \in \left\{ M_{1,2,3}, m_{Q}^2, m_{U}^2, m_{D}^2, m_{H_u}^2, m_{H_d}^2, \mu, A_t, B\mu, \ldots \right\}$ (all at $M_U$). The + points have $m_h < 114$ GeV, and are experimentally excluded. The $\times$ points have $m_h \geq 114$ GeV. Plot is for $\tan \beta = 10, M_{1,2,3} = 100, 200, 300$ GeV (at scale $m_Z$). All parameters scanned, but $|A_t| < 500$ GeV is imposed.
Essentially all the blue points fail LEP limits due to $m_h < 114$ GeV. Note that if $m_h \sim 100$ GeV were ok, then smallest $F$ occurs there. One can do better by taking very large $A_t$ values.

**Figure 5:** $F$ in the MSSM. The + points have $m_h < 114$ GeV. The $\times$ points have $m_h \geq 114$ GeV. Plot is for $\tan \beta = 10$, $M_{1,2,3} = 100, 200, 300$ GeV (at scale $m_Z$). All other parameters were scanned over, with $|A_t| < 4$ TeV imposed.
The figure shows clearly that large negative $A_t(m_Z)$ is required to get anything like reasonable $F$ for allowed $m_h \geq 114$ GeV points, and even then $F \gtrsim 30$.

**So, what direction should one head in?**

- CP-violating MSSM, e.g. CPX-like scenarios?
  These don’t solve the $\mu$ issue, and nature has shown very little inclination for CP-violation as large as that needed to significantly alter the CP-conserving situation.
- Large extra dimensions, little Higgs, Higgsless, ....
  All worth exploring, but these models are complicated and typically have problems of one kind or another, especially precision EW data.
- Hints from string theory.
  In particular, it is very clear that extra singlet superfields are common in string models.
  Let’s make use of singlets and let’s do it in the simplest possible way (i.e. no associated gauge group and no dimensionful superpotential parameters) $\Rightarrow$ the NMSSM.
The NMSSM

- The NMSSM introduces just one extra singlet superfield, with superpotential $\lambda \hat{S} \hat{H}_u \hat{H}_d$. The $\mu$ parameter is then automatically generated by $\langle S \rangle$ leading to $\mu_{\text{eff}} \hat{H}_u \hat{H}_d$ with $\mu_{\text{eff}} = \lambda \langle S \rangle$. The only requirement is that $\langle S \rangle$ be of order the SUSY-breaking scale at $\sim 1$ TeV.

- However, $\lambda \hat{S} \hat{H}_u \hat{H}_d$ cannot be the end.

  Including Yukawa $W$ terms, there is a PQ symmetry that will spontaneously break when the Higgs scalars gain vevs, and a pseudo\(^2\)-Nambu-Goldstone boson, known as the PQ axion (it is actually one of the pseudoscalar Higgs bosons), will be generated.

  For values of $\lambda \sim \mathcal{O}(1)$, this axion would have been detected in experiment and this model ruled out.

- Gauging the $U(1)_{\text{PQ}}$ (so that axion is absorbed in $Z'$ mass) typically leads to FCNC problems.

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\(^2\)The axion is only a “pseudo”-Nambu-Goldstone boson since the PQ symmetry is explicitly broken by the QCD triangle anomaly. The axion then acquires a small mass from its mixing with the pion.
In the NMSSM, the PQ symmetry is explicitly broken by $W \ni \frac{1}{3} \kappa \hat{S}^3$. Other possible superpotential terms with dimensionful parameters are absent if one demands that the superpotential be invariant under a $Z_3$ symmetry.

If the $Z_3$ is applied also to soft SUSY breaking terms, only $\frac{1}{3} \kappa A \kappa S^3$ is allowed in addition to $\lambda A \lambda S H_u H_d$.

However, this $Z_3$ symmetry cannot be completely unbroken. If it were, a cosmological "domain wall problem" would arise.

To avoid this problem (Panagiotakopoulos and Tamvakis), one introduces a $Z_2^R$ symmetry that is broken by the soft-SUSY breaking terms, giving rise to harmless tadpoles of order $\frac{1}{(16\pi^2)^n} M^3_{\text{SUSY}}$, with $2 \leq n \leq 4$. For example, a superpotential term of form $\hat{S}^7/M^4_\phi$ (which is ok under $Z_2^R$) generates at 4-loops (Abel) the tadpole form $\delta V \sim (\frac{1}{16\pi^2})^4 m^3_{\text{SUSY}} (S + S^*)$.

Although these terms are phenomenologically irrelevant, they are entirely sufficient to break the global $Z_3$ symmetry and make the domain walls collapse.

Net Result
Since the only relevant superpotential terms that are introduced have dimensionless couplings, the scale of the vevs (i.e. the scale of EWSB) is determined by the scale of SUSY-breaking.

- Further, all the good properties of the MSSM (coupling unification and RGE EWSB, in particular) are preserved under singlet addition.

- New Particles

The single extra singlet superfield of the NMSSM contains an extra neutral gaugino (the singlino) ($\tilde{\chi}_0^{1,2,3,4,5}$), an extra CP-even Higgs boson ($h_{1,2,3}$) and an extra CP-odd Higgs boson ($a_{1,2}$).

- The parameters of the NMSSM

Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

$$\lambda \hat{S} H_u H_d + \frac{\kappa}{3} \hat{S}^3$$

depending on two dimensionless couplings $\lambda, \kappa$ beyond the MSSM. The associated trilinear soft terms are

$$\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3.$$
The final two input parameters are

$$\tan \beta = h_u/h_d, \quad \mu_{\text{eff}} = \lambda s,$$

where $h_u \equiv \langle H_u \rangle$, $h_d \equiv \langle H_d \rangle$ and $s \equiv \langle S \rangle$. These, along with $m_Z$, can be viewed as determining the three SUSY breaking masses squared for $H_u$, $H_d$ and $S$ (denoted $m_{H_u}^2$, $m_{H_d}^2$ and $m_S^2$) through the three minimization equations of the scalar potential. (From the model building point of view, we emphasize the reverse — i.e. the SUSY-breaking scales $m_{H_u}^2$, $m_{H_d}^2$ and $m_S^2$, along with $A_\lambda$ and $A_\kappa$ determine the EWSB vevs, $\lambda$ and $\kappa$ being dimensionless.)

Thus, as compared to the three independent parameters needed in the MSSM context (often chosen as $\mu$, $\tan \beta$ and $M_A$), the Higgs sector of the NMSSM is described by the six parameters

$$\lambda, \kappa, A_\lambda, A_\kappa, \tan \beta, \mu_{\text{eff}}.$$

In addition, values must be input for the gaugino masses and for the soft terms related to the (third generation) squarks and sleptons that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths.
The NMSSM is much less constrained than the MSSM, and is not necessarily forced into awkward/fine-tuned corners of parameter space either by LEP limits or by theoretical reasoning.

⇒ the NMSSM should be adopted as the more likely benchmark minimal SUSY model and it should be explored in detail.

- To further this study, Ellwanger, Hugonie and I constructed NMHDECAY

  http://www.th.u-psud.fr/NMHDECAY/nmhdecay.html

  http://higgs.ucdavis.edu/nmhdecay/nmhdecay.html

  It computes all aspects of the Higgs sector and checks against many (but, as we shall see, not all) LEP limits and various other constraints.

- We also developed a program to examine the LHC observability of Higgs signals in the NMSSM.

  A significant hole in the LHC no-lose theorem for Higgs discovery emerges: only if we avoid that part of parameter space for which $h \rightarrow aa$ and similar decays are present is there a guarantee for finding a Higgs boson at the LHC in one of the nine “standard” channels (e.g. $h \rightarrow \gamma\gamma$, $t\bar{t}h$, $a \rightarrow t\bar{t}b\bar{b}$, $t\bar{t}h$, $a \rightarrow t\bar{t}\gamma\gamma$, $b\bar{b}h$, $a \rightarrow b\bar{b}\tau^+\tau^-$, $WW \rightarrow h \rightarrow \tau^+\tau^-$, ...
A series of papers (beginning with JFG+Haber+Moroi at Snowmass 1996 and continued by JFG, Ellwanger, Hugonie, Moretti, Miller, .. .) has demonstrated the general nature of this LHC no-lose theorem “hole”.

- The portion of parameter space with $h \rightarrow aa, \ldots$ is small ⇒ one is tempted to ignore it were it not for the fact that it is where fine-tuning can be absent.

As before, the canonical measure of fine-tuning employed is

$$F = \text{Max}_p F_p \equiv \text{Max}_p \left| \frac{d \log m_Z}{d \log p} \right| ,$$

where the parameters $p$ comprise the GUT-scale values of $\lambda, \kappa, A_{\lambda}, A_{\kappa}$, and the usual soft-SUSY-breaking gaugino, squark, slepton, . . . masses.

- How do we get small fine-tuning?
  1. $F$ is minimum for $m_{h_1} \sim 100 \div 104$ GeV (in a totally unconstrained scan of parameter space this is just what one finds for moderate $\tan \beta$). Neither lower nor higher!
For $m_{h_1} \sim 100$ GeV, $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \sim 350$ GeV.

**Figure 6:** $F$ vs. $m_{h_1}$ for $M_{1,2,3} = 100, 200, 300$ GeV and $\tan \beta = 10$. Small $\times =$ no constraints other than global and local minimum, no Landau pole before $M_U$ and neutralino LSP. The O’s = stop and chargino limits imposed, but NO Higgs limits. The □’s = all single channel Higgs limits imposed. The large FANCY CROSSES are after requiring $m_{\tilde{a}_1} < 2m_b$. 
2. \( m_{h_1} \sim 100 \text{ GeV} \) is only LEP-allowed if \( h_1 \rightarrow a_1 a_1 \) and \( a_1 \rightarrow \tau^+ \tau^- \) \((2m_\tau < m_{a_1} < 2m_b)\) or \( gg, q\bar{q} \) \((m_{a_1} < 2m_\tau)\) so as to hide the \( h_1 \) in this mass range (more later).

3. **How natural is a light \( a_1 \)?** In fact, \( a_1 \) is a pseudo-Nambu-Goldstone boson associated with a \( U(1)_R \) symmetry of the superpotential, whose spontaneous breaking by the vevs of \( H_u, H_d \) and \( S \) would yield \( m_{a_1} = 0 \) were it not that the \( U(1)_R \) is explicitly broken by the \( A_\kappa \) and \( A_\lambda \) soft-SUSY-breaking terms, implying \( m_{a_1} \rightarrow 0 \) for \( A_\kappa, A_\lambda \rightarrow 0 \) (ignoring the small one-loop contributions to \( U(1)_R \) breaking from gaugino masses).

\[
m^2_{a_1} \simeq 3s \left( \kappa A_\kappa \sin^2 \theta_A + \frac{3\lambda A_\lambda \cos^2 \theta_A}{2 \sin 2\beta} \right) \quad \text{where}
\]
\[
a_1 \equiv \cos \theta_A a_{MSSM} + \sin \theta_A a_S, \quad \text{with} \quad \cos \theta_A \simeq \frac{2v}{s \tan \beta}.
\]  

What do we expect for \( A_\kappa \) and \( A_\lambda \)? The RGE's

\[
16\pi^2 \frac{dA_\lambda}{dt} = -6A_t \lambda_t^2 + 8\lambda^2 A_\lambda - 4\kappa^2 A_\kappa - 6g_2^2 M_2 - \frac{6}{5}g_1^2 M_1
\]
\[
16\pi^2 \frac{dA_\kappa}{dt} = 12(-\lambda^2 A_\lambda + \kappa^2 A_\kappa)
\]

imply that if \( A_\kappa(M_U), A_\lambda(M_U) \sim 0 \) then \( A_\kappa(m_Z) \ll A_\lambda(m_Z) \sim M_2. \)
Since $\cos \theta_A$ is small, the contributions of the $A_\kappa$ and $A_\lambda$ terms to $m_{a_1}^2$ are comparable for $A_\kappa(m_Z) \ll A_\lambda(m_Z)$.

4. One finds that correlated $A_\kappa, A_\lambda \neq 0$ are needed for large $B(h_1 \rightarrow a_1a_1)$ and small enough $m_{a_1}$ that $a_1 \rightarrow \tau^+\tau^-$ or jets, both being required for $h_1$ with $m_{h_1} \sim 100$ GeV to escape LEP limits on $Zh_1 \rightarrow Zb\bar{b}$.

![Figure 7: $A_\kappa(m_Z)$ vs. $A_\lambda(m_Z)$ for $F < 25$ fully ok $m_{a_1} < 2m_b$ solutions in case of $M_{1,2,3} = 100, 200, 300$ GeV and $\tan \beta = 10$. Note: $A_\kappa, A_\lambda$ exactly 0 is not ok.](image)

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How does $B(h_1 \rightarrow a_1a_1)$ depend on $A_\kappa$ and $A_\lambda$?

$M_{\text{SUSY}}=300$ GeV, $\mu=200$ GeV, $\tan \beta=10$, $m_{h_1}>90$ GeV, $m_{a_1}<2m_b$

Figure 8: $B(h_1 \rightarrow a_1a_1)$ vs. $A_\kappa$ and $A_\lambda$ for $M_{\text{SUSY}}=300$ GeV, $\tan \beta=10$, $\mu=150$ GeV after scanning over $\lambda$ and $\kappa$ values. All points have $m_{a_1}<2m_b$. Light cyan $\times$ points have large enough $B(h_1 \rightarrow a_1a_1)$ to escape LEP limits on $Zh_1 \rightarrow Zb\bar{b}$.

We see again that $|A_\kappa|$ of order a few GeV is needed for large enough $B(h_1 \rightarrow a_1a_1)$ to escape LEP limits.
Is there a new fine-tuning associated with getting both a very light $m_{a_1}$ and large enough $B(h_1 \to a_1a_1)$? Define

$$F_{\text{MAX}} = \text{Max} \left\{ \left| \frac{A_\lambda}{m_{a_1}^2} \frac{d^2m_{a_1}}{dA_\lambda} \right|, \left| \frac{A_\kappa}{m_{a_1}^2} \frac{d^2m_{a_1}}{dA_\kappa} \right| \right\}.$$

(10)

Figure 9: $F_{\text{MAX}}$ vs. $A_\kappa$ and $A_\lambda$ for $M_{\text{SUSY}} = 300$ GeV, $\tan\beta = 10$, $\mu = 150$ GeV after scanning over $\lambda$ and $\kappa$ values. All points have $m_{a_1} < 2m_b$. Light cyan $\times$ points have large enough $B(h_1 \to a_1a_1)$ to escape LEP limits on $Zh_1 \to Zbb$.

$\Rightarrow$ 5% to 10% fine tuning to get $m_{a_1} < 2m_b$ and ok $B(h_1 \to a_1a_1)$. 
5. Small $F$ is associated with small values for $m_{H_u}^2(M_U)$, $m_{H_d}^2(M_U)$ and $m_S^2(M_U)$.

Figure 10: $F$ vs. $m_{H_u,H_d,S}^2(M_U)$ for $M_{1,2,3} = 100, 200, 300$ GeV and $\tan \beta = 10$ for fully ok $m_{a_1} < 2m_b$ solutions.
More Details on Fine-Tuning and LEP limits

- Dermisek and I find that fine-tuning is absent in the NMSSM for precisely those parameter choices for which $m_{h_1} \sim 100$ GeV (and is SM-like) and yet the $h_1$ escapes LEP limits due to the presence of $h_1 \rightarrow a_1a_1$ decays. (There is little improvement in $F$ per se associated with $m_{a_1} < 2m_b$, but, as we now review, LEP limits require this, and many of the low-$F$ solutions do the job.)

We illustrate LEP constrained results for $\tan \beta = 10$, and $M_{1,2,3} = 100, 200, 300$ GeV.

After incorporating the latest LEP single-channel limits (to be discussed), we find the results shown in the following figure after doing a large scan. The + points have $m_{h_1} < 114$ GeV and the × points have $m_{h_1} \geq 114$ GeV.

As already noted, for $m_{h_1} < 114$ GeV, and in particular $m_{h_1} \sim 100$ GeV, one can achieve very low $F$ values.

An $h_1$ with $m_{h_1} \sim 100$ GeV and SM-like couplings to gauge bosons and fermions is, of course, exactly the value preferred by precision electroweak constraints.
Figure 11: \( F \) as a function of root mean stop mass after latest single-channel LEP limits. Both \( m_{h_1} < 114 \text{ GeV} \) (+) and \( m_{h_1} \geq 114 \text{ GeV} \) (×) points are allowed.
Figure 12: $F$ as a function of $m_{h_1}$ after latest *single-channel* LEP limits.
Figure 13: $F$ as a function of $B(h_1 \rightarrow a_1a_1)$ after latest single-channel LEP limits. Note that $h_1 \rightarrow a_1a_1$ can be dominant even when $m_{h_1}$ is large enough that the decay is not needed to escape LEP limits.
Among the points shown in the preceding few plots with low $F$, there are ones with $m_{a_1} > 2m_b$ and ones with $m_{a_1} < 2m_b$. The former have problems unless $m_{h_1} \gtrsim 110$ GeV.

In particular, the $Z2b$ and $Z4b$ channels are not actually independent.

- Putting the $F < 10$ scenarios with $m_{a_1} > 2m_b$ through the full LHWG analysis, one finds that all are excluded at somewhat more than the 99% CL.

In fact, all the $m_{a_1} > 2m_b$ scenarios with $m_{h_1} \lesssim 108 \div 110$ GeV are ruled out at a similar level. What is happening is that you can change the $h_1 \rightarrow b\bar{b}$ direct decay branching ratio and you can change the $h_1 \rightarrow a_1a_1 \rightarrow 4b$ branching ratio, but roughly speaking $B(h_1 \rightarrow b's) \gtrsim 0.85$ (a kind of sum rule). So, if the $ZZh_1$ coupling is full strength (as is the case in all the scenarios with any kind of reasonable $F$) there is no escape except high enough $m_{h_1}$.

- The only way to achieve really low $F$, which comes with low $m_{h_1} \sim 100$ GeV, and remain consistent with LEP is to have $m_{a_1} < 2m_b$.

The relevant limit from LEP is then only that from the $Z2b$ channel. (It turns out that LEP has never placed limits on the $Z4\tau$ channel for $h$ masses larger than about 87 GeV.)
• Note: Such a light to very light $a_1$ is not excluded by $\Upsilon, \ldots$ precision decay measurements since, as remarked earlier, the $a_1$ is very singlet-like for all the low-$F$ scenarios.

Figure 14: Observed LEP limits on $C_{eff}^{2b}$ for the low-$F$ points with $m_{a_1} < 2m_b$. 
So just how consistent are the $F < 10$ points with the observed event excess. Although it is slightly misleading, a good place to begin is to recall the famous $1 - CL_b$ plot for the $Z2b$ channel. (Recall: the smaller $1 - CL_b$ the less consistent is the data with expected background only.)

![](image)

**Figure 15:** Plot of $1 - CL_b$ for the $Zbb$ final state.
There is an observed vs. expected discrepancy exactly where we want it! And because $B(h_1 \rightarrow b\bar{b})$ is 1/10 the SM value, the discrepancy is of about the right size.

Are there other relevant limits on the kind of scenario we envision?

If the $a_1 a_1 \rightarrow 4\tau$ decay is the relevant scenario, the LEP limits run out for $m_h > 87$ GeV.

Figure 16: Contours of limits on $C^2 = \left[ g_{Zh}^2 / [g_{Zh}^2]_{SM} \right] \times BR(h \rightarrow aa) \times [BR(a \rightarrow \tau^+ \tau^-)]^2$ at $C^2 = 0.2, 0.4, 0.5, 0.6, 0.8$ and 1 (red, blue, green, yellow, magenta, and black, respectively). For example, if $C^2 > 0.2$, then the region below the $C^2 = 0.2$ contour is excluded at 95% CL.
If the $a_1 a_1 \rightarrow (gg, q \bar{q}) + (gg, q \bar{q})$ decay is relevant, then we have the hadronic decay limits. They run out for $m_h > 80 \text{ GeV}$.

Figure 17: Plot of the 95% CL limit on $C^2 = \left[ \frac{g_{Zh}^2}{g_{Zh}^2_{SM}} \right] \times BR(h \rightarrow \text{hadrons})$, where $h$ is only assumed to decay to hadrons, not any specific number of jets.
Or, if we say that the $gg$ or $q\bar{q}$ from each $a_1$ overlap to form a single 'jet', then we have the limits in the 'jet-jet' channel. They give $C^2 \lesssim 0.4$ for $m_h \sim 100$ GeV and might be relevant. A detailed analysis is needed.

Figure 18: 95% CL upper limit on $C^2 = \left[ \frac{g^2_{Zh}}{[g^2_{Zh}]_{SM}} \right] \times BR(h \rightarrow jj)$ from LEP analyses.
To see how well the $F < 10$, $m_{a_1} < 2m_b$ points describe the LEP excesses we have to run them through the full LHWG code. Well, we didn’t do it, but Philip Bechtle did it for us.

In Table 1, we give the precise masses and branching ratios of the $h_1$ and $a_1$ for all the $F < 10$ points.

We also give the number of standard deviations, $n_{\text{obs}}$ ($n_{\text{exp}}$) by which the observed rate (expected rate obtained for the predicted signal+background) exceeds the predicted background. The numbers are obtained after full processing of all $Zh$ final states using the preliminary LHWG analysis code (thanks to P. Bechtle). They are derived from $(1 - CL_b)_{\text{observed}}$ and $(1 - CL_b)_{\text{expected}}$ using the usual tables: e.g. $(1 - CL_b) = 0.32, 0.045, 0.0027$ correspond to $1\sigma, 2\sigma, 3\sigma$ excesses, respectively.

The quantity $s_{95}$ is the factor by which the signal predicted in a given case would have to be multiplied in order to exceed the 95% CL. All these quantities are obtained by processing each scenario through the full preliminary LHWG confidence level/likelihood analysis.
<table>
<thead>
<tr>
<th>$m_{h_1}/m_{a_1}$ (GeV)</th>
<th>Branching Ratios</th>
<th>$n_{\text{obs}}/n_{\text{exp}}$</th>
<th>$s95$</th>
<th>$N_{SD}^{LHC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.0/2.6</td>
<td>$h_1 \rightarrow b\bar{b}$</td>
<td>0.062</td>
<td>0.926</td>
<td>0.000</td>
</tr>
<tr>
<td>100.0/9.3</td>
<td>$h_1 \rightarrow a_1a_1$</td>
<td>0.075</td>
<td>0.910</td>
<td>0.852</td>
</tr>
<tr>
<td>100.2/3.1</td>
<td>$a_1 \rightarrow \tau\bar{\tau}$</td>
<td>0.141</td>
<td>0.832</td>
<td>0.000</td>
</tr>
<tr>
<td>102.0/7.3</td>
<td>$h_1 \rightarrow b\bar{b}$</td>
<td>0.095</td>
<td>0.887</td>
<td>0.923</td>
</tr>
<tr>
<td>102.2/3.6</td>
<td>$h_1 \rightarrow a_1a_1$</td>
<td>0.177</td>
<td>0.789</td>
<td>0.814</td>
</tr>
<tr>
<td>102.4/9.0</td>
<td>$a_1 \rightarrow \tau\bar{\tau}$</td>
<td>0.173</td>
<td>0.793</td>
<td>0.875</td>
</tr>
<tr>
<td>102.5/5.4</td>
<td>$h_1 \rightarrow b\bar{b}$</td>
<td>0.128</td>
<td>0.848</td>
<td>0.938</td>
</tr>
<tr>
<td>105.0/5.3</td>
<td>$h_1 \rightarrow a_1a_1$</td>
<td>0.062</td>
<td>0.926</td>
<td>0.938</td>
</tr>
</tbody>
</table>

Table 1: Some properties of the $h_1$ and $a_1$ for the eight allowed points with $F < 10$ and $m_{a_1} < 2m_b$ from our $\tan\beta = 10$, $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV NMSSM scan. $N_{SD}^{LHC}$ is the statistical significance of the best “standard” LHC Higgs detection channel for integrated luminosity of $L = 300$ fb$^{-1}$.

**Comments**

- If $n_{\text{exp}}$ is larger than $n_{\text{obs}}$ then the excess predicted by the signal plus background Monte Carlo is larger than the excess actually observed and vice versa.
- The points with $m_{h_1} \lesssim 100$ GeV have the largest $n_{\text{obs}}$. 
– Point 2 gives the best consistency between $n_{\text{obs}}$ and $n_{\text{exp}}$, with a predicted excess only slightly smaller than that observed.
– Points 1 and 3 also show substantial consistency.
– For the 4th and 7th points, the predicted excess is only modestly larger (roughly within $1\sigma$) compared to that observed.
– The 5th and 6th points are very close to the 95% CL borderline and have a predicted signal that is significantly larger than the excess observed.
– LEP is not very sensitive to point 8.

Thus, a significant fraction of the $F < 10$ points are very consistent with the observed event excess.

• In our scan there are many, many points that satisfy all constraints and have $m_{a_1} < 2m_b$. The remarkable result is that those with $F < 10$ have a substantial probability that they predict the Higgs boson properties that would imply a LEP $Zh \rightarrow Z + b$’s excess of the sort seen.
Collider Implications

- An important question is the extent to which the type of $h \rightarrow aa$ Higgs scenario (whether NMSSM or other) described here can be explored at the Tevatron, the LHC and a future $e^+e^-$ linear collider.

At the first level of thought, the $h_1 \rightarrow a_1a_1$ decay mode renders inadequate the usual Higgs search modes that might allow $h_1$ discovery at the LHC.

Since the other NMSSM Higgs bosons are rather heavy and have couplings to $b$ quarks that are not greatly enhanced, they too cannot be detected at the LHC. The last column of Table 1 shows the statistical significance of the most significant signal for any of the NMSSM Higgs bosons in the “standard” SM/MSSM search channels for the eight $F < 10$ NMSSM parameter choices.

For the $h_1$ and $a_1$, the most important detection channels are $h_1 \rightarrow \gamma\gamma$, $Wh_1 + t\bar{t}h_1 \rightarrow \gamma\gamma\ell^+\ell^-$, $t\bar{t}h_1/a_1 \rightarrow t\bar{t}b\bar{b}$, $b\bar{b}h_1/a_1 \rightarrow b\bar{b}\tau^+\tau^-$ and $WW \rightarrow h_1 \rightarrow \tau^+\tau^-$. Even after $L = 300$ fb$^{-1}$ of accumulated luminosity, the typical maximal
signal strength is at best $3.5\sigma$. For the eight points of Table 1, this largest signal derives from the $W h_1 + t\bar{t} h_1 \rightarrow \gamma \gamma \ell^\pm X$ channel.

There is a clear need to develop detection modes sensitive to the $h_1 \rightarrow a_1 a_1 \rightarrow \tau^+ \tau^- \tau^+ \tau^-$ and (unfortunately) $4j$ decay channels.

I will focus on $4\tau$ in my discussion of possibilities below, but keep in mind the $4j$ case.

**Hadron Colliders**

1. **An obvious possibility** is $WW \rightarrow h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$.

   Study under way with Schumacher. Looks moderately promising but far from definitive results at this time.

2. **Another mode** is $t\bar{t} h_1 \rightarrow t\bar{t} a_1 a_1 \rightarrow t\bar{t} \tau^+ \tau^- \tau^+ \tau^-$. Study begun.

3. **A third possibility**: $\tilde{\chi}_2^0 \rightarrow h_1 \tilde{\chi}_1^0$ with $h_1 \rightarrow a_1 a_1 \rightarrow 4\tau$. 

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(Recall that the $\tilde{\chi}_2^0 \to h_1\tilde{\chi}_1^0$ channel provides a signal in the MSSM when $h_1 \to b\bar{b}$ decays are dominant.)

4. Last, but definitely not least: diffractive production $pp \to pph_1 \to ppX$.

The mass $M_X$ can be reconstructed with roughly a $1 - 2$ GeV resolution, potentially revealing a Higgs peak, independent of the decay of the Higgs.

Preliminary results are that one expects about 3 clean, i.e. reconstructed and tagged, events per $30$ fb$^{-1}$ of luminosity. ⇒ clearly a high luminosity game.

Tevatron

1. It is possible that $Zh_1$ and $Wh_1$ production, with $h_1 \to a_1a_1 \to 4\tau$, will provide a small signal. (Wacker et.al; JG, McElrath, Conway).

   Backgrounds can be made small, but efficiencies are low and one must simply accumulate enough events.

2. Wacker et. al. and JG+McElrath have considered $gg \to h_1 \to a_1a_1 \to 4\tau$ which would have substantially larger rate. But cuts etc. imply low efficiencies.
Wacker et al. suggest hints are possible in the all lepton channel with $6 \text{ fb}^{-1}$. We estimated $15 \text{ fb}^{-1}$ would be needed for believable signal.

**Further points**

- If supersymmetry is detected at the Tevatron, but no Higgs is seen, and if LHC discovery of the $h_1$ remains uncertain, the question will arise of whether Tevatron running should be extended so as to allow eventual discovery of $h_1 \rightarrow 4\tau$.

However, rates imply that the $h_1$ signal could only be seen if Tevatron running is extended until $L > 10 - 15 \text{ fb}^{-1}$ (our estimates) has been accumulated.

And, there is the risk that $m_{a_1} < 2m_\tau$, in which case Tevatron backgrounds to $a_1a_1 \rightarrow 4 - jet$ would be impossibly large regardless of how the $h_1$ is produced.

- Of course, even if the LHC is unable to see any of the NMSSM Higgs bosons, it would observe numerous supersymmetry signals and would confirm that $WW \rightarrow WW$ scattering is perturbative, implying that something like a light Higgs boson must be present.
• Of course, discovery of the \( h_1 \) will be straightforward at an \( e^+e^- \) linear collider via the inclusive \( Zh \rightarrow \ell^+\ell^-X \) reconstructed \( M_X \) approach (which allows Higgs discovery independent of the Higgs decay mode).

Figure 19: Decay-mode-independent Higgs \( M_X \) peak in the \( Zh \rightarrow \mu^+\mu^-X \) mode for \( L = 500 \text{ fb}^{-1} \) at \( \sqrt{s} = 350 \) GeV, taking \( m_h = 120 \) GeV.
There are lots of events in just the $\mu^+\mu^-$ channel (which you may want to restrict to since it has the best mass resolution).

- Although the $h \rightarrow b\bar{b}$ and $h \rightarrow \tau^+\tau^-$ rates are 1/10 of the normal, the number of Higgs produced will be such that you can certainly see $Zh \rightarrow Zb\bar{b}$ and $Zh \rightarrow Z\tau^+\tau^-$ in a variety of $Z$ decay modes.

This is quite important, as it will allow you to subtract these modes off and get a determination of $B(h_1 \rightarrow a_1a_1)$, which will provide unique information about $\lambda, \kappa, A_\lambda, A_\kappa$.

- Presumably direct detection in the $Zh \rightarrow Za_1a_1 \rightarrow Z4\tau$ mode will also be possible although I am unaware of any actual studies.

This would give a direct measurement of $B(h_1 \rightarrow a_1a_1 \rightarrow \tau^+\tau^-\tau^+\tau^-)$. Error?

- Coupled with the indirect measurement of $B(h_1 \rightarrow a_1a_1)$ from subtracting the direct $b\bar{b}$ and $\tau^+\tau^-$ modes would give a measurement of $B(a_1 \rightarrow \tau^+\tau^-)$.

This would allow a first unfolding of information about the $a_1$ itself.
Of course, the above assumes we have accounted for all modes.

- Maybe, given the large event rate, one could even get a handle on modes such as $h_1 \rightarrow a_1 a_1 \rightarrow \tau^+ \tau^- jj$ ($j = c, g$), thereby getting still more cross checks.

- At a $\gamma \gamma$ collider, the $\gamma \gamma \rightarrow h_1 \rightarrow 4\tau$ signal will be easily seen (Gunion, Szleper).

  This could help provide still more information about the $h$.

- In contrast, since (as already noted) the $a_1$ in these low-$F$ NMSSM scenarios is fairly singlet in nature, its direct (i.e. not in $h_1$ decays) detection will be very challenging even at the ILC.

- Further, the low-$F$ points are all such that the other Higgs bosons are fairly heavy, typically above 400 GeV in mass, and essentially inaccessible at both the LHC and all but a $\gtrsim 1$ TeV ILC.
A few notes on $m_{a_1} > 2m_b$.

- We should perhaps also not take describing the LEP excess and achieving extremely low fine tuning overly seriously.

Indeed, scenarios with $m_{h_1} > 114$ GeV (automatically out of the reach of LEP) begin at a still modest (relative to the MSSM) $F \gtrsim 25$.

In fact, one can probably push down to as low as $m_{h_1} \gtrsim 108 \div 110$ GeV when $m_{a_1} > 2m_b$.

$\Rightarrow$ must be on the lookout for the $4b$ and $2b2\tau$ final states from $h_1$ decay, with $h_1 \rightarrow 4b$ being the largest when $m_{a_1} > 2m_b$.

- At the LHC, the modes that seem to hold some promise are:

1. $WW \rightarrow h_1 \rightarrow a_1a_1 \rightarrow b\bar{b}\tau^+\tau^-$. 
   Our (JFG, Ellwanger, Hugonie, Moretti) work suggested some hope. Experimentalists (esp. D. Zerwas) are working on a fully realistic evaluation but are not that optimistic.

2. $t\bar{t}h_1 \rightarrow t\bar{t}a_1a_1 \rightarrow t\bar{t}4b$.
   This I imagine will be viable.
3. Gluino cascades containing $\tilde{\chi}^0_2 \rightarrow h_1\tilde{\chi}^0_1$.

4. Doubly diffractive $pp \rightarrow pp h_1$ followed by $h_1 \rightarrow a_1 a_1 \rightarrow 4b$ or $2b2\tau$. These modes are also under consideration by JFG, Khoze, ... .

- At the Tevatron, perhaps the lack of overlapping events and lower background rates might allow some sign of a signal in modes such as $Wh_1$ and $Zh_1$ production with $h_1 \rightarrow a_1 a_1 \rightarrow 4b$ or $2b2\tau$. There is a study underway by G. Huang, Tao Han and collaborators.

**General Considerations**

- We should note that much of the discussion above regarding Higgs discovery is quite generic. Whether the $a$ is truly the NMSSM CP-odd $a_1$ or just a lighter Higgs boson into which the SM-like $h$ pair-decays, hadron collider detection of the $h$ in its $h \rightarrow aa$ decay mode will be very challenging — only an $e^+e^-$ linear collider can currently guarantee its discovery.
Conclusions

• If low fine-tuning is imposed for an acceptable SUSY model, we should expect:
  
  – a $m_{h_1} \sim 100$ GeV Higgs decaying via $h_1 \rightarrow a_1a_1$.

  Higgs detection will be quite challenging at a hadron collider. Higgs detection at the ILC is easy using the missing mass $e^+e^- \rightarrow ZX$ method of looking for a peak in $M_X$.

  Higgs detection in $\gamma\gamma \rightarrow h_1 \rightarrow a_1a_1$ will be easy.

  – The very smallest $F$ values are attained when:
    * $h_2$ and $h_3$ have “moderate” mass, i.e. in the 300 GeV to 700 GeV mass range;
    * the $a_1$ mass is $< 2m_b$ and the $a_1$ has a substantial singlet component.
    * the stops and other squarks are light;
    * the gluino, and, by implication assuming conventional mass orderings, the wino and bino all have modest mass;

• Detailed studies of the $WW \rightarrow h_1 \rightarrow a_1a_1$, $t\bar{t}h_1 \rightarrow t\bar{t}a_1a_1$, diffractive $pp \rightarrow pph_1$ and $\tilde{g}$ cascades with $\tilde{\chi}_2^0 \rightarrow h_1\tilde{\chi}_1^0$ channels (with $h_1 \rightarrow 4b$ or $4\tau$) by the experimental groups at both the Tevatron and the LHC should receive significant priority.
• Even if the LHC sees the Higgs $h_1 \rightarrow a_1 a_1$ directly, it will not be able to get much detail. Only the ILC can do the detailed measurements needed to verify the model in detail.

• It is likely that other models in which the MSSM $\mu$ parameter is generated using additional scalar fields can achieve small fine-tuning in a manner similar to the NMSSM.

• In general, very natural solutions to the fine-tuning and little hierarchy problems are possible in relatively simple extensions of the MSSM.

One does not have to employ more radical approaches or give up on small fine-tuning!

Further, small fine-tuning probably requires a light SUSY spectrum in all such models and SUSY should be easily explored at both the LHC (and very possibly the Tevatron) and the ILC and $\gamma\gamma$ colliders.

Only Higgs detection at the LHC will be a real challenge.

Ability to check perturbativity of $WW \rightarrow WW$ at the LHC might prove to be very crucial to make sure that there really is a light Higgs accompanying light SUSY.
A light $a_1$ allows for a light $\tilde{\chi}_1^0$ to be responsible for dark matter of correct relic density: annihilation would be via $\tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow a_1$. Only the ILC could have any chance of measuring the properties of the very light $\tilde{\chi}_1^0$ and of the $a_1$ in sufficient detail to verify that it all fits together.