Unique Physics Probes Using an e^-e^- Collider

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Outstanding Problems

- **Electroweak Symmetry Breaking: Is it the SM, an effective 2HDM, technicolor, . . .**
- **Supersymmetry and Unification: Sparticle masses and properties, is there a desert, . . .**
- **Flavor Physics: Masses, mixings, CP violation, neutrino mass, . . .**
- **Space-time Structure: Large extra dimensions, branes, . . .**

In all cases, an $e^- e^-$ collider would provide unique and important probes.
Especially important: high luminosity, $L = 300 - 500 \mathrm{fb}^{-1}$ per year, and high polarization, $P_e \sim 80 - 90\%$.

$e^- e^-$ Collisions

Unique capabilities arise for

- Contact interactions.
- SUSY studies, R-parity conserving.
- SUSY studies, R-Parity violating.
- Understanding or eliminating sources of neutrino masses and mixing.
- Looking for flavor violation via $e^- \to \mu^-$.
- Higgs studies related to above and to unification.

We provide only a few examples.

Moller Scattering — Czarnecki+Marciano, Barklow

$$\frac{N_{\rm LL}+N_{\rm LR}-N_{\rm RL}-N_{\rm RR}}{N_{\rm LL}+N_{\rm LR}+N_{\rm RL}+N_{\rm RR}} = P_1 A^{(1)}_{\rm LR}(y), \quad (1)$$

$$\frac{N_{\rm RR}+N_{\rm LR}-N_{\rm RL}-N_{\rm LL}}{N_{\rm RR}+N_{\rm LR}+N_{\rm RL}+N_{\rm LL}} = -P_2 A^{(1)}_{\rm LR}(y), \quad (2)$$

$$\frac{N_{\rm LL}-N_{\rm RR}}{N_{\rm LL}+N_{\rm RR}} = P_{\rm eff} A^{(2)}_{\rm LR}(y)\left(\frac{1}{1+\frac{1-P_1P_2}{1+P_1P_2}\frac{\sigma_{\rm LR}+\sigma_{\rm RL}}{\sigma_{\rm LL}+\sigma_{\rm RR}}}\right), \quad (3)$$

$$y=\frac{1-\cos\theta}{2} \qquad P_{\rm eff}=\frac{P_1+P_2}{1+P_1P_2}.$$

For $P_1=P_2=0.9\pm 0.005$, $P_{\rm eff}=0.9945\pm 0.0004$, i.e. $P_{\rm eff}$ is very large and has negligible error. It is $P_{\rm eff}$ that is important.

In the above,

$$A^{(1)}_{LR} \equiv \frac{d\sigma_{LL}+d\sigma_{LR}-d\sigma_{RL}-d\sigma_{RR}}{d\sigma_{LL}+d\sigma_{LR}+d\sigma_{RL}+d\sigma_{RR}} \quad (4)$$

$$A_{LR}^{(2)} \equiv \frac{d\sigma_{LL} - d\sigma_{RR}}{d\sigma_{LL} + d\sigma_{RR}}, \quad (5)$$

where $d\sigma$'s are for $e_i^- e_j^- \to e^- e^-$. Since $d\sigma_{RL} = d\sigma_{LR}$, $A_{LR}^{(2)}$ differs from $A_{LR}^{(1)}$ only in the denominator. $A_{LR}^{(2)}$ requires double polarization. Assuming dominance by γ, Z exchange, we find for $ys, (1-y)s \gg m_Z^2$

$$A_{\rm LR}^{(1)}(y) = \frac{(1-4s_W^2)(1+4s_W^2)}{1+16s_W^4 + 8\left[y^4 + (1-y)^4\right] s_W^4}, \quad (6)$$

$$A_{\rm LR}^{(2)}(y) = \frac{(1-4s_W^2)(1+4s_W^2)}{1+16s_W^4}. \quad (7)$$

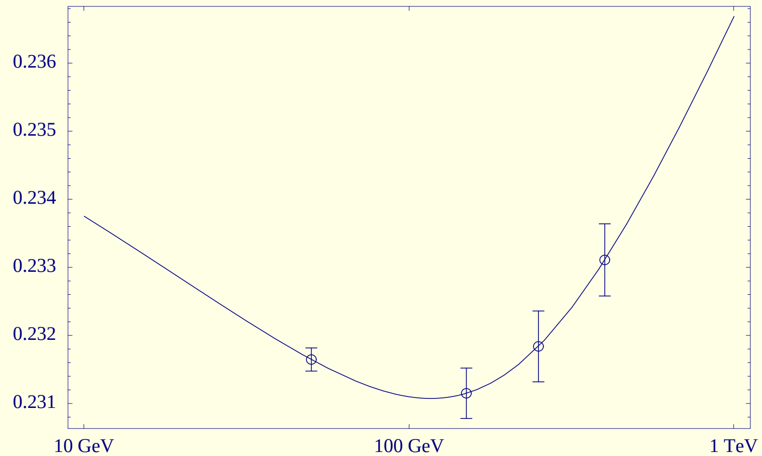
where factor of $(1-4s_W^2)$ means great sensitivity to s_W^2 since $s_W^2 \sim 1/4$. Note, that despite apparent y-independence of $A_{\rm LR}^{(2)}$, in fact s_W^2 depends on y, actually on the momentum transfer squared $Q^2 = ys$. This opens up the possibility of measuring Q dependence of s_W^2.

- Use the above and 'sufficiently' (e.g. from Z pole data) known value of $A_{LR}^{(1)}$ to simultaneously determine P_1, P_2 and $A_{LR}^{(2)}$.

- For $P_1 = P_2 = 0.9$, the correction term in parentheses of Eq. (3) is small but must be accounted for.

- Expected accuracy: $\delta s_W^2 \sim \pm 0.0003$ at $\sqrt{s} = 1$ TeV and modest $\mathcal{L}$.

- $A_{LR}^{(2)}$ can probe running of $\sin^2\theta_W$ with unprecedented accuracy.



- A deviation in Moller scattering from expectations would signal "new physics." For example, deviations in angular dependence of cross section would probe

$$\mathcal{L}_{\rm eff} = \frac{2\pi}{\Lambda^2}\bar{e}_L\gamma^\mu e_L \bar{e}_L \gamma_\mu e_L \,. \quad (8)$$

Need to assume you have a precise determination of P_1 and P_2 from above.

With $\sqrt{s} = 1$ TeV and $82 \mathrm{fb}^{-1} \Rightarrow$ probe $\Lambda = 150$ TeV.

- Bhabha scattering at e^+e^- with same $L \Rightarrow$ probes $\Lambda = 100$ TeV. Moller is better because of u vs. s dependence of Moller vs. Bhabha.

$$\frac{\Lambda_{e^-e^-}}{\Lambda_{e^+e^-}} = 2^{-1/4} \langle \left(\frac{s}{u} \right)^{3/2} \rangle \, . \tag{9}$$

- Roughly, luminosity and $\sqrt{s}$ scaling dependence of maximum value of Λ probed is

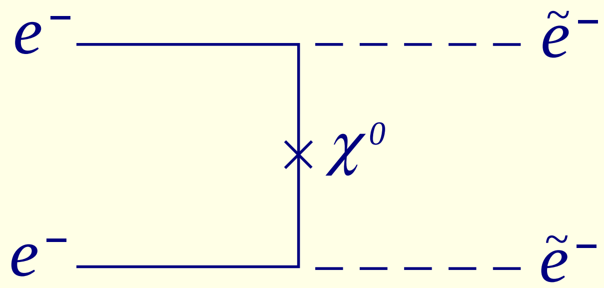
$$\Lambda^{\mathrm{limit}} \propto \mathcal{L}^{1/4} \sqrt{s} \, . \tag{10}$$

assuming that one does not encounter systematic error problems and such.

Note that weak $\mathcal{L}^{1/4}$ dependence of Λ^{limit} on $\mathcal{L}$ implies that if e^-e^- collisions have 1/3 as much luminosity as e^+e^- because of beam disruption, e^-e^- will still do better.

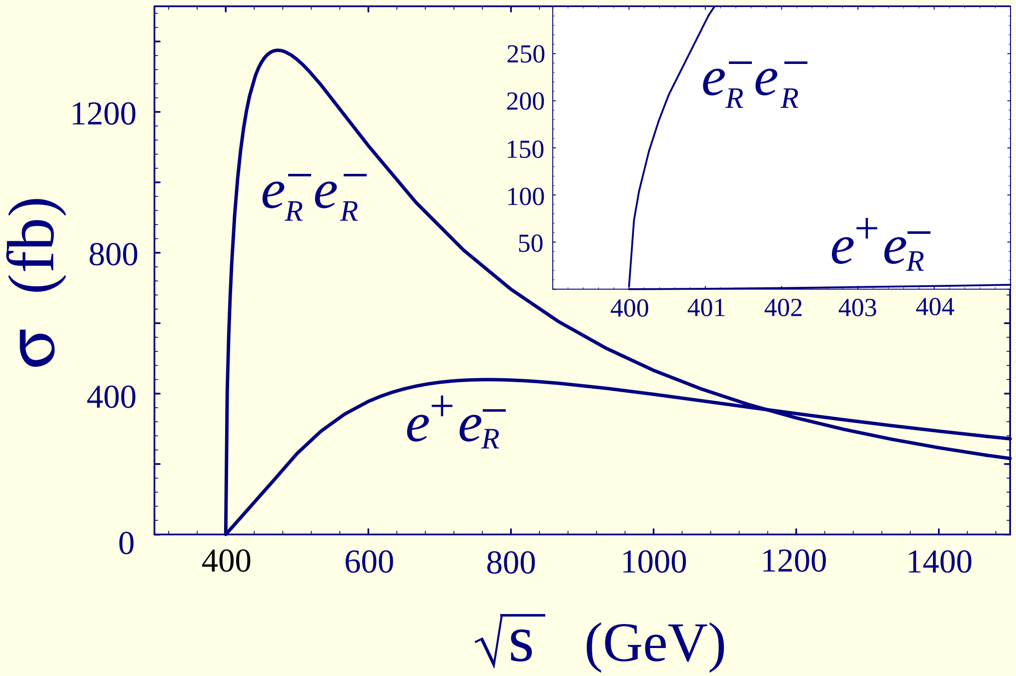
- The impact of reduced $\mathcal{L}$ due to beam disruption must be evaluated for each new physics case.

RPC SUSY Peskin + Feng



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 M_1^2}{2\cos^4\theta_W}\left(\frac{1}{t-M_1^2}+\frac{1}{u-M_1^2}\right)^2 \tag{11}$$

Very sensitive to M_1 as well as to $m_{\widetilde{e}^-}$ (through threshold turn on in S-wave).



- S-wave β turn on of $e^-e^- \to \widetilde{e}_R^-\widetilde{e}_R^- \Rightarrow$ uniquely precise measurement of $m_{\widetilde{e}_R^-}$. About $100\times$ as much L required for same precision in e^+e^- where turn on is β^3.

- $m_{\widetilde{e}_R^-}$-optimized mode: $L = 1(10)\mathrm{fb}^{-1} \Rightarrow \Delta m_{\widetilde{e}_R^-} = 70(20)$ MeV assuming $m_{\widetilde{\chi}_1^0}$ is well-determined from kinematic end-point measurements elsewhere (e.g. e^+e^-). Backgrounds very small, unlike e^+e^-.

Such precision could be crucial for evolving up to GUT scale with adequate precision to really determine soft-SUSY-breaking boundary conditions.

- A 2-point scan determines M_1 to $\pm 5\ \mathrm{GeV}$ unlike e^+e^- where M_1 comes only from end-point game.

- Even if M_1 is very large, we get a determination of M_1 from cross section size.

- $e^+e^- \to \widetilde{e}^{\pm}_R \widetilde{e}^{\mp}_L \Rightarrow m_{\widetilde{e}^-_L}$ and then $m_{\widetilde{e}^-_L} - m_{\widetilde{e}^-_R} \Rightarrow$ model independent determination of $\tan\beta$ (at low to moderate $\tan\beta$).

- Check equality $\widetilde{g} = g$ of $\widetilde{g}e\widetilde{e}\widetilde{B}$ vs. $geeB_\mu$ couplings to much better than 1%.

  Non-decoupling of corrections to $\widetilde{g}/g = 1$ means that this ratio would be sensitive to sparticles of arbitrarily large mass.

Looking for slepton flavor oscillations (J. Feng, S. Thomas, G. Kribs, ...)

- In general, the matrix that diagonalizes lepton flavor does not diagonalize slepton flavor. $\Rightarrow$, for example,

$$\mathcal{M}^2_{\rm slepton} = \begin{pmatrix} m^2_{ee} & m^2_{e\mu} \\ m^2_{e\mu} & m^2_{\mu\mu} \end{pmatrix} \tag{12}$$

where it is very possible that $m^2_{e\mu}$ is comparable to $m^2_{ee} - m^2_{\mu\mu}$.

- This $\Rightarrow e^-e^- \to e^-\mu^- + \not{E}_T$ final states in 2 ways.

  1. Direct $\widetilde{\mu}^-$ production: $e^-e^- \to \widetilde{e}^-\widetilde{\mu}^-$ via $\widetilde{\chi}^0_1$ exchange.
     The sleptons then decay ($\widetilde{e}^- \to e^-\widetilde{\chi}^0_1$, $\widetilde{\mu}^- \to \mu^-\widetilde{\chi}^0_1$), yielding $e^-e^- \to e^-\mu^- + \not{E}_T$ events.
     The cross section could be small if $m_{\widetilde{\chi}^0_1}$ is large.
  2. Lepton number violating decay: $e^-e^- \to \widetilde{e}^-\widetilde{e}^-$ followed by $\widetilde{e}^- \to \mu^-\widetilde{\chi}^0_1$ decay.
     This mechanism might have little phase space.

- These events are much more background free in e^-e^- than corresponding events in e^+e^- collisions, especially if we have ability to turn off W^-W^-

production using e_R^- polarized beams.

1. $e^-e^- \to W^-W^-$ **forbidden by lepton number conservation.**
2. $e^-e^- \to \nu\nu W^-W^-$ **and** $e^-e^- \to e^-\nu W^-$ **are both turned off for double** e_R^- **polarization.**
3. $\gamma\gamma \to W^+W^-$ **is not a background since** $\Rightarrow$ **opposite (vs. same sign) dileptons.**

RPV SUSY

If there are R-parity violating interactions of the leptonic variety, an e^-e^- collider could provide some truly unique information. Consider, for example,

$$\lambda_{132} L_1 L_3 \mu^c + \lambda_{231} L_2 L_3 e^c \,.$$

Murayama (e-e-97) provides the following summary.

- These interactions do not $\Rightarrow \mu \to e\gamma$ or μ-e conversion because they conserve $L_e + L_\mu$.

- Best bounds on the λ's come from muonium conversion, $\mu^+ e^- \leftrightarrow \mu^- e^+$, and e-μ-τ universality.
  Net bound: $\lambda < 0.1$ for $m_{\tilde L} \sim 200$ GeV.

- The above superpotential $\Rightarrow e^-_L e^-_R \to \mu^-_L \mu^-_R$ via $\tilde\nu_\tau$ exchange.
  There is essentially no background. The event rate is given roughly by

$$\frac{\#(\text{event})}{20\ \text{fb}^{-1}} \simeq 10^6 \times \lambda^4 \times \left(\frac{200\ \text{GeV}}{m_{\tilde L}}\right)^4 \left(\frac{\sqrt{s}}{200\ \text{GeV}}\right)^2 .$$

$\Rightarrow\ \geq 5$ events if $\lambda \geq 0.05$, which is below the current limits.

Bileptons and Doubly Charged Higgs Bosons Gunion, Frampton, Littlest Higgs Model . . .

Even within SM context, should consider extended Higgs sector possibilities.

- **Frampton considered bilepton gauge bosons. Briefly,**

$$\mathcal{L} \sim \begin{pmatrix} \ell^- & \nu & \ell^+ \end{pmatrix}^*_L \begin{pmatrix} & & Y^{--} \\ & & Y^- \\ Y^{++} & Y^+ & \end{pmatrix} \begin{pmatrix} \ell^- \\ \nu \\ \ell^+ \end{pmatrix}_L , \qquad (13)$$

  where Y are new gauge bosons. Y^{--} are produced as an s-channel resonance at e^-e^- colliders, and $\Rightarrow$ background-free events like $e^-e^- \to Y^{--} \to \mu^-\mu^-$.

- **For Higgs, adding triplets or higher reps. is a possibility.**

  If neutral vev $= 0$, then no EWSB impact and $\rho = 1$ is natural.

- **Triplets very desirable for neutrino mass game in L/R symmetric models.**

Usual notation is

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} . \tag{14}$$

Introduce Δ_R triplet for see-saw with $\langle \Delta_R^0 \rangle$ =large.

L/R symmetry requires Δ_L and $\langle \Delta_L^0 \rangle \equiv v_\Delta = 0$ is natural.

- Triplets are good for unification without SUSY, but at lower scale than usual (maybe desirable for large-scale extra dimensions, . . .).

  Use notation $N_{T,Y}$ for number of Higgs reps. of given T, Y. $T, Y = 1, 2$ and $T, Y = 1/2, 3$ both imply Δ^{--} state.

  $N_{\frac{1}{2},1} = 1$, $N_{1,2} = 2 \Rightarrow M_U \sim 1.5 \times 10^{13}$ GeV.

  $N_{\frac{1}{2},1} \geq 1$ and $N_{\frac{1}{2},3} \neq 0$ solutions $\Rightarrow M_U \lesssim 10^{13}$ GeV.

- Mass limits from LEP are model dependent, but certainly pair production pretty much excludes masses below 100 GeV.

- For $v_\Delta = 0$,
  - $W_L W_L$ couplings to Higgs bosons of triplet are absent.
  - Most $\Delta\Delta^{(\prime)}V$ couplings are non-zero.
  - Trilinear couplings of three Δ's are zero.
  - $H\Delta\Delta$ couplings involving one doublet H and two triplet Δ's are non-zero if the doublet vev is non-zero.
  - Δ couplings to $f^{(\prime)}\overline{f}$ are zero.

- There is a possibility of non-zero bi-lepton couplings of Higgs bosons. For the standard SU(2)$_L$ case, with $Q = T_3 + \frac{Y}{2} = -2$, the allowed doubly-charged cases are:

$$\begin{aligned} e^-_R e^-_R &\to \Delta^{--}(T=0, T_3=0, Y=-4)\,, \\ e^-_L e^-_R &\to \Delta^{--}(T=\tfrac{1}{2}, T_3=-\tfrac{1}{2}, Y=-3)\,, \\ e^-_L e^-_L &\to \Delta^{--}(T=1, T_3=-1, Y=-2)\,. \end{aligned} \tag{15}$$

  Note that the above cases do not include the $T=3, Y=-4$ representation that yields $\rho = 1$, nor the $T=1, Y=-4$ triplet with no neutral member, but do include the $T=1/2, Y=-3$ doublet representation with no neutral member, and the $T=1, Y=-2$ triplet representation.

In the case of a $|Y|=2$ triplet representation the lepton-number-violating coupling to (left-handed) leptons is specified in the lepton-number violating Lagrangian form:

$$\mathcal{L}_Y = i h_{ij} \psi_i^T C \tau_2 \Delta \psi_j + \mathrm{h.c.}\,, \quad i,j = e,\mu,\tau\,. \tag{16}$$

Limits on the h_{ij} by virtue of the $\Delta^{--} \to \ell^-\ell^-$ couplings: writing $|h_{\ell\ell}^{\Delta^{--}}|^2 \equiv c_{\ell\ell} m_{\Delta^{--}}^2(\,\mathrm{GeV})$, strongest limits (no limits on $c_{\tau\tau}$) are:

- $c_{ee} < 10^{-5}$ (Bhabha),
- $c_{\mu\mu} < 5 \times 10^{-7}$ ($(g-2)_\mu$ – predicted contribution has wrong sign) and
- $\sqrt{c_{ee} c_{\mu\mu}} < 10^{-7}$ (muonium-antimuonium).

- If $v_\Delta = 0$, most decay channels are eliminated.

  If allowed, $\Delta \to \Delta' V$ might dominate, but since the Δ's of typical model are approximately degenerate, all these modes might be virtual.

  $\Rightarrow$ dominance of bi-lepton decay modes is possible if bi-lepton couplings are non-zero.

Since there are currently no limits on $c_{\tau\tau}$, the $\tau^-\tau^-$ channel could easily have the largest partial width and be the dominant decay of the Δ^{--}.

In any case, for $v_\Delta = 0$, $\Gamma^T_{\Delta^{--}}$ is likely to be small. $\Rightarrow$ possibly very large s-channel e^-e^- and $\mu^-\mu^-$ production rates.

- Note:

  For $v_\Delta = 0$, the Δ^0 (generally the lightest) will be stable unless it has bi-lepton couplings to $\nu_L\nu_L$.

  Such a Δ^0 would be a candidate for the dark matter.

Detection and Study:

- Discover Δ^{--} in $p\overline{p} \to \Delta^{--}\Delta^{++}$ with $\Delta^{--} \to \ell^-\ell^-, \Delta^{++} \to \ell^+\ell^+$ ($\ell = e, \mu, \tau$) at TeV33 or LHC (J.G., Loomis, Pitts: hep-ph/9610237). LHC reach up to $m_{\Delta^{--}} \sim 1~\mathrm{TeV}$.

  $\Rightarrow$ TeV33 + LHC will tell us if such a Δ^{--} exists in the mass range accessible to NLC and FMC and how it decays.

  – If no W^-W^- decays are detected, it is very probable that $v_\Delta = 0$.

- If Δ^{--} decays to $\ell^-\ell^-$ ($\ell = e, \mu, \tau$), then $\Rightarrow$ the $\ell^-\ell^-$ coupling clearly exists.
- For any observed $\ell^-\ell^-$ mode(s), you know the $\Delta^{--} \to \ell^-\ell^-$ coupling(s) are non-zero, but you do not know that the others are zero — only that they are relatively smaller.
- Even if only the $\Delta^- W^-$ mode is observed, that does not mean that the $c_{\ell\ell}$'s are zero, only that they are quite small.
  $\Rightarrow$ can still probe $c_{\ell\ell}$'s that are too small for $\ell^-\ell^-$ to appear in decays if total width of the Δ^{--} is small.
- If the $\Delta^- W^-$ mode is disallowed and the $c_{\ell\ell}$'s are all very small ($\lesssim 10^{-16}$), the Δ^{--} could be sufficiently long-lived to escape the detector.
  The hadron colliders would see the corresponding 'stable' particle highly ionizing tracks.

- Bottom line: from LHC will know if Δ^{--} exists and may know if some $c_{\ell\ell}$'s are non-zero, but will not have measured any $c_{\ell\ell}$.

- The $c_{\ell\ell}$'s are of fundamental importance. $\Rightarrow$ Look for (and study if found) in $e^-e^- \to \Delta^{--}$ s-channel collisions. (From Tevatron and/or LHC, we will know $m_{\Delta^{--}}$ quite precisely.)

This should be done no matter how the Δ^{--} is seen to decay at the hadron colliders.

Event rates can be enormous (see JFG, hep-ph/9803222 and hep-ph/9510350): equivalently can probe to very small $c_{\ell\ell}$.

– Using the Gaussian approximation, the effective cross section for Δ^{--} production in the s-channel is obtained by convoluting the standard s-channel pole form with the Gaussian distribution in $\sqrt{s}$ of rms width $\sigma_{\sqrt{s}}$.
  A useful mnemonic for $\sigma_{\sqrt{s}}$ is

$$\sigma_{\sqrt{s}} \sim 0.2 \text{ GeV} \left(\frac{m_{\Delta^{--}}}{100 \text{ GeV}}\right)\left(\frac{R}{0.2\%}\right), \quad (17)$$

  where R is the beam energy resolution in percent. The crucial issue is how $\sigma_{\sqrt{s}}$ compares to $\Gamma^T_{\Delta^{--}}$.
  The resulting cross section is denoted by $\overline{\sigma}_{\Delta^{--}}$. For $\Gamma^T_{\Delta^{--}} \gg \sigma_{\sqrt{s}}$,

$\Gamma^T_{\Delta^{--}} \ll \sigma_{\sqrt{s}}$, $\overline{\sigma}_{\Delta^{--}}$ at $\sqrt{s} = m_{\Delta^{--}}$ is given by:

$$\overline{\sigma}_{\Delta^{--}} = \begin{cases} \frac{4\pi BR(\Delta^{--}\to e^- e^-)}{m^2_{\Delta^{--}}}, & \Gamma^T_{\Delta^{--}} \gg \sigma_{\sqrt{s}}; \\ \frac{\sqrt{\pi}}{2\sqrt{2}} \frac{4\pi \frac{\Gamma(\Delta^{--}\to e^- e^-)}{\sigma_{\sqrt{s}}}}{m^2_{\Delta^{--}}}, & \Gamma^T_{\Delta^{--}} \ll \sigma_{\sqrt{s}} \,. \end{cases} \tag{18}$$

We compute rates as $L\overline{\sigma}_{\Delta^{--}}$ with $L = 50\mathrm{fb}^{-1}$ assumed under Gaussian peak (after accounting for losses in radiative tail).

In the following, we assume $\Gamma^T_{\Delta^{--}} \ll \sigma_{\sqrt{s}}$. As discussed above, this is likely if $v_\Delta = 0$.

– Taking $L = 50\mathrm{fb}^{-1}$, and using Eq. (17) for $\sigma_{\sqrt{s}}$ and the analytical form for $\Gamma(\Delta^{--} \to e^- e^-)$, we find from Eq. (18) an event rate of

$$N(\Delta^{--}) \sim 3 \times 10^{10} \left(\frac{c_{ee}}{10^{-5}}\right) \left(\frac{0.2\%}{R}\right) \,. \tag{19}$$

An enormous event rate results if c_{ee} is within a few orders of magnitude of its upper bound.

– For 100 events, Eq. (19) ⇒ we probe

$$c_{ee}\big|_{100\ \text{events}} \sim 3.3 \times 10^{-14} \left(\frac{R}{0.2\%}\right) \left(\frac{50\text{fb}^{-1}}{L}\right), \quad \Gamma^T_{\Delta^{--}} \ll \sigma_{\sqrt{s}}, \tag{20}$$

independent of $m_{\Delta^{--}}$.

– If $\Delta^{--} \to \mu^-\mu^-$ primarily, 10 events might → a viable signal.

– If $\Delta^{--} \to e^-e^-$ or Δ^-W^-, 1000 events might be needed because of backgrounds. A better study is needed.

– ⇒ dramatic sensitivity — at least factor of $10^8 - 10^9$ improvement over current limits.

– Observation ⇒ actual measurement of c_{ee} at level relevant to neutrino mass generation by a right-handed partner Δ_R representation (with $\langle\Delta^0_R\rangle \neq 0$) of the left-handed Δ_L representation .

Other processes:

- $\Delta^{--}Z$ and $\Delta^{--}\gamma$ production in e^-e^- collisions.

This is essentially equivalent to using the bremsstrahlung tail in $\sqrt{s}$ at the e^-e^- collider to self-scan for the Δ^{--}. Depending on c_{ee}, it can allow discovery and mass measurement, which in turn allows centering in $\sqrt{s}$ for direct resonance production as above.

The total number of events is proportional to c_{ee} as in the $\Gamma^T_{\Delta^{--}} \ll \sigma_{\sqrt{s}}$ limit of on-resonance production, but observable rates are only possible if c_{ee} is not too far below current bounds.

- $e^-\gamma \to e^+\Delta^{--} \to e^+\mu^-\mu^-$ (Godfrey etal.)

  Can probe quite small h_{ee} if the energy is sufficient for the Δ^{--} to be resonantly produced.

  There is even some sensitivity if the Δ^{--} is massive and only appears as a virtually exchanged particle.

- $\Delta^- W^-$ production in e^-e^- collisions.

  This process relies on a $\Delta^- \to e^-\nu_e$ coupling, and would be an interesting way of both observing any Δ^- with such a coupling and a way of determining the magnitude of the coupling.

  Observable rates require substantial coupling.

Littlest Higgs Model

- This model has a triplet Higgs of the classic $T = 1, Y = 2$ type, called Φ in the model.

- The interesting point from the $e^- e^-$ point of view is that $v' \equiv \langle \Phi^0 \rangle \neq 0$ does not present any particular problems below the ultraviolate completion scale of $4\pi f$.

  In fact, it is very awkward to have $v' = 0$ in the littlest Higgs model since this would imply very large non-oblique radiative corrections to precision EW observables. (The limit in which $v' = 0$ corresponds to the case where the gauge coupling constants for the two SU(3)'s are equal: $g_1 = g_2$, whereas small non-oblique requires $g_2 \gg g_1$.)

  It is conventional to define

$$v' \equiv x \frac{v^2}{4f} . \tag{21}$$

  $x \sim 1$ is expected if $g_2 \gg g_1$.

- Consistency requires $v'/v \lesssim v/4f$, i.e. $x \lesssim \mathcal{O}(1)$.

- Precision electroweak at 5% level requires $f \gtrsim \frac{1}{2}v/\sqrt{0.05} \sim 2.3v$ and $v' \lesssim \frac{1}{2}\sqrt{0.05}v \sim 0.1v$. The latter is completely consistent with $x \lesssim \mathcal{O}(1)$ for $f \gtrsim 2.3v$. $x \sim 1$ would imply $v' \sim 10~\mathrm{GeV}$.

- There is nothing to prevent $\ell^-\ell^- \to \Phi^{--}$ couplings for example from $h_{\ell\ell}^{\Phi^{--}} L\Phi L$ lepton-number violating coupling.

- However, there are some strong constraints on the model.

1. The magnitude of $h_{\ell\ell}^{\Phi^{--}}$ cannot be very large if it is related by SU(2) invariance to the $h_{\nu\nu}^{\Phi^0}$ coupling since the latter will give a Majorana mass contribution to the left-handed neutrinos. We require

$$h_{\nu\nu}^{\Phi^0} v' \lesssim 1~\mathrm{eV}\,, \tag{22}$$

which converts to

$$h_{\nu\nu}^{\Phi^0} \lesssim 10^{-9} x\,. \tag{23}$$

This could be regarded as an unnaturally small coupling. Maybe we should not allow it, but for purposes of discussion, let us suppose that it is there.

2. Another important relation implied by the model is

$$m_\Phi \gtrsim \sqrt{2}\frac{m_h f}{v} \gtrsim 4 m_h \,. \tag{24}$$

Thus, the Φ^{--} would not be very light.

3. The large mass means that very high energy would be required to to produce the Φ^{--} on-shell either in e^-e^- collisions through the lepton-number violating coupling or through $e^-e^- \to \nu\nu W^{*-}W^{*-} \to \nu\nu\Phi^{--}$ via the coupling proportional to gv'.
   The first possibility does not look very promising. Taking $m_\Phi \sim 1~\mathrm{TeV}$ would give a corresponding $c_{\ell\ell} = h^2_{\ell\ell}/m^2_\Phi(GeV) \sim 10^{-24}$, well below the maximum sensitivity estimated for an e^-e^- collider for direct s-channel production.
   As regards the latter possibility, Wacker estimates that this will be difficult to see in the presence of backgrounds.
4. At a low energy e^-e^- collider, one could only look for virtual effects in $e^-e^- \to \Phi^{--*} \to e^-e^-, \mu^-\mu^-, \tau^-\tau^-$ for the L violating coupling or $e^-e^- \to \nu\nu W^{*-}W^{*-} \to \nu\nu\Phi^{--*} \to \nu\nu W^-W^-$ for the gv'-induced coupling.
   Our preliminary estimates are that the backgrounds are too large for such

virtual exchanges to be detectable.

Heavy Majorana Neutrinos

- Minkowski and collaborators have shown that e^-e^- colliders are uniquely able to probe for the existence of massive Majorana neutrinos.

  Their virtual exchange in the t-channel leads to $e^-e^- \to W^-W^-$.

- Masses as high as $\sim 12~\mathrm{TeV}$ can be probed (for mixing angle near its current upper bound) at $\sqrt{s} \sim 1~\mathrm{TeV}$ with $L = 100\mathrm{fb}^{-1}$.

Strong W scattering

- If strong W scattering is nature's choices, we will have a situation very analogous to pion scattering in the early days of QCD.

- We will wish to probe every possible channel.

- Only e^-e^- collisions provide access to the $W^-W^- \to W^-W^-$ channel required to complete the survey of all possible weak-isospin states in the WW system.

- **Of course, very high energy $\sqrt{s} \gtrsim 3~\mathrm{TeV}$ is required to do a complete job. (Barger+Berger+Gunion+Han)**

- **But, even at low energy, $\sqrt{s} = 0.5 - 1~\mathrm{TeV}$, the e^-e^- channel would provide important information. (See, e.g. the summary by Murayama for e-e-97.)**