Higgs Bosons

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Introductory Outline

- In Yang-Mills theories, an explicit mass term for the gauge vector bosons in the Lagrangian is forbidden by gauge invariance.

$$\mathcal{L} \ni m^2 A_\mu A^\mu \tag{1}$$

  is clearly not invariant under the standard gauge transformation of $A_\mu \to A_\mu + \partial_\mu \Lambda$.

- While this is acceptable for theories like QED (Quantum Electrodynamics) and QCD (Quantum Chromodynamics), where both photons and gluons are massless, it is unacceptable for the gauge theory of weak interactions, since both the charged ($W^\pm$) and neutral (Z) gauge bosons have very heavy masses ($M_W \simeq 81$ GeV, $M_Z \simeq 91$ GeV), and simply inserting corresponding mass terms into the Lagrangian destroys the gauge invariance of the theory and leads to non-renormalizabiity and violations of unitarity.

- A possible solution to this problem, inspired by similar phenomena found in the study of spin systems, was proposed by Englert, Higgs and Guralnik in 1964, and it is known today simply as **the Higgs mechanism**.

- These lectures will begin with a review of the basic ideas. Following this, we will recall how the Higgs mechanism is implemented in the Standard Model and we will discuss **the Higgs boson**, the only physical scalar particle predicted by the model and how the SM expectations compare to current observations.

- Finally, we will generalize our discussion to models such as the MSSM, NMSSM and 2HDM to illustrate how differently the Higgs mechanism can be implemented in extensions of the SM and how the current deviations in the 126 GeV data from SM predictions compare to predictions for the Higgs bosons of these BSM models.

A brief introduction to the Higgs mechanism

The essence of the Higgs mechanism can be very easily illustrated considering the case of a classical abelian Yang-Mills theory. In this case, it is realized by adding to the Yang-Mills Lagrangian

$$\mathcal{L}_A = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad \textbf{with} \quad F^{\mu\nu} = (\partial^\mu A^\nu - \partial^\nu A^\mu) \ , \tag{2}$$

a complex scalar field with Lagrangian

$$\mathcal{L}_\phi = (D^\mu \phi)^* D_\mu \phi - V(\phi) = (D^\mu \phi)^* D_\mu \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \ , \tag{3}$$

where $D^\mu = \partial^\mu + igA^\mu$, and $\lambda > 0$ for the scalar potential to be bounded from below. The full Lagrangian

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi \tag{4}$$

is invariant under a $U(1)$ gauge transformation acting on the fields as:

$$\phi(x) \to e^{i\alpha(x)}\phi(x) \ , \ A^\mu(x) \to A^\mu(x) + \frac{1}{g}\partial^\mu\alpha(x) \ , \tag{5}$$

while a gauge field mass term (i.e., a term quadratic in the fields A^μ) would not be gauge invariant and cannot be added to $\mathcal{L}$ if the $U(1)$ gauge symmetry has to be preserved.

Although the Lagrangian in Eq. (4) does not have an explicit mass term for the gauge field, it can still describe the physics of a massive gauge boson, provided the potential $V(\phi)$ develops a non trivial minimum ($\langle\phi\rangle \neq 0$).

The occurrence of a non trivial minimum, or, better, of a continuum of degenerate of minima only depends on the sign of the μ^2 parameter in $V(\phi)$.

For $\mu^2 > 0$ there is a unique minimum at $\phi = 0$, while for $\mu^2 < 0$ the potential develops a continuum of degenerate minima satisfying the equation $\phi^*\phi = -\mu^2/(2\lambda)$.

This is illustrated in Fig. 1, where the potential $V(\phi)$ is plotted as a function of the real and imaginary parts of the field $\phi = \phi_1 + i\phi_2$.

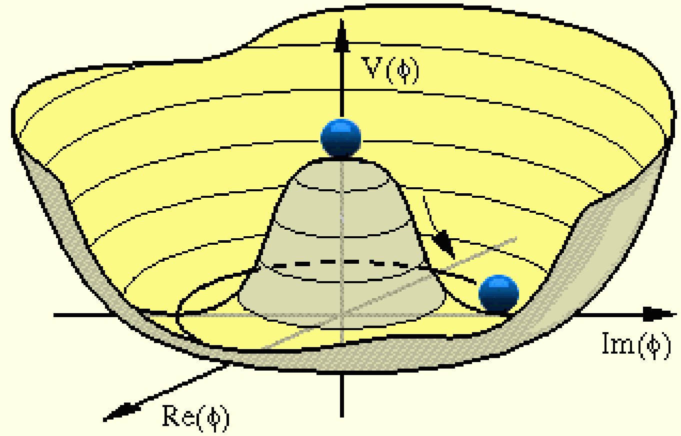


Figure 1: The potential $V(\phi)$ with $\phi = \mathrm{Re}(\phi) + i\mathrm{Im}(\phi)$ is plotted for an arbitrary positive value of λ and for an arbitrary negative value of μ^2.

In the case of a unique minimum at $\phi^*\phi = 0$ the Lagrangian in Eq. (4) describes the physics of a massless vector boson (e.g. the photon, in electrodynamics, with $g = -e$) interacting with a massive charged scalar particle.

On the other hand, something completely different takes place when $\mu^2 < 0$.

Choosing the ground state of the theory to be a particular ϕ among the many satisfying the equation of the minimum, and expanding the potential in the vicinity of the chosen minimum, transforms the Lagrangian in such a way that the original gauge symmetry is now hidden or spontaneously broken, and new interesting features emerge.

To be more specific, let's pick the following ϕ_0 minimum (along the direction of the real part of ϕ, as traditional) and shift the ϕ field accordingly:

$$\phi_0 = \left(-\frac{\mu^2}{2\lambda}\right)^{1/2} = \frac{v}{\sqrt{2}} \longrightarrow \phi(x) = \phi_0 + \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x)) \quad . \tag{6}$$

The Lagrangian in Eq. (4) can then be rearranged as follows:

$$\mathcal{L} = \underbrace{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}g^2v^2A^\mu A_\mu}_{\text{massive vector field}} + \underbrace{\frac{1}{2}(\partial^\mu\phi_1)^2 + \mu^2\phi_1^2}_{\text{massive scalar field}} + \underbrace{\frac{1}{2}(\partial^\mu\phi_2)^2 + gvA_\mu\partial^\mu\phi_2}_{\text{Goldstone boson}} + \ldots \tag{7}$$

and now contains the correct terms to describe a massive vector field A^μ with mass $m_A^2 = g^2v^2$ (originating from the kinetic term of $\mathcal{L}_\phi$), a massive real scalar field ϕ_1 with mass $m_{\phi_1} = -2\mu^2$, that will become a Higgs boson, and a massless scalar field ϕ_2, a so called Goldstone boson which couples to the gauge vector boson A^μ.

The terms omitted contain couplings between the ϕ_1 and ϕ_2 fields irrelevant to this discussion.

The gauge symmetry of the theory allows us to make the particle content more transparent. Indeed, if we parameterize the complex scalar field ϕ as:

$$\phi(x) = \frac{e^{i\frac{\chi(x)}{v}}}{\sqrt{2}}(v + H(x)) \quad \xrightarrow{U(1)} \quad \frac{1}{\sqrt{2}}(v + H(x)) \ , \tag{8}$$

the χ degree of freedom can be rotated away, as indicated in Eq. (8), by enforcing the $U(1)$ gauge invariance of the original Lagrangian. With this gauge choice, known as unitary gauge or unitarity gauge, the Lagrangian becomes:

$$\mathcal{L} = \mathcal{L}_A + \frac{g^2 v^2}{2} A^\mu A_\mu + \frac{1}{2}\left(\partial^\mu H \partial_\mu H + 2\mu^2 H^2\right) + \ldots \tag{9}$$

which unambiguously describes the dynamics of a massive vector boson A^μ of mass $m_A^2 = g^2 v^2$, and a massive real scalar field of mass $m_H^2 = -2\mu^2 = 2\lambda v^2$, the Higgs field.

It is interesting to note that the total counting of degrees of freedom (d.o.f.) before the original $U(1)$ symmetry is spontaneously broken and after the breaking has occurred is the same. Indeed, one goes from a theory with one massless vector field (two d.o.f.) and one complex scalar field (two d.o.f.) to a theory with one massive vector field (three d.o.f.) and one real scalar field (one d.o.f.), for a total of four d.o.f. in both cases. This is what is colorfully described by saying that each gauge boson has eaten up one scalar degree of freedom, becoming massive.

We can now easily generalize the previous discussion to the case of a non-abelian Yang-Mills theory. $\mathcal{L}_A$ in Eq. (4) now becomes:

$$\mathcal{L}_A = -\frac{1}{4}F^{a,\mu\nu}F^a_{\mu\nu} \quad \text{with} \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \ , \tag{10}$$

where the latin indices are group indices and f^{abc} are the structure constants of the Lie Algebra associated to the non abelian gauge symmetry Lie group, defined by the commutation relations of the Lie Algebra generators t^a: $[t^a, t^b] = i f^{abc} t^c$.

Let us also generalize the scalar Lagrangian to include several scalar fields ϕ_i which we will in full generality consider as real:

$$\mathcal{L}_\phi = \frac{1}{2}(D^\mu \phi_i)^2 - V(\phi) \quad \text{where} \quad V(\phi) = \mu^2 \phi_i^2 + \frac{\lambda}{2}\phi_i^4 \ , \tag{11}$$

where the sum over the index i is understood and $D_\mu = \partial_\mu - i g t^a A^a_\mu$. The Lagrangian of Eq. (4) is invariant under a non-abelian gauge transformation of the form:

$$\begin{aligned}\phi_i(x) &\rightarrow (1+i\alpha^a(x)t^a)_{ij}\phi_j \ , \\ A^a_\mu(x) &\rightarrow A^a_\mu(x)+\frac{1}{g}\partial_\mu\alpha^a(x)+f^{abc}A^b_\mu(x)\alpha^c(x) \ .\end{aligned} \tag{12}$$

When $\mu^2 < 0$ the potential develops a degeneracy of minima described by the minimum condition: $\phi^2 = \phi_0^2 = -\mu^2/\lambda$, which only fixes the magnitude of the vector $\phi_0 = (\phi_1, \phi_2, \ldots,)$.

By arbitrarily choosing the direction of ϕ_0, the degeneracy is removed. The Lagrangian can be expanded in a neighborhood of the chosen minimum and mass terms for the gauge vector bosons emerge as in the abelian case, i.e.:

$$\begin{aligned}\frac{1}{2}(D_\mu\phi_i)^2 &\longrightarrow \ldots + \frac{1}{2}g^2(t^a\phi)_i(t^b\phi)_i A^a_\mu A^{b\mu} + \ldots \\ &\xrightarrow{\phi_{min}=\phi_0} \ldots + \frac{1}{2}\underbrace{g^2(t^a\phi_0)_i(t^b\phi_0)_i}_{m^2_{ab}} A^a_\mu A^{b\mu} + \ldots\end{aligned} \tag{13}$$

Upon diagonalization of the mass matrix m^2_{ab} in Eq. (13), all gauge vector bosons A^a_μ for which $t^a\phi_0 \neq 0$ become massive, and to each of them corresponds a Goldstone particle, i.e. an unphysical massless particle like the χ field of the abelian example. The remaining scalar degrees of freedom become massive, and correspond to the Higgs field H of the abelian example.

The Higgs mechanism can be very elegantly generalized to the case of a quantum field theory when the theory is quantized via the path integral method[1]. In this context, the quantum analog of the potential $V(\phi)$ is the effective potential $V_{eff}(\varphi_{cl})$, defined in term of the effective action $\Gamma[\phi_{cl}]$ (the generating functional of the 1PI connected correlation functions) as:

$$V_{eff}(\varphi_{cl}) = -\frac{1}{VT}\Gamma[\phi_{cl}] \quad \text{for} \quad \phi_{cl}(x) = \text{constant} = \varphi_{cl} \ , \tag{14}$$

where VT is the space-time extent of the functional integration and $\phi_{cl}(x)$ is the vacuum expectation value of the field configuration $\phi(x)$:

$$\phi_{cl}(x) = \langle\Omega|\phi(x)|\Omega\rangle \ . \tag{15}$$

The stable quantum states of the theory are defined by the variational

[1]Here I assume some familiarity with path integral quantization and the properties of various generating functionals introduced in that context. The detailed explanation of the formalism used would take us too far away from our main track

condition:

$$\frac{\delta}{\delta\phi_{cl}}\Gamma[\phi_{cl}]\Big|_{\phi_{cl}=\varphi_{cl}} = 0 \quad \longrightarrow \quad \frac{\partial}{\partial\varphi_{cl}} V_{eff}(\varphi_{cl}) = 0\ , \tag{16}$$

which identifies in particular the states of minimum energy of the theory, i.e. the stable vacuum states. A system with spontaneous symmetry breaking has several minima, all with the same energy. Specifying one of them, as in the classical case, breaks the original symmetry of the vacuum. The relation between the classical and quantum case is made even more transparent by the perturbative form of the effective potential. Indeed, $V_{eff}(\varphi_{cl})$ can be organized as a loop expansion and calculated systematically order by order in $\hbar$:

$$V_{eff}(\varphi_{cl}) = V(\varphi_{cl}) + \text{loop effects}\ , \tag{17}$$

with the lowest order being the classical potential in Eq. (3). Quantum corrections to $V_{eff}(\varphi_{cl})$ affect some of the properties of the potential and therefore have to be taken into account in more sophisticated studies of the Higgs mechanism for a spontaneously broken quantum gauge theory. Time permitting, will see how this can be important when we discuss how the mass of the SM Higgs boson is related to the energy scale at which we expect new physics effects to become relevant in the SM.

Finally, let us observe that at the quantum level the choice of gauge becomes a delicate issue. For example, in the unitarity gauge of Eq. (8) the particle content of the theory becomes transparent but the propagator of a massive vector field A^μ turns out to be:

$$\Pi^{\mu\nu}(k) = -\frac{i}{k^2 - m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_A^2}\right) \quad , \tag{18}$$

and has a problematic ultra-violet behavior, which makes it more difficult to consistently define and calculate ultraviolet-stable scattering amplitudes and cross sections. Indeed, for the very purpose of studying the renormalizability of quantum field theories with spontaneous symmetry breaking, the so called renormalizable or renormalizability gauges (R_ξ gauges) are introduced. If we consider the abelian Yang-Mills theory of Eqs. (2)-(4), the renormalizable gauge choice is implemented by quantizing with a gauge condition G of the form:

$$G = \frac{1}{\sqrt{\xi}}(\partial_\mu A^\mu + \xi g v \phi_2) \quad , \tag{19}$$

in the generating functional

$$Z[J] = C\int DA\, D\phi_1\, D\phi_2 \exp\left[i\int(\mathcal{L} - \frac{1}{2}G^2)\right] \det\left(\frac{\delta G}{\delta\alpha}\right) \quad , \tag{20}$$

where C is an overall factor independent of the fields, ξ is an arbitrary parameter, and α is the gauge transformation parameter in Eq. (5). After having reduced the determinant in Eq. (20) to an integration over ghost fields (c and $\bar{c}$), the gauge plus scalar fields Lagrangian looks like:

$$\begin{aligned}\mathcal{L}-\frac{1}{2}G^2+\mathcal{L}_{ghost} &= -\frac{1}{2}A_\mu\left(-g^{\mu\nu}\partial^2+\left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu-(gv)^2g^{\mu\nu}\right)A_\nu \\ &+ \frac{1}{2}(\partial_\mu\phi_1)^2-\frac{1}{2}m_{\phi_1}^2\phi_1^2+\frac{1}{2}(\partial_\mu\phi_2)^2-\frac{\xi}{2}(gv)^2\phi_2^2+\cdots \\ &+ \bar{c}\left[-\partial^2-\xi(gv)^2\left(1+\frac{\phi_1}{v}\right)\right]c\ , \end{aligned} \tag{21}$$

such that:

$$\begin{aligned}\langle A^\mu(k)A^\nu(-k)\rangle &= \frac{-i}{k^2-m_A^2}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right)+\frac{-i\xi}{k^2-\xi m_A^2}\left(\frac{k^\mu k^\nu}{k^2}\right)\ , \\ \langle\phi_1(k)\phi_1(-k)\rangle &= \frac{-i}{k^2-m_{\phi_1}^2}\ , \\ \langle\phi_2(k)\phi_2(-k)\rangle &= \langle c(k)\bar{c}(-k)\rangle=\frac{-i}{k^2-\xi m_A^2}\ , \end{aligned} \tag{22}$$

where the vector field propagator now has a safe ultraviolet behavior. Moreover we notice that the ϕ_2 propagator has the same denominator as the longitudinal component of the gauge vector boson propagator. This shows in a more formal way the relation between the ϕ_2 degree of freedom and the longitudinal component of the massive vector field A^μ, upon spontaneous symmetry breaking.

The Higgs sector of the Standard Model

The Standard Model is a spontaneously broken Yang-Mills theory based on the $SU(2)_L \times U(1)_Y$ non-abelian symmetry group. The Higgs mechanism is implemented in the Standard Model by introducing a complex scalar field ϕ which is a doublet of $SU(2)$ with hypercharge $Y_\phi = 1/2$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \ , \tag{23}$$

with Lagrangian

$$\mathcal{L}_\phi = (D^\mu \phi)^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2 \ , \tag{24}$$

where $D_\mu \phi = (\partial_\mu - igA^a_\mu \tau^a - ig' Y_\phi B_\mu)$, and $\tau^a = \sigma^a/2$ (for $a = 1, 2, 3$) are the $SU(2)$ Lie Algebra generators, proportional to the Pauli matrix σ^a. The gauge symmetry of the Lagrangian is broken to $U(1)_{em}$ when a particular vacuum expectation value is chosen, e.g.:

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{with} \quad v = \left(\frac{-\mu^2}{\lambda}\right)^{1/2} \quad (\mu^2 < 0,\, \lambda > 0)\ . \tag{25}$$

Upon spontaneous symmetry breaking the kinetic term in Eq. (24) gives origin to the SM gauge boson mass terms. Indeed, specializing Eq. (13) to the present case, and using Eq. (25), one gets:

$$\begin{aligned} (D^\mu\phi)^\dagger D_\mu\phi \quad &\longrightarrow \quad \cdots + \frac{1}{8}(0\ \ v)\left(gA^a_\mu\sigma^a + g'B_\mu\right)\left(gA^{b\mu}\sigma^b + g'B^\mu\right)\begin{pmatrix} 0 \\ v \end{pmatrix} + \cdots \\ &\longrightarrow \quad \cdots + \frac{1}{2}\frac{v^2}{4}\left[g^2(A^1_\mu)^2 + g^2(A^2_\mu)^2 + (-gA^3_\mu + g'B_\mu)^2\right] + \cdots \end{aligned} \tag{26}$$

One recognizes in Eq. (26) the mass terms for the charged gauge bosons $W^\pm_\mu$:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \pm A^2_\mu) \quad \longrightarrow \quad M_W = g\frac{v}{2}\ , \tag{27}$$

and for the neutral gauge boson Z_μ:

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}(gA^3_\mu - g'B_\mu) \quad \longrightarrow \quad M_Z = \sqrt{g^2+g'^2}\frac{v}{2} \ , \tag{28}$$

while the orthogonal linear combination of A^3_μ and B_μ remains massless and corresponds to the photon field (A_μ):

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'A^3_\mu + gB_\mu) \quad \longrightarrow \quad M_A = 0 \ , \tag{29}$$

the gauge boson of the residual $U(1)_{em}$ gauge symmetry.

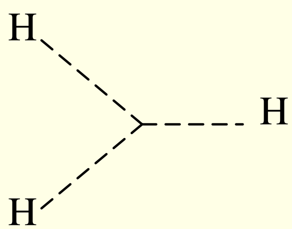
The content of the scalar sector of the theory becomes more transparent if one works in the unitary gauge and eliminates the unphysical degrees of freedom using gauge invariance. In analogy to what we wrote for the abelian case in Eq. (8), this amounts to parametrize and rotate the $\phi(x)$ complex scalar field as follows:

$$\phi(x) = \frac{e^{\frac{i}{v}\vec{\chi}(x)\cdot\vec{\tau}}}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v+H(x) \end{array}\right) \quad \xrightarrow{SU(2)} \quad \phi(x) = \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v+H(x) \end{array}\right) \ , \tag{30}$$

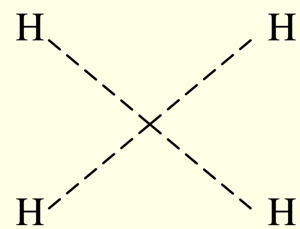
after which the scalar potential in Eq. (24) becomes:

$$\mathcal{L}_\phi = \mu^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4 = -\frac{1}{2} M_H^2 H^2 - \sqrt{\frac{\lambda}{2}} M_H H^3 - \frac{1}{4} \lambda H^4 \ . \quad (31)$$

Three degrees of freedom, the $\chi^a(x)$ Goldstone bosons, have been reabsorbed into the longitudinal components of the $W_\mu^\pm$ and Z_μ weak gauge bosons. One real scalar field remains, the Higgs boson H, with mass $M_H^2 = -2\mu^2 = 2\lambda v^2$ and self-couplings:

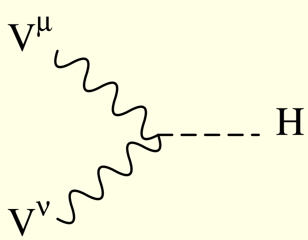


$$= -3i \frac{M_H^2}{v}$$

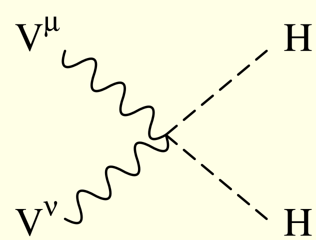


$$= -3i \frac{M_H^2}{v^2}$$

Furthermore, some of the terms that we omitted in Eq. (26), the terms linear in the gauge bosons $W_\mu^\pm$ and Z_μ, define the coupling of the SM Higgs boson to the weak gauge fields:

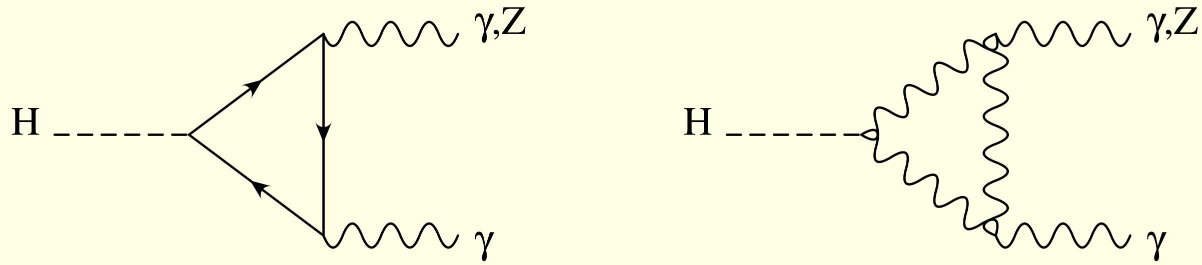


$$= 2i \frac{M_V^2}{v} g^{\mu\nu}$$

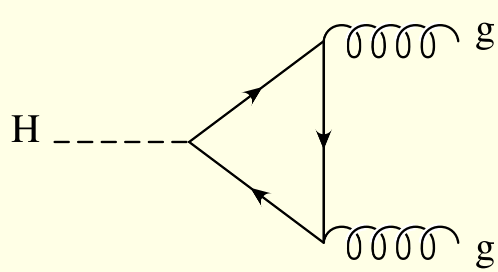


$$= 2i \frac{M_V^2}{v^2} g^{\mu\nu}$$

We notice that the couplings of the Higgs boson to the gauge fields are proportional to their mass. Therefore H does not couple to the photon at tree level. It is important, however, to observe that couplings that are absent at tree level may be induced at higher order in the gauge couplings by loop corrections. Particularly relevant to the SM Higgs boson phenomenology that will be discussed later are the couplings of the SM Higgs boson to pairs of photons, and to a photon and a Z_μ weak boson:



as well as the coupling to pairs of gluons:



The analytical expressions for the $H\gamma\gamma$, $H\gamma Z$, and Hgg one-loop vertices are somewhat involved and will be given later. As far as the Higgs boson tree level couplings go, we observe that they are all expressed in terms of just two parameters, either λ and μ appearing in the scalar potential of $\mathcal{L}_\phi$ (see Eq. 24)) or, equivalently, M_H and v, the Higgs boson mass and the scalar field vacuum expectation value. Since v is measured in muon decay to be $v=(\sqrt{2}G_F)^{-1/2}=246$ GeV, the physics of the SM Higgs boson is actually just a function of its mass M_H.

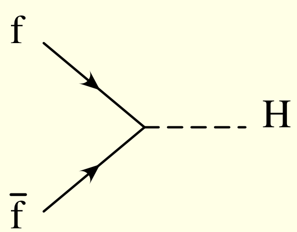
The Standard Model gauge symmetry also forbids explicit mass terms for the fermionic degrees of freedom of the Lagrangian, since mass terms mix right- and left-handed spinors which are in doublet and singlet representations of the $SU(2)$ group respectively. One needs to bring in another doublet somehow and the Higgs doublet is perfect for the job. The fermion mass terms can be (and in the SM are assumed to be) generated via gauge invariant renormalizable Yukawa couplings to the scalar field ϕ:

$$\mathcal{L}_{Yukawa} = -\Gamma_u^{ij}\bar{Q}_L^i\phi^c u_R^j - \Gamma_d^{ij}\bar{Q}_L^i\phi d_R^j - \Gamma_e^{ij}\bar{L}_L^i\phi l_R^j + h.c. \tag{32}$$

where $\phi^c=-i\sigma^2\phi^\star$, and Γ_f ($f=u,d,l$) are matrices of couplings arbitrarily introduced to realize the Yukawa coupling between the field ϕ and the fermionic fields of the SM. Q_L^i and L_L^i (where $i=1,2,3$ is a generation index) represent quark and lepton left handed doublets of $SU(2)_L$, while u^i_R, d^i_R and l^i_R are the corresponding right handed singlets. When the scalar fields ϕ acquires a non zero vacuum expectation value through spontaneous symmetry breaking, each fermionic degree of freedom coupled to ϕ develops a mass term with mass parameter

$$m_f = \Gamma_f \frac{v}{\sqrt{2}} \ , \tag{33}$$

where the process of diagonalization from the current eigenstates in Eq. (32) to the corresponding mass eigenstates is understood, and Γ_f are therefore the elements of the diagonalized Yukawa matrices corresponding to a given fermion f. The Yukawa couplings of the f fermion to the Higgs boson (y_f) is proportional to Γ_f:



$$= -i\frac{m_f}{v} = -i\frac{\Gamma_f}{\sqrt{2}} = -iy_f$$

As long as the origin of fermion masses is not better understood in some more general context beyond the Standard Model, the Yukawa couplings y_f represent free parameters of the SM Lagrangian.

The mechanism through which fermion masses are generated in the Standard Model, although related to the mechanism of spontaneous symmetry breaking, clearly requires further assumptions and involves a larger degree of arbitrariness as compared to the gauge boson sector of the theory. So, one has to take with a grain of salt the often made public statement that the Higgs boson is responsible for the masses of all the elementary particles.

Unitarity and the Higgs

Unitarity plays a crucial role in our understanding of why we need a Higgs boson. Let us first discuss the general partial wave formulation of unitarity.

For $2 \to 2$ scattering we have the standard formula

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|\mathcal{M}|^2 . \tag{34}$$

If we do the partial wave decomposition (the form employed assumes I am dealing with spin-0 particles — effectively the longitudinally polarized W's that I will be discussing can be thought of as spin-0 objects)

$$\mathcal{M} = 16\pi \sum_{J=0}^{\infty}(2J+1)P_J(\cos\theta)a_J \tag{35}$$

and recall the standard orthogonality relation

$$\int_{-1}^{1} dx P_J(x)P_{J'}(x) = \frac{2}{2J+1}\partial_{J,J'} \tag{36}$$

we obtain

$$\begin{aligned}\sigma &= \frac{8\pi}{s}\sum_{J=0}^{\infty}(2J+1)\sum_{J'=0}^{\infty}(2J'+1)a_J a^*_{J'}\int_{-1}^{1} d\cos\theta P_J(\cos\theta)P_{J'}(\cos\theta) \\ &= \frac{16\pi}{s}\sum_{J=0}^{\infty}(2J+1)|a_J|^2 \,. \end{aligned} \quad (37)$$

Meanwhile, the optical theorem says that

$$s\sigma = \mathrm{Im}\mathcal{M}(\theta=0) = 16\pi\sum_{\mathrm{J}=0}^{\infty}(2\mathrm{J}+1)\mathrm{Im}\mathrm{a}_{\mathrm{J}} \,. \quad (38)$$

If a single partial wave is dominant, we end up with the requirement that

$$\mathrm{Im}\mathrm{a}_{\mathrm{J}} = |\mathrm{a}_{\mathrm{J}}|^2 \,. \quad (39)$$

In fact, this equation applies for every value of J even if more than one partial wave is present (requires more work to show explicitly — see short appendix — but is basically a statement of angular momentum conservation for the elastic process). However, the above equality requires that the only process contributing to the total cross section be the $2 \to 2$ process. Normally, there are many inelastic channels such as $2 \to 2'$, $2 \to 3, 2 \to 4, \ldots.$. The $\mathrm{Im a_J}$ must take into account all these extra channels. In short, what we really have is

$$\mathrm{Im a_J} \geq |\mathrm{a_J}|^2 = (\mathrm{Im a_J})^2 + (\mathrm{Re a_J})^2\,, \tag{40}$$

When the equality holds, we can write

$$a_J = e^{i\partial_J} \sin \partial_J\,, \tag{41}$$

which form automatically satisfies $\mathrm{Im a_J} = |\mathrm{a_J}|^2$ and makes it clear that $|a_J|^2 = \sin^2 \partial_J \leq 1$ with maximum value of 1 when a_J is purely imaginary, $\partial_J = \pi/2$.

It is useful to have a graphical picture. On the unitarity circle, we can rewrite $\mathrm{Im a_J} = |\mathrm{a_J}|^2$ as

$$\frac{1}{4} = (\mathrm{Im a_J} - \tfrac{1}{2})^2 + (\mathrm{Re a_J})^2\,. \tag{42}$$

so that in the complex plane a_J will lie on a "unitarity" circle of radius $1/2$.

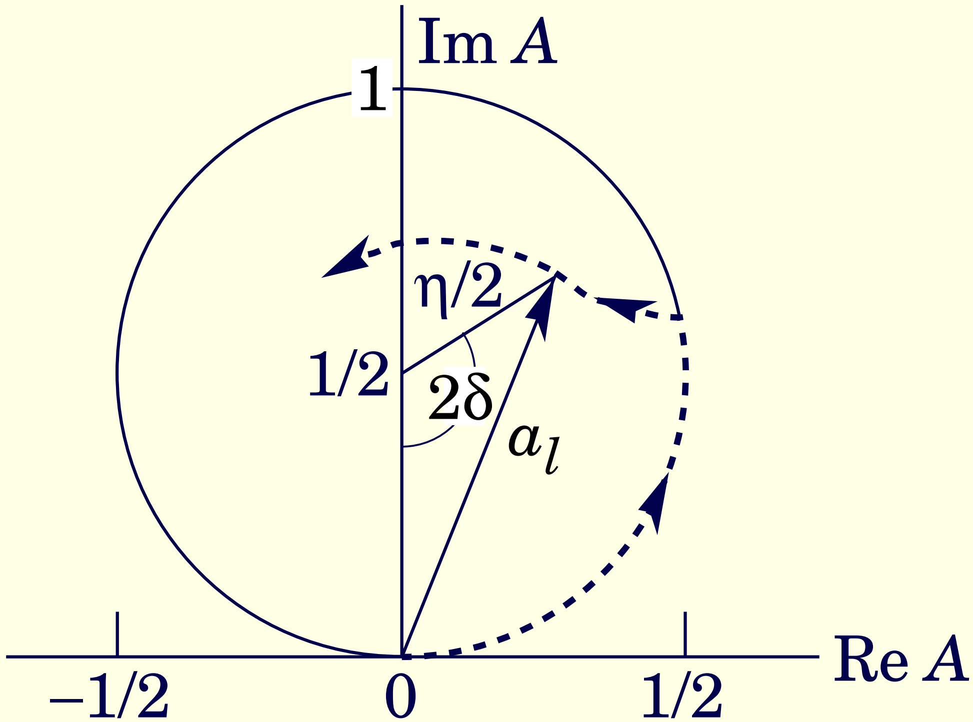


Figure 2: **The partial wave unitarity circle.**

Allowing for the presence of inelasticity, we can write

$$\frac{1}{4} \geq (\mathrm{Im} a_J - \tfrac{1}{2})^2 + (\mathrm{Re} a_J)^2 \,, \tag{43}$$

implying that a_J will lie inside the unitarity circle. In this case, we can write

$$a_J = \frac{\eta_J e^{2i\partial_J} - 1}{2i}, \tag{44}$$

since $\mathrm{Ima_J} - \frac{1}{2} = -\frac{1}{2}\eta_\mathrm{J}\cos 2\partial_\mathrm{J}$ and $\mathrm{Rea_J} = \frac{1}{2}\eta_\mathrm{J}\sin 2\partial_\mathrm{J}$ so that

$$(\mathrm{Ima_J} - \tfrac{1}{2})^2 + (\mathrm{Rea_J})^2 = \frac{1}{4}\eta_\mathrm{J}^2 \tag{45}$$

which will be $\leq \frac{1}{4}$ if $\eta_J \leq 1$. The quantity η_J is called the inelasticity. To repeat, $\eta_J \leq 1$ is required in order for a_J to be within the unitarity circle.

As a point of reference, you are presumably familiar with the standard spin-J resonance form of

$$a_J = a = \frac{-m\Gamma_{el}}{s - m^2 + im\Gamma_{tot}} \tag{46}$$

which saturates the unitarity circle when $\Gamma_{el} = \Gamma_{tot}$. You will notice that when the latter is true then at $s = m^2$ one finds $a_J = i$ so that one is at the top of the circle. As one starts from low $s \ll m^2$ with $\mathrm{Rea_J} > 0$, passes

through $s = m^2$ with $\mathrm{Rea_J} = 0$ and on to $s \gg m^2$ with $Rea_J < 0$ one is rotating in a counter-clockwise sense about the unitarity circle.

The important final result is that the largest value of $|\mathrm{Rea_J}|$ that is possible if on the "unitarity" circle is $|\mathrm{Rea_J}| = \frac{1}{2}$. And, if there is inelasticity then we have our final constraint of

$$|\mathrm{Rea_J}| \leq \tfrac{1}{2} \,. \tag{47}$$

Usually it is the $J = 0$ constraint that is the strongest for a typical process of interest to us.

We now wish to apply this to $WW \to WW$ scattering where the W's all have longitudinal polarization.

In the SM, the partial wave amplitudes take the asymptotic form

$$a_J = A_J \left(\frac{s}{m_W^2}\right)^2 + B_J \left(\frac{s}{m_W^2}\right) + C_J \,, \tag{48}$$

where s is the center-of-mass energy squared. Contributions that are divergent in the limit $s \to \infty$ appear only for $J = 0, 1, 2$. The A-terms vanish by virtue of gauge invariance, while, and here enters the Higgs boson, the B-term for $J = 1$ and 0 ($B_2 = 0$) arising from gauge interaction diagrams is canceled by Higgs-boson exchange diagrams. In the high-energy limit, the result is that

a_J asymptotes to an m_h-dependent constant. Imposing the unitarity limit of $|\mathrm{Re a_J}| < 1/2$ implies the Lee-Quigg-Thacker bound for the Higgs boson mass: $m_h \lesssim \mathbf{870}\ \mathrm{GeV}$.

What happens if m_h is increased beyond $\mathbf{870}\ \mathrm{GeV}$ is that the perturbatively calculated a_J violates unitarity at lower and lower s values. This is not to say that the theory actually violates unitarity. When a_J approaches the unitarity bound, the theory becomes strongly interacting and we are no longer able to calculate the consequences of the theory perturbatively. At the moment we do not have any means of computing what actually happens. Nonetheless, the theory cannot actually violate unitarity since unitarity is guaranteed simply by the hermiticity of the Hamiltonian that we began with. In other words, $m_h = \mathbf{870}\ \mathrm{GeV}$ is the largest value for which a perturbative approach to computing in the theory is possible.

Alternatively, if a_J reaches the unitarity limit at some value of s, we can hope that some beyond-the-SM physics enters that will be such that we can compute perturbatively in the extended model. The value of s at which the SM violates unitarity then sets the upper bound on the scale at which such new BSM physics must enter.

Anyway, let us give some more details on the perturbative SM calculation. In detail, we have the following. (In my normalization, $v = \mathbf{246}\ \mathrm{GeV}$ and

$m_W = \frac{gv}{2}$. And, $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{1}{2v^2}$.) The various contributions to the amplitude are given in Table 1. From the table, we see that the gauge boson contributions and Higgs exchange contributions cancel at $\mathcal{O}(s^2)$ and $\mathcal{O}(s^1)$.

diagram	$\mathcal{O}(\frac{s^2}{v^4})$	$\mathcal{O}(\frac{s^1}{v^2})$
γ, Z s-channel	$-\frac{s^2}{g^2v^4}4\cos\theta$	$-\frac{s}{v^2}\cos\theta$
γ, Z t-channel	$-\frac{s^2}{g^2v^4}(-3+2\cos\theta+\cos^2\theta)$	$-\frac{s}{v^2}\frac{3}{2}(1-5\cos\theta)$
$WWWW$ contact	$-\frac{s^2}{g^2v^4}(3-6\cos\theta-\cos^2\theta)$	$-\frac{s}{v^2}2(-1+3\cos\theta)$
h s-channel	0	$-\frac{s}{v^2}$
h t-channel	0	$-\frac{s}{v^2}\frac{-1+\cos\theta}{2}$
Sum	0	0

Table 1: The leading contributions to $\mathcal{M}(W_L^+W_L^- \to W_L^+W_L^-)$ amplitude — where $\mathcal{M}$ is defined in the convention $S_{fi} = \partial_{fi} + i(2\pi)^4\partial^4(p_f - p_i)\mathcal{M}_{fi}$.

The cancellation of the $\mathcal{O}(s^2)$ contributions in Table 1 between the contact term and s- and t-channel gauge-boson exchange diagrams is guaranteed by gauge invariance. The Higgs or something like it is required for cancelling the s^1 terms.

The easiest of the amplitudes above to derive is that from the $WWWW$

contact interaction. First, we need to write down the polarization vector for a massive, longitudinally-polarized W of momentum p. If we start with a massive vector boson with momentum $k^\mu = (m, 0, 0, 0)$, then the 3 orthgonal polarizations are

$$\epsilon^\mu = (0, 1, 0, 0), \quad (0, 0, 1, 0), \quad (0, 0, 0, 1)\,. \tag{49}$$

If we now boost along the z direction, the first two (the transverse polarizations) remain unchanged. The third (the longitudinal polarization) boosts to ($k \equiv k_z$)

$$\epsilon^\mu_L(k) = \left(\frac{k}{m}, 0, 0, \frac{E_k}{m}\right) \tag{50}$$

which generalizes for arbitrary direction to

$$\epsilon^\mu_L(\vec{k}) = \left(\frac{|\vec{k}|}{m}, \frac{\vec{k}}{|\vec{k}|}\frac{E_{\vec{k}}}{m}\right) \sim \frac{k^\mu}{m} + \mathcal{O}\left(\frac{m}{E_{\vec{k}}}\right) \tag{51}$$

(Note that the first, exact form on the rhs of the equation is explicitly orthogonal to $k^\mu = (E_{\vec{k}}, \vec{k})$.) It is the proportionality of the longitudinal polarization to the momentum which makes longitudinally polarized W's so "dangerous". Of course, for our case of W's, $m \to m_W$.

Meanwhile, the $WWWW$ contact interaction takes the familiar form in the all outgoing convention (and $S_{fi} = \partial_{fi} + (2\pi)^4\partial^4(p_f - p_i)i\mathcal{M}_{fi}$ convention):

$$i\mathcal{M}^{abcd}_{\alpha\beta\gamma\partial}(p,q,r,s) = -ig^2\Big\{c^{eab}c^{ecd}\left(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}\right) + c^{eac}c^{edb}\left(g^{\alpha\delta}g^{\gamma\beta} - g^{\alpha\beta}g^{\gamma\delta}\right) + c^{ead}c^{ebc}\left(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta}\right)\Big\}, \tag{52}$$

which, for the $SU(2)$ group and a,b,c,d index choices corresponding to $(a = W^+)(d = W^-) \to (b = W^+)(c = W^-)$ scattering reduces to[2] (after noting that the incoming W^+W^- should be converted to outgoing $(a = W^-)(d = W^+)$ to apply the "all-outgoing" vertex)

$$-ig^2(g^{\alpha\partial}g^{\beta\gamma} + g^{\gamma\partial}g^{\alpha\beta} - 2g^{\beta\partial}g^{\alpha\gamma}) \tag{53}$$

where the first two terms are s and t channel-like (outgoing W^-W^+) and

[2] A convenient reference for electroweak theory Feynman rules is the book "Gauge Theories of the Strong, Weak and Electromagnetic Interactions" by Chris Quigg, pages 113-16, or Appendix B of Cheng and Li.

the last is u channel-like (outgoing W^-W^-) and thus comes with a larger coefficient and opposite sign.

To see how this works in the case of the $WWWW$ vertex starting with the general result of Eq. (52), we first recall that $W^\pm = \frac{W^1 \mp iW^2}{\sqrt{2}}$. So, what we are interested in is (using a shorthand)

$$a = \frac{1+i2}{\sqrt{2}}, \quad b = \frac{1-i2}{\sqrt{2}}, \quad c = \frac{1+i2}{\sqrt{2}}, \quad d = \frac{1-i2}{\sqrt{2}}, \tag{54}$$

where we used the fact that, for example,

$$\begin{aligned} W^a a \ni W^1 1 + W^2 2 &= \frac{W^1 - iW^2}{\sqrt{2}} \frac{1+i2}{\sqrt{2}} + \frac{W^1 + iW^2}{\sqrt{2}} \frac{1-i2}{\sqrt{2}} \\ &= W^+ \frac{1+i2}{\sqrt{2}} + W^- \frac{1-i2}{\sqrt{2}}, \end{aligned} \tag{55}$$

where the W^- field operator either annihilates a W^- or creates a W^+. So, if we want to create an outgoing $b = W^+$ we connect to $b = \frac{1-i2}{\sqrt{2}}$ and if we want to connect to an outgoing $a = W^-$ we should use $a = \frac{1+i2}{\sqrt{2}}$. Thus, the general vertex of Eq. (52) reduces for the case of interest to

$$\begin{aligned} i\mathcal{M}^{abcd}_{\alpha\beta\gamma\partial}(p,q,r,s) &= -ig^2\Big\{ c^{e\frac{1+i2}{\sqrt{2}}\frac{1-i2}{\sqrt{2}}} c^{e\frac{1+i2}{\sqrt{2}}\frac{1-i2}{\sqrt{2}}} \left(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}\right) \\ &\quad + c^{e\frac{1+i2}{\sqrt{2}}\frac{1+i2}{\sqrt{2}}} c^{e\frac{1-i2}{\sqrt{2}}\frac{1-i2}{\sqrt{2}}} \left(g^{\alpha\delta}g^{\gamma\beta} - g^{\alpha\beta}g^{\gamma\delta}\right) \\ &\quad + c^{e\frac{1+i2}{\sqrt{2}}\frac{1-i2}{\sqrt{2}}} c^{e\frac{1-i2}{\sqrt{2}}\frac{1+i2}{\sqrt{2}}} \left(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta}\right)\Big\}, \end{aligned} \tag{56}$$

Now, for $SU(2)$ we have $c^{abc} = \epsilon^{abc}$ so that for the 1st term we have the form

$$c^{e\frac{1+i2}{\sqrt{2}}\frac{1-i2}{\sqrt{2}}} c^{e\frac{1+i2}{\sqrt{2}}\frac{1-i2}{\sqrt{2}}} = \left(\frac{-2i}{(\sqrt{2})^2}\epsilon^{e12}\right)\left(\frac{-2i}{(\sqrt{2})^2}\epsilon^{e12}\right) = -1\,. \tag{57}$$

The 2nd term is 0 because of the antisymmetry of the ϵ's and the 3rd term has the order switched on the last c and thus gives $+1$. The net result for the vertex is then:

$$\begin{aligned} i\mathcal{M} &= -ig^2\left[-\left(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}\right) + \left(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta}\right)\right] \\ &= -ig^2(g^{\alpha\partial}g^{\beta\gamma} + g^{\gamma\partial}g^{\alpha\beta} - 2g^{\beta\partial}g^{\alpha\gamma}) \end{aligned} \tag{58}$$

Now we can return to using this form for the scattering process of interest. We simply multiply $\mathcal{M}$ by the polarization vectors which in leading order means we multiply by

$$\frac{p^\alpha}{m_W}\frac{q^\beta}{m_W}\frac{r^\gamma}{m_W}\frac{s^\partial}{m_W}, \tag{59}$$

yielding

$$\mathcal{M}(W_L^+W_L^- \to W_L^+W_L^-) \sim -\frac{g^2}{m_W^4}(p\cdot sq\cdot r + r\cdot sp\cdot q - 2p\cdot rq\cdot s)\,. \tag{60}$$

Now, one goes to the center of mass frame where (all outgoing convention)

$$\begin{aligned} p &= -\frac{\sqrt{s}}{2}(1,0,0,\beta_W) \\ s &= -\frac{\sqrt{s}}{2}(1,0,0,-\beta_W) \\ q &= \frac{\sqrt{s}}{2}(1,0,\beta_W\sin\theta,\beta_W\cos\theta) \\ r &= \frac{\sqrt{s}}{2}(1,0,-\beta_W\sin\theta,-\beta_W\cos\theta) \end{aligned} \tag{61}$$

with $\beta_W = \sqrt{1 - 4m_W^2/s}$. In this frame, one easily finds

$$
\begin{aligned}
&2p \cdot s = 2q \cdot r = s - 2m_W^2\,, \\
&2p \cdot q = 2r \cdot s = t - 2m_W^2 = -\frac{s}{2}(1 - \cos\theta) - 2m_W^2 \cos\theta\,, \\
&2p \cdot r = 2q \cdot s = u - 2m_W^2 = -\frac{s}{2}(1 + \cos\theta) + 2m_W^2 \cos\theta
\end{aligned}
\tag{62}
$$

Substituting and collecting the leading terms of order s^2 one obtains

$$
\mathcal{M}(W_L^+ W_L^- \to W_L^+ W_L^-) \ni -\frac{g^2}{m_W^4}\frac{s^2}{16}(3 - 6\cos\theta - \cos^2\theta) \tag{63}
$$

which agrees with the $\mathcal{O}(s^2)$ result in the table for the contact interaction term after substituting $m_W = gv/2$.

To get the $\mathcal{O}(s)$ term of the table, the exact forms of the polarization vectors must be used.

The Higgs exchange diagrams are, of course, easier. For example for the s-channel Higgs exchange diagram, remembering that the Feynman rule for the $hW_\mu W_\nu$ vertex is $igm_W g_{\mu\nu}$, we find

$$
\begin{aligned}
i\mathcal{M} &= \frac{i}{s-m_h^2}(igm_W)^2\epsilon_p\cdot\epsilon_s\,\epsilon_q\cdot\epsilon_r \\
&\simeq -ig^2m_W^2\frac{1}{s-m_h^2}\left(\frac{p\cdot s\, q\cdot r}{m_W^4}\right) \\
&\simeq \frac{-i4m_W^4}{v^2}\frac{1}{s-m_h^2}\left(\frac{\frac{1}{4}s^2}{m_W^4}\right) \\
&= -i\frac{1}{v^2}\frac{s^2}{s-m_h^2} \\
&= -i\frac{1}{v^2}\left(s+\frac{s\,m_h^2}{s-m_h^2}\right)\,. \qquad (64)
\end{aligned}
$$

The first term is listed as the $\mathcal{O}(s)$ term for this diagram and the 2nd term will appear below

In any case, after including the Higgs diagrams and summing everything together, we are left with a constant behavior for the amplitude. In the limit where $s, m_h^2 \gg m_W^2, m_Z^2$ we find:

$$
\mathcal{M}(W_L^+W_L^- \to W_L^+W_L^-) = -\frac{m_h^2}{v^2}\left[\frac{s}{s-m_h^2}+\frac{t}{t-m_h^2}\right]\,. \qquad (65)
$$

This is too naive if s is near m_h^2; one must include the Higgs width appropriately. For now, we only consider $s \ll m_h^2$ or $s \gg m_h^2$.

The above result can also be derived using the Goldstone Equivalence theorem. It states that in the limit of $m_h^2 \gg m_W^2$ interactions of enhanced strength, $\mathcal{O}\left(G_F m_h^2\right)$, arise only from diagrams in which the $internal$ particles are also Goldstone bosons or the Higgs boson. The relevant interactions are summarized by the Lagrangian:

$$\mathcal{L} = -\lambda \left(w^+ w^- + \tfrac{1}{2} z^2 + \tfrac{1}{2} h^2 + vh + \tfrac{1}{2} v^2 - \frac{\mu^2}{2\lambda} \right)^2 \tag{66}$$

where h is the Higgs field, the w's, z are the charged and neutral Goldstone bosons, respectively, v is the usual vev related to m_W as above and λ is the bare coupling of the $\lambda\phi^4$ theory.

Crudely, one can arrive at this result as follows. First, we note that using $m_h^2 = 2\lambda v^2$ and $m_W = gv/2$ one finds $\lambda = g^2 \frac{m_h^2}{8m_W^2}$. When $m_h^2 \gg m_W^2$, λ is large and the Higgs self coupling term in $V(\phi) = \lambda(\phi^\dagger\phi)^2 - \mu^2(\phi^\dagger\phi)$ is the source of the strongest interactions. Writing $\phi = (h^+, \frac{1}{\sqrt{2}}(v + h + ia))$ the two terms of $V(\phi)$ are

$$\lambda(\phi^\dagger\phi)^2 \quad = \quad \lambda \left((h^-, \frac{1}{\sqrt{2}}(v + h - ia)), (h^+, \frac{1}{\sqrt{2}}(v + h + ia)) \right)$$

$$
\begin{aligned}
&= \lambda\left((h^+h^-) + \tfrac{1}{2}[(v+h)^2 + a^2]\right)^2 \\
-\mu^2(\phi^\dagger\phi) &= -\mu^2\left((h^+h^-) + \tfrac{1}{2}[(v+h)^2 + a^2]\right) .
\end{aligned} \tag{67}
$$

It is the $h^\pm$ and a fields that are eaten and become the longitudinal modes of the $W^\pm$ and Z. Thus, we denote them by $w^\pm$ and z. In this notation, $\mathcal{L} \ni -V = \mu^2(\phi^\dagger\phi) - \lambda(\phi^\dagger\phi)^2$ takes the form of Eq. (66) after including a conventional constant such that $V = 0$ at the tree-level minimum where $\frac{\partial V}{\partial h}|_{h=0} = 0$ requiring $\lambda v^2 = \mu^2$.

Although the last two terms in Eq. (66) cancel at tree level, they more generally yield a tadpole counterterm which is fixed at each order in perturbation theory in such a way that the physical Higgs field has zero vev. This extra counter term can be ignored for our present purposes.

The Lagrangian of Eq. (66) generates the Feynman rules of Fig. 3. For example, the upper left Feynman rule comes from the $-2\lambda v w^+ w^- h$ cross term coming from the square using

$$
-2\lambda v = -2\frac{g^2 m_h^2}{8m_W^2}\frac{2m_W}{g} = -\frac{g m_h^2}{2m_W} \tag{68}
$$

and supplying the usual i from expanding $\exp[i\mathcal{L}]$.

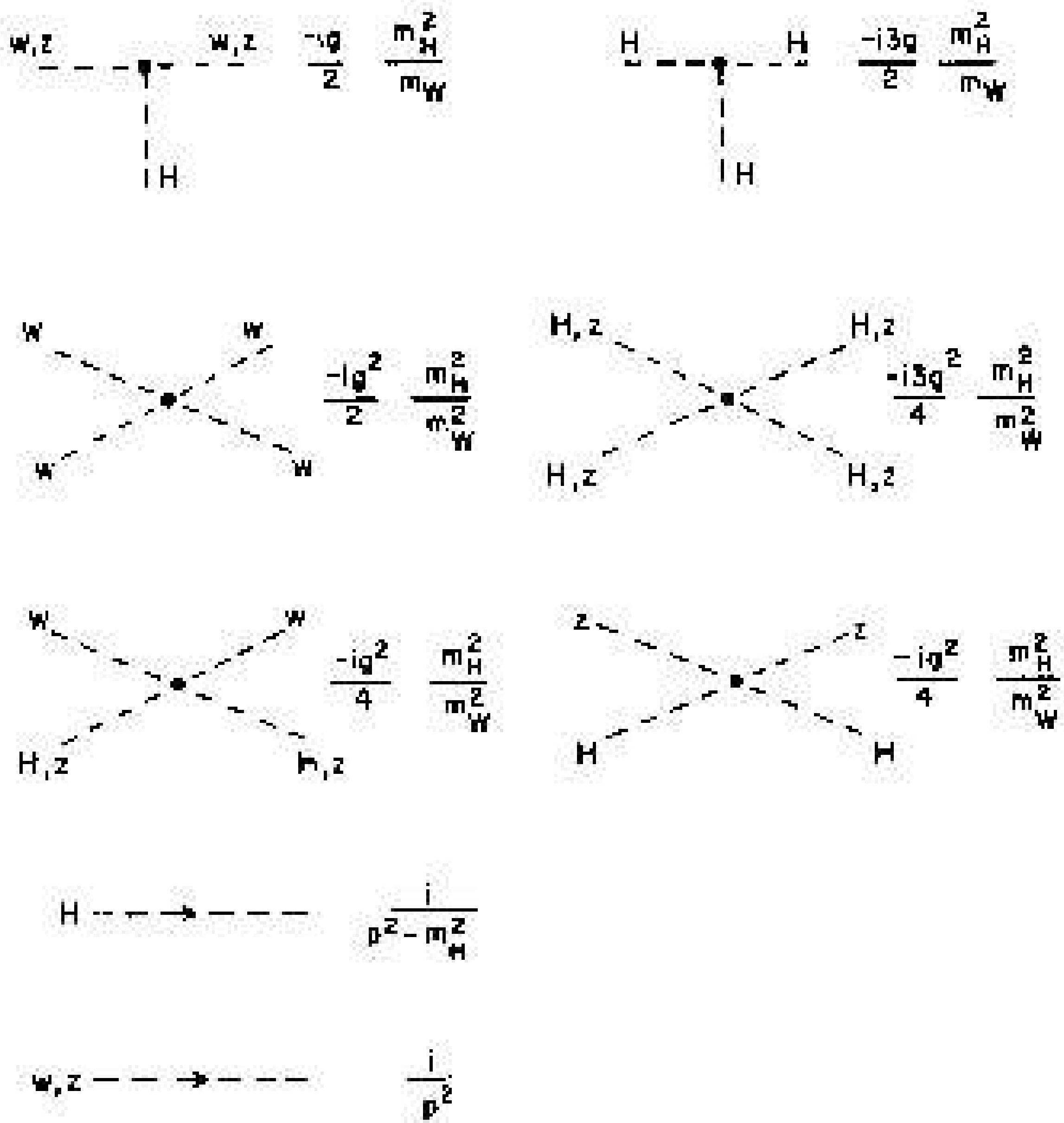


FIG. 1. Feynman rules for the Higgs-Goldstone scalar theory. Closed loops containing identical particles must be multiplied by $\frac{1}{2}$.

Figure 3: **Feynman rules from Eq. (66).**

The Feynman rule just below comes from the $-\lambda w^+ w^- w^+ w^-$ term of $\mathcal{L}$ after substituting $\lambda = \frac{g^2 m_h^2}{8 m_W^2}$, taking account of the fact that there are two possible ways to contract the $w^+ w^+$ with two such external states and similarly for $w^- w^-$ — thereby leading to a net contraction counting factor of 4, and supplying the standard i.

In these Feynman rules, we have effectively chosen to work in the Landau gauge where the $w^\pm$ and z propagators have zero mass and the $W^\pm$ and Z propagators (not given explicitly) are proportional to $g^{\mu\nu} - k^\mu k^\nu / k^2$. In this way, gauge-boson-scalar mixing is avoided, since any such interaction is proportional to the gauge-boson four momentum k^μ. Furthermore, we can neglect diagrams with internal $W^\pm$ and Z propagators since they are suppressed by m_W^2 / m_h^2 in this gauge. Thus, the Landau gauge is the simplest and most natural gauge in which to employ the Goldstone-boson equivalence theorem.

For $ww \to ww$ scattering there are diagrams with s-channel h exchange, t-channel h exchange and the $wwww$ contact interaction diagram. The result for the amplitude is (in agreement with the earlier-stated result)

$$i\mathcal{M}(ww \to ww) = \left(-ig\frac{m_h^2}{2m_W}\right)^2 \frac{i}{s - m_h^2} + \left(-ig\frac{m_h^2}{2m_W}\right)^2 \frac{i}{t - m_h^2} - ig^2 \frac{m_h^2}{2m_W^2}$$

$$
\begin{aligned}
&= -i\frac{g^2 m_h^2}{4m_W^2}\left(\frac{m_h^2}{s-m_h^2}+\frac{m_h^2}{t-m_h^2}+2\right)\\
&= -i\frac{m_h^2}{v^2}\left(\frac{s}{s-m_h^2}+\frac{t}{t-m_h^2}\right) . \qquad (69)
\end{aligned}
$$

Let us now take the partial wave projection of $\mathcal{M}(ww \to ww)$,

$$
a_J = \frac{1}{32\pi(2J+1)}\int d\cos\theta P_J(\cos\theta)\mathcal{M} \qquad (70)
$$

to obtain (using $t = -\frac{s}{2}(1-\cos\theta)$ implying $\frac{2}{s}dt = d\cos\theta$ and the next to last form for the parenthesis in $\mathcal{M}$ above):

$$
\begin{aligned}
a_0 &= \frac{1}{16\pi s}\int_{-s}^{0} dt\mathcal{M}(W_L^+W_L^- \to W_L^+W_L^-)\\
&= -\frac{1}{16\pi}\frac{m_h^2}{v^2}\left[2+\frac{m_h^2}{s-m_h^2}-\frac{m_h^2}{s}\log\left(1+\frac{s}{m_h^2}\right)\right] . \qquad (71)
\end{aligned}
$$

Now let us consider some interesting limits:

1. $s \gg m_h^2 \gg m_W^2, m_Z^2$:

We get

$$a_0 \to -\frac{m_h^2}{8\pi v^2}\,. \tag{72}$$

Requiring $|\mathrm{Re} a_0| \leq \frac{1}{2}$ this gives

$$m_h^2 \leq 4\pi v^2 = (872~\mathrm{GeV})^2,. \tag{73}$$

This is the absolute upper limit on m_h in order that unitarity hold for all s. If we consider a full coupled channel analysis (which includes $Z_L Z_L \to W_L^+ W_L^-$, $hh \to hh$, $hh \to W_L^+ W_L^-$, $\ldots$) then this limit gets reduced to

$$m_h^2 \lesssim (700~\mathrm{GeV})^2\,. \tag{74}$$

2. $m_W^2, m_Z^2 \ll s \ll m_h^2$:

By carefully expanding the $\log$ to 2nd order in the (small) ratio s/m_h^2 we find

$$a_0 \to \frac{s}{32\pi v^2}\,. \tag{75}$$

Requiring $|\mathrm{Re} a_0| \leq \frac{1}{2}$ this gives

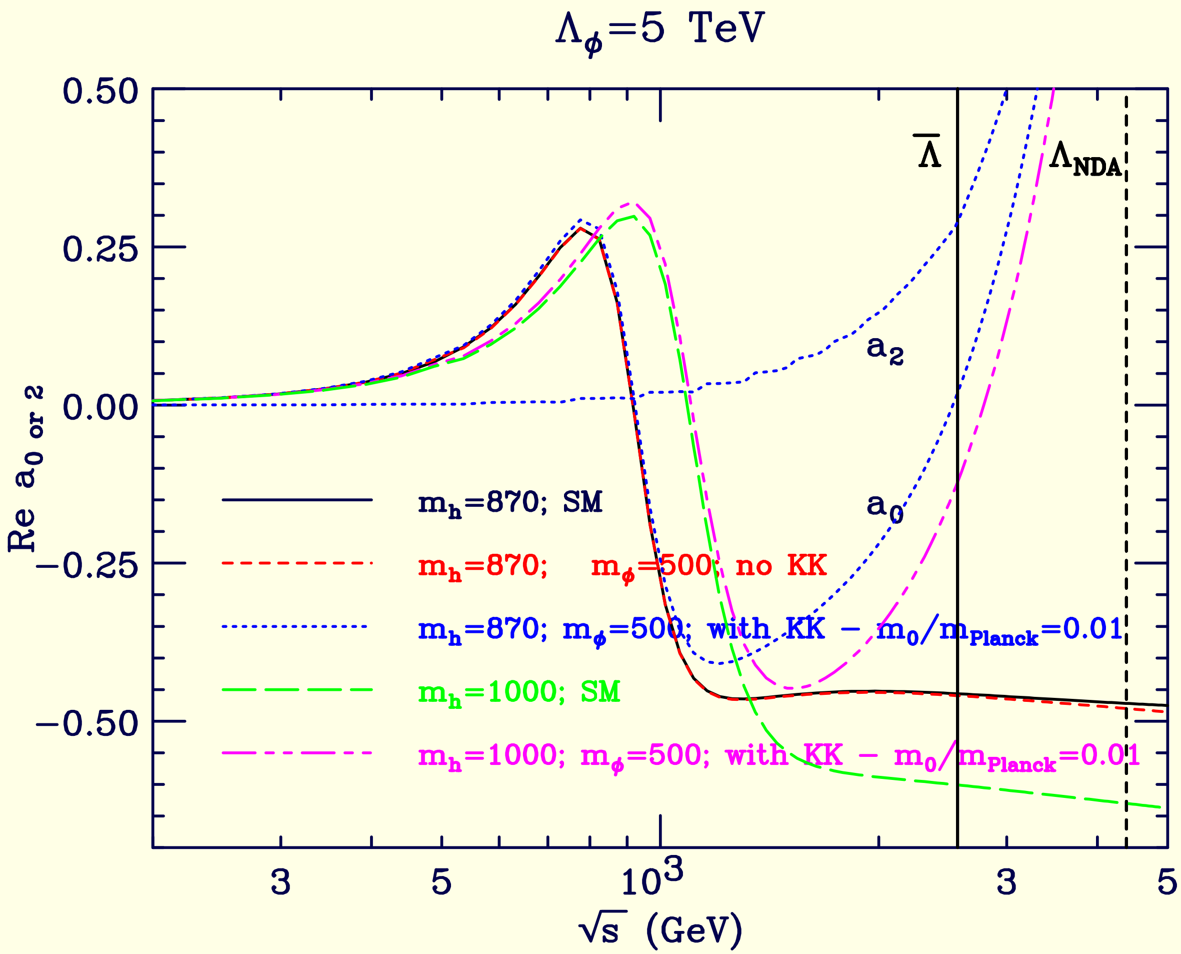
$$s \leq 16\pi v^2 \,. \tag{76}$$

An even better bound emerges by considering the (properly normalized) isospin zero channel $\sqrt{1/6}(2W_L^+W_L^- + Z_L Z_L)$ and is

$$s \leq 8\pi v^2 = (1233~\mathrm{GeV})^2 \,. \tag{77}$$

The interpretation of this limit is that if the Higgs is very heavy, then the SM can only be valid (in the sense of satisfying unitarity perturbatively) if $\sqrt{s} < 1.23~\mathrm{TeV}$. After that energy, new physics *must* enter or the $W_L^+W_L^- \to W_L^+W_L^-, \ldots$ sector must become strongly interacting.

If one does a full treatment, then the kind of plot for $\mathrm{Re} a_0(\mathrm{W_L^+W_L^-} \to \mathrm{W_L^+W_L^-})$ as a function of $\sqrt{s}$ that emerges is that below (only look at the SM curves which are for $m_h = 870~\mathrm{GeV}$ and $1000~\mathrm{GeV}$). Note how near the Higgs resonance $\mathrm{Re} a_0$ goes positive, but not so positive as to violate the unitarity limit. This is the result obtained after including the Higgs width in the formula for a_0; basically $m_h^2 \to m_h^2 - i m_h \Gamma_h$. Near the resonance, one is inside the Argand circle that typifies a resonance. $\eta_0 < 1$ in the case of $W_L^+W_L^- \to W_L^+W_L^-$ because Γ_h includes "inelastic" channels such as $h \to Z_LZ_L$ as well as $h \to W_L^+W_L^-$.



However, for $s \gg m_h^2$, $\mathrm{Re} a_0$ asymptotes towards $-\frac{1}{2}$ in the $m_h = 870~\mathrm{GeV}$ case and falls much below $-\frac{1}{2}$ for $m_h = 1000~\mathrm{GeV}$. As m_h is increased further, the $\sqrt{s}$ value at which a_0 falls below $-1/2$ decreases slowly, ultimately reaching the value of $\sqrt{s} = \sqrt{2} \times 1.233~\mathrm{TeV}$ (the $\sqrt{2}$ because this figure does not include the full coupled channel analysis). If m_h is decreased below $m_h = 870~\mathrm{GeV}$, the SM a_0 curve for $W_L^+ W_L^- \to W_L^+ W_L^-$ asymptotes to a less negative value for $\sqrt{s} \to \infty$ following Eq. (72).

Well, this is only a brief introduction to all this. There are endless ramifications of unitarity in the context of every new physics model. It might turn out that the $W_L W_L$, hh, $Z_L Z_L$ sector simply becomes strongly interacting and the LHC job will be to sort out exactly what kind of theory is describing these strong interactions.

This is closely analogous to the old situation in which we had strong interactions in the $\pi\pi$ scattering and related channels and only after much misery figured out that the π's were bound states of quarks.

In the strongly interacting WW-scattering scenario, the LHC will have a very tough time in sorting things out. The energy and luminosity are not quite up to the job. The old SSC was designed at higher energy precisely to cover adequately this unpleasant scenario. The problem is that there is a

huge background coming from the scattering of transversely polarized W's, such as $W_TW_T \to W_TW_T$, $W_LW_L \to W_TW_T$, $W_TW_T \to W_LW_L$, ... from which it is very difficult to extract the $W_L^+W_L^- \to W_L^+W_L^-$ scattering of interest.

Partial Wave Appendix

Starting from the most general unitarity statement, Eq. (??) repeated below,

$$
\begin{aligned}
& [-i\mathcal{M}(k_1 k_2 \to p_1 p_2) + i\mathcal{M}^*(p_1 p_2 \to k_1 k_2)] \\
= \; & \sum_n \left(\prod_{i=1}^{n} \int \frac{d^3 q_i}{(2\pi)^3} \frac{1}{2E_i} \right) \\
& \quad \times \mathcal{M}^*(p_1 p_2 \to \{q_i\}) \mathcal{M}(k_1 k_2 \to \{q_i\}) (2\pi)^4 \partial^4 (k_1 + k_2 - \sum_i q_i)
\end{aligned}
\tag{78}
$$

we consider only the same two-body final state as the initial state (i.e. $n = 2$). We imagine that $\vec{k}_1$ $(= -\vec{k}_2)$ in the com is along the z axis and that $\vec{p}_1$ is in the $x - z$ plane at location $\theta, \phi = 0$. The two particle intermediate state is defined to have $\vec{q}_1$ $(= -\vec{q}_2)$ defined by location θ', ϕ'. The angle between $\vec{p}_1$ and $\vec{q}_1$ we define to be γ.

Two-body phase space in the massless limit reduces to

$$\int \frac{d^3q_1}{(2\pi)^3 2E_1}\frac{d^3q_2}{(2\pi)^3 2E_2}(2\pi)^4\partial^4(k_1+k_2-q_1-q_2) = \frac{1}{32\pi^2}\int d\cos\theta' d\phi' \,. \quad (79)$$

So, now let us do partial wave expansions of all the $\mathcal{M}$'s (implicitly I am assuming spin-0 equivalent as appropriate for W_L scattering)

$$\begin{aligned}
\mathcal{M}(k_1k_2 \to p_1p_2) &= 16\pi\sum_J (2J+1)P_J(\cos\theta)a_J \\
\mathcal{M}^*(p_1p_2 \to k_1k_2) &= 16\pi\sum_J (2J+1)P_J(\cos\theta)a_J^* \\
\mathcal{M}(k_1k_2 \to q_1q_2) &= 16\pi\sum_J (2J+1)P_J(\cos\theta')a_J \\
\mathcal{M}^*(p_1p_2 \to q_1q_2) &= 16\pi\sum_J (2J+1)P_J(\cos\gamma)a_J^*
\end{aligned} \quad (80)$$

Inserting into Eq. (78), we get

$$\begin{aligned}
&32\pi\sum_J (2J+1)P_J(\cos\theta)\mathrm{Im a_J} = \frac{1}{32\pi^2}\int \mathrm{d}\cos\theta'\mathrm{d}\phi' \\
&\times(256\pi^2)\sum_{J'}(2J'+1)P_{J'}(\cos\theta')a_{J'}\sum_{J''}(2J''+1)P_{J''}(\cos\gamma)a^*_{J''}
\end{aligned} \quad (81)$$

So, now let us write (the standard addition theorem from Jackson)

$$P_{J''}(\cos\gamma) = \frac{4\pi}{2J''+1}\sum_m Y^*_{J''m}(\theta',\phi')Y_{J''m}(\theta,\phi=0) \tag{82}$$

The $\int d\phi'$ simply gives

$$\begin{aligned}\int d\phi' P_{J''}(\cos\gamma) &= (2\pi)\frac{4\pi}{2J''+1}Y^*_{J''0}(\theta')Y_{J''0}(\theta) \\ &= (2\pi)\frac{4\pi}{2J''+1}\left[\sqrt{\frac{2J''+1}{4\pi}}\right]^2 P_{J''}(\cos\theta')P_{J''}(\cos\theta) \\ &= (2\pi)P_{J''}(\cos\theta')P_{J''}(\cos\theta)\,. \end{aligned} \tag{83}$$

Eq. (81) then reduces to

$$\begin{aligned} &\sum_J (2J+1)P_J(\cos\theta)\mathrm{Ima_J} \\ =\ & \tfrac{1}{2}\int d\cos\theta' \sum_{J'}(2J'+1)P_{J'}(\cos\theta')a_{J'}\sum_{J''}(2J''+1)P_{J''}(\cos\theta')P_{J''}(\cos\theta)a^*_{J''} \\ =\ & \tfrac{1}{2}\sum_{J'}(2J'+1)a_{J'}\sum_{J''}(2J''+1)\frac{2}{2J'+1}\partial_{J',J''}P_{J''}(\cos\theta)a^*_{J''} \\ =\ & \sum_{J'}(2J'+1)P_{J'}(\cos\theta)a_{J'}a^*_{J'} \end{aligned} \tag{84}$$

from which we immediately conclude that

$$\mathrm{Im}\,a_J = a_J a_J^* \tag{85}$$

if only the same two-body intermediate state is present as in the initial and final states (equivalent to there being no inelastic scattering).

Triviality and vacuum stability

The argument of triviality in a $\lambda\phi^4$ theory goes as follows. The dependence of the quartic coupling λ on the energy scale (Q) is regulated by the renormalization group equation

$$\frac{d\lambda(Q)}{dQ^2} = \frac{3}{4\pi^2}\lambda^2(Q) \ . \tag{86}$$

This equation states that the quartic coupling λ decreases for small energies and increases for large energies.

Therefore,

- in the low energy regime the coupling vanishes and the theory becomes trivial, i.e. non-interactive.

- In the large energy regime, on the other hand, the theory becomes non-perturbative, since λ grows, and it can remain perturbative only if λ is set to zero, i.e. only if the theory is made trivial.

The situation in the Standard Model is more complicated, since the running of λ is governed by more interactions. Including the lowest orders in all the relevant couplings, we can write the equation for the running of $\lambda(Q)$ with the energy scale as follows:

$$32\pi^2\frac{d\lambda}{dt}=24\lambda^2-(3g'^2+9g^2-24y_t^2)\lambda+\frac{3}{8}g'^4+\frac{3}{4}g'^2g^2+\frac{9}{8}g^4-24y_t^4+\cdots \quad (87)$$

where $t=\ln(Q^2/Q_0^2)$ is the logarithm of the ratio of the energy scale and some reference scale Q_0 square, $y_t=m_t/v$ is the top-quark Yukawa coupling, and the dots indicate the presence of higher order terms that have been omitted. We see that when M_H becomes large, λ also increases (since $M_H^2=2\lambda v^2$) and the first term in Eq. (87) dominates. The evolution equation for λ can then be easily solved and gives:

$$\lambda(Q)=\frac{\lambda(Q_0)}{1-\frac{3}{4\pi^2}\lambda(Q_0)\ln\left(\frac{Q^2}{Q_0^2}\right)} \quad . \quad (88)$$

When the energy scale Q grows, the denominator in Eq. (88) may vanish, in which case $\lambda(Q)$ hits a pole, becomes infinite, and a triviality condition needs to be imposed. This is avoided imposing that the denominator in Eq. (88) never vanishes, i.e. that $\lambda(Q)$ is always finite and $1/\lambda(Q) > 0$. This condition gives an explicit upper bound on M_H. Requiring

$$\frac{3}{4\pi^2}\lambda(Q_0)\ln\left(\frac{Q^2}{Q_0^2}\right) < 1\,, \quad \text{with} \quad Q^2 = \Lambda^2, \;\; Q_0 = v, \;\; \lambda(v) = \frac{M_H^2}{2v^2} \tag{89}$$

we obtain the constraint

$$M_H^2 < \frac{8\pi^2 v^2}{3\log\left(\frac{\Lambda^2}{v^2}\right)} \;. \tag{90}$$

Λ is the scale at which new physics must enter and alter the evolution of λ.

- If we wanted to retain perturbativity all the way up to $\Lambda = 10^{16}~\mathrm{GeV}$, this would require $m_{h_{\rm SM}} \lesssim 160~\mathrm{GeV}$.

- For perturbativity only up to $\Lambda = 3~\mathrm{TeV}$, we would have the much weaker bound of $m_{h_{\rm SM}} \lesssim 600~\mathrm{GeV}$.

- If we keep the terms of order λ in the RGE equation Eq. (87), then we have

$$\frac{d\lambda}{dt} \sim \frac{\lambda}{16\pi^2}\left[12\lambda + 12y_t^2 - \frac{3}{2}(3g^2 + {g'}^2)\right] \tag{91}$$

We see that there is a critical value of the coupling, and associated $m_{h_{\rm SM}}^c \equiv \sqrt{2\lambda_c}v$,

$$\lambda_c = \frac{1}{8}(3g^2 + {g'}^2) - y_t^2 \tag{92}$$

for which the quartic coupling stops evolving. If $m_{h_{\rm SM}} > m_{h_{\rm SM}}^c$ then quartic coupling becomes infinite at some scale and the theory goes non-perturbative. If we require that the theory be perturbative ($i.e.$ the Higgs quartic coupling be finite) then an upper bound on on the Higgs mass is obtained as a function of m_t.

- To get a fairly precise value for $m_{h_{\rm SM}}^c$, the evolution of the gauge coupling constants and of y_t must also be included in a series of coupled differential equations.

  One finds that for $m_t = 175~\mathrm{GeV}$, the bound is $m_{h_{\rm SM}} < 170~\mathrm{GeV}$.

  If the Higgs boson had been found above this bound, it would have required that there be some new physics below the unification scale.

However, since $m_{h_{\rm SM}} \sim 126~{\rm GeV}$, this upper bound does not come into play.

- On the other hand, for small λ, $i.e.$ small M_H, the RGE equation takes the approximate form

$$\frac{d\lambda}{dt} \simeq \frac{1}{16\pi^2}\left[-12y_t^2 + \frac{3}{16}(2g^4 + (g^2 + {g'}^2)^2)\right] \tag{93}$$

which is easily solved to give

$$\lambda(\Lambda) = \lambda(v) + \frac{1}{16\pi^2}\left[-12y_t^4 + \frac{3}{16}(2g^4 + (g^2 + {g'}^2)^2)\right]\ln\left(\frac{\Lambda^2}{v^2}\right) \quad . \tag{94}$$

- To assure the stability of the vacuum state of the theory we need to require that $\lambda(\Lambda) > 0$ and this gives a lower bound for M_H. Using $\lambda(v) = \frac{M_H^2}{2v^2}$ and keeping the dominant y_t^4 term, we find:

$$\lambda(\Lambda) > 0 \ \longrightarrow \ M_H^2 > \frac{3v^2}{2\pi^2}y_t^4 \log\left(\frac{\Lambda^2}{v^2}\right) \quad . \tag{95}$$

More accurate analyses include higher order quantum corrections in the scalar potential and use a 2-loop renormalization group improved effective potential, V_{eff}, whose nature and meaning was briefly sketched earlier in these notes. The net result is that for $\Lambda = 10^{16}~\mathrm{GeV}$ we require

$$m_{h_{\rm SM}}(~\mathrm{GeV}) > 130.5 + 2.1(m_t - 174)\,. \tag{96}$$

If the SM need only be valid up to $1~\mathrm{TeV}$, then the limit is much weaker:

$$m_{h_{\rm SM}}(~\mathrm{GeV}) > 71 + 0.74(m_t - 174)],. \tag{97}$$

- Note that the observed mass, $\sim 126~\mathrm{GeV}$, is such that the SM could be valid as an effective theory almost all (maybe all) the way up to the Planck scale once a mestable vacuum at high Λ is allowed for.

  After all the important higher order corrections and coupled channel analysis is done, we end up with the plot of Fig. 4.

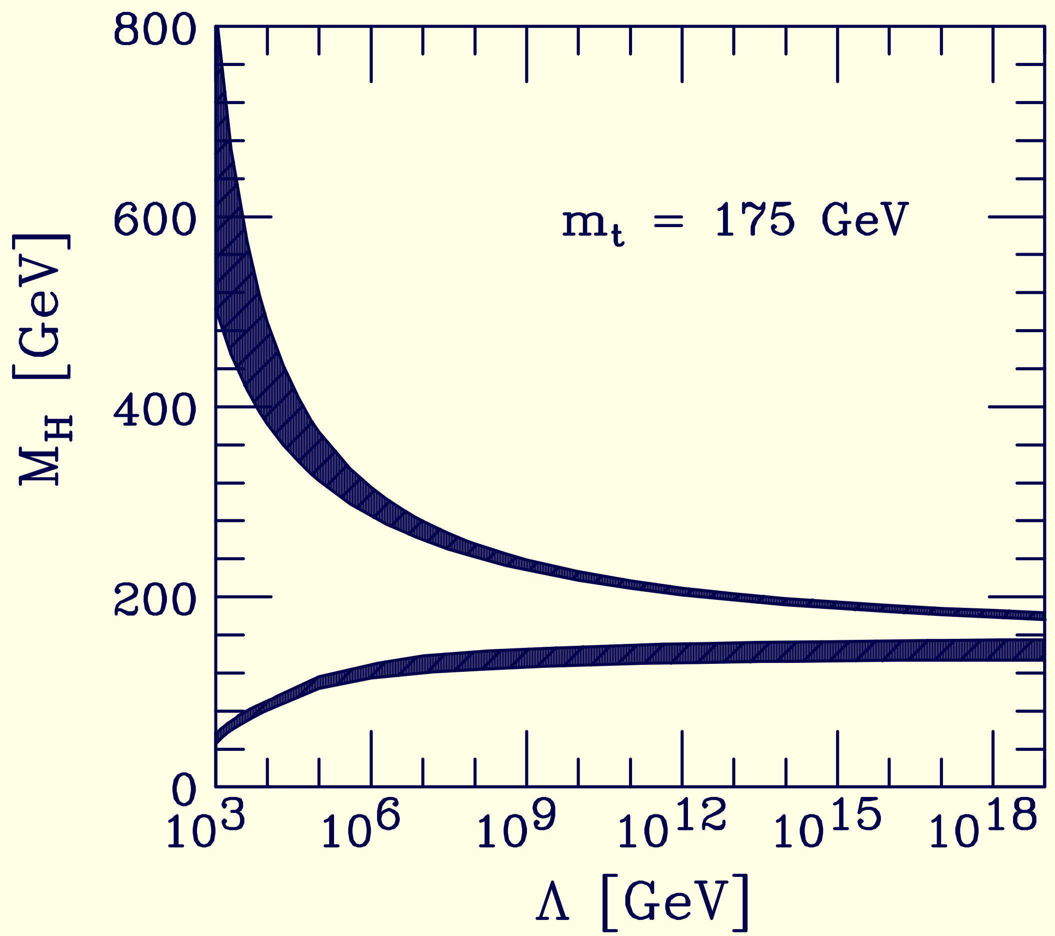


Figure 4: **Triviality and stability bounds on $m_{h_{\rm SM}}$ as a function of the energy scale up to which we hope the SM is valid as an effective theory.**

Indirect bounds from electroweak precision measurements

- Once a Higgs field is introduced in the Standard Model, its virtual excitations contribute to several physical observables, from the mass of the W boson, to various leptonic and hadronic asymmetries, to many other electroweak observables that are usually considered in precision tests of the Standard Model.

  In particular, there is a correction to the ρ parameter that looks like

$$\rho = 1 - \frac{11 g^2}{96\pi^2} \tan^2\theta_W \ln\left(\frac{m_{h_{\rm SM}}}{m_W}\right) . \qquad (98)$$

  Note that the dependence on $m_{h_{\rm SM}}$ is only logarithmic, implying that limits on $m_{h_{\rm SM}}$ are very sensitive to the precision of the electroweak fits to ρ and other quantities.

- In practice, it is the relationship between m_W and m_t arising from radiative corrections that depend somewhat sensitively on $m_{h_{\rm SM}}$ that gives the best constraint. We have

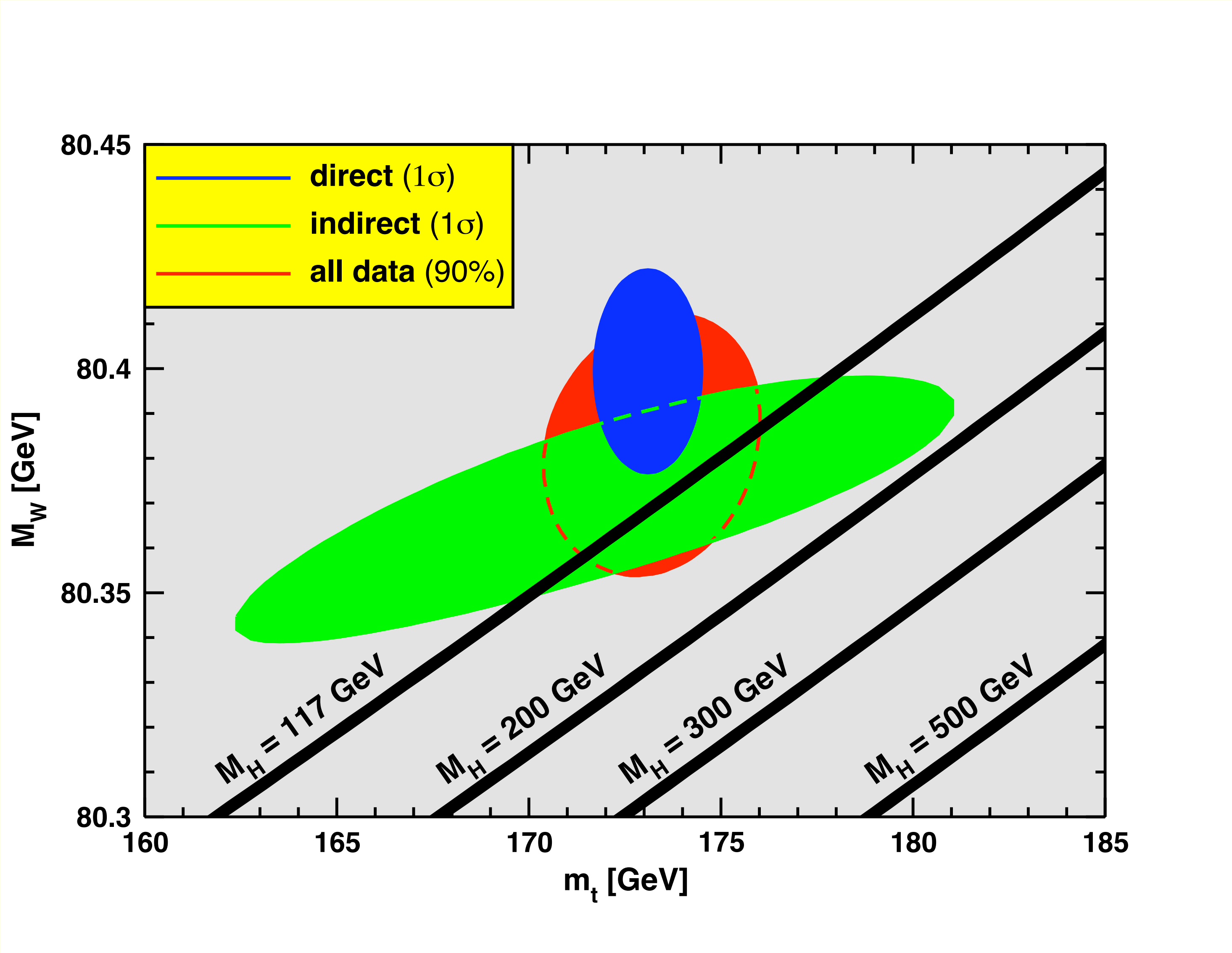
$$\left(\frac{m_W^2}{m_Z^2\cos^2\theta_W}\right) = 1 - \frac{\pi\alpha}{\sqrt{2}G_F m_W^2(1-\delta r)} \tag{99}$$

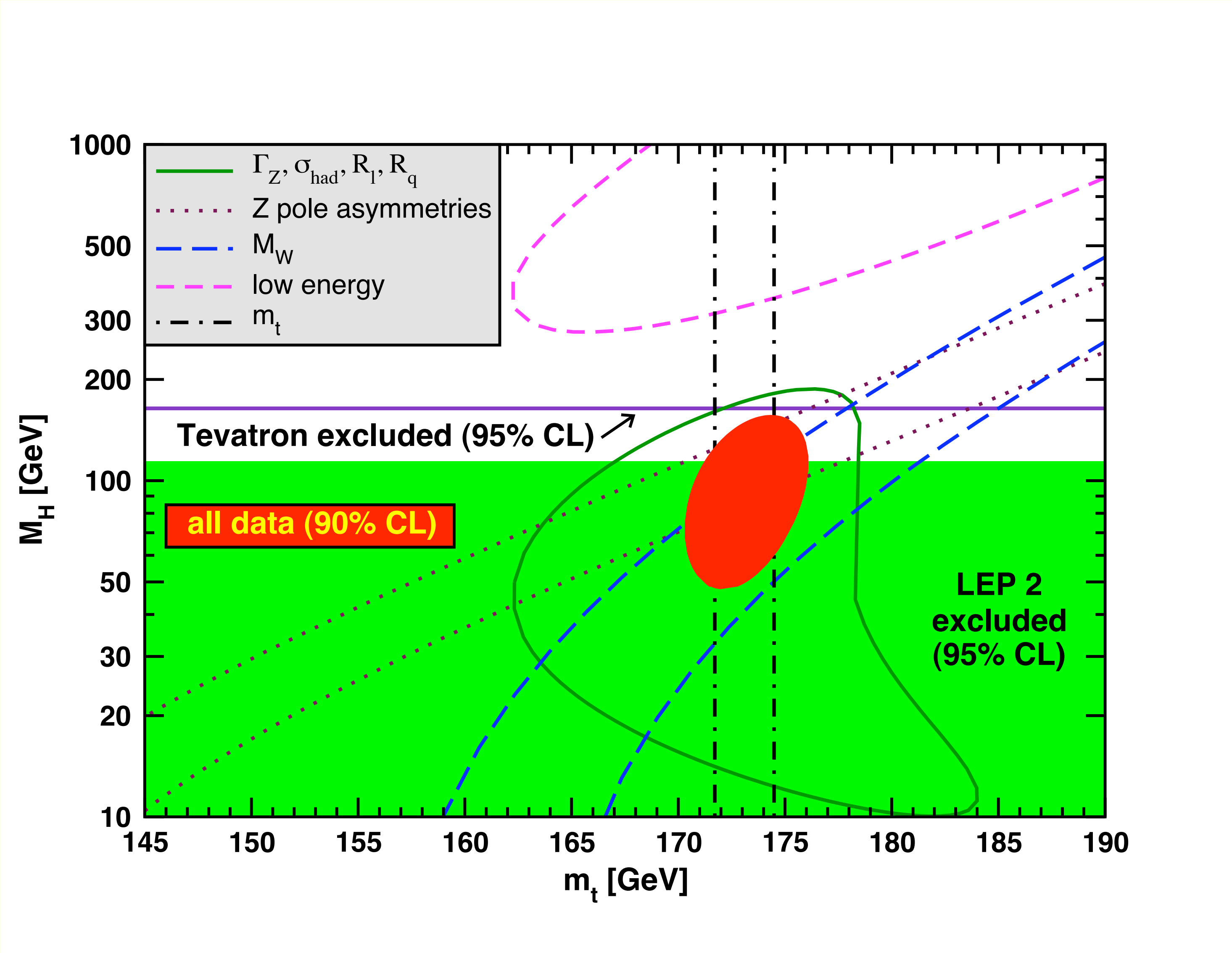
where δr is a function of m_t^2 and $\ln(m_{h_{\rm SM}})$. The result, see below, is that quite a low value of $m_{h_{\rm SM}}$ is preferred.

- Now that the Higgs boson mass is directly determined it is interesting to ask how consistent the observed $126~\mathrm{GeV}$ value is with that extracted indirectly from precision fits of all the measured electroweak observables, within the fit uncertainty.

  This is actually one of the most important results that was obtained from precision tests of the Standard Model and greatly illustrates the predictivity of the Standard Model itself.

- I show some plots available from the PDG web page. For more details on this, see the mini-review there. Basically, in the SM context we expected a somewhat lighter Higgs mass than that seen. The discrepancy can be resolved within the Supersymmetric context.


direct (1σ)
indirect (1σ)
all data (90%)
80.45
80.4
80.35
80.3
160
165
170
175
180
185
M_W [GeV]
m_t [GeV]
M_H = 117 GeV
M_H = 200 GeV
M_H = 300 GeV
M_H = 500 GeV

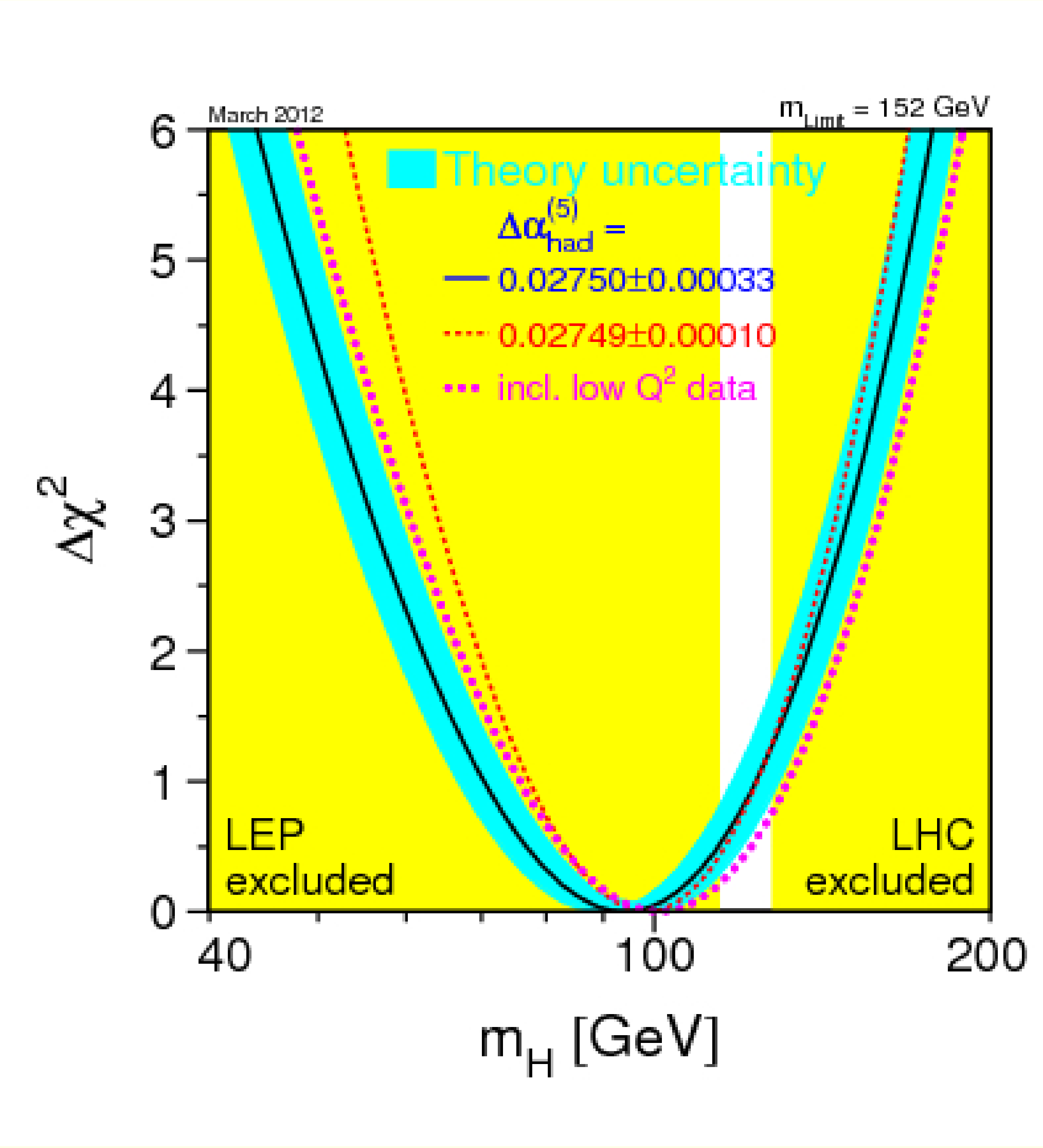


The correlation between the Higgs boson mass M_H, the W boson mass M_W, the top-quark mass m_t, and the precision data is illustrated in the above figures. Apart from the impressive agreement existing between the

indirect determination of M_W and m_t and their experimental measurements we see that the 68% CL contours, from just the LEP, SLD, and Tevatron measurements, selected a SM Higgs boson mass region roughly below 200 GeV. Therefore, assuming no physics beyond the Standard Model at the weak scale, all available electroweak data demanded a light Higgs boson.

This is summarized by the famous "blue-band" plot showing the $\Delta\chi^2$ (relative to the minimum) for fitting all electroweak constraints as a function of M_H. We see a distinct minimum at about $95 - 100$ GeV. We also observe that at 95% CL M_H was predicted to lie below 152 GeV.

Thus, it was no surprise that the Higgs was discovered at a modest mass of 126 GeV, which is only a little way up the $\Delta\chi^2$ curve.



March 2012
m_{Limit} = 152 GeV
Theory uncertainty
$\Delta\alpha_{had}^{(5)}$ =
0.02750±0.00033
0.02749±0.00010
incl. low Q^2 data
$\Delta\chi^2$
LEP excluded
LHC excluded
m_H [GeV]

Fine-tuning

- One aspect of the Higgs sector of the Standard Model that is traditionally perceived as problematic is that higher order corrections to the square of the Higgs boson mass contain quadratic ultraviolet divergences.

- This is expected in a $\lambda\phi^4$ theory and it does not pose a renormalizability problem, since a $\lambda\phi^4$ theory is renormalizable.

- However, although per se renormalizable, these quadratic divergences leave the inelegant feature that the square of the Higgs boson renormalized mass has to result from the adjusted or fine-tuned balance between a bare Higgs boson mass squared and a counterterm that is proportional to the ultraviolet cutoff squared.

  If the physical Higgs mass has to live at the electroweak scale, this can cause a fine-tuning of several orders of magnitude when the scale of new physics Λ (the ultraviolet cutoff of the Standard Model interpreted as an effective low energy theory) is well above the electroweak scale.

- Ultimately this is related to a symmetry principle, or better to the absence of a symmetry principle.

  Indeed, setting to zero the masses of the scalar fields in the Lagrangian of the Standard Model does not restore any symmetry to the model. Hence, the masses of the scalar fields are not protected against large corrections.

- Models of new physics beyond the Standard Model should address this fine-tuning problem and propose a more satisfactory mechanism to obtain the mass of the Higgs particle(s) around the electroweak scale.

  Supersymmetric models, for instance, have the remarkable feature that fermionic and bosonic degrees of freedom conspire to cancel the Higgs mass quadratic loop divergence, when the symmetry is exact.

  Other non supersymmetric models, like little Higgs models, address the problem differently, by interpreting the Higgs boson as a Goldstone boson of some global approximate symmetry.

  In both cases the Higgs mass turns out to be proportional to some small deviation from an exact symmetry principle, and therefore intrinsically small.

- As just stated, quantum corrections to the Higgs mass-squared lead to severe quadratic-divergence fine-tuning unless new physics enters at a low scale.

- **Let us gives some details:**

After including the one loop corrections we have

$$m_{h_{\rm SM}}^2 = \mu^2 + \frac{3\Lambda^2}{32\pi^2 v^2}(2m_W^2 + m_Z^2 + m_{h_{\rm SM}}^2 - 4m_t^2) \tag{100}$$

where $\mu^2 = 2\lambda v_{SM}^2$, and λ is the quartic coupling in the Higgs potential.

The μ^2 and Λ^2 terms have entirely different sources, and so a value of $m_{h_{\rm SM}} \sim m_Z$ should not arise by fine-tuned cancellation between the two terms.

And, even if you do have a fine-tuned cancellation the theory is out of control for large Λ since large μ^2 requires large λ.

Although you can never cure the quadratic fine-tuning problem without new physics, there are some tactics for delaying it to quite large Λ values.

Purely Higgs sector approaches for delaying fine-tuning from quadratic divergences

1. $m_{h_{\rm SM}}$ could obey the "Veltman" condition,

$$m_{h_{\rm SM}}^2 = 4m_t^2 - 2m_W^2 - m_Z^2 \sim (317 \text{ GeV})^2 . \tag{101}$$

At higher loop order, one must carefully coordinate the value of $m_{h_{\rm SM}}$ with the value of Λ.

Just as we do not want to have a fine-tuned cancellation of the two terms in Eq. (100), we also do not want to insist on too fine-tuned a choice for $m_{h_{\rm SM}}$ (in the SM, there is no symmetry that predicts this value).

$\Rightarrow$ cannot continue the game to too high a Λ.

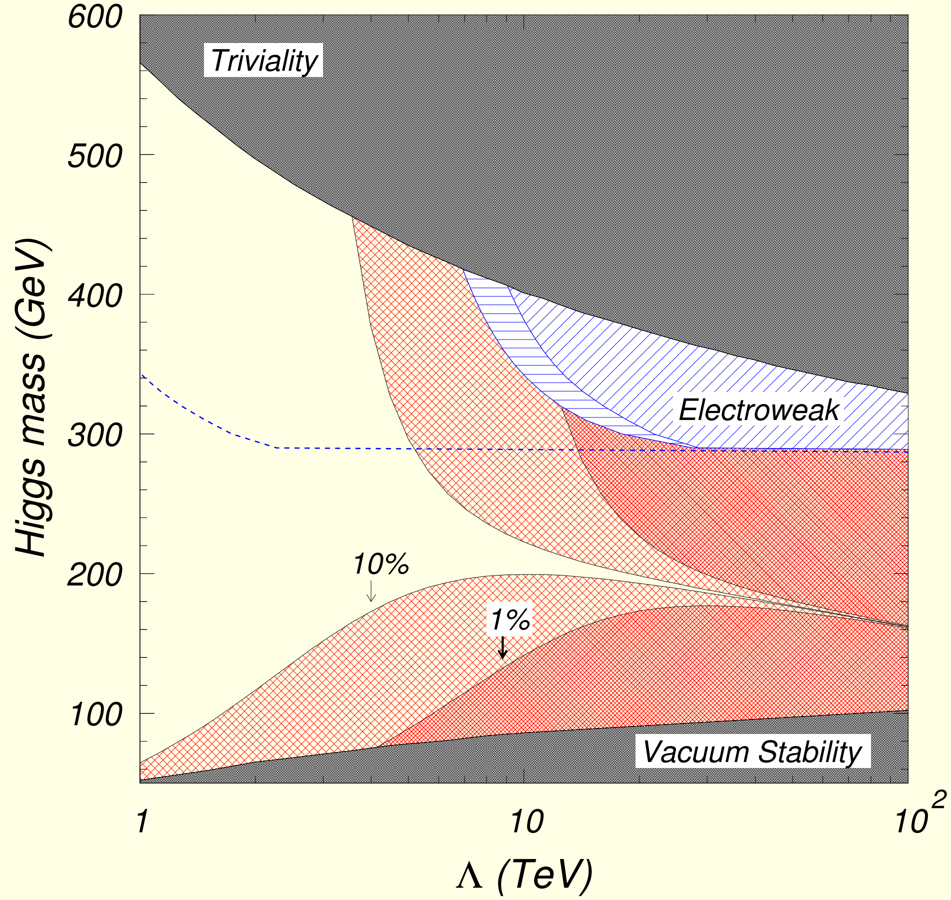


Figure 5: Fine-tuning constraints on Λ, from Kolda + Murayama, hep-ph/0003170.

The upper bound for Λ at which new physics must enter is largest for $m_{h_{\rm SM}} \sim 200~\mathrm{GeV}$ where the SM fine-tuning would be 10% if $\Lambda \sim 30~\mathrm{TeV}$. At this point, one would have to introduce some kind of new physics.

However, we already know that there is a big problem with this approach — namely the Higgs mass is about 126 GeV.

2. **There is the "multi-doublet" approach.**

   In the simplest case where all h_i have the same top quark Yukawa, but rescaled by v_i/v_{SM}, each h_i has its top quark loop mass correction scaled by $f_i^2 \equiv \frac{v_i^2}{v_{SM}^2}$ and thus

$$F_t^i = f_i^2 F_t(m_i) = K f_i^2 \frac{\Lambda_t^2}{m_i^2} \tag{102}$$

   $i.e.$ **significantly reduced.**

   Thus, multiple mixed Higgs allow a much larger Λ_t for a given maximum acceptable common F_t^i.

   One should note one possibly good feature of delaying new physics:

   large Λ_t implies significant corrections to low-E phenomenology from Λ_t-scale physics are less likely.

A model with 4 doublets can allow $\Lambda_t \sim 5~\mathrm{TeV}$ before the hierarchy fine-tuning problem becomes significant.

- However, in the end, there is always going to be a Λ or Λ_t for which we get into trouble.

  $\Rightarrow$ Ultimately we will need new physics.

  So, why not have it right away ($i.e.$ at $\Lambda \lesssim 1~\mathrm{TeV}$) and avoid the above somewhat ad hoc games.

  This is the approach of supersymmetry, which (unlike Little Higgs or UED or ....) solves the hierarchy problem once and for all.

- As we have seen, following in more detail Ref. [28], the no fine-tuning condition in the Standard Model can be softened and translated into a maximum amount of allowed fine-tuning, that can be directly related to the scale of new physics.

- As derived earlier, upon spontaneous breaking of the electroweak symmetry, the SM Higgs boson mass at tree level is given by $M_H^2 = -2\mu^2$, where μ^2 is the coefficient of the quadratic term in the scalar potential.

Higher order corrections to M_H^2 can therefore be calculated as loop corrections to μ^2, i.e. by studying how the effective potential in Eq. (17) and its minimum condition are modified by loop corrections.

If we interpret the Standard Model as the electroweak scale effective limit of a more general theory living at a high scale Λ, then the most general form of μ^2 including all loop corrections is:

$$\bar{\mu}^2 = \mu^2 + \Lambda^2 \sum_{n=0}^{\infty} c_n(\lambda_i) \log^n(\Lambda/Q) \ , \tag{103}$$

where Q is the renormalization scale, λ_i are a set of input parameters (couplings) and the c_n coefficients can be deduced from the calculation of the effective potential at each loop order. h

- As noted originally by Veltman, there would be no fine-tuning problem if the coefficient of Λ^2 in Eq. (103) were zero, i.e. if the loop corrections to μ^2 had to vanish.

  This condition, known as Veltman condition, is usually over constraining, since the number of independent c_n (set to zero by the Veltman condition) can be larger than the number of inputs λ_i.

However the Veltman condition can be relaxed, by requiring that only the sum of a finite number of terms in the coefficient of Λ^2 is zero, i.e. requiring that:

$$\sum_0^{n_{max}} c_n(\lambda_i)\log^n(\Lambda/M_H) = 0 \ , \tag{104}$$

where the renormalization scale μ has been arbitrarily set to M_H and the order n has been set to n_{max}, fixed by the required order of loop in the calculation of V_{eff}.

This is based on the fact that higher orders in n come from higher loop effects and are therefore suppressed by powers of $(16\pi^2)^{-1}$.

Limiting n to n_{max}, Eq. (104) can now have a solution. Indeed, if the scale of new physics Λ is not too far from the electroweak scale, then the Veltman condition in Eq. (104) can be softened even more by requiring that:

$$\sum_0^{n_{max}} c_n(\lambda_i)\log^n(\Lambda/M_H) < \frac{v^2}{\Lambda^2} \ . \tag{105}$$

This condition determines a value of Λ_{max} such that for $\Lambda \leq \Lambda_{max}$ the stability of the electroweak scale does not require any dramatic cancellation in $\bar{\mu}^2$.

In other words, for $\Lambda \leq \Lambda_{max}$ the renormalization of the SM Higgs boson mass does not require any fine-tuning.

As an example, for $n_{max}=0$,

$$c_0 = (32\pi^2 v^2)^{-1} 3(2M_W^2 + M_Z^2 + M_H^2 - 4m_t^2)\,, \tag{106}$$

and the stability of the electroweak scale is assured up to Λ of the order of $4\pi v \simeq 2$ TeV.

For $n_{max} = 1$ the maximum Λ is pushed up to $\Lambda \simeq 15$ TeV and for $n_{max}=2$ up to $\Lambda \simeq 50$ TeV.

So, just going up to 2-loops assures us that we can consider the SM Higgs sector free of fine-tuning up to scales that are well beyond where we would hope to soon discover new physics.

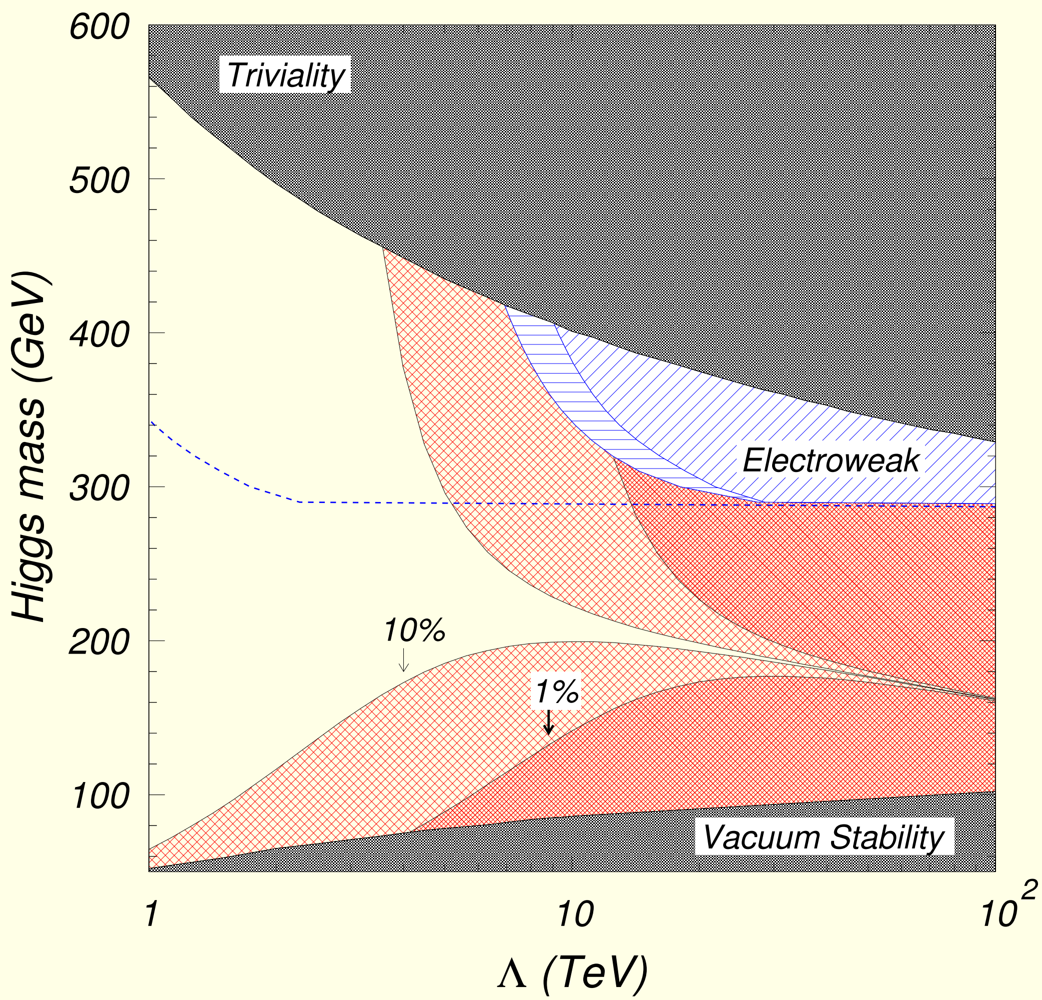


Figure 6: **The SM Higgs boson mass M_H as a function of the scale of new physics Λ, with all the constraints derived from unitarity, triviality, vacuum stability, electroweak precision fits, and the requirement of a limited fine-tuning. The empty region is consistent with all the constraints and less than 1 part in 10 fine-tuning. From Ref. [28]. But, it is not consistent with $M_H = 126$ GeV unless finetuning much worse than 1% is allowed.**

For each value of n_{max}, and for each corresponding Λ_{max}, M_H becomes a function of the cutoff Λ, and the amount of fine-tuning allowed in the theory limits the region in the (Λ, M_H) plane allowed to $M_H(\Lambda)$.

This is well represented in Fig. 6, where also the constraints from the conditions of unitarity, triviality, vacuum stability and electroweak precision fits are summarized.

- Finally, the main lesson we take away from this plot is that if a Higgs boson is discovered new physics is just around the corner and should manifest itself at the LHC.

  But, we have not yet seen new physics at the expected scale — these considerations are pushing us into an uncomfortable corner.

The Higgs sector of the MSSM

- In the supersymmetric extension of the Standard Model, the electroweak symmetry is spontaneously broken via the Higgs mechanism.

  However, in supersymmetry the Higgs sector **must** contain an even number of doublets, as required by holomorphy and anomaly cancellation.

  The minimal number of 2 doublets defines the MSSM.

- The dynamics of the Higgs mechanism works pretty much unchanged with respect to the Standard Model case, although the form of the scalar potential is more complex and its minimization more involved.

- As a result, the $W^{\pm}$ and Z weak gauge bosons acquire masses that depend on the parameterization of the supersymmetric model at hand.

- At the same time, fermion masses are generated by coupling the two scalar doublets to the fermions via Yukawa interactions.

- **A supersymmetric model is therefore a natural reference to compare the Standard Model to, since it is a theoretically sound extension of the Standard Model, still fundamentally based on the same electroweak symmetry breaking mechanism.**

- **Far from being a simple generalization of the SM Higgs sector, the scalar sector of a supersymmetric model can be theoretically more satisfactory because:**

  1. **spontaneous symmetry breaking is radiatively induced (i.e. the sign of the quadratic term in the Higgs potential is driven from positive to negative) mainly by the evolution of the top-quark Yukawa coupling from the scale of supersymmetry-breaking to the electroweak scale, and**
  2. **higher order corrections to the Higgs mass do not contain quadratic divergences, since they cancel when the contribution of both scalars and their super-partners is considered.**

- **At the same time, the fact of having a supersymmetric theory and two scalar doublets modifies the phenomenological properties of the supersymmetric physical scalar fields dramatically.**

- **In what follows, we will review only the most important properties of the**

Higgs sector of the MSSM, so that we can compare the physics of the SM Higgs boson to that of the MSSM Higgs bosons.

- Let us start by recalling some general properties of a Two Higgs Doublet Model and then specify the discussion to the case of the MSSM.

- Following this, we will review the form of the couplings of the MSSM Higgs bosons to the SM gauge bosons and fermions, including the impact of the most important supersymmetric higher order corrections.

1. **About Two Higgs Doublet Models**

The most popular and simplest extension of the Standard Model is obtained by considering a scalar sector made of two instead of one complex scalar doublets.

These models, dubbed Two Higgs Doublet Models (2HDM), have a richer spectrum of physical scalar fields.

Indeed, after spontaneous symmetry breaking, only three of the eight original scalar degrees of freedom (corresponding to two complex doublets) are reabsorbed in transforming the originally massless vector bosons into massive ones.

The remaining five degrees of freedom correspond to physical degrees of freedom in the form of: two neutral scalars, one neutral pseudoscalar, and two charged scalar fields.

At the same time, having multiple scalar doublets in the Yukawa Lagrangian (see Eq. (32)) allows for scalar flavor changing neutral currents.

Indeed, when generalized to the case of two scalar doublet ϕ^1 and ϕ^2, Eq. (32) becomes (quark case only):

$$\mathcal{L}_{Yukawa} = -\sum_{k=1,2} \Gamma^u_{ij,k} \bar{Q}^i_L \Phi^{k,c} u^j_R - \sum_{k=1,2} \Gamma^d_{ij,k} \bar{Q}^i_L \Phi^k d^j_R + \mathrm{h.c.} \ , \quad (107)$$

where each pair of fermions (i, j) couple to a linear combination of the scalar fields ϕ^1 and ϕ^2.

When, upon spontaneous symmetry breaking, the fields ϕ^1 and ϕ^2 acquire vacuum expectation values

$$\langle \Phi^k \rangle = \frac{v^k}{\sqrt{2}} \quad \text{for} \quad k = 1, 2 \ , \quad (108)$$

the reparameterization of $\mathcal{L}_{Yukawa}$ of Eq. (107) in the vicinity of the minimum of the scalar potential, with $\Phi^k = \Phi'^k + v^k$ (for $k = 1, 2$), gives:

$$\mathcal{L}_{Yukawa} = -\bar{u}_L^i \underbrace{\sum_k \Gamma_{ij,k}^u \frac{v^k}{\sqrt{2}}}_{M_{ij}^u} u_R^j - \bar{d}_L^i \underbrace{\sum_k \Gamma_{ij,k}^d \frac{v^k}{\sqrt{2}}}_{M_{ij}^d} d_R^j + \mathrm{h.c.} + \text{FC couplings} \quad , \tag{109}$$

where the fermion mass matrices M_{ij}^u and M_{ij}^d are now proportional to a linear combination of the vacuum expectation values of ϕ^1 and ϕ^2.

The diagonalization of M_{ij}^u and M_{ij}^d does not imply the diagonalization of the couplings of the ϕ'^k fields to the fermions, and Flavor Changing (FC) couplings arise.

This is perceived as a problem in view of the absence of experimental evidence to support neutral flavor changing effects.

If present, these effects have to be tiny in most processes involving in particular the first two generations of quarks, and a safer way to build a 2HDM is to forbid them all together at the Lagrangian level.

This is traditionally done by requiring either that u-type and d-type quarks couple to the same doublet (Model I) or that u-type quarks couple to one scalar doublet while d-type quarks to the other (Model II).

Indeed, these two different realizations of a 2HDM can be justified by enforcing on $\mathcal{L}_{Yukawa}$ the following ad hoc discrete symmetry:

$$\begin{cases} \Phi^1 \to -\Phi^1 \quad \text{and} \quad \Phi^2 \to \Phi^2 \\ d^i \to -d^i \quad \text{and} \quad u^j \to \pm u^j \end{cases} \tag{110}$$

The case in which FC scalar neutral currents are not forbidden (Model III) has also been studied in detail.

In this case both up and down-type quarks can couple to both scalar doublets, and strict constraints have to be imposed on the FC scalar couplings, in particular between the first two generations of quarks.

2HDMs have a very rich phenomenology that has been extensively studied.

In what follows here, however, I will only compare the SM Higgs boson phenomenology to the phenomenology of the Higgs bosons of the MSSM, a particular kind of 2HDM that will be discussed in the following.

2. The MSSM Higgs sector: introduction

The Higgs sector of the MSSM is actually a Model II 2HDM. It contains two complex $SU(2)_L$ scalar doublets:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \quad , \quad \Phi_2 = \begin{pmatrix} \phi_2^0 \\ \phi_2^- \end{pmatrix} \ , \qquad (111)$$

with opposite hypercharge ($Y = \pm 1$), as needed to make the theory anomaly-free[3]. Φ_1 couples to the up-type and Φ_2 to the down-type quarks respectively. Correspondingly, the Higgs part of the potential can be written as:[4]

$$\begin{aligned} V_H &= (|\mu|^2 + m_1^2)|\Phi_1|^2 + (|\mu|^2 + m_2^2)|\Phi_2|^2 - \mu B \epsilon_{ij}(\Phi_1^i \Phi_2^j + h.c.) \\ &+ \frac{g^2 + g'^2}{8}\left(|\Phi_1|^2 - |\Phi_2|^2\right)^2 + \frac{g^2}{2}|\Phi_1^\dagger \Phi_2|^2 \ , \end{aligned} \qquad (112)$$

[3]Another reason for the choice of a 2HDM is that in a supersymmetric model the superpotential should be expressed just in terms of superfields, not their conjugates. So, one needs to introduce two doublets to give mass to fermion fields of opposite weak isospin. The second doublet plays the role of ϕ^c in the Standard Model (see Eq. (32)), where ϕ^c has opposite hypercharge and weak isospin with respect to ϕ.

[4] For further details, see the Appendix which gives a more theoretical treatment of the MSSM Higgs sector.

in which we can identify three different contributions [29, 298]:

(i) the so called D terms, containing the quartic scalar interactions, which for the Higgs fields Φ_1 and Φ_2 correspond to:

$$\frac{g^2+g'^2}{8}\left(|\Phi_1|^2-|\Phi_2|^2\right)^2+\frac{g^2}{2}|\Phi_1^\dagger\Phi_2|^2 \ , \tag{113}$$

with g and g' the gauge couplings of $SU(2)_L$ and $U(1)_Y$ respectively;

(ii) the so called F terms, corresponding to:

$$|\mu|^2(|\Phi_1|^2+|\Phi_2|^2) \ ; \tag{114}$$

(iii) the soft SUSY-breaking scalar Higgs mass and bilinear terms, corresponding to:

$$m_1^2|\Phi_1|^2+m_2^2|\Phi_2|^2-\mu B\epsilon_{ij}(\Phi_1^i\Phi_2^j+h.c.) \ . \tag{115}$$

Overall, the scalar potential in Eq. (112) depends on three independent combinations of parameters, $|\mu|^2+m_1^2$, $|\mu|^2+m_2^2$, and μB. One basic difference with respect to the SM case is that the quartic coupling has been replaced by gauge couplings. This reduced arbitrariness will play an important role in the following.

Upon spontaneous symmetry breaking, the neutral components of Φ_1 and Φ_2 acquire vacuum expectation values

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad , \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_2 \\ 0 \end{pmatrix} \ , \tag{116}$$

and the Higgs mechanism proceeds as in the Standard Model except that now one starts with eight degrees of freedom, corresponding to the two complex doublets Φ_1 and Φ_2. Three degrees of freedom are absorbed in making the $W^\pm$ and the Z massive.

Note: The convention for v_1 and v_2 above is opposite the Higgs Hunters Guide. What is important is that in the present notation v_1 is responsible for up quark masses and v_2 responsible for down quark and lepton masses — see upcoming material.

The W mass is given by: $M_W^2 = g^2(v_1^2 + v_2^2)/4 = g^2v^2/4$, and this fixes the normalization of $v_1^2 + v_2^2$, leaving only two independent parameters to describe the entire MSSM Higgs sector. The remaining five degrees of freedom are physical and correspond to two neutral scalar fields

$$h \;=\; -(\sqrt{2}\mathrm{Re}\phi_2^0 - v_2)\sin\alpha + (\sqrt{2}\mathrm{Re}\phi_1^0 - v_1)\cos\alpha \tag{117}$$

$$H = (\sqrt{2}\mathrm{Re}\phi_2^0 - v_2)\cos\alpha + (\sqrt{2}\mathrm{Re}\phi_1^0 - v_1)\sin\alpha \ ,$$

one neutral pseudoscalar field

$$A = \sqrt{2}\left(\mathrm{Im}\phi_2^0 \sin\beta + \mathrm{Im}\phi_1^0 \cos\beta\right) \ , \tag{118}$$

and two charged scalar fields

$$H^\pm = \phi_2^\pm \sin\beta + \phi_1^\pm \cos\beta \ , \tag{119}$$

where α and β are mixing angles, and $\tan\beta = v_1/v_2$. At tree level, the masses of the scalar and pseudoscalar degrees of freedom satisfy the following relations:

$$M_{H^\pm}^2 = M_A^2 + M_W^2 \ , \tag{120}$$

$$M_{H,h}^2 = \frac{1}{2}\left(M_A^2 + M_Z^2 \pm ((M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta)^{1/2}\right) \ ,$$

making it natural to pick M_A and $\tan\beta$ as the two independent parameters of the Higgs sector.

Eq. (120) provides the famous tree level upper bound on the mass of one of the neutral scalar Higgs bosons, h:

$$M_h^2 \leq M_Z^2 \cos 2\beta \leq M_Z^2 \ , \tag{121}$$

which contradicted the experimental lower bound set by LEP II: $M_h \gtrsim 114$ GeV, and is in clear disagreement with the measured $126~\mathrm{GeV}$ value.

The contradiction is lifted by including higher order radiative corrections to the Higgs spectrum, in particular by calculating higher order corrections to the neutral scalar mass matrix.

A huge effort has been dedicated to the calculation of the full one-loop corrections and of several leading and sub-leading sets of two-loop corrections, including resummation of leading and sub-leading logarithms via appropriate renormalization group equation (RGE) methods.

A detailed discussion of this topic can be found in the reviews [8, 30, 31] and in the original literature referenced therein.

For the purpose of these lectures, let us just observe that, qualitatively, the impact of radiative corrections on M_h^{max} can be seen by just including the leading two-loop corrections proportional to y_t^2, the square of the top-quark

Yukawa coupling, and applying RGE techniques to resum the leading orders of logarithms.

In this case, the upper bound on the light neutral scalar in Eq. (121) is modified as follows:

$$M_h^2 \le M_Z^2 + \frac{3g^2 m_t^2}{8\pi^2 M_W^2}\left[\log\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12 M_S^2}\right)\right] \ , \tag{122}$$

where $M_S^2 = (M_{\tilde{t}_1}^2 + M_{\tilde{t}_2}^2)/2$ is the average of the two top-squark masses, m_t is the running top-quark mass (to account for the leading two-loop QCD corrections), and X_t is the top-squark mixing parameter defined by the top-squark mass matrix:

$$\begin{pmatrix} M_{Q_t}^2 + m_t^2 + D_L^t & m_t X_t \\ m_t X_t & M_{R_t}^2 + m_t^2 + D_R^t \end{pmatrix} \ , \tag{123}$$

with $X_t \equiv A_t - \mu \cot\beta$ (A_t being one of the top-squark soft SUSY breaking trilinear coupling), $D_L^t = (1/2 - 2/3\sin\theta_W) M_Z^2 \cos 2\beta$, and $D_R^t = 2/3 \sin^2\theta_W M_Z^2 \cos 2\beta$.

Examination of the result Eq. (122) shows that the largest value of M_h as a function of X_t is achieved for $X_t/M_S = \sqrt{6}$, although some studies taking two-loop effects into account suggest that the largest mixing contribution occurs closer to $|X_t/M_S| \sim 2$. Values larger than about $\sqrt{6} = 2.45$ will induce charge and and color-breaking minima in the scalar potential and should be avoided.

Fig. 7 illustrates the behavior of M_h as a function of $\tan\beta$, in the case of minimal and maximal mixing. For large $\tan\beta$ a plateau (i.e. an upper bound) is clearly reached. The green bands represent the variation of M_h as a function of m_t when $m_t = 175 \pm 5$ GeV.

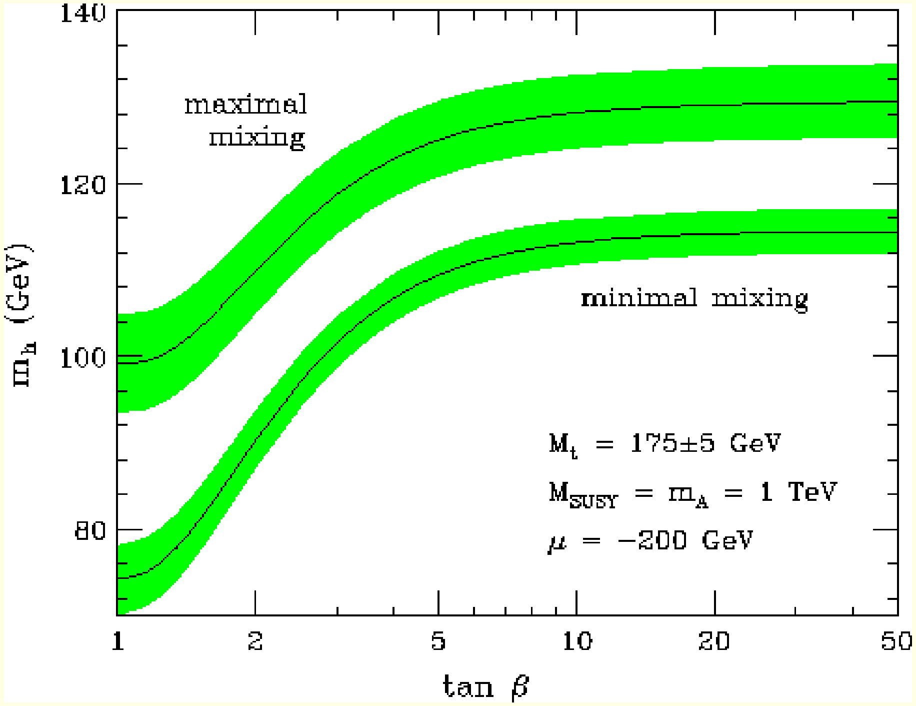


Figure 7: The mass of the light neutral scalar Higgs boson, h, as a function of $\tan\beta$, in the minimal mixing and maximal mixing scenario. The green bands are obtained by varying the top-quark mass in the $m_t = 175 \pm 5$ GeV range. The plot is built by fixing $M_A = 1$ TeV and $M_{SUSY} \equiv M_Q = M_U = M_D = 1$ TeV. From Ref. [8].

If top-squark mixing is maximal, the upper bound on M_h is approximately $M_h^{max} \simeq 135$ GeV[5]. The behavior of both $M_{h,H}$ and $M_{H^\pm}$ as a function of M_A and $\tan\beta$ is summarized in Fig. 8, always for the case of maximal mixing. It is interesting to notice that for all values of M_A and $\tan\beta$ one

[5] This limit is obtained for $m_t = 175$ GeV, and it can go up to $M_h^{max} \simeq 144$ GeV for $m_t = 178$ GeV.

has $M_H > M_h^{max}$. Also we observe that, in the limit of large $\tan\beta$,

i) for $M_A < M_h^{max}$: $M_h \simeq M_A$ and $M_H \simeq M_h^{max}$, while

ii) for $M_A > M_h^{max}$: $M_H \simeq M_A$ and $M_h \simeq M_h^{max}$.

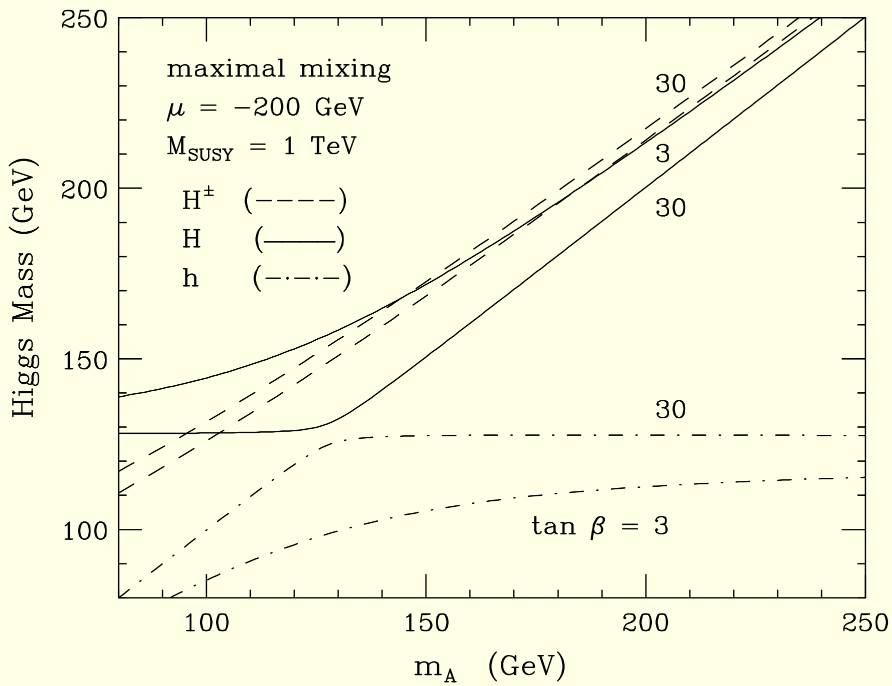


Figure 8: The mass of the light (h) and heavy (H) neutral scalar Higgs bosons, and of the charged scalar Higgs boson ($H^{\pm}$) as a function of the neutral pseudoscalar mass M_A, for two different values of $\tan\beta$ ($\tan\beta = 3, 30$). The top-quark mass is fixed to $m_t = 174.3$ GeV and $M_{SUSY} \equiv M_Q = M_U = M_D = 1$ TeV. The maximal mixing scenario is chosen. From Ref. [8].

3. MSSM Higgs boson couplings to electroweak gauge bosons

The Higgs boson couplings to the electroweak gauge bosons are obtained from the kinetic term of the scalar Lagrangian, in strict analogy to what we have explicitly seen in the case of the SM Higgs boson.

Here, we would like to state the form of the H_iVV and H_iH_jV couplings (for $H_i = h, H, A, H^\pm$, and $V = W^\pm, Z$) that are most important in order to understand the main features of the MSSM plots that will be shown when we discuss phenomenology.

First of all, the couplings of the neutral scalar Higgs bosons to both $W^\pm$ and Z can be written as:

$$g_{hVV} = g_V M_V \sin(\beta - \alpha) g^{\mu\nu} \quad , \quad g_{HVV} = g_V M_V \cos(\beta - \alpha) g^{\mu\nu} \ , \tag{124}$$

where $g_V = 2M_V/v$, while the AVV and $H^\pm VV$ couplings vanish because of CP-invariance. As in the SM case, since the photon is massless, there are no tree level $\gamma\gamma H_i$ and $\gamma Z H_i$ couplings.

Moreover, in the neutral Higgs sector, only the HAZ and HAZ couplings are allowed and they are given by:

$$g_{hAZ} = \frac{g\cos(\beta-\alpha)}{2\cos\theta_W}(p_h - p_A)^\mu \quad , \quad g_{HAZ} = -\frac{g\sin(\beta-\alpha)}{2\cos\theta_W}(p_H - p_A)^\mu \ , \tag{125}$$

where all momenta are incoming. We also have several H_iH_jV couplings involving the charged Higgs boson, namely:

$$\begin{aligned} g_{H^+H^-Z} &= -\frac{g}{2\cos\theta_W}\cos 2\theta_W (p_{H^+} - p_{H^-})^\mu \ , \\ g_{H^+H^-\gamma} &= -ie(p_{H^+} - p_{H^-})^\mu \ , \\ g_{H^\mp hW^\pm} &= \mp i\frac{g}{2}\cos(\beta-\alpha)(p_h - p_{H^\mp})^\mu \ , \\ g_{H^\mp HW^\pm} &= \pm i\frac{g}{2}\sin(\beta-\alpha)(p_H - p_{H^\mp})^\mu \ , \\ g_{H^\mp AW^\pm} &= \frac{g}{2}(p_A - p_{H^\pm})^\mu \ . \end{aligned} \tag{126}$$

At this stage it is interesting to introduce the so called decoupling limit, i.e. the limit of $M_A \gg M_Z$, and to analyze how masses and couplings behave in this particular limit. $M_{H^\pm}$ in Eq. (120) is unchanged, while $M_{h,H}$ become:

$$M_h \simeq M_h^{max} \quad \text{and} \quad M_H \simeq M_A^2 + M_Z^2 \sin^2 2\beta \ . \tag{127}$$

Moreover, as one can derive from the diagonalization of the neutral scalar Higgs boson mass matrix:

$$\cos^2(\beta-\alpha) = \frac{M_h^2(M_Z^2 - M_h^2)}{M_A^2(M_H^2 - M_h^2)} \xrightarrow{M_A^2 \gg M_Z^2} \frac{M_Z^4 \sin^2 4\beta}{4M_A^4} \ . \tag{128}$$

From the previous equations we then deduce that, in the decoupling limit, the only light Higgs boson is h with mass $M_h \simeq M_h^{max}$, while $M_H \simeq M_{H^\pm} \simeq M_A \gg M_Z$, and because $\cos(\beta-\alpha) \to 0$ ($\sin(\beta-\alpha) \to 1$)), the couplings of h to the gauge bosons tend to the SM Higgs boson limit. This is to say that, in the decoupling limit, the light MSSM Higgs boson will be hardly distinguishable from the SM Higgs boson.

Finally, we need to remember that the tree level couplings may be modified by radiative corrections involving both loops of SM and MSSM particles, among which loops of third generation quarks and squarks dominate. The very same radiative corrections that modify the Higgs boson mass matrix, thereby changing the definition of the mass eigenstates, also affect the couplings of the corrected mass eigenstates to the gauge bosons. This can be reabsorbed into the definition of a renormalized mixing angle α or a radiatively corrected value for $\cos(\beta-\alpha)$ ($\sin(\beta-\alpha)$). Using the notation of Ref. [8], the radiatively corrected $\cos(\beta-\alpha)$ can be written as:

$$\cos(\beta-\alpha) = K\left[\frac{M_Z^2 \sin 4\beta}{2M_A^2} + \mathcal{O}\left(\frac{M_Z^4}{M_A^4}\right)\right] \ , \tag{129}$$

where

$$K \equiv 1 + \frac{\delta\mathcal{M}_{11}^2 - \delta\mathcal{M}_{22}^2}{2M_Z^2 \cos 2\beta} - \frac{\delta\mathcal{M}_{12}^2}{M_Z^2 \sin 2\beta} \ , \tag{130}$$

and $\delta\mathcal{M}_{ij}$ are the radiative corrections to the corresponding elements of the CP-even Higgs squared-mass matrix (see Ref. [8]).

It is interesting to notice that on top of the traditional decoupling limit introduced above ($M_A \gg M_Z$), there is now also the possibility that $\cos(\beta-\alpha) \to 0$ if $K \to 0$, and this happens independently of the value of M_A.

4. MSSM Higgs boson couplings to fermions

As anticipated, Φ_1 and Φ_2 have Yukawa-type couplings to the up-type and down-type components of all $SU(2)_L$ fermion doublets. For example, the Yukawa Lagrangian corresponding to the third generation of quarks reads:

$$\mathcal{L}_{Yukawa} = -h_t \left[\bar{t}_R \phi_1^0 t_L - \bar{t}_R \phi_1^+ b_L\right] - h_b \left[\bar{b}_R \phi_2^0 b_L - \bar{b}_R \phi_2^- t_L\right] + \mathrm{h.c.} \tag{131}$$

Upon spontaneous symmetry breaking $\mathcal{L}_{Yukawa}$ leads to corresponding quark masses:

$$m_t = h_t \frac{v_1}{\sqrt{2}} = h_t \frac{v \sin\beta}{\sqrt{2}} \quad \text{and} \quad m_b = h_b \frac{v_2}{\sqrt{2}} = h_b \frac{v \cos\beta}{\sqrt{2}} \ , \tag{132}$$

and the corresponding Higgs-quark couplings:

$$\begin{aligned} g_{ht\bar{t}} &= \frac{\cos\alpha}{\sin\beta} y_t = \left[\sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)\right] y_t \ , \qquad (133) \\ g_{hb\bar{b}} &= -\frac{\sin\alpha}{\cos\beta} y_b = \left[\sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)\right] y_b \ , \end{aligned}$$

$$
\begin{aligned}
g_{Ht\bar{t}} &= \frac{\sin\alpha}{\sin\beta} y_t = [\cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha)]\, y_t \ , \\
g_{Hb\bar{b}} &= \frac{\cos\alpha}{\cos\beta} y_b = [\cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha)]\, y_b \ , \\
g_{At\bar{t}} &= \cot\beta\, y_t \ , \quad g_{Ab\bar{b}} = \tan\beta\, y_b \ , \\
g_{H^\pm t\bar{b}} &= \frac{g}{2\sqrt{2}M_W} [m_t \cot\beta(1-\gamma_5) + m_b \tan\beta(1+\gamma_5)] \ ,
\end{aligned}
$$

where $y_q = m_q/v$ (for $q = t, b$) are the SM couplings. It is interesting to notice that in the $M_A \gg M_Z$ decoupling limit, as expected, all the h couplings in Eq. (133) reduce to the SM limit, while all the H couplings become like their A counterparts.

The Higgs boson-fermion couplings are also modified directly by one-loop radiative corrections (squarks-gluino loops for quarks couplings and slepton-neutralino loops for lepton couplings). A detailed discussion can be found in Ref. [8, 298] and in the literature referenced therein. Of particular relevance are the corrections to the couplings of the third quark generation. These can be parameterized at the Lagrangian level by writing the radiatively corrected effective Yukawa Lagrangian as:

$$
\begin{aligned}
\mathcal{L}^{eff}_{Yukawa} &= -\epsilon_{ij}\left[(h_b+\delta h_b)\bar{b}_R Q^j_L \Phi^i_2 + (h_t+\delta h_t)\bar{t}_R Q^i_L \Phi^j_1\right] \\
&- \Delta h_t \bar{t}_R Q^k_L \Phi^{k*}_2 - \Delta h_b \bar{b}_R Q^k_L \Phi^{k*}_1 + \mathrm{h.c.}\ ,
\end{aligned}
\tag{134}
$$

where we notice that radiative corrections induce a small coupling between Φ_1 and down-type fields and between Φ_2 and up-type fields. Moreover the tree level relation between h_b, h_t, m_b and m_t are modified as follows:

$$
\begin{aligned}
m_b &= \frac{h_b v}{\sqrt{2}}\cos\beta\left(1+\frac{\delta h_b}{h_b}+\frac{\Delta h_b \tan\beta}{h_b}\right) \equiv \frac{h_b v}{\sqrt{2}}\cos\beta(1+\Delta_b), \\
m_t &= \frac{h_t v}{\sqrt{2}}\sin\beta\left(1+\frac{\delta h_t}{h_t}+\frac{\Delta h_t \tan\beta}{h_t}\right) \equiv \frac{h_t v}{\sqrt{2}}\sin\beta(1+\Delta_t)\ ,
\end{aligned}
\tag{135}
$$

where the leading corrections are proportional to Δh_b and turn out to also be $\tan\beta$ enhanced. Meanwhile, the couplings between Higgs mass eigenstates and third generation quarks given in Eq. (133) are corrected as follows:

$$
g_{ht\bar{t}} = \frac{\cos\alpha}{\sin\beta} y_t \left[1 - \frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta+\tan\alpha)\right]\ , \tag{136}
$$

$$
\begin{aligned}
g_{hb\bar{b}} &= -\frac{\sin\alpha}{\cos\beta} y_b \left[1 + \frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b} - \Delta_b\right)(1+\cot\alpha\cot\beta)\right] \quad , \\
g_{Ht\bar{t}} &= \frac{\sin\alpha}{\sin\beta} y_t \left[1 - \frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta - \cot\alpha)\right] \quad , \\
g_{Hb\bar{b}} &= \frac{\cos\alpha}{\cos\beta} y_b \left[1 + \frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b} - \Delta_b\right)(1-\tan\alpha\cot\beta)\right] \quad , \\
g_{At\bar{t}} &= \cot\beta\, y_t \left[1 - \frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta + \tan\beta)\right] \quad , \\
g_{Ab\bar{b}} &= \tan\beta\, y_b \left[1 + \frac{1}{(1+\Delta_b)\sin^2\beta}\left(\frac{\delta h_b}{h_b} - \Delta_b\right)\right] \quad , \\
g_{H^\pm t\bar{b}} &\simeq \frac{g}{2\sqrt{2}M_W}\left\{ m_t\cot\beta \left[1 - \frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta+\tan\beta)\right](1+\gamma_5) \right. \\
&+ \left. m_b\tan\beta\left[1 + \frac{1}{(1+\Delta_b)\sin^2\beta}\left(\frac{\delta h_b}{h_b} - \Delta_b\right)\right](1-\gamma_5)\right\} \quad ,
\end{aligned}
$$

where the last coupling is given in the approximation of small isospin breaking effects, since interactions of this kind have been neglected in the Lagrangian of Eq. (134).

Fine-tuning II and Hints suggesting extending the MSSM

- Returning to the scalar potential involving only the neutral Higgs fields, but with a switch of notation to H_u for the higgs associated with giving the top quark mass and H_d the Higgs responsible for the bottom quark and lepton masses, we have:

$$\begin{aligned} V &= (|\mu|^2 + m^2_{H_u})|H^0_u|^2 + (|\mu|^2 + m^2_{H_d})|H^0_d|^2 - (b\, H^0_u H^0_d + \mathrm{c.c.}) \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H^0_u|^2 - |H^0_d|^2)^2, \end{aligned} \tag{137}$$

  let us write

$$v_u = \langle H^0_u \rangle, \qquad v_d = \langle H^0_d \rangle. \tag{138}$$

  These VEVs are related to the known mass of the Z boson and the electroweak gauge couplings (I have changed my convention so that $v = v_{previous}/\sqrt{2}$.):

$$v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \approx (174\,\mathrm{GeV})^2. \tag{139}$$

The ratio of the VEVs is traditionally written as

$$\tan\beta \equiv v_u/v_d. \tag{140}$$

The value of $\tan\beta$ is not fixed by present experiments, but it depends on the Lagrangian parameters of the MSSM in a calculable way.

Since $v_u = v\sin\beta$ and $v_d = v\cos\beta$ were taken to be real and positive by convention, we have $0 < \beta < \pi/2$, a requirement that will be sharpened below.

Now one can write down the conditions $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$ under which the potential Eq. (363) will have a minimum satisfying Eqs. (365) and (366):

$$m_{H_u}^2 + |\mu|^2 - b\cot\beta - (m_Z^2/2)\cos(2\beta) \;=\; 0, \tag{141}$$

$$m_{H_d}^2 + |\mu|^2 - b\tan\beta + (m_Z^2/2)\cos(2\beta) \;=\; 0. \tag{142}$$

These equations allow us to eliminate two of the Lagrangian parameters b and $|\mu|$ in favor of $\tan\beta$, but do not determine the phase of μ.

Taking $|\mu|^2$, b, $m^2_{H_u}$ and $m^2_{H_d}$ as input parameters, and m^2_Z and $\tan\beta$ as output parameters obtained by solving these two equations, one obtains:

$$\sin(2\beta) = \frac{2b}{m^2_{H_u} + m^2_{H_d} + 2|\mu|^2}, \tag{143}$$

$$m^2_Z = \frac{|m^2_{H_d} - m^2_{H_u}|}{\sqrt{1 - \sin^2(2\beta)}} - m^2_{H_u} - m^2_{H_d} - 2|\mu|^2. \tag{144}$$

(Note that $\sin(2\beta)$ is always positive. If $m^2_{H_u} < m^2_{H_d}$, as is usually assumed, then $\cos(2\beta)$ is negative; otherwise it is positive.)

Eqs. (369) and (370) highlight what is called the "μ problem".

– Without miraculous cancellations, all of the input parameters ought to be within an order of magnitude or two of m^2_Z.
– However, in the MSSM, μ is a supersymmetry-respecting parameter appearing in the superpotential, while b, $m^2_{H_u}$, $m^2_{H_d}$ are supersymmetry-breaking parameters.
– This has lead to a widespread belief that the MSSM must be extended at very high energies to include a mechanism that relates the effective value of μ to the supersymmetry-breaking mechanism in some way.

– Even if the value of μ is set by soft supersymmetry breaking, the cancellation needed by Eq. (370) is often very substantial ($\Rightarrow$ finetuning) when evaluated in specific model frameworks, after constraints from direct searches for the Higgs bosons and superpartners are taken into account.

– For example, expanding for large $\tan\beta$, Eq. (370) becomes

$$m_Z^2 = -2(m_{H_u}^2 + |\mu|^2) + \frac{2}{\tan^2\beta}(m_{H_d}^2 - m_{H_u}^2) + \mathcal{O}(1/\tan^4\beta). \quad (145)$$

Typical viable solutions for the MSSM have $-m_{H_u}^2$ and $|\mu|^2$ each much larger than m_Z^2, so that significant cancellation is needed.

– In particular, large top squark squared masses, needed to avoid having the Higgs boson mass turn out too small compared to $126~\mathrm{GeV}$ will feed into $m_{H_u}^2$.

The cancellation needed in the minimal model may therefore be at the fraction of a per cent level. It is impossible to objectively characterize whether this should be considered worrisome, but it could be taken as a weak hint in favor of non-minimal models.

The NMSSM

1. Introduction

Supersymmetric extensions of the Standard Model (SM) are motivated by a solution of the hierarchy problem [75, 76, 77, 78, 79], an automatic unification of the running gauge couplings at a Grand Unified (GUT) scale $M_{\rm GUT}$ [80, 81, 82, 83], and the possibility to explain the dark matter relic density in terms of a stable neutral particle [84, 85].

It is well known that a supersymmetric extension of the Higgs sector of the SM [86, 87] requires the introduction of two Higgs $SU(2)$-doublets H_u and H_d, where vacuum expectation values (vevs) of H_u and H_d generate masses for up-type quarks and down-type quarks and charged leptons, respectively. The model with this minimal field content in the Higgs sector is denoted as the Minimal Supersymmetric Standard Model (MSSM) (for reviews see, e. g., [88, 89, 90]). The Lagrangian of the MSSM must contain a supersymmetric (SUSY) mass term μ for H_u and H_d, which has to be of the order of the SUSY breaking scale $M_{\rm SUSY}$ for phenomenological reasons (see below). This spoils a potentially attractive property of supersymmetric extensions of the SM: the electroweak scale generated by the Higgs vevs could depend only on $M_{\rm SUSY}$, which would be the only scale asking for an explanation to why it is far below $M_{\rm GUT}$ or the Planck scale $M_{\rm Planck}$. The question how a supersymmetric mass parameter μ can assume a value of the order of $M_{\rm SUSY}$ is denoted as the "μ-problem" [91] of the MSSM.

A simple and elegant way to solve this problem consists in generating an effective (supersymmetric) mass term μ in a way similar to the generation of quark and lepton masses in the SM: the mass term μ is replaced by a Yukawa coupling of H_u and H_d to a scalar field, and the vev of the scalar field – induced by the soft SUSY breaking terms – is of the desired order. Since the μ parameter carries no $SU(3) \times SU(2) \times U(1)_Y$ quantum numbers, the field to be introduced has to be a singlet S (the complex scalar component of a chiral superfield $\widehat{S}$), and the resulting model is the Next-to-Minimal Supersymmetric Standard Model (NMSSM), sometimes also denoted as the (M+1)SSM.

In fact, already the first attempts to construct supersymmetric extensions of the SM employed such a singlet field [86, 87, 92]. A singlet was also present in most of the first globally supersymmetric GUT models [79, 93, 94, 95, 96]. Then one realised that spontaneous supersymmetry breaking in the framework of supergravity (SUGRA) leads in a simple way to the desired soft SUSY breaking terms in the Lagrangian; see [88] for an early review. Within SUGRA, a μ term of the order of $M_{\rm SUSY}$ can actually be generated if one assumes the presence of a particular Higgs-dependent structure in the Kähler potential [97]. Still, the first locally supersymmetric extensions of the SM [98, 99, 100] as well as most GUT models within SUGRA [101, 102, 103, 104, 105, 106, 107, 108] used a singlet field in the Higgs sector leading to variants of the NMSSM at the weak or SUSY breaking scale $\lesssim 1$ TeV. (See also SUGRA models motivated by string theory [109, 110, 111, 112, 113, 114, 115, 116, 117, 118].)

Expanding around the vacuum with non-vanishing vevs of the neutral CP-even components of H_u, H_d and S, one finds that the scalar components of $\widehat{S}$ mix with the neutral scalar components of $\widehat{H}_u$ and $\widehat{H}_d$ leading, in the absence of complex parameters (corresponding to the absence of explicit CP violation), to three CP-even and two CP-odd neutral scalars (see [119, 120, 121] for some reviews). Likewise, the fermionic superpartner of $\widehat{S}$ mixes with the neutral fermionic superpartners of $\widehat{H}_u$, $\widehat{H}_d$ (and the neutral electroweak gauginos) leading to five neutralinos. As a consequence, both the Higgs and the neutralino sectors of the NMSSM can get considerably modified compared to the MSSM.

In the Higgs sector, important alterations with respect to the MSSM are a possibly larger mass of the Higgs scalar with SM-like couplings to gauge bosons, and additional possibly light states with reduced couplings to gauge bosons. Notably a light CP-odd scalar with vanishing couplings to two gauge bosons like all CP-odd scalars (but with possibly even enhanced couplings to quarks and leptons) can appear in the Higgs spectrum, allowing for new Higgs-to-Higgs decays. Under these circumstances, the detection of Higgs bosons at colliders can become considerably more complicated. A priori this means that it is not even guaranteed that a single Higgs scalar can be observed at the LHC within the NMSSM. However, now that we have seen the $126~\mathrm{GeV}$ Higgs, we know that this kind of situation is not dominant (but might still be present at some level). In addition, a light CP-odd scalar can affect "low energy" observables in B physics, Υ physics and the anomalous magnetic moment of the muon.

The modifications within the neutralino sector are particularly relevant if the additional singlet-like neutralino is the lightest one and, simultaneously, the lightest supersymmetric particle (LSP). This would have an important impact on all decay chains of supersymmetric particles (sparticles), and hence on their signatures at colliders. For instance, the next-to-lightest supersymmetric particle (NLSP) can have a long life time leading to displaced vertices. Also, the LSP relic density has to be reconsidered in this case.

Given the strong theoretical motivations for the NMSSM, its phenomenological consequences must be worked out in order not to miss (or misinterpret) both Higgs and sparticles signals – or the absence thereof – at past, present and future colliders.

Here, we review theoretical and phenomenological aspects of the NMSSM: Most notably, those related to the Higgs sector. There are many other aspects of the NMSSM of interest, but these must be deferred to a different course.

2. The μ problem

Let us go into more detail regarding the arguments for a μ parameter of the order of $M_{\rm SUSY}$, whose necessity constitutes the main motivation for the NMSSM: both complex Higgs scalars H_u and H_d of the MSSM have to be components of chiral superfields which contain, in addition, fermionic $SU(2)$-doublets ψ_u and ψ_d. The Lagrangian of the MSSM can contain supersymmetric mass terms for these fields, i.e. identical positive masses squared μ^2 for $|H_u|^2$ and $|H_d|^2$, and a Dirac mass μ for ψ_u and ψ_d. In the presence of a SUSY mass term $\sim \mu$ in the Lagrangian, a soft SUSY breaking mass term $B\mu\, H_u H_d$ can also appear, where the soft SUSY breaking parameter B has the dimension of a mass.

For various reasons the mass parameter μ cannot vanish. First, a Dirac mass μ for ψ_u and ψ_d is required for phenomenological reasons: both fermionic $SU(2)$-doublets ψ_u and ψ_d contain electrically charged components. Together with with the fermionic superpartners of the $W^\pm$ bosons, they constitute the so-called chargino sector (two charged Dirac fermions) of SUSY extensions of the SM. Due to the fruitless searches for a chargino at LEP, the lighter chargino has to have a mass above ~ 103 GeV [122]. Analysing the chargino mass matrix, one finds that this lower limit implies that the Dirac mass μ for ψ_u and ψ_d – for arbitrary values of the other parameters – has to satisfy the constraint $|\mu| \gtrsim 100$ GeV.

Second, an analysis of the Higgs potential shows that a non-vanishing term $B\mu\, H_u H_d$ is a necessary condition for that both neutral components of H_u and H_d are non-vanishing at the minimum. This, in turn, is required in order to generate masses for up-type quarks, down-type quarks and leptons by the Higgs mecanism. Moreover, the numerical value of the product $B\mu$ should be roughly of the order of the electroweak scale (M_Z^2).

Third, $\mu = 0$ would generate a Peccei-Quinn symmetry in the Higgs sector, and hence an unacceptable massless axion [91].

However, $|\mu|$ must not be too large: the Higgs potential must be unstable at its origin $H_u = H_d = 0$ in order to generate the electroweak symmetry breaking. Whereas the soft SUSY breaking mass terms for H_u and H_d of the order of the SUSY breaking scale $M_{\rm SUSY}$ can generate such a desired instability, the μ-induced masses squared for H_u and H_d are always positive, and must not dominate the negative soft SUSY breaking mass terms. Consequently the μ parameter must obey $|\mu| \lesssim M_{\rm SUSY}$. Hence, both "natural" values $\mu = 0$ and very large μ ($\sim M_{\rm GUT}$ or $\sim M_{\rm Planck}$) are ruled out, and the need for an explanation of $\mu \approx M_{\rm SUSY}$ is the μ-problem.

Within the NMSSM, where μ is generated by the vev $\langle S\rangle$ of a singlet S, $\langle S\rangle$ has to be of the order of $M_{\rm SUSY}$; this is easy to obtain with the help of soft SUSY breaking negative masses squared (or trilinear couplings) of the order of $M_{\rm SUSY}$ for S. Then, $M_{\rm SUSY}$ is the only scale in the theory. In this sense, the NMSSM is the simplest supersymmetric extension of the SM in which the weak scale is generated by the supersymmetry breaking scale $M_{\rm SUSY}$ only.

3. Lagrangian of the general NMSSM

The Next to Minimal Supersymmetric Standard Model (NMSSM [612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623]) provides a very elegant solution to the μ problem of the MSSM via the introduction of a singlet superfield $\widehat{S}$. For the simplest possible scale invariant form of the superpotential, the scalar component of $\widehat{S}$ acquires naturally a vacuum expectation value of the order of the SUSY breaking scale, giving rise to a value of μ of order the electroweak scale. The NMSSM is actually the simplest supersymmetric extension of the standard model in which the electroweak scale originates from the SUSY breaking scale only.

In addition, the NMSSM renders the "little fine tuning problem" of the MSSM, originating from the non-observation of a neutral CP-even Higgs boson at LEP II, less severe [613].

A possible cosmological domain wall problem [614] can be avoided by introducing suitable non-renormalizable operators [615] that do not generate dangerously large singlet tadpole diagrams [616].

Hence, the phenomenology of the NMSSM deserves to be studied at least as fully and precisely as that of the MSSM. Its particle content differs from the MSSM by the addition of one CP-even and one CP-odd state in the neutral Higgs sector (assuming CP conservation), and one additional neutralino. Thus, the physics of the Higgs bosons – masses, couplings and branching ratios [612, 617, 618, 619, 620, 621, 622, 623] can differ significantly from the MSSM. The purpose of the Fortran code NMHDECAY (Non Minimal Higgs Decays), that accompanies the present paper, is an accurate computation of these properties of the Higgs bosons in the NMSSM in terms of the parameters in the Lagrangian. As its name suggests, the Fortran code uses to some extent – for MSSM-like processes – parts of the code HDECAY that is applicable to the Higgs sector of the MSSM [624].

We define the NMSSM in terms of its parameters at the SUSY breaking scale. No assumption on the soft terms (like universal soft terms at a GUT scale) are made. The parameters in the Higgs sector are chosen as follows:

a) Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

$$\lambda\, \widehat{S}\widehat{H_u}\widehat{H_d} + \frac{\kappa}{3}\, \widehat{S}^3 \tag{146}$$

depending on two dimensionless couplings λ, κ beyond the MSSM. (Hatted capital letters denote superfields, and unhatted capital letters will denote their scalar components).

b) The associated trilinear soft terms are

$$\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3 \,. \tag{147}$$

c) The final two input parameters are

$$\tan\beta = \langle H_u \rangle / \langle H_d \rangle \;, \; \mu_{\rm eff} = \lambda \langle S \rangle \;. \tag{148}$$

These, along with M_Z, can be viewed as determining the three SUSY breaking masses squared for H_u, H_d and S through the three minimization equations of the scalar potential.

Thus, as compared to two independent parameters in the Higgs sector of the MSSM (often chosen as $\tan\beta$ and M_A), the Higgs sector of the NMSSM is described by the six parameters

$$\lambda \;, \; \kappa \;, \; A_\lambda \;, \; A_\kappa, \; \tan\beta \;, \; \mu_{\rm eff} \;. \tag{149}$$

We will choose sign conventions for the fields such that λ and $\tan\beta$ are positive, while κ, A_λ, A_κ and $\mu_{\rm eff}$ should be allowed to have either sign.

For any choice of these parameters – as well as of the values for the gaugino masses and of the soft terms related to the squarks and sleptons that contribute to the radiative corrections in the Higgs sector

4. General Conventions for the NMSSM

Below, we define our conventions for the tree level Lagrangian of the NMSSM. The superpotential for the Higgs fields, the quarks and the leptons of the 3rd generation is

$$W = h_t \widehat{Q} \cdot \widehat{H}_u \widehat{T}_R^c - h_b \widehat{Q} \cdot \widehat{H}_d \widehat{B}_R^c - h_\tau \widehat{L} \cdot \widehat{H}_d \widehat{L}_R^c + \lambda \widehat{S} \widehat{H}_u \cdot \widehat{H}_d + \frac{1}{3} \kappa \widehat{S}^3 \,. \quad (150)$$

Hereafter, hatted capital letters denote superfields, and unhatted capital letters the corresponding (complex) scalar components. The $SU(2)$ doublets are

$$\widehat{Q} = \begin{pmatrix} \widehat{T}_L \\ \widehat{B}_L \end{pmatrix}, \ \widehat{L} = \begin{pmatrix} \widehat{\nu}_{\tau L} \\ \widehat{\tau}_L \end{pmatrix}, \ \widehat{H}_u = \begin{pmatrix} \widehat{H}_u^+ \\ \widehat{H}_u^0 \end{pmatrix}, \ \widehat{H}_d = \begin{pmatrix} \widehat{H}_d^0 \\ \widehat{H}_d^- \end{pmatrix}. \quad (151)$$

Products of two $SU(2)$ doublets are defined as, e.g.,

$$\widehat{H}_u \cdot \widehat{H}_d = \widehat{H}_u^+ \widehat{H}_d^- - \widehat{H}_u^0 \widehat{H}_d^0 \, . \tag{152}$$

For the soft SUSY breaking terms we take

$$\begin{aligned}
-\mathcal{L}_{\rm soft} &= m_{\rm H_u}^2 |H_u|^2 + m_{\rm H_d}^2 |H_d|^2 + m_{\rm S}^2 |S|^2 + m_Q^2 |Q^2| + m_T^2 |T_R^2| \\
&\quad + m_B^2 |B_R^2| + m_L^2 |L^2| + m_\tau^2 |L_R^2| \\
&\quad + (h_t A_t \; Q \cdot H_u T_R^c - h_b A_b \; Q \cdot H_d B_R^c - h_\tau A_\tau \; L \cdot H_d L_R^c \\
&\quad + \lambda A_\lambda \; H_u \cdot H_d S + \tfrac{1}{3} \kappa A_\kappa \; S^3 + \mathrm{h.c.}) \, .
\end{aligned} \tag{153}$$

5. Higgs Sector at Tree Level

For completeness, we list here the Higgs potential, tree level Higgs masses and our conventions for the mixing angles. The tree level Higgs potential is given by

$$
\begin{aligned}
V &= \lambda^2(|H_u|^2|S|^2 + |H_d|^2|S|^2 + |H_u \cdot H_d|^2) + \kappa^2|S^2|^2 \\
&+\lambda\kappa(H_u \cdot H_d S^{*2} + \mathrm{h.c.}) + \frac{1}{4}g^2(|H_u|^2 - |H_d|^2)^2 \\
&+\frac{1}{2}g_2^2|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 + m_{\mathrm{H_u}}^2|H_u|^2 + m_{\mathrm{H_d}}^2|H_d|^2 + m_S^2|S|^2 \\
&+(\lambda A_\lambda H_u \cdot H_d S + \frac{1}{3}\kappa A_\kappa\, S^3 + \mathrm{h.c.}) \qquad (154)
\end{aligned}
$$

where

$$
g^2 = \frac{1}{2}(g_1^2 + g_2^2)\ . \qquad (155)
$$

Assuming vevs h_u, h_d and s such that

$$
H_u^0 = h_u + \frac{H_{uR} + iH_{uI}}{\sqrt{2}}, \quad H_d^0 = h_d + \frac{H_{dR} + iH_{dI}}{\sqrt{2}}, \quad S = s + \frac{S_R + iS_I}{\sqrt{2}} \qquad (156)
$$

eq. (154) simplifies to

$$
\begin{aligned}
V &= \lambda^2(h_u^2 s^2 + h_d^2 s^2 + h_u^2 h_d^2) + \kappa^2 s^4 - 2\lambda\kappa h_u h_d s^2 - 2\lambda A_\lambda\, h_u h_d s \\
&\quad + \frac{2}{3}\kappa A_\kappa s^3 + m_{\mathrm{H_u}}^2 h_u^2 + m_{\mathrm{H_d}}^2 h_d^2 + m_S^2 s^2 + \frac{1}{4} g^2 (h_u^2 - h_d^2)^2\ . \qquad (157)
\end{aligned}
$$

The sign conventions for the fields can be chosen such that the Yukawa couplings λ, h_t, h_b, the vevs h_u, h_d (and hence $\tan\beta$) as well as the soft gaugino masses M_i are all positive. Then, the Yukawa coupling κ, the trilinear soft terms A_i, and the vev s (and hence μ_{eff}) can all be either positive or negative.

CP-even neutral states

In the basis $S^{bare} = (H_{uR}, H_{dR}, S_R)$ and using the minimization equations in order to eliminate the soft masses squared, one obtains the following mass-squared matrix entries:

$$
\mathcal{M}_{S,11}^2 = g^2 h_u^2 + \lambda s \frac{h_d}{h_u}(A_\lambda + \kappa s),
$$

$$
\begin{aligned}
\mathcal{M}^2_{S,22} &= g^2 h_d^2 + \lambda s \frac{h_u}{h_d}(A_\lambda + \kappa s), \\
\mathcal{M}^2_{S,33} &= \lambda A_\lambda \frac{h_u h_d}{s} + \kappa s(A_\kappa + 4\kappa s), \\
\mathcal{M}^2_{S,12} &= (2\lambda^2 - g^2) h_u h_d - \lambda s(A_\lambda + \kappa s), \\
\mathcal{M}^2_{S,13} &= 2\lambda^2 h_u s - \lambda h_d(A_\lambda + 2\kappa s), \\
\mathcal{M}^2_{S,23} &= 2\lambda^2 h_d s - \lambda h_u(A_\lambda + 2\kappa s).
\end{aligned} \tag{158}
$$

After diagonalization by an orthogonal matrix S_{ij} one obtains 3 CP-even states (ordered in mass) $h_i = S_{ij} S_j^{bare}$, with masses denoted by m_{h_i}.

In the MSSM limit (λ, $\kappa \to 0$, and parameters such that $h_3 \sim S_R$) the elements of the first 2×2 sub-matrix of S_{ij} are related to the MSSM angle α as

$$
\begin{aligned}
S_{11} &\sim \cos\alpha \,, & S_{21} &\sim \sin\alpha \,, \\
S_{12} &\sim -\sin\alpha \,, & S_{22} &\sim \cos\alpha \,.
\end{aligned} \tag{159}
$$

Rotating the upper left 2×2 submatrix by an angle β, one finds that one of its diagonal elements reads

$$M_Z^2 \left(\cos^2 2\beta + \frac{\lambda^2}{g^2} \sin^2 2\beta \right) \tag{160}$$

which constitutes an upper bound on the lightest eigenvalue of $\mathcal{M}_S^2$. The additional positive contribution $\sim \lambda^2 \sin^2 2\beta$ (as compared to the MSSM) in the NMSSM is highly welcome in view of the observed mass of ~ 126 GeV.

However, this additional contribution is relevant only for not too large $\tan\beta$; in fact, the expression inside the parenthesis in (160) is larger than one only for $\lambda^2 > g^2$, in which case it is maximal for small $\tan\beta$. Moreover, the actual lightest eigenvalue of $\mathcal{M}_S^2$ is smaller than the value given in (160) in general.

CP-odd neutral states

In the basis $P^{bare} = (H_{uI}, H_{dI}, S_I)$ and using the minimization equations in order to eliminate the soft masses squared, one obtains the following

mass-squared matrix entries:

$$
\begin{aligned}
\mathcal{M}^2_{P,11} &= \lambda s \frac{h_d}{h_u}(A_\lambda + \kappa s), \\
\mathcal{M}^2_{P,22} &= \lambda s \frac{h_u}{h_d}(A_\lambda + \kappa s), \\
\mathcal{M}^2_{P,33} &= 4\lambda\kappa h_u h_d + \lambda A_\lambda \frac{h_u h_d}{s} - 3\kappa A_\kappa s, \\
\mathcal{M}^2_{P,12} &= \lambda s(A_\lambda + \kappa s), \\
\mathcal{M}^2_{P,13} &= \lambda h_d(A_\lambda - 2\kappa s), \\
\mathcal{M}^2_{P,23} &= \lambda h_u(A_\lambda - 2\kappa s).
\end{aligned}
\tag{161}
$$

The diagonalization of this mass matrix is performed in two steps. First, one rotates into a basis ($\tilde{A}, \tilde{G}, S_I$), where $\tilde{G}$ is a massless Goldstone mode:

$$\begin{pmatrix} H_{uI} \\ H_{dI} \\ S_I \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{A} \\ \tilde{G} \\ S_I \end{pmatrix} \tag{162}$$

where $\tan\beta = h_u/h_d$. Dropping the Goldstone mode, the remaining 2×2 mass matrix in the basis $(\tilde{A}, S_I)$ has the matrix elements

$$\begin{aligned}
\mathcal{M}^2_{P,11} &= \lambda s \frac{h_u^2 + h_d^2}{h_u h_d}(A_\lambda + \kappa s), \\
\mathcal{M}^2_{P,22} &= 4\lambda\kappa h_u h_d + \lambda A_\lambda \frac{h_u h_d}{s} - 3\kappa A_\kappa s, \\
\mathcal{M}^2_{P,12} &= \lambda\sqrt{h_u^2 + h_d^2}(A_\lambda - 2\kappa s).
\end{aligned} \tag{163}$$

It can be diagonalized by an orthogonal 2×2 matrix P'_{ij} such that the physical CP-odd states a_i (ordered in mass) are

$$a_1 = P'_{11}\tilde{A} + P'_{12}S_I$$

$$
\begin{aligned}
&= P'_{11}(\cos\beta H_{uI} + \sin\beta H_{dI}) + P'_{12} S_I, \\
a_2 &= P'_{21}\tilde{A} + P'_{22} S_I \\
&= P'_{21}(\cos\beta H_{uI} + \sin\beta H_{dI}) + P'_{22} S_I, \qquad (164)
\end{aligned}
$$

and, for completeness,

$$
\tilde{G} = -\sin\beta H_{uI} + \cos\beta H_{dI} \ . \qquad (165)
$$

The decomposition of the bare states in terms of physical states reads

$$
\begin{aligned}
H_{uI} &= \cos\beta(P'_{11}a_1 + P'_{21}a_2) - \sin\beta\tilde{G} \ , \\
H_{dI} &= \sin\beta(P'_{11}a_1 + P'_{21}a_2) + \cos\beta\tilde{G} \ , \\
S_I &= P'_{12}a_1 + P'_{22}a_2 \ . \qquad (166)
\end{aligned}
$$

(In principle, since the matrix P'_{ij} is orthogonal, it could be parameterized by one angle.) Eqs. (166) suggest the introduction of a 2×3 matrix P_{ij}

with

$$P_{i1} = \cos\beta P'_{i1}, P_{i2} = \sin\beta P'_{i1}, P_{i3} = P'_{i2} \tag{167}$$

such that, omitting the Goldstone boson,

$$\begin{aligned} H_{uI} &= P_{11}a_1 + P_{21}a_2 \ , \\ H_{dI} &= P_{11}a_1 + P_{21}a_2 \ , \\ S_I &= P_{13}a_1 + P_{23}a_2 \ . \end{aligned} \tag{168}$$

Charged states

In the basis $(H_u^+, [H_d^-]^* = H_d^+)$, the charged Higgs mass matrix is given by

$$\mathcal{M}^2_\pm = \left(\lambda s(A_\lambda + \kappa s) + h_u h_d(\frac{g_2^2}{2} - \lambda^2)\right) \begin{pmatrix} \cot\beta & 1 \\ 1 & \tan\beta \end{pmatrix} . \tag{169}$$

This gives one eigenstate $H^\pm$ of mass Tr$\mathcal{M}^2_\pm$ and one massless goldstone mode $G^\pm$ with

$$\begin{aligned} H_u^\pm &= \cos\beta H^\pm - \sin\beta G^\pm \,, \\ H_d^\pm &= \sin\beta H^\pm + \cos\beta G^\pm \,. \end{aligned} \tag{170}$$

The physical charged Higgs eigenstate then has mass

$$\mathcal{M}_\pm^2 = M_A^2 + v^2(\frac{g_2^2}{2} - \lambda^2) = M_A^2 + m_W^2 - \lambda^2 v^2 \,. \tag{171}$$

where

$$M_A^2 \equiv \mathcal{M}_{P,11}^2 = \frac{2\lambda s(A_\lambda + \kappa s)}{\sin 2\beta} \tag{172}$$

is usually close to the mass of the heavier a_2.

Due to the term $\sim -\lambda^2$, the charged Higgs mass in the NMSSM can be somewhat smaller than in the MSSM. In contrast to the MSSM it is not even guaranteed within the NMSSM that $U(1)_{\rm em}$ remains unbroken: the expression for the charged Higgs mass squared becomes negative for $s = \mu_{\rm eff} = 0$, $\lambda^2 > g_2^2/2$, indicating a possible minimum in field space where the charged Higgs has a vev. Although radiative corrections have to be added and the depth of this minumum has to be compared to the physical one with $s \neq 0$, the above implies that λ is bounded from above by the absence of a charged Higgs vev.

6. SUSY Particles

Neutralinos

Denoting the $U(1)_Y$ gaugino by λ_1 and the neutral $SU(2)$ gaugino by λ_2^3, the mass terms in the Lagrangian read

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{2}M_1\lambda_1\lambda_1 + \frac{1}{2}M_2\lambda_2^3\lambda_2^3 \\ & +\lambda(s\psi_u^0\psi_d^0 + h_u\psi_d^0\psi_s + h_d\psi_u^0\psi_s) - \kappa s\psi_s\psi_s \\ & +\frac{ig_1}{\sqrt{2}}\lambda_1(h_u\psi_u^0 - h_d\psi_d^0) - \frac{ig_2}{\sqrt{2}}\lambda_2^3(h_u\psi_u^0 - h_d\psi_d^0).\end{aligned} \tag{173}$$

In the basis $\psi^0 = (-i\lambda_1, -i\lambda_2, \psi_u^0, \psi_d^0, \psi_s)$ one can rewrite

$$\mathcal{L} = -\frac{1}{2}(\psi^0)^T\mathcal{M}_0(\psi^0) + \mathrm{h.c.} \tag{174}$$

where

$$\mathcal{M}_0 = \begin{pmatrix} M_1 & 0 & \frac{g_1 h_u}{\sqrt{2}} & -\frac{g_1 h_d}{\sqrt{2}} & 0 \\ & M_2 & -\frac{g_2 h_u}{\sqrt{2}} & \frac{g_2 h_d}{\sqrt{2}} & 0 \\ & & 0 & -\mu & -\lambda h_d \\ & & & 0 & -\lambda h_u \\ & & & & 2\kappa s \end{pmatrix}. \tag{175}$$

(Recall that here $\mu = \mu_{\rm eff} = \lambda s$). One obtains 5 eigenstates (ordered in mass) $\chi_i^0 = N_{ij}\psi_j^0$, with N_{ij} real, with masses $m_{\chi_{\rm i}^0}$ that are real, but not necessarily positive.

Charginos

The charged $SU(2)$ gauginos are $\lambda^- = \frac{1}{\sqrt{2}}\left(\lambda_2^1 + i\lambda_2^2\right)$, $\lambda^+ = \frac{1}{\sqrt{2}}\left(\lambda_2^1 - i\lambda_2^2\right)$.

Defining

$$\psi^+ = \begin{pmatrix} -i\lambda^+ \\ \psi_u^+ \end{pmatrix}, \qquad \psi^- = \begin{pmatrix} -i\lambda^- \\ \psi_d^- \end{pmatrix} \tag{176}$$

the Lagrangian can be written as

$$\mathcal{L} = -\frac{1}{2}(\psi^+, \psi^-) \left(\begin{array}{cc} 0 & X^T \\ X & 0 \end{array} \right) \left(\begin{array}{c} \psi^+ \\ \psi^- \end{array} \right) + \text{h.c.} \tag{177}$$

with

$$X = \left(\begin{array}{cc} M_2 & g_2 h_u \\ g_2 h_d & \mu \end{array} \right) . \tag{178}$$

The mass eigenstates are $\chi^+ = V\psi^+,\ \chi^- = U\psi^-$, with

$$U = \left(\begin{array}{cc} \cos\theta_U & \sin\theta_U \\ -\sin\theta_U & \cos\theta_U \end{array} \right), \qquad V = \left(\begin{array}{cc} \cos\theta_V & \sin\theta_V \\ -\sin\theta_V & \cos\theta_V \end{array} \right). \tag{179}$$

Defining

$$\gamma = \sqrt{\text{Tr}(X^T X) - 4\text{det}(X^T X)} \tag{180}$$

one has

$$\tan\theta_U \;=\; \frac{g_2^2(h_d^2 - h_u^2) + \mu^2 - M_2^2 - \gamma}{2g_2(M_2 h_u + \mu h_d)},$$

$$\tan\theta_V = \frac{g_2^2(h_u^2 - h_d^2) + \mu^2 - M_2^2 - \gamma}{2g_2(M_2 h_d + \mu h_u)} \tag{181}$$

where $-\pi/2 \le \theta_U, \theta_V \le \pi/2$ are such that $M_D = UXV^T$ is diagonal, but not necessarily positive. The masses with $|m_{\widetilde{\chi}_1}| < |m_{\widetilde{\chi}_2}|$ are given by

$$m_{\widetilde{\chi}_1} = \cos\theta_U(M_2\cos\theta_V + g_2 h_u \sin\theta_V) + \sin\theta_U(g_2 h_d \cos\theta_V + \mu\sin\theta_V),$$
$$m_{\widetilde{\chi}_2} = \sin\theta_U(M_2\sin\theta_V - g_2 h_u \cos\theta_V) - \cos\theta_U(g_2 h_d \sin\theta_V - \mu\cos\theta_V).$$

(1

In terms of 4 component Dirac spinors $\Psi_i = \begin{pmatrix} \chi_i^+ \\ \overline{\chi_i}^- \end{pmatrix}$ one can rewrite the Lagrangian as

$$\mathcal{L} = -\chi^- M_D \chi^+ + \mathrm{h.c.} = -m_{\widetilde{\chi}_1}\overline{\Psi}_1\Psi_1 - m_{\widetilde{\chi}_2}\overline{\Psi}_2\Psi_2 \,. \tag{183}$$

Top and Bottom Squarks

To complete the consequences of our conventions above, we give here the top and bottom squark mass-squared matrices (without the D-term contributions). Below, t_L, t^c_R, b_L and b^c_R denote the two component quark spinors.

Top squarks:

$$\begin{array}{cc} & \begin{array}{cc} T_R & T_L \end{array} \\ \begin{array}{c} T_R^* \\ T_L^* \end{array} & \left(\begin{array}{cc} m_T^2 + h_t^2 h_u^2 & h_t(A_t h_u - \lambda s h_d) \\ h_t(A_t h_u - \lambda s h_d) & m_Q^2 + h_t^2 h_u^2 \end{array} \right) \end{array} \tag{184}$$

Bottom squarks:

$$\begin{array}{cc} & \begin{array}{cc} B_R & B_L \end{array} \\ \begin{array}{c} B_R^* \\ B_L^* \end{array} & \left(\begin{array}{cc} m_B^2 + h_b^2 h_d^2 & h_b(A_b h_d - \lambda s h_u) \\ h_b(A_b h_d - \lambda s h_u) & m_Q^2 + h_b^2 h_d^2 \end{array} \right) \end{array} \tag{185}$$

7. Feynman rules for the Higgs Couplings

Higgs-Quarks

The couplings are obtained by expanding the quark mass matrices in the (properly normalized) physical Higgs fields h_i, a_i and $H^\pm$. Below, we use $v^2 = h_u^2 + h_d^2$, and consider the quarks of the third generation.

$$
\begin{aligned}
h_i t_L t_R^c &: \quad \frac{m_t}{\sqrt{2} v \sin\beta} S_{i1} \\
h_i b_L b_R^c &: \quad \frac{m_b}{\sqrt{2} v \cos\beta} S_{i2} \\
a_i t_L t_R^c &: \quad i \frac{m_t}{\sqrt{2} v \sin\beta} P_{i1} \\
a_i b_L b_R^c &: \quad i \frac{m_b}{\sqrt{2} v \cos\beta} P_{i2} \\
H^+ b_L t_R^c &: \quad -\frac{m_t}{v} \cot\beta \\
H^- t_L b_R^c &: \quad -\frac{m_b}{v} \tan\beta
\end{aligned}
\tag{186}
$$

Higgs-Gauge Bosons

These couplings are obtained from the kinetic terms in the Lagrangian:

$$
\begin{aligned}
h_i Z_\mu Z_\nu &: \quad g_{\mu\nu}\frac{g_1^2+g_2^2}{\sqrt{2}}(h_u S_{i1}+h_d S_{i2}) \\
h_i W_\mu^+ W_\nu^- &: \quad g_{\mu\nu}\frac{g_2^2}{\sqrt{2}}(h_u S_{i1}+h_d S_{i2}) \\
h_i(p) H^+(p') W_\mu^- &: \quad \frac{g_2}{2}(\cos\beta S_{i1}-\sin\beta S_{i2})(p-p')_\mu \\
a_i(p) H^+(p') W_\mu^- &: \quad i\frac{g_2}{2}(\cos\beta P_{i1}+\sin\beta P_{i2})(p-p')_\mu \\
h_i(p) a_j(p') Z_\mu &: \quad i\frac{g}{\sqrt{2}}(S_{i1}P_{j1}-S_{i2}P_{j2})(p-p')_\mu \\
H^+(p) H^-(p') Z_\mu &: \quad \frac{g_1^2-g_2^2}{\sqrt{g_1^2+g_2^2}}(p-p')_\mu
\end{aligned}
\tag{187}
$$

Higgs-Neutralinos/Charginos

As in the case of the Higgs-Quark couplings, these couplings are obtained by expanding the corresponding mass matrices:

$$
\begin{aligned}
h_a\chi_i^+\chi_j^- &: \quad \frac{\lambda}{\sqrt{2}}S_{a3}U_{i2}V_{j2} + \frac{g_2}{\sqrt{2}}(S_{a1}U_{i1}V_{j2} + S_{a2}U_{i2}V_{j1}) \\
a_a\chi_i^+\chi_j^- &: \quad i\left(\frac{\lambda}{\sqrt{2}}P_{a3}U_{i2}V_{j2} - \frac{g_2}{\sqrt{2}}(P_{a1}U_{i1}V_{j2} + P_{a2}U_{i2}V_{j1})\right) \\
H^+\chi_i^-\chi_j^0 &: \quad \lambda\cos\beta U_{i2}N_{j5} - \frac{\sin\beta}{\sqrt{2}}U_{i2}(g_1N_{j1} + g_2N_{j2}) + g_2\sin\beta U_{i1}N_{j4} \\
H^-\chi_i^+\chi_j^0 &: \quad \lambda\sin\beta V_{i2}N_{j5} + \frac{\cos\beta}{\sqrt{2}}V_{i2}(g_1N_{j1} + g_2N_{j2}) + g_2\cos\beta V_{i1}N_{j3} \\
h_a\chi_i^0\chi_j^0 &: \quad \frac{\lambda}{\sqrt{2}}(S_{a1}\Pi_{ij}^{45} + S_{a2}\Pi_{ij}^{35} + S_{a3}\Pi_{ij}^{34}) - \sqrt{2}\kappa S_{a3}N_{i5}N_{j5} \\
&\quad -\frac{g_1}{2}(S_{a1}\Pi_{ij}^{13} - S_{a2}\Pi_{ij}^{14}) + \frac{g_2}{2}(S_{a1}\Pi_{ij}^{23} - S_{a2}\Pi_{ij}^{24}) \\
a_a\chi_i^0\chi_j^0 &: \quad i\left(\frac{\lambda}{\sqrt{2}}(P_{a1}\Pi_{ij}^{45} + P_{a2}\Pi_{ij}^{35} + P_{a3}\Pi_{ij}^{34}) - \sqrt{2}\kappa P_{a3}N_{i5}N_{j5}\right. \\
&\quad \left. +\frac{g_1}{2}(P_{a1}\Pi_{ij}^{13} - P_{a2}\Pi_{ij}^{14}) - \frac{g_2}{2}(P_{a1}\Pi_{ij}^{23} - P_{a2}\Pi_{ij}^{24})\right) \qquad (188)
\end{aligned}
$$

where $\Pi_{ij}^{ab} = N_{ia}N_{jb} + N_{ib}N_{ja}$.

Triple Higgs Interactions

The trilinear Higgs self-couplings are obtained by expanding the scalar potential.

$$
\begin{aligned}
h_a h_b h_c \quad : \quad & \frac{\lambda^2}{\sqrt{2}}\left(h_u(\Pi_{abc}^{122} + \Pi_{abc}^{133}) + h_d(\Pi_{abc}^{211} + \Pi_{abc}^{233}) + s(\Pi_{abc}^{311} + \Pi_{abc}^{322})\right) \\
& - \frac{\lambda\kappa}{\sqrt{2}}(h_u\Pi_{abc}^{323} + h_d\Pi_{abc}^{313} + 2s\Pi_{abc}^{123}) + \sqrt{2}\kappa^2 s\Pi_{abc}^{333} \\
& - \frac{\lambda A_\lambda}{\sqrt{2}}\Pi_{abc}^{123} + \frac{\kappa A_\kappa}{3\sqrt{2}}\Pi_{abc}^{333} \\
& + \frac{g^2}{2\sqrt{2}}\left(h_u(\Pi_{abc}^{111} - \Pi_{abc}^{122}) - h_d(\Pi_{abc}^{211} - \Pi_{abc}^{222})\right) \qquad (189)
\end{aligned}
$$

where

$$
\Pi_{abc}^{ijk} \quad = \quad S_{ai}S_{bj}S_{ck} + S_{ai}S_{cj}S_{bk} + S_{bi}S_{aj}S_{ck}
$$

$$+S_{bi}S_{cj}S_{ak}+S_{ci}S_{aj}S_{bk}+S_{ci}S_{bj}S_{ak}\,. \tag{190}$$

$$\begin{aligned} h_a a_b a_c \quad : \quad & \frac{\lambda^2}{\sqrt{2}}\left(h_u(\Pi^{122}_{abc}+\Pi^{133}_{abc})+h_d(\Pi^{211}_{abc}+\Pi^{233}_{abc})+s(\Pi^{311}_{abc}+\Pi^{322}_{abc})\right) \\ & +\frac{\lambda\kappa}{\sqrt{2}}\left(h_u(\Pi^{233}_{abc}-2\Pi^{323}_{abc})+h_d(\Pi^{133}_{abc}-2\Pi^{313}_{abc})\right. \\ & \left.+2s(\Pi^{312}_{abc}-\Pi^{123}_{abc}-\Pi^{213}_{abc})\right)+\sqrt{2}\kappa^2 s\Pi^{333}_{abc} \\ & +\frac{\lambda A_\lambda}{\sqrt{2}}(\Pi^{123}_{abc}+\Pi^{213}_{abc}+\Pi^{312}_{abc})-\frac{\kappa A_\kappa}{\sqrt{2}}\Pi^{333}_{abc} \\ & +\frac{g^2}{2\sqrt{2}}\left(h_u(\Pi^{111}_{abc}-\Pi^{122}_{abc})-h_d(\Pi^{211}_{abc}-\Pi^{222}_{abc})\right) \end{aligned} \tag{191}$$

where

$$\Pi^{ijk}_{abc}=S_{ai}(P_{bj}P_{ck}+P_{cj}P_{bk})\,. \tag{192}$$

$$
\begin{aligned}
h_a H^+ H^- \quad : \quad & \frac{\lambda^2}{\sqrt{2}}(s(\Pi_a^{311} + \Pi_a^{322}) - h_u \Pi_a^{212} - h_d \Pi_a^{112}) \\
& + \sqrt{2}\lambda\kappa s \Pi_a^{312} + \frac{\lambda A_\lambda}{\sqrt{2}} \Pi_a^{312} \\
& + \frac{g_1^2}{4\sqrt{2}} \left(h_u(\Pi_a^{111} - \Pi_a^{122}) + h_d(\Pi_a^{222} - \Pi_a^{211}) \right) \\
& + \frac{g_2^2}{4\sqrt{2}} \left(h_u(\Pi_a^{111} + \Pi_a^{122} + 2\Pi_a^{212}) + h_d(\Pi_a^{211} + \Pi_a^{222} + 2\Pi_a^{112}) \right.
\end{aligned}
$$
(193

where

$$
\Pi_a^{ijk} = 2 S_{ai} C_j C_k \qquad (194)
$$

with $C_1 = \cos\beta$, $C_2 = \sin\beta$.

Radiative corrections and the mass of the lightest CP-even NMSSM Higgs boson

An approximate formula for the mass $M_{\rm SM}$ of the SM-like Higgs scalar in the NMSSM in the limit $\kappa s \gg |A_\kappa|, |A_\lambda|$ (corresponding to a heavy singlet-like scalar), including the dominant top/stop radiative corrections, is given by

$$\begin{aligned} M_{\rm SM}^2 &\simeq M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta - \frac{\lambda^2}{\kappa^2} v^2 (\lambda - \kappa \sin 2\beta)^2 \\ &+ \frac{3m_t^4}{4\pi^2 v^2} \left(\ln \left(\frac{m_T^2}{m_t^2} \right) + \frac{A_t^2}{m_T^2} \left(1 - \frac{A_t^2}{12 m_T^2} \right) \right) \end{aligned} \quad (195)$$

where $v = 174~\mathrm{GeV}$, the soft SUSY breaking stop masses squared in (??) are assumed to satisfy $m_T^2 \sim m_{Q_3}^2 \gg m_t^2$, A_t is the stop trilinear coupling assumed to satisfy $|A_t| \gg m_t,~\mu_{\rm eff}$; the terms $\sim \lambda^2$ are specific to the NMSSM, and the last term in the first line originates from the mixing with the singlet-like scalar.

In the MSSM, where $\lambda = 0$, the $126~\mathrm{GeV}$ observed value of $M_{\rm SM}$ implies that $\tan\beta$ has to be large such that $\cos 2\beta \sim 1$, and m_T must be above

~ 600 GeV for maximal mixing ($A_t^2 \sim 6\, m_T^2$, maximizing the second line in (195)), or $\gtrsim 1$ TeV otherwise.

In order to maximize $M_{\rm SM}$ in the NMSSM, λ should be as large as possible, and $\tan\beta$ should be small in order to avoid a suppression from $\sin^2 2\beta$. (As discussed before, λ is bounded from above by $\lambda \lesssim 0.7 - 0.8$ if one requires the absence of a Landau singularity below the GUT scale.)

However, the negative contribution from the mixing with the singlet-like scalar should vanish. If we keep terms involving A_λ, the relevant mixing term is proportional to $(\lambda - \sin 2\beta(\kappa + A_\lambda/(2s)))^2$ [178]. If this expression is not small, a larger value of λ can even generate a decrease of the mass of the Higgs scalar with SM-like couplings to the Z boson in the NMSSM.

The resulting upper bound on the lightest CP-even Higgs mass in the NMSSM has been studied in the leading log approximation in [177, 179, 156, 157, 158, 180, 181, 182, 183]. Full one-loop calculations of the corresponding upper bound involving top/bottom quark/squark loops have been carried out in [184, 159, 160, 185, 186, 187, 146, 188, 189, 190, 191]. (Analyses at large values of $\tan\beta$ have been performed in [192, 193, 194], and upper bounds for more general supersymmetric Higgs sectors have been considered in [195, 196, 197].)

At present, additional known radiative corrections to the Higgs mass matrices in the NMSSM include MSSM-like electroweak together with the

NMSSM-specific Higgs one-loop contributions [198, 199] and dominant two-loop terms [200, 147, 161, 201, 199]. In order to discuss these in detail, it is convenient to separate the quantum corrections involving scales Q^2 with $Q^2 \gtrsim M^2_{\rm SUSY}$ from those with scales $Q^2 \lesssim M^2_{\rm SUSY}$.

The electroweak and NMSSM specific Higgs one-loop contributions, and the two-loop contributions $\propto h_t^2 \alpha_s$, have recently been computed in [199] without an expansion in large logarithms.

Taking many one and two loop corrections into account, and requiring perturbative running Yukawa couplings h_t, λ and κ below the GUT scale, the upper bound on the lightest CP-even Higgs mass has been studied in [178] as a function of $\tan\beta$ and for different values of m_t in the NMSSM, and compared to the MSSM with the result shown in Fig. 9. (In Fig. 9, the upper bound is denoted as $m_{\rm max}$.)

The squark mass terms (and hence $M_{\rm SUSY}$) have been chosen as 1 TeV; the upper bound would still increase slowly (logarithmically) with $M_{\rm SUSY}$.

In order to maximize the one-loop top/bottom (s)quark contributions to the lightest CP-even Higgs mass for these squark masses, the trilinear soft couplings are chosen as $A_t = A_b = 2.5$ TeV.

The threshold effects depend somewhat on the gaugino masses, which are $M_1 = 150$ GeV, $M_2 = 300$ GeV and $M_3 = 1$ TeV; the remaining parameters λ, κ, A_λ, A_κ and $\mu_{\rm eff}$ of the Z_3-invariant NMSSM have been chosen such that the upper bound is maximized, subject to avoiding Landau singularities. The maximum is achieved for the largest λ which is possible without singularities being encountered, the latter requiring that κ be as small as possible, subject to constraints such as an unstable potential, which forbids $\kappa \to 0$.

The lower dashed lines in Fig. 9 refer to the MSSM, where the mass of the CP-odd scalar M_A – which can be chosen as the other independent parameter in the Higgs sector apart from $\tan\beta$ – is set to $M_A = 1$ TeV. The other parameters (and approximations) are the same as described above. In the MSSM, the increase of the upper bound with $\tan\beta$ originates from the tree level term (the first term $\sim \cos^2 2\beta$ in (195)), according to which $m_{\max}$ is maximised for large $\tan\beta$. Due to the one-loop top (s)quark contributions, the upper bound $m_{\max}$ increases with m_t. Numerically, a variation Δm_t of m_t implies nearly the same variation $\Delta m_{\max}$ for large $\tan\beta$.

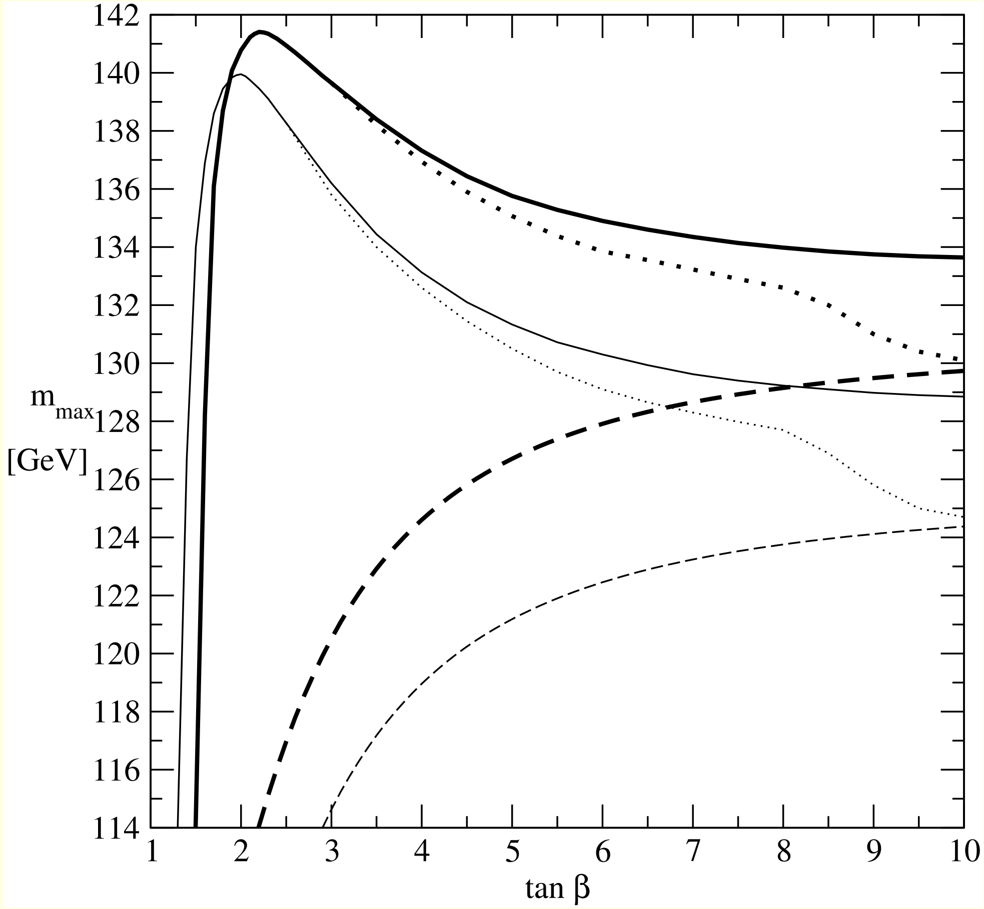


Figure 9: **Upper bound on the lightest Higgs mass in the NMSSM as a function of $\tan\beta$ for $m_t = 178$ GeV (M_A arbitrary: thick full line, $M_A = 1$ TeV: thick dotted line) and $m_t = 171.4$ GeV (thin full line: M_A arbitrary, thick dotted line: $M_A = 1$ TeV) and in the MSSM (with $M_A = 1$ TeV) for $m_t = 178$ GeV (thick dashed line) and $m_t = 171.4$ GeV (thin dashed line). Squark and gluino masses are 1 TeV and $A_t = A_b = 2.5$ TeV. (From [178].)**

In the NMSSM, the second term $\sim \sin^2 2\beta$ in the tree level expression (195) dominates the first one for sufficiently large λ, and accordingly $m_{\max}$ is maximal for low values of $\tan\beta$.

On the other hand, the absence of a Landau singularity for λ below the GUT scale implies a decrease of the maximally allowed value of λ at M_{SUSY} with increasing h_t, i. e. with increasing m_t and decreasing $\tan\beta$.

(At large $\tan\beta$, arbitrary variations of the NMSSM parameters λ, κ, A_λ, A_κ and μ_{eff} can imply a mass M_A of the MSSM-like CP-odd scalar far above 1 TeV. For comparison with the MSSM, $m_{\max}$ in the NMSSM with $M_A \leq 1$ TeV is depicted as dotted lines in Fig. 9.)

Finally, $m_{\max}$ in the NMSSM obviously increases if one allows for larger values of λ [202, 203]. This would have implied a Landau singularity below M_{GUT} for the particle content of the NMSSM. But, for the observed Higgs mass of $126~\mathrm{GeV}$, we don't need really large λ values. Thus, we can avoid having to consider modifications of the theory at larger energy scales below M_{GUT}.

Phenomenology of the Higgs Boson

1. Standard Model Higgs boson decay branching ratios

In this section, we approach the physics of the SM Higgs boson by considering its branching ratios for various decay modes. Earlier, we have derived the SM Higgs couplings to gauge bosons and fermions.

Therefore we know that, at the tree level, the SM Higgs boson can decay into pairs of electroweak gauge bosons ($H \to W^+W^-, ZZ$), and into pairs of quarks and leptons ($H \to Q\bar{Q}, l^+l^-$); while at one-loop it can also decay into two photons ($H \to \gamma\gamma$), two gluons ($H \to gg$), or a γZ pair ($H \to \gamma Z$).

Fig. 10 shows all the decay branching ratios of the SM Higgs boson as functions of its mass M_H.

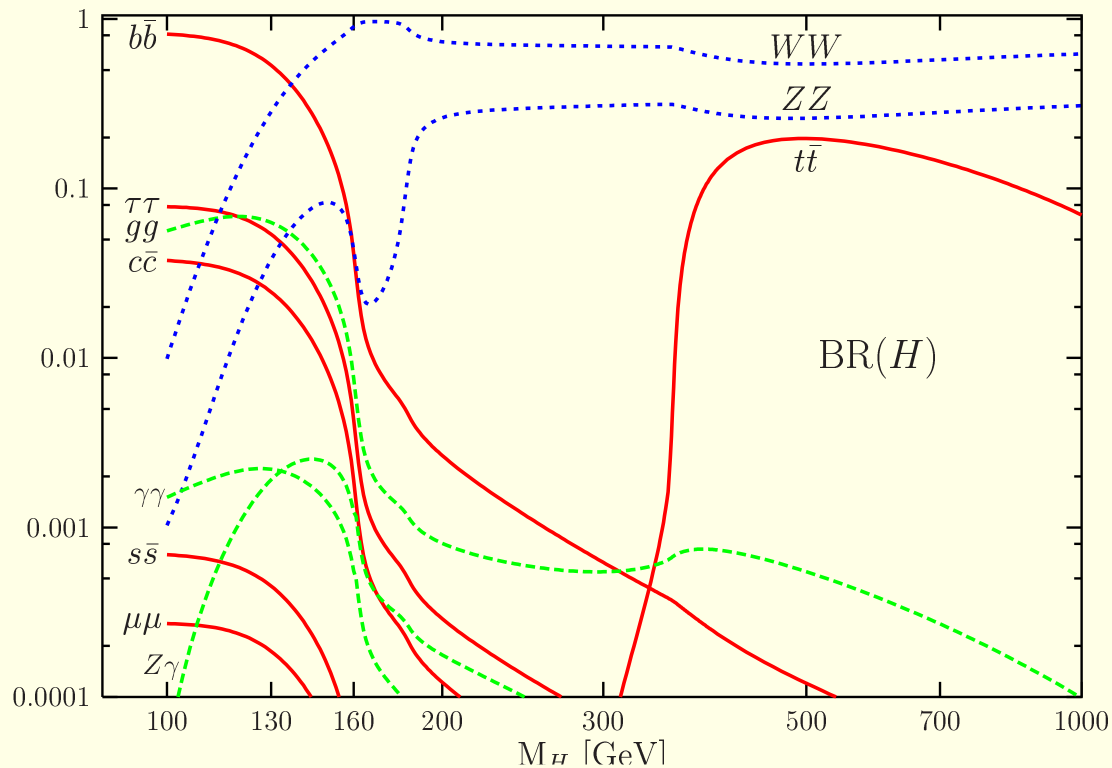


Figure 10: SM Higgs decay branching ratios as a function of M_H. The blue curves represent tree-level decays into electroweak gauge bosons, the red curves tree level decays into quarks and leptons, the green curves one-loop decays. From Ref. [6].

Fig. 10 shows that a light Higgs boson ($M_H \leq 130 - 140$ GeV) behaves very differently from a heavy Higgs boson ($M_H \geq 130 - 140$ GeV).

Indeed, a light SM Higgs boson mainly decays into a $b\bar{b}$ pair, followed hierarchically by all other pairs of lighter fermions.

Loop-induced decays also play a role in this region. $H \to gg$ is dominant among them, and it is actually larger than many tree level decays. Unfortunately, this decay mode is almost useless, in particular at hadron colliders, because of background limitations.

Among radiative decays, $H \to \gamma\gamma$ is tiny, but it is actually phenomenologically very important because the two photon signal can be seen over large hadronic backgrounds.

On the other hand, for larger Higgs masses, the decays to W^+W^- and ZZ dominates.

All decays into fermions or loop-induced decays are suppressed, except $H \to t\bar{t}$ for Higgs masses above the $t\bar{t}$ production threshold.

Note the intermediate region, from around $120~\mathrm{GeV}$ to $130~\mathrm{GeV}$, i.e. well below the W^+W^- and ZZ threshold, where many different decays are significant, including the decays into WW^* and ZZ^* (i.e. one of the two gauge bosons is off-shell).

These three-body decays of the Higgs boson start to dominate over the $H \to b\bar{b}$ two-body decay mode when the large sizes of the HWW or HZZ couplings compensate for their phase space suppression[6].

[6] Actually, even four-body decays, corresponding to $H \to W^*W^*, Z^*Z^*$ may become important in the intermediate

The different decay pattern of a light vs a heavy Higgs boson influences the role played, in each mass region, by different Higgs production processes at hadron and lepton colliders.

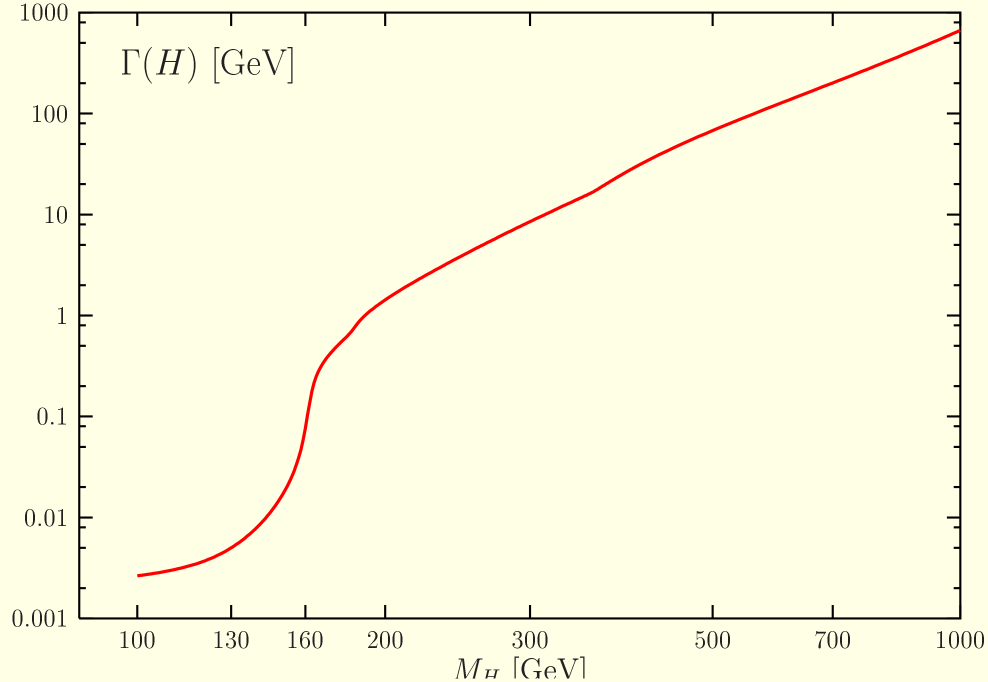


Figure 11: **SM Higgs total decay width as a function of M_H. From Ref. [6].**

The SM Higgs boson total width, i.e. the sum of all the partial widths $\Gamma(H \to XX)$, is represented in Fig. 11. Note how small the width is for $M_H \sim 126$ GeV. $\Rightarrow$ a non-SM channel can have a big impact: e.g. $H \to aa$, $H \to LSP + LSP$.

mass region and are indeed accounted for in Fig. 10.

The curves in Fig. 10 are obtained by including all available QCD and electroweak (EW) radiative corrections. Indeed, the problem of computing the relevant orders of QCD and EW corrections for Higgs decays has been thoroughly explored and the results are nowadays available in public codes like HDECAY [32], which has been used to produce Fig. 10.

It would actually be more accurate to represent each curve as a band, obtained by varying the parameters that enters both at tree level and in particular through loop corrections within their uncertainties. This is shown, for a light and intermediate mass Higgs boson, in Fig. 12 where each band has been obtained including the uncertainty from the quark masses and from the strong coupling constant.

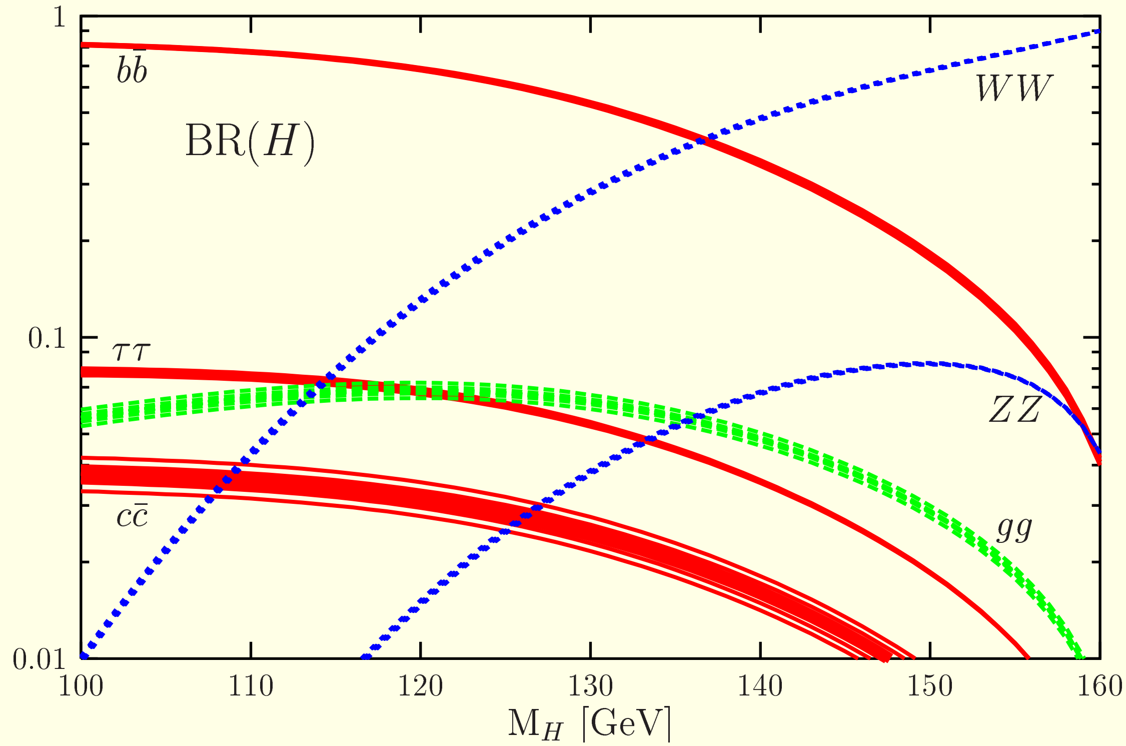


Figure 12: **SM Higgs boson decay branching ratios in the low and intermediate Higgs boson mass range including the uncertainty from the quark masses $m_t = 178 \pm 4.3$ GeV, $m_b = 4.88 \pm 0.07$ GeV, and $m_c = 1.64 \pm 0.07$ GeV, as well as from $\alpha_s(M_Z) = 0.1172 \pm 0.002$. From Ref. [6].**

In the following, we will briefly review the various SM Higgs decay channels. Giving a schematic but complete list of all available radiative corrections goes beyond the purpose of these lectures. Therefore we will only discuss those aspects that can be useful as a general background.

In particular, below are some comments on the general structure of radiative corrections to Higgs decays and more details on QCD corrections to $H \to Q\bar{Q}$ ($Q =$ heavy quark).

For a detailed review of QCD corrections in Higgs decays see Ref. [33]. Ref. [6] also contains an excellent summary of both QCD and EW radiative corrections to Higgs decays.

General properties of radiative corrections to Higgs decays

All Higgs boson decay rates are modified by both EW and QCD radiative corrections.

QCD corrections are particularly important for $H \to Q\bar{Q}$ decays, where they mainly amount to a redefinition of the Yukawa coupling by shifting the mass parameter in it from the pole mass value to the running mass value, and for $H \to gg$.

EW corrections can be further separated into:

i) corrections due to fermion loops,

ii) corrections due to the Higgs boson self-interaction, and

iii) other EW corrections.

Both corrections of type (ii) and (iii) are in general very small except for large Higgs boson masses, i.e. for $M_H \gg M_W$.

On the other hand, corrections of type (i) are very important over the entire Higgs mass range, and are particularly relevant for $M_H \ll 2m_t$, where the top-quark loop corrections play a leading role.

Indeed, for $M_H \ll 2m_t$, the dominant corrections for both Higgs decays into fermion and gauge bosons come from the top-quark contribution to the renormalization of the Higgs wave function and vacuum expectation value.

Several higher order radiative corrections to Higgs decays have been calculated in the large m_t limit, specifically in the limit when $M_H \ll 2m_t$.

In this limit, results can be derived by applying some very powerful low energy theorems. The idea is that, for an on-shell Higgs field ($p_H^2 = M_H^2$), the limit of small masses ($M_H \ll 2m_t$) is equivalent to a $p_H \to 0$ limit, in which case the Higgs couplings to the fermion fields can be simply obtained by substituting

$$m_i^0 \to m_i^0 \left(1 + \frac{H}{v^0}\right) \tag{196}$$

in the (bare) Yukawa Lagrangian, for each massive particle i. In Eq. (196) H is a constant field and the upper zero indices indicate that all formal manipulations are done on bare quantities. This induces a simple relation between the bare matrix element for a process with $(X \to Y + H)$ and without $(X \to Y)$ a Higgs field, namely

$$\lim_{p_H \to 0} \mathcal{A}(X \to Y + H) = \frac{1}{v^0} \sum_i m_i^0 \frac{\partial}{\partial m_i^0} \mathcal{A}(X \to Y) \ . \tag{197}$$

When the theory is renormalized, the only actual difference is that the derivative operation in Eq. (197) needs to be modified as follows

$$m_i^0 \frac{\partial}{\partial m_i^0} \longrightarrow \frac{m_i}{1 + \gamma_{m_i}} \frac{\partial}{\partial m_i} \tag{198}$$

where γ_{m_i} is the mass anomalous dimension of fermion f_i. This accounts for the fact that the renormalized Higgs-fermion Yukawa coupling is determined through the Z_2 and Z_m counterterms, and not via the $Hf\bar{f}$ vertex function at zero momentum transfer (as used in the $p_H \to 0$ limit above).

The theorem summarized by Eq. (197) is valid also when higher order radiative corrections are included. Therefore, outstanding applications of

Eq. (197) include the determination of the one-loop Hgg and $H\gamma\gamma$ vertices from the gluon or photon self-energies, as well as the calculation of several orders of their QCD and EW radiative corrections.

Indeed, in the $m_t \to \infty$ limit, the loop-induced $H\gamma\gamma$ and Hgg interactions can be seen as effective vertices derived from an effective Lagrangian of the form:

$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi} F^{(a)\mu\nu} F^(a)_{\mu\nu} \frac{H}{v} (1 + O(\alpha_s)) \ , \qquad (199)$$

where $F^{(a)}_{\mu\nu}$ is the field strength tensor of QED (for the $H\gamma\gamma$ vertex) or QCD (for the Hgg vertex). The calculation of higher order corrections to the $H \to \gamma\gamma$ and $H \to gg$ decays is then reduced by one order of loops! Since these vertices start as one-loop effects, the calculation of the first order of corrections would already be a strenuous task, and any higher order effect would be a formidable challenge. Thanks to the low energy theorem above, QCD NNLO corrections have indeed been calculated.

Higgs boson decays to gauge bosons: $H \to W^+W^-, ZZ$

The tree level decay rate for $H \to VV$ $(V = W^\pm, Z)$ can be written as:

$$\Gamma(H \to VV) = \frac{G_F M_H^3}{16\sqrt{2}\pi}\delta_V \left(1 - \tau_V + \frac{3}{4}\tau_V^2\right)\beta_V \ , \quad (200)$$

where $\beta_V = \sqrt{1-\tau_V}$, $\tau_V = 4M_V^2/M_H^2$, and $\delta_{W,Z} = 2, 1$.

Below the W^+W^- and ZZ threshold, the SM Higgs boson can still decay via three (or four) body decays mediated by WW^* (W^*W^*) or ZZ^* (Z^*Z^*) intermediate states.

As we can see from Fig. 10, the off-shell decays $H \to WW^*$ and $H \to ZZ^*$ are relevant in the intermediate mass region around $M_H \simeq 160$ GeV, where they compete and overcome the $H \to b\bar{b}$ decay mode.

The decay rates for $H \to VV^* \to Vf_i\bar{f}_j$ $(V = W^\pm, Z)$ are given by:

$$\Gamma(H \to WW^*) = \frac{3g^4 M_H}{512\pi^3} F\left(\frac{M_W}{M_H}\right) \ , \quad (201)$$

$$\Gamma(H \to ZZ^*) = \frac{g^4 M_H}{2048(1-s_W^2)^2\pi^3}\left(7 - \frac{40}{3}s_W^2 + \frac{160}{9}s_W^4\right) F\left(\frac{M_Z}{M_H}\right) ,$$

where $s_W = \sin\theta_W$ is the sine of the Weinberg angle and the function $F(x)$ is given by

$$\begin{aligned} F(x) &= -(1-x^2)\left(\frac{47}{2}x^2 - \frac{13}{2} + \frac{1}{x^2}\right) - 3\left(1 - 6x^2 + 4x^4\right)\ln(x) \\ &+ 3\,\frac{1-8x^2+20x^4}{\sqrt{4x^2-1}}\arccos\left(\frac{3x^2-1}{2x^3}\right) . \end{aligned} \quad (202)$$

Higgs boson decays to fermions: $H \to Q\bar{Q}, l^+l^-$

The tree level decay rate for $H \to f\bar{f}$ ($f = Q, l$, Q =quark, l =lepton) can be written as:

$$\Gamma(H \to f\bar{f}) = \frac{G_F M_H}{4\sqrt{2}\pi} N_c^f m_f^2 \beta_f^3 , \quad (203)$$

where $\beta_f = \sqrt{1-\tau_f}$, $\tau_f = 4m_f^2/M_H^2$, and $(N_c)^{l,Q} = 1, 3$. QCD corrections dominate over other radiative corrections and they modify the rate as follows:

$$\Gamma(H \to Q\bar{Q})_{QCD} = \frac{3G_F M_H}{4\sqrt{2}\pi}\bar{m}_Q^2(M_H)\beta_q^3\left[\Delta_{QCD} + \Delta_t\right] \quad , \tag{204}$$

where Δ_t represents specifically QCD corrections involving a top-quark loop. Δ_{QCD} and Δ_t have been calculated up to three loops and are given by:

$$\begin{aligned}
\Delta_{QCD} &= 1 + 5.67\frac{\alpha_s(M_H)}{\pi} + (35.94 - 1.36N_F)\left(\frac{\alpha_s(M_H)}{\pi}\right)^2 + \\
&\quad (164.14 - 25.77N_F + 0.26N_F^2)\left(\frac{\alpha_s(M_H)}{\pi}\right)^3 \quad , \\
\Delta_t &= \left(\frac{\alpha_s(M_H)}{\pi}\right)^2\left[1.57 - \frac{2}{3}\ln\frac{M_H^2}{m_t^2} + \frac{1}{9}\ln^2\frac{\bar{m}_Q^2(M_H)}{M_H^2}\right] \quad ,
\end{aligned} \tag{205}$$

where $\alpha_s(M_H)$ and $\bar{m}_Q(M_H)$ are the renormalized running QCD coupling and quark mass in the $\overline{MS}$ scheme. It is important to notice that using the

$\overline{MS}$ running mass in the overall Yukawa coupling square of Eq. (204)is very important in Higgs decays, since it reabsorbs most of the QCD corrections, including large logarithms of the form $\ln(M_H^2/m_Q^2)$. Indeed, for a generic scale μ, $\bar{m}_Q(\mu)$ is given at leading order by:

$$\begin{aligned} \bar{m}_Q(\mu)_{LO} &= \bar{m}_Q(m_Q)\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{\frac{2b_0}{\gamma_0}} \\ &= \bar{m}_Q(m_Q)\left(1-\frac{\alpha_s(\mu)}{4\pi}\ln\left(\frac{\mu^2}{m_Q^2}\right)+\cdots\right) \;, \end{aligned} \tag{206}$$

where b_0 and γ_0 are the first coefficients of the β and γ functions of QCD, while at higher orders it reads:

$$\bar{m}_Q(\mu)=\bar{m}_Q(m_Q)\frac{f\left(\alpha_s(\mu)/\pi\right)}{f\left(\alpha_s(m_Q)/\pi\right)} \;, \tag{207}$$

where, from renormalization group techniques, the function $f(x)$ is of the form:

$$f(x) = \left(\frac{25}{6}x\right)^{\frac{12}{25}}[1+1.014x+\ldots] \quad \text{for} \quad m_c < \mu < m_b \ , \tag{208}$$

$$f(x) = \left(\frac{23}{6}x\right)^{\frac{12}{23}}[1+1.175x+\ldots] \quad \text{for} \quad m_b < \mu < m_t \ ,$$

$$f(x) = \left(\frac{7}{2}x\right)^{\frac{4}{7}}[1+1.398x+\ldots] \quad \text{for} \quad \mu > m_t \ .$$

As we can see from Eqs. (207) and (208), by using the $\overline{MS}$ running mass, leading and subleading logarithms up to the order of the calculation are actually resummed at all orders in α_s.

The overall mass factor coming from the quark Yukawa coupling square is actually the only place where we want to employ a running mass.

For quarks like the b quark this could indeed have a large impact, since, in going from $\mu \simeq M_H$ to $\mu \simeq m_b$, $\bar{m}_n(\mu)$ varies by almost a factor of two, which yields almost a factor of four reduction at the rate level.

All other mass corrections, in the matrix element and phase space entering the calculation of the $H \to Q\bar{Q}$ decay rate, can in first approximation be safely neglected.

Loop induced Higgs boson decays: $H \to \gamma\gamma, \gamma Z, gg$

As mentioned earlier, the $H\gamma\gamma$ and $H\gamma Z$ couplings are induced at one loop via both a fermion loop and a W-loop. At the lowest order the decay rate for $H \to \gamma\gamma$ can be written as:

$$\Gamma(H \to \gamma\gamma) = \frac{G_F \alpha^2 M_H^3}{128\sqrt{2}\pi^3} \left| \sum_f N_c^f Q_f^2 A_f^H(\tau_f) + A_W^H(\tau_W) \right|^2 , \qquad (209)$$

where $N_c^f = 1, 3$ (for $f = l, q$ respectively), Q_f is the charge of the f fermion species, $\tau_f = 4m_f^2/M_H^2$, the function $f(\tau)$ is defined as:

$$f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4}\left[\ln \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 & \tau < 1 \ , \end{cases} \qquad (210)$$

and the form factors A_f^H and A_W^H are given by:

$$A_f^H = 2\tau\left[1+(1-\tau)f(\tau)\right] \to 4/3 \quad \text{for} \quad \tau\to\infty \ , \tag{211}$$
$$A_W^H(\tau) = -\left[2+3\tau+3\tau(2-\tau)f(\tau)\right] \to -7 \quad \text{for} \quad \tau\to\infty, \ .$$

The decay rate for $H \to \gamma Z$ is given by:

$$\Gamma(H\to\gamma Z) = \frac{G_F^2 M_W^2 \alpha M_H^3}{64\pi^4}\left(1-\frac{M_Z^2}{M_H^2}\right)^3 \left|\sum_f A_f^H(\tau_f,\lambda_f) + A_W^H(\tau_W,\lambda_W)\right|^2 \ , \tag{212}$$

where $\tau_i = 4M_i^2/M_H^2$ and $\lambda_i = 4M_i^2/M_Z^2$ ($i = f, W$), and the form factors $A_f^H(\tau,\lambda)$ and $A_W^H(\tau,\lambda)$ are given by:

$$A_f^H(\tau,\lambda) = 2N_c^f \frac{Q_f(I_{3f}-2Q_f\sin^2\theta_W)}{\cos\theta_W}\left[I_1(\tau,\lambda)-I_2(\tau,\lambda)\right] \tag{213}$$
$$A_W^H(\tau,\lambda) = \cos\theta_W\left\{\left[\left(1+\frac{2}{\tau}\right)\tan^2\theta_W - \left(5+\frac{2}{\tau}\right)\right] I_1(\tau,\lambda)\right.$$

$$+4\left(3-\tan^2\theta_W\right) I_2(\tau,\lambda)\Big\} \quad , \tag{214}$$

where N_c^f and Q_f are defined after Eq. (209), and I_3^f is the weak isospin of the f fermion species. Moreover:

$$\begin{aligned}
I_1(\tau,\lambda) &= \frac{\tau\lambda}{2(\tau-\lambda)}+\frac{\tau^2\lambda^2}{2(\tau-\lambda)^2}[f(\tau)-f(\lambda)]+\frac{\tau^2\lambda}{(\tau-\lambda)^2}[g(\tau)-g(\lambda)] \\
I_2(\tau,\lambda) &= -\frac{\tau\lambda}{2(\tau-\lambda)}[f(\tau)-f(\lambda)] \quad ,
\end{aligned} \tag{215}$$

and

$$g(\tau)=\begin{cases} \sqrt{\tau-1}\arcsin\frac{1}{\sqrt{\tau}} & \tau\geq 1 \\ \frac{\sqrt{1-\tau}}{2}\left[\ln\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}-i\pi\right] & \tau<1 \end{cases} \tag{216}$$

while $f(\tau)$ is defined in Eq. (210). QCD and EW corrections to both $\Gamma(H\to\gamma\gamma)$ and $\Gamma(H\to\gamma Z)$ are pretty small and for their explicit expression we refer the interested reader to the literature [33, 6].

As far as $H\to gg$ is concerned, this decay can only be induced by a fermion loop, and therefore its rate, at the lowest order, can be written as:

$$\Gamma(H \to gg) = \frac{G_F \alpha_s^2 M_H^3}{64\sqrt{2}\pi^3} \left| \sum_q A_q^H(\tau_q) \right|^2 , \tag{217}$$

where $\tau_q = 4m_q^2/M_H^2$, $f(\tau)$ is defined in Eq.(210) and the form factor $A_q^H(\tau)$ is given in Eq. (213). QCD corrections to $H \to gg$ have been calculated up to NNLO in the $m_t \to \infty$ limit, as explained in Section 1. At NLO the expression of the corrected rate is remarkably simple

$$\Gamma(H \to gg(g), q\bar{q}g) = \Gamma_{LO}(H \to gg) \left[1 + E(\tau_Q) \frac{\alpha_s^{(N_L)}}{\pi} \right] , \tag{218}$$

where

$$E(\tau_Q) \stackrel{M_H^2 \ll 4m_q^2}{\longrightarrow} \frac{95}{4} - \frac{7}{6} N_L + \frac{33 - 2N_F}{6} \log \left(\frac{\mu^2}{M_H^2} \right) . \tag{219}$$

When compared with the fully massive NLO calculation (available in this case), the two calculations display an impressive 10% agreement, as illustrated in Fig. 13, even in regions where the light Higgs approximation is not justified. This is actually due to the presence of large constant factors in the first order of QCD corrections.

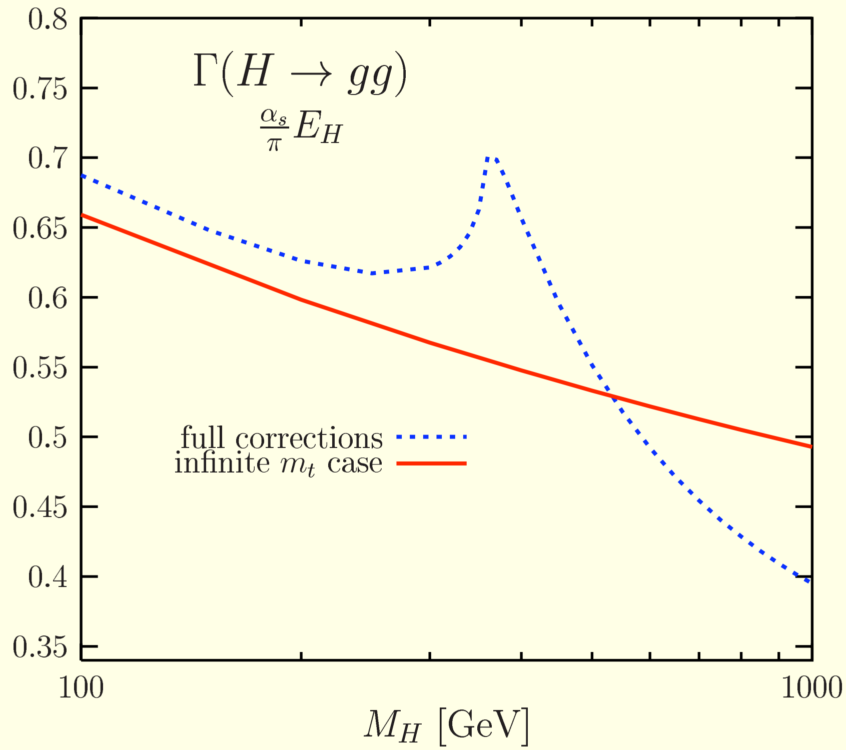


Figure 13: The QCD correction factor for the partial width $\Gamma(H \to gg)$ as a function of the Higgs boson mass, in the full massive case with $m_t = 178$ GeV (dotted line) and in the heavy-top-quark limit (solid line). The strong coupling constant is normalized at $\alpha_s(M_Z) = 0.118$. From Ref. [6].

We also observe that the first order of QCD corrections has quite a large impact on the lowest order cross section, amounting to more than 50% of Γ_{LO} on average. This has been indeed the main reason to prompt for a NNLO QCD calculation of $\Gamma(H \to gg)$. The result, obtained in the heavy-top approximation, has shown that NNLO QCD corrections amount to only 20% of the NLO cross section, therefore pointing to a convergence of the $\Gamma(H \to gg)$ perturbative series. We will refer to this discussion when dealing with the $gg \to H$ production mode, since its cross section can be easily related to $\Gamma(H \to gg)$.

2. MSSM Higgs boson branching ratios

The decay patterns of the MSSM Higgs bosons are many and diverse, depending on the specific choice of supersymmetric parameters.

In particular they depend on the choice of M_A and $\tan\beta$, which parameterize the MSSM Higgs sector, and they are clearly sensitive to the choice of other supersymmetric masses (gluino masses, squark masses, etc.) since this determines the possibility for the MSSM Higgs bosons to decay into pairs of supersymmetric particles and for the radiative induced decay channels ($h, H \to gg, \gamma\gamma, \gamma Z$) to receive supersymmetric loop contributions.

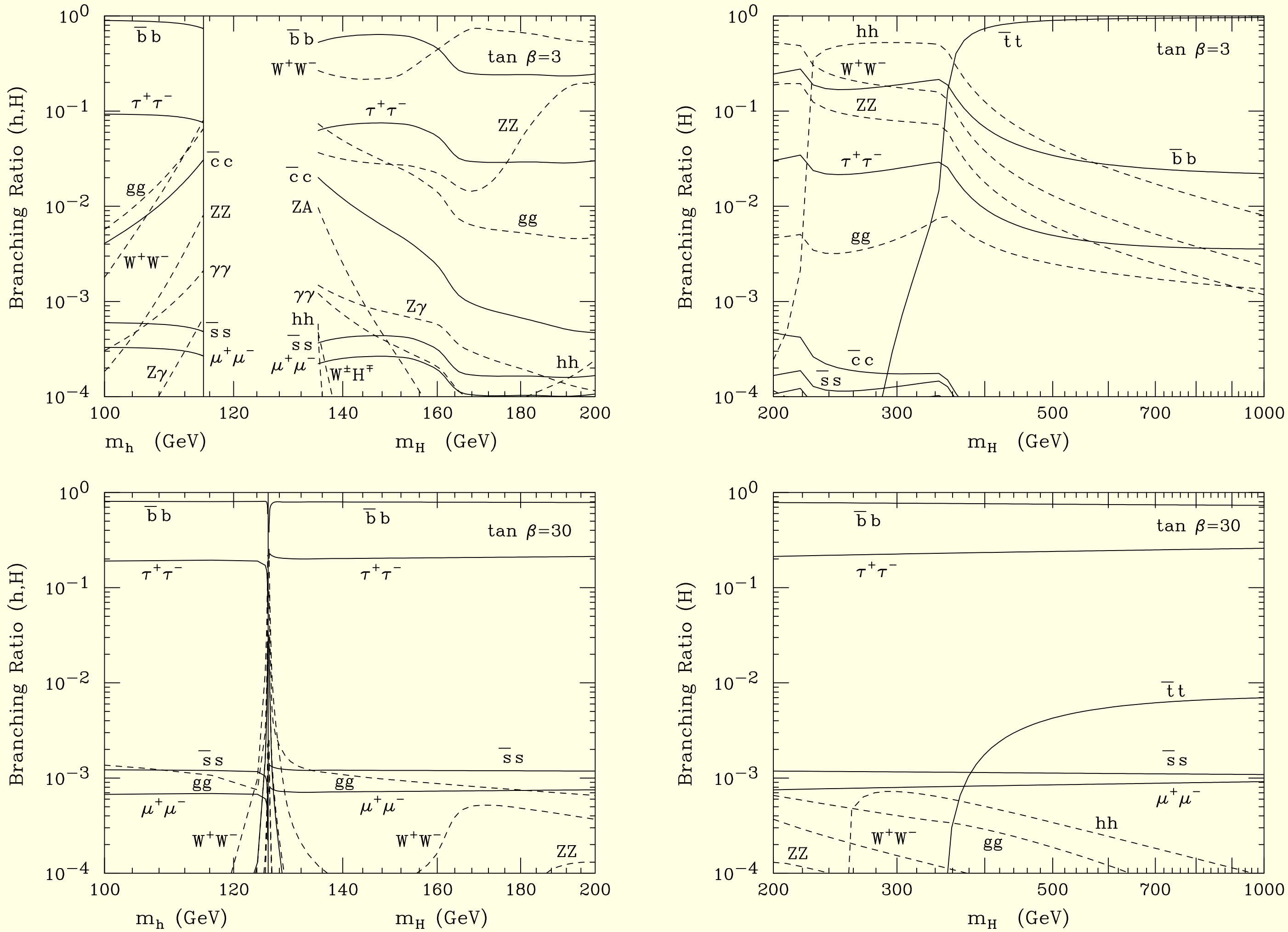


Figure 14: **Branching ratios for the h and H MSSM Higgs bosons, for $\tan\beta = 3, 30$. The range of M_H corresponds to $M_A = 90$ GeV $- 1$ TeV, in the MSSM scenario discussed in the text, with maximal top-squark mixing. The vertical line in the left hand side plots indicates the upper bound on M_h, which, for the given scenario is $M_h^{max} = 115$ GeV ($\tan\beta = 3$) or $M_h^{max} = 125.9$ GeV ($\tan\beta = 30$). So, only the latter is relevant in practice. From Ref. [8].**

In order to be more specific, let us assume that all supersymmetric masses are large enough to prevent the decay of the MSSM Higgs bosons into pairs of supersymmetric particles (a good choice could be $M_{\tilde{g}} = M_Q = M_U = M_D = 1$ TeV).

Then, we only need to examine the decays into SM particles and compare with the decay patterns of a SM Higgs boson to identify any interesting difference.

From the study of the MSSM Higgs boson couplings in Sections 3 and 4, we expect that:

i) in the decoupling regime, when $M_A \gg M_Z$, the properties of the h neutral Higgs boson are very much the same as the SM Higgs boson; while away from the decoupling limit

ii) the decay rates of h and/or H to electroweak gauge bosons are suppressed with respect to the SM case, in particular for large Higgs masses (M_H),

iii) the $A \to VV$ ($V = W^{\pm}, Z$) decays are absent,

iv) the decay rates of h and/or H to $\tau^+\tau^-$ and $b\bar{b}$ are enhanced for large $\tan\beta$,

v) even for not too large values of $\tan\beta$, due to ii) above, the $h, H \to \tau^+\tau^-$ and $h, H \to b\bar{b}$ decay are large up to the $t\bar{t}$ threshold, when the decay $H \to t\bar{t}$ becomes dominant,

vi) for the charged Higgs boson, the decay $H^+ \to \tau^+\nu_\tau$ dominates over $H^+ \to t\bar{b}$ below the $t\bar{b}$ threshold, and vice versa above it.

As far as QCD and EW radiative corrections go, what we have seen in Sections 1-1 for the SM case applies to the corresponding MSSM decays too.

Moreover, the truly MSSM corrections discussed in Sections 3 and 4 need to be taken into account and are included in Figs. 14 and 15.

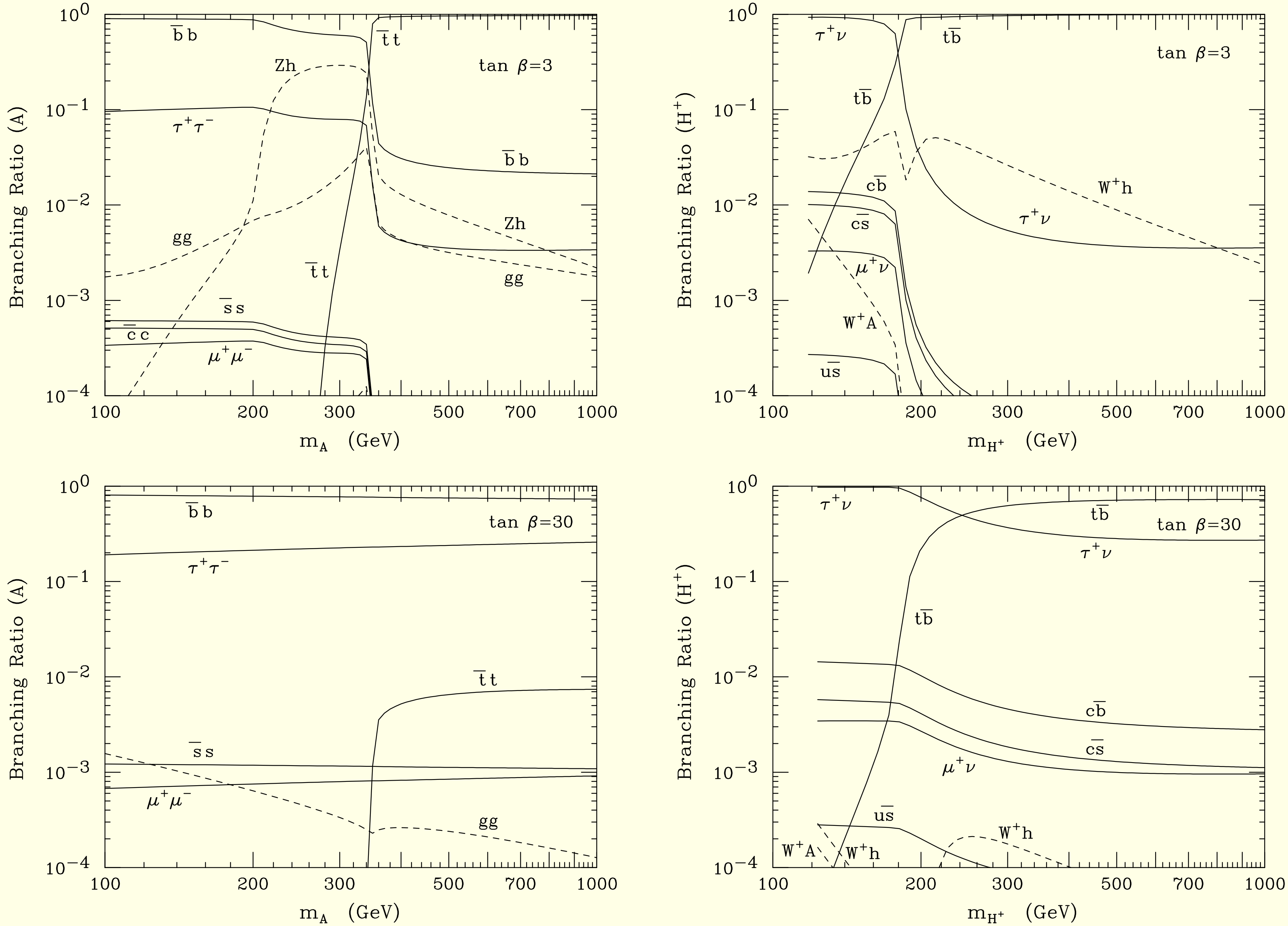


Figure 15: **Branching ratios for the A and H^+ MSSM Higgs bosons, for $\tan\beta = 3, 30$. The range of $M_{H^\pm}$ corresponds to $M_A = 90$ GeV $- 1$ TeV, in the MSSM scenario discussed in the text, with maximal top-squark mixing. From Ref. [8].**

How do we see the Higgs boson?

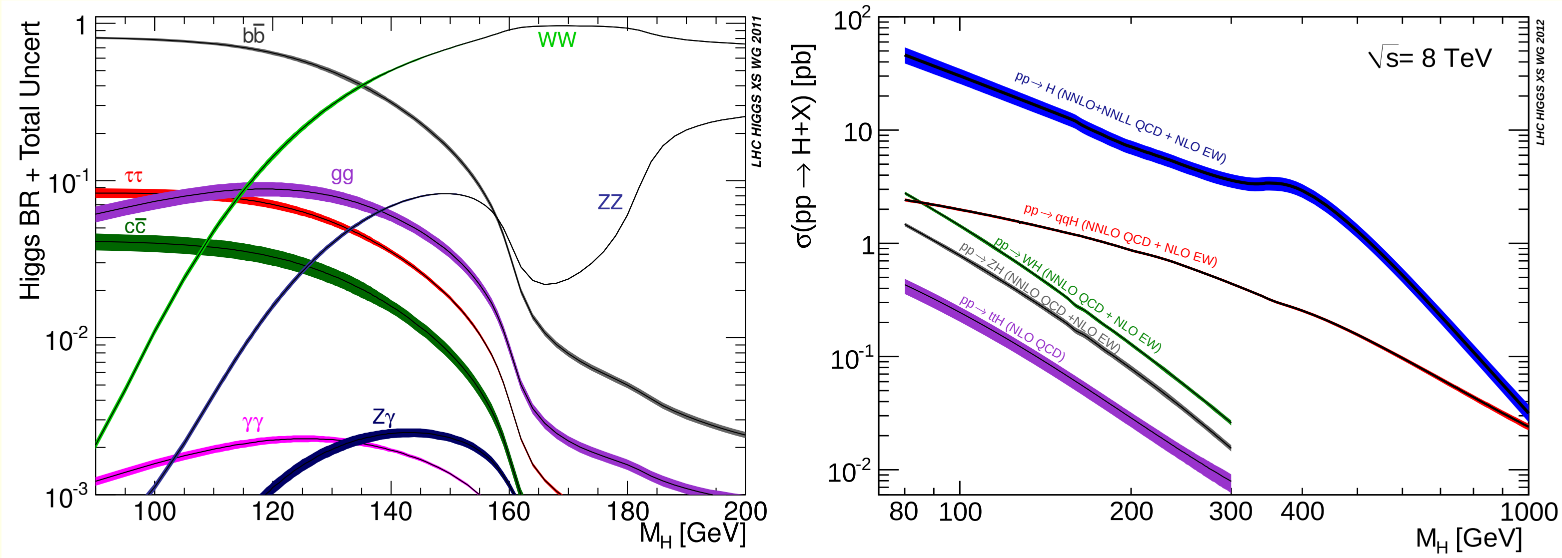
- **Since the Higgs field gives mass, the Higgs boson H couples to elementary particles proportionally to their mass:**

$$\begin{aligned}\mathcal{L} &= g\Big[C_V\left(m_W W_\mu W^\mu + \frac{m_Z}{\cos\theta_W} Z_\mu Z^\mu\right) \\ &\quad -C_U\frac{m_t}{2m_W}\bar{t}t - C_D\frac{m_b}{2m_W}\bar{b}b - C_D\frac{m_\tau}{2m_W}\bar{\tau}\tau\Big]H\,. \qquad (220)\end{aligned}$$

where $C_U = C_D = C_V = 1$ in the SM.

- **In addition to these "tree-level" couplings there are also loop-induced couplings $gg \to H$ and $\gamma\gamma \to H$, the former dominated by the top-quark loop and the latter dominated by the W loop with a smaller and opposite contribution from a top-quark loop.**

- **Because of the Higgs mass being $\sim 125~\mathrm{GeV}$, there is a remarkable mixture of observable Higgs decays and observable cross sections.**



The most important ones for the initial discovery were those with very excellent mass resolution — the $H \to \gamma\gamma$ final state and the $H \to ZZ \to 4\ell$ final state.

In these final states you can actually see the resonance peak — see later.

- **The four key production and decay processes for the initial discovery were:**

$$\begin{array}{rl} gg \text{ fusion: ggF} & gg \to H \to \gamma\gamma; \quad gg \to H \to ZZ \to 4\ell \\ WW, ZZ \text{ fusion: VBF} & WW \to H \to \gamma\gamma; \quad WW \to H \to ZZ \to 4\ell \end{array} \tag{221}$$

the gg induced processes having the highest rate. Sample diagrams for two of these processes are given below.

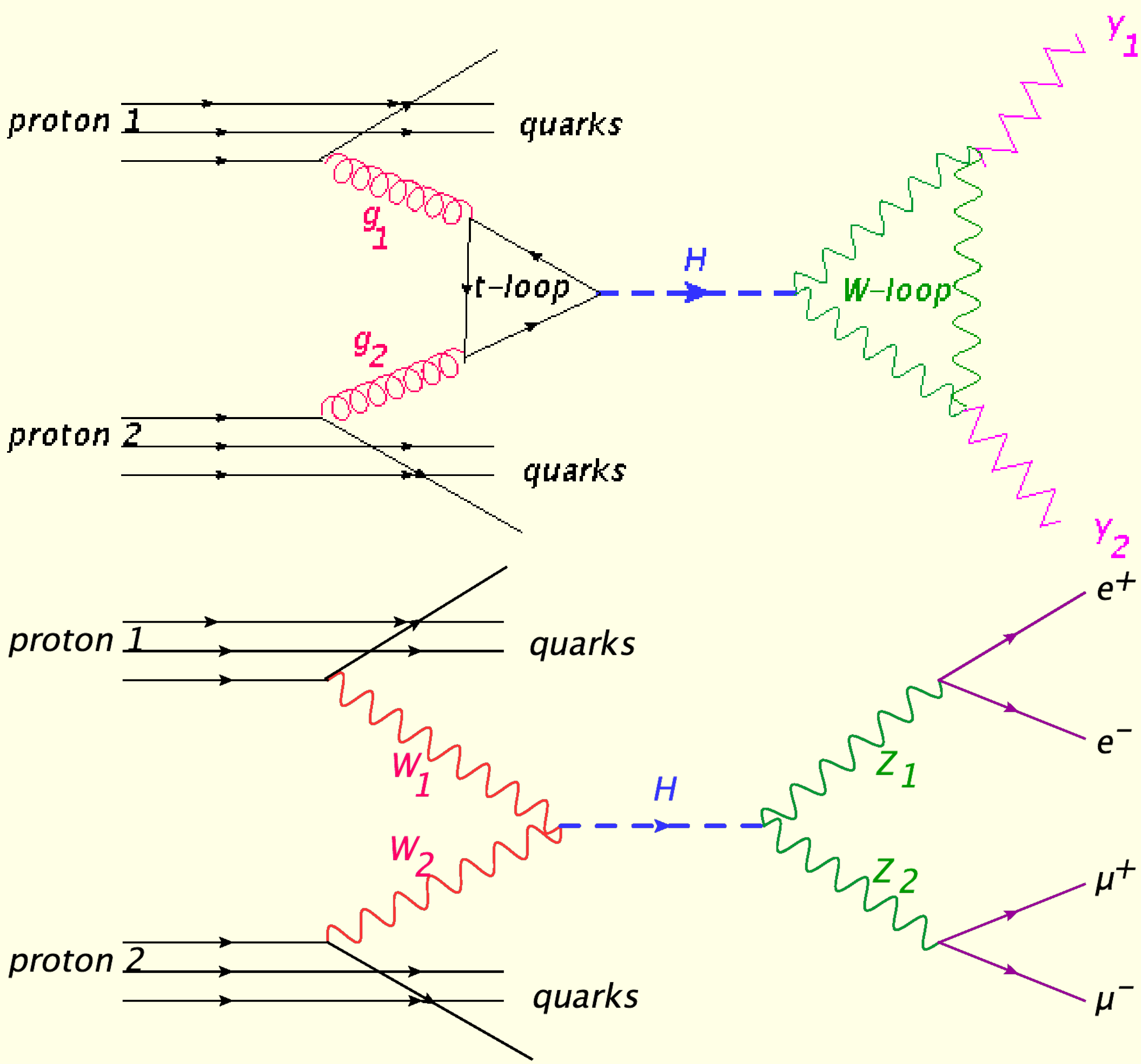
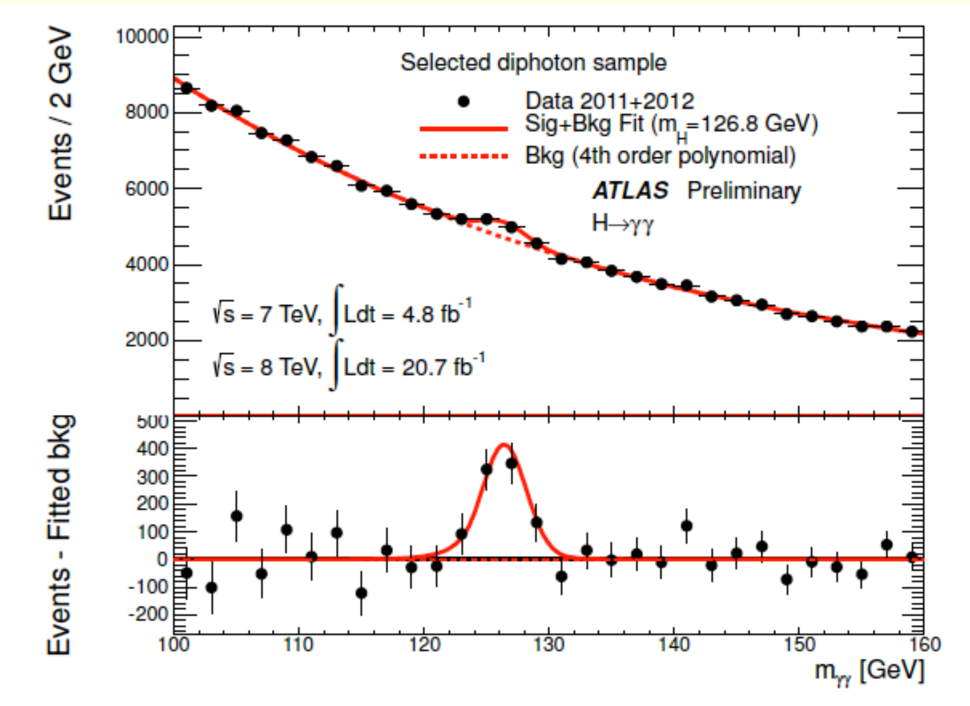
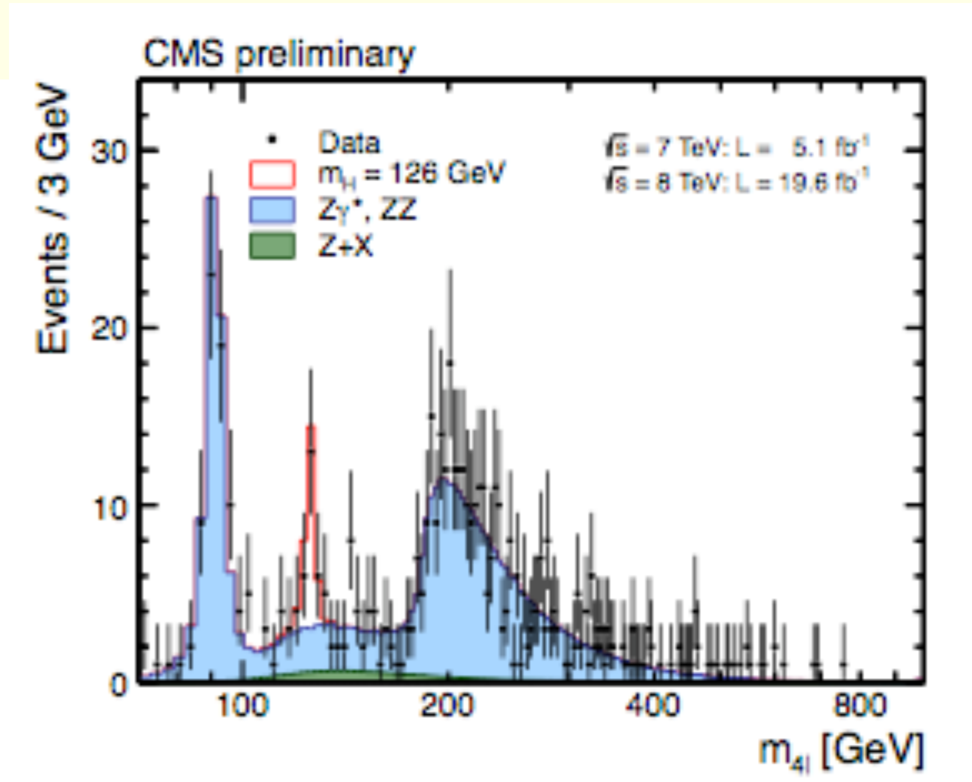


Figure 16: **Note the loops for the gg and $\gamma\gamma$ couplings in the upper figure.**

Gordy Kane, Jose Wudka and I anticipated the importance of these channels back in 1985 and pushed the detectors to have excellent electromagnetic calorimeters so that they could actually see the resonance peaks. 80 million dollars later we see:





Diagnosing the Nature of the 125-126 GeV LHC Higgs-like signal

Higgs couplings Collaborators: Belanger, Beranger, Ellwanger, Kraml

1. *"Status of invisible Higgs decays"* **G. Belanger, B. Dumont, U. Ellwanger, J. F. Gunion and S. Kraml. arXiv:1302.5694 [hep-ph]**
2. *"Higgs Couplings at the End of 2012"* **G. Belanger, B. Dumont, U. Ellwanger, J. F. Gunion and S. Kraml. arXiv:1212.5244 [hep-ph]**

NMSSM Collaborators: G. Belanger, U. Ellwanger, Y. Jiang, S. Kraml, J. Schwarz

1. *"Higgs Bosons at 98 and 125 GeV at LEP and the LHC"* **G. Belanger, U. Ellwanger, J. F. Gunion, Y. Jiang, S. Kraml and J. H. Schwarz. arXiv:1210.1976 [hep-ph]**
2. *"Two Higgs Bosons at the Tevatron and the LHC?"* **G. Belanger, U. Ellwanger, J. F. Gunion, Y. Jiang and S. Kraml. arXiv:1208.4952 [hep-ph]**
3. *"Diagnosing Degenerate Higgs Bosons at 125 GeV"* **J. F. Gunion, Y. Jiang and S. Kraml. arXiv:1208.1817 [hep-ph]**
4. *"Could two NMSSM Higgs bosons be present near 125 GeV?"* **J. F. Gunion, Y. Jiang and S. Kraml. arXiv:1207.1545 [hep-ph]**
5. *"The Constrained NMSSM and Higgs near 125 GeV"* **J. F. Gunion, Y. Jiang and S. Kraml. arXiv:1201.0982 [hep-ph] Phys. Lett. B 710, 454 (2012)**

2HDM Collaborators: Alexandra Drozd, Bohdan Grzadkowski, Yun Jiang

1. *"Two-Higgs-Doublet Models and Enhanced Rates for a 125 GeV Higgs"* **A. Drozd, B. Grzadkowski, J. F. Gunion and Y. Jiang. arXiv:1211.3580 [hep-ph]**

Basic Features of the Higgs-like LHC Excesses at $125 - 126$ GeV

- It is conventional to reference the SM expectations by defining the R ratios, called μ ratios by the experimentalists:

$$R^h_Y(X) = \frac{\sigma(pp \to Y \to h)\mathrm{BR}(h \to X)}{\sigma(pp \to Y \to h_{\mathrm{SM}})\mathrm{BR}(h_{\mathrm{SM}} \to X)}\,, \quad R^h(X) = \sum_Y R^h_Y\,, \tag{222}$$

  where $Y = gg$, VV, Vh or $t\bar{t}h$. The notation $\mu \equiv R$ is employed by the experimental groups.

  A brief summary:

  – ATLAS sees $\mu_{\mathrm{ggF}}(\gamma\gamma) > 1$ and $\mu_{\mathrm{VBF}}(\gamma\gamma) > 1$.

  – CMS MVA analysis finds $\mu_{\mathrm{ggF}}(\gamma\gamma) < 1$, although still within errors of the SM value of 1. However, they do find $\mu_{\mathrm{VBF}}(\gamma\gamma) > 1$.

  – ATLAS sees $\mu_{\mathrm{ggF}}(4\ell) > 1$ and $\mu_{\mathrm{VBF}}(4\ell) > 1$.

  – CMS MVA analysis yields very SM-like values for the $ZZ \to 4\ell$ rates in ggF and VBF.

  Sample plots are:

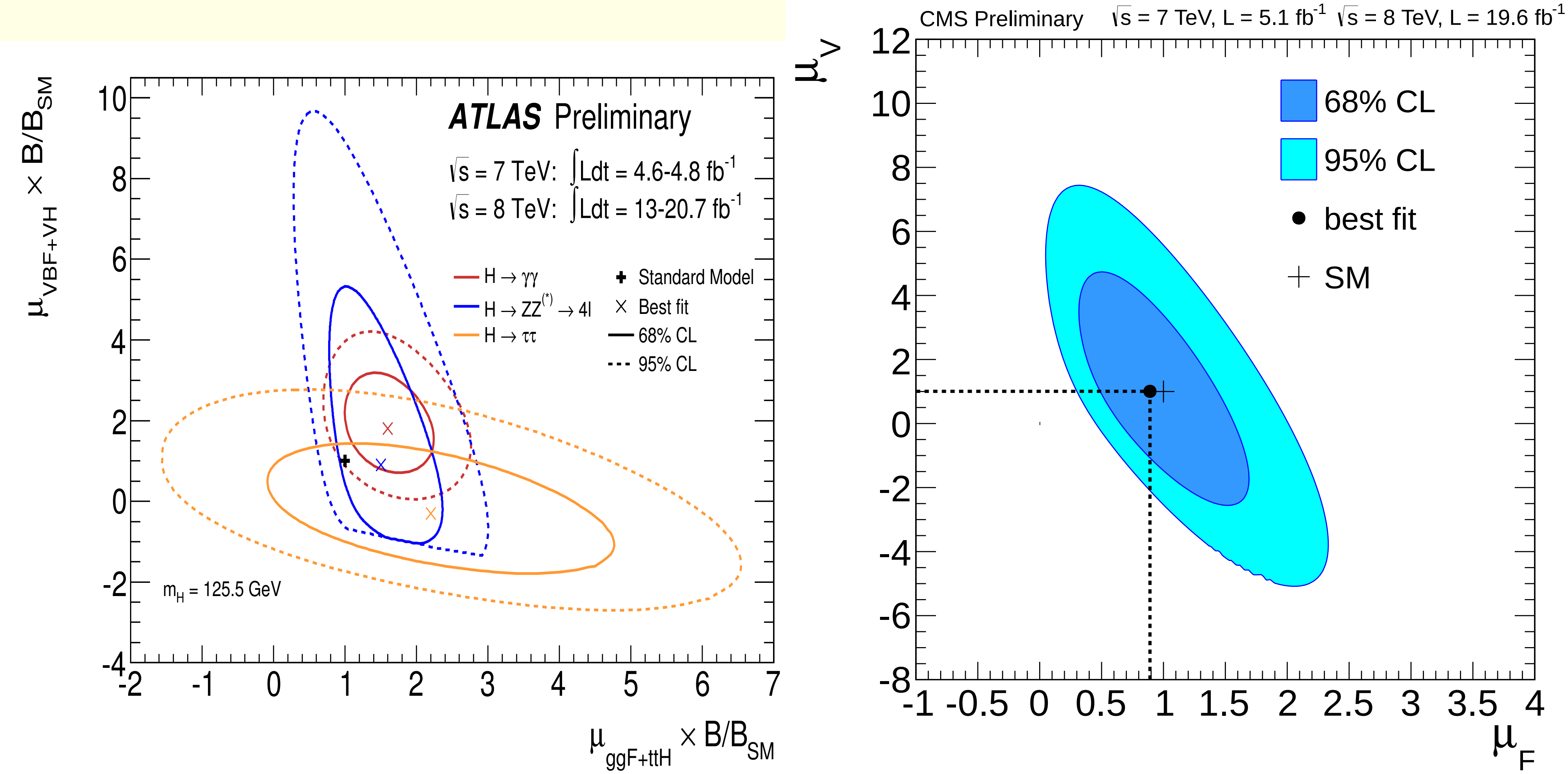


Figure 17: **Left: ATLAS results, including 4ℓ and $\gamma\gamma$. Right: CMS results for 4ℓ. Vertical axis is VBF and horizontal is ggF.**

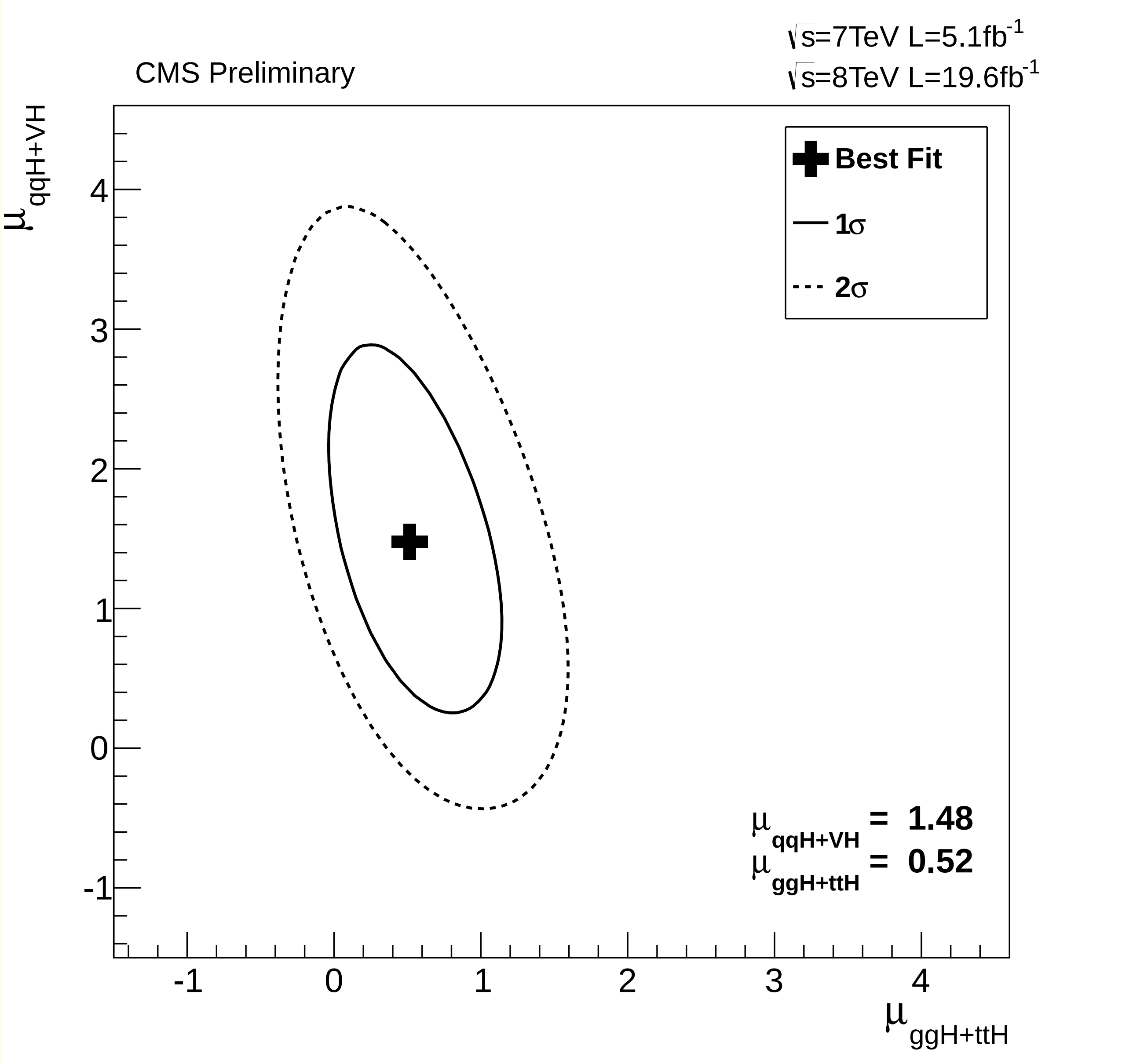


Figure 18: CMS MVA results for $\gamma\gamma$. Vertical axis is VBF and horizontal is ggF. Note $\mu_{\rm ggF} < 1$ bu $\mu_{\rm VBF} > 1$.

On the other hand there is a 2nd CMS analysis that gives a larger $\gamma\gamma$ signal. Compare:

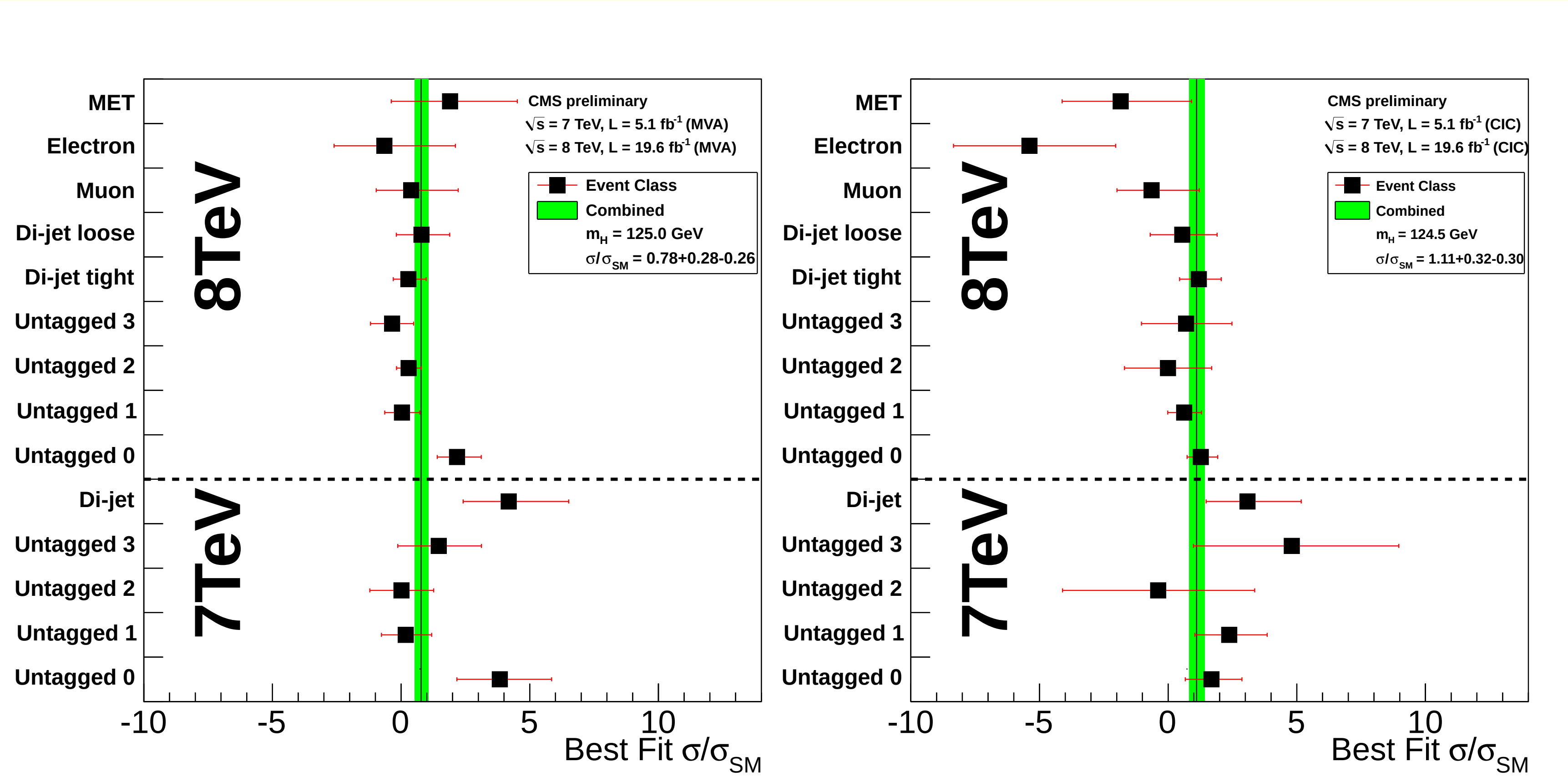


Figure 19: Left: MVA analysis results with overall $\mu = 0.78$. Right: CiC (cut-based) analysis results. CiC analysis shows overall enhancement in $\gamma\gamma$ of $\mu = 1.11$. CMS quotes a discrepancy of 1.8σ between the two analyses.

- **The big questions:**
  1. **If the deviations from a single SM Higgs survive what is the model?**

     And, how far beyond the "standard" model must we go to describe them?
  2. **If all results become SM-like, how can we be sure that we are seeing just a SM-like Higgs boson?**

     Yes, there are complicated Higgs models that can give SM-like rates for most, or even all, channels.

Higgs coupling fits

- Suppose the signal derives from just one Higgs boson — we assume 0^+.
- The structure we will test is that given earlier:

$$\begin{aligned}\mathcal{L} &= g\Big[C_V\left(m_W W_\mu W^\mu + \frac{m_Z}{\cos\theta_W} Z_\mu Z^\mu\right) \\ &\quad - C_U \frac{m_t}{2m_W}\bar{t}t - C_D \frac{m_b}{2m_W}\bar{b}b - C_D \frac{m_\tau}{2m_W}\bar{\tau}\tau\Big] H \,. \qquad (223)\end{aligned}$$

  In general, the C_I can take on negative as well as positive values; there is one overall sign ambiguity which we fix by taking $C_V > 0$.
- We will be fitting the data summarized earlier (using CMS MVA analysis results for $\gamma\gamma$).

- **In addition to the tree-level couplings given above, the H has couplings to gg and $\gamma\gamma$ that are first induced at one loop and are completely computable in terms of C_U, C_D and C_V if only loops containing SM particles are present.**

  We define $\overline{C}_g$ and $\overline{C}_\gamma$ to be the ratio of these couplings so computed to the SM ($i.e.$ $C_U = C_D = C_V = 1$) values.
- **However, in some of our fits we will also allow for additional loop contributions ΔC_g and ΔC_γ from new particles; in this case $C_g = \overline{C}_g + \Delta C_g$ and $C_\gamma = \overline{C}_\gamma + \Delta C_\gamma$.**
- **The largest set of independent parameters that we might wish to consider is thus:**

$$C_U,\ C_D,\ C_V,\ \Delta C_g,\ \Delta C_\gamma\,. \tag{224}$$

- **Fit I: $C_U = C_D = C_V = 1$, ΔC_g and ΔC_γ free.**

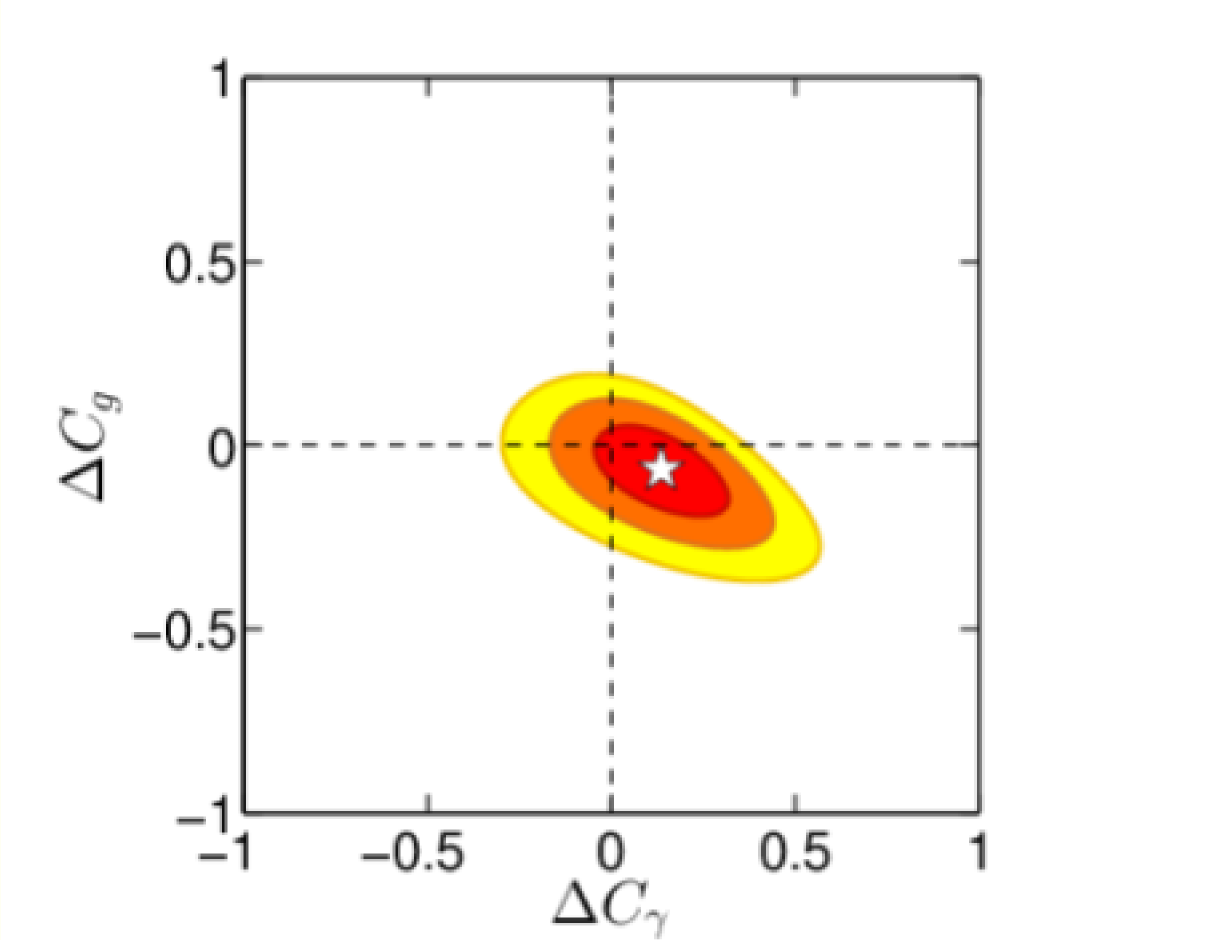


Figure 20: Two parameter fit of ΔC_γ and ΔC_g, assuming $C_U = C_D = C_V = 1$ (Fit I). The red, orange and yellow ellipses show the 68%, 95% and 99.7% CL regions, respectively. The white star marks the best-fit point. Looking quite SM-like when all ATLAS and CMS data are combined.

- **Fit II: varying C_U, C_D and C_V ($\Delta C_\gamma = \Delta C_g = 0$)**

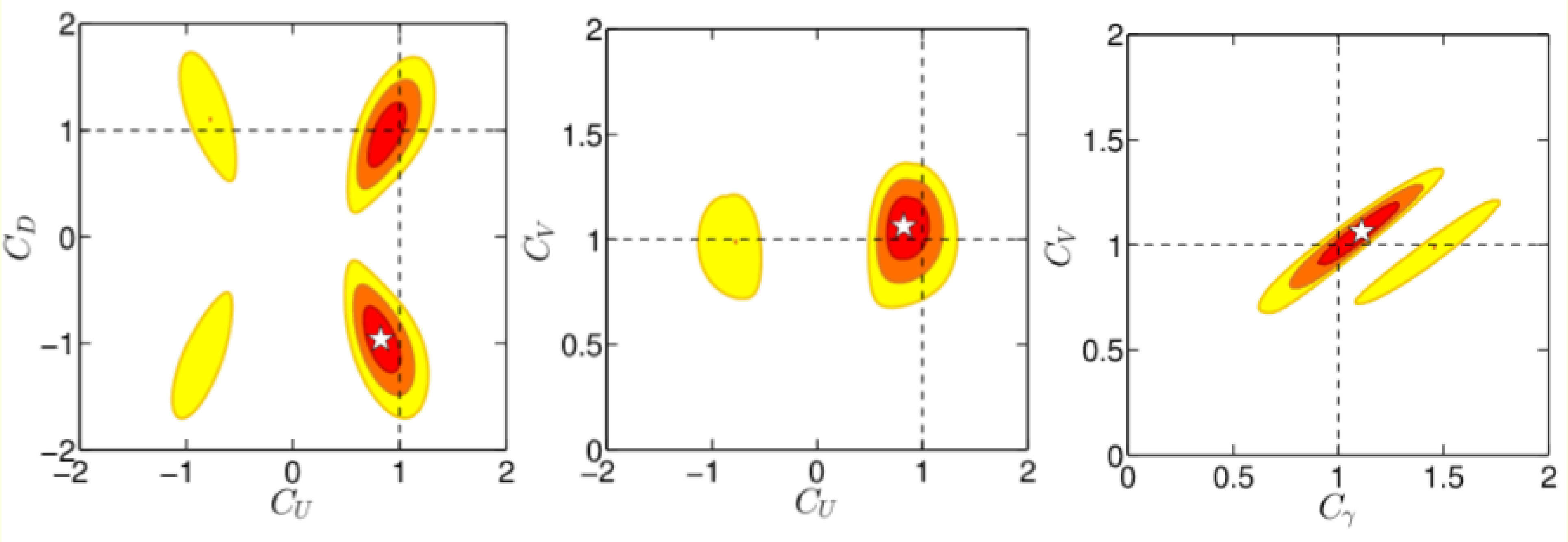


Figure 21: Two-dimensional χ^2 distributions for the three parameter fit, Fit II, of C_U, C_D, C_V with $C_\gamma = \overline{C}_\gamma$ and $C_g = \overline{C}_g$ as computed in terms of C_U, C_D, C_V. Unlike earlier fits that did not include CMS MVA $\gamma\gamma$ results, $C_U > 0$ is now preferred since overall there is no $\gamma\gamma$ enhancement in ggF after "averaging" ATLAS and CMS.

Fit	Standard Model	ΔC_γ, ΔC_g	C_U, C_D, C_V
$\chi^2_{\rm min}$ $\chi^2_{\rm min}$/d.o.f.	21.6 0.90	20.1 0.91	19.9 0.94
dominant contributions to $\chi^2_{\rm min}$	ATLAS $\gamma\gamma$ Tevatron $\gamma\gamma$ ATLAS ZZ	ATLAS ZZ CMS $\gamma\gamma$ ATLAS $\gamma\gamma$	ATLAS $\gamma\gamma$ CMS WW VBF Tevatron $\gamma\gamma$

- **There is no improvement in $\chi^2/d.o.f.$ as freedom is introduced, $i.e.$ the lowest p value is achieved in the SM!**

  Allowing all five parameters, $C_U, C_D, C_V, \Delta C_\gamma, \Delta C_g$ to vary again worsens the p value, unlike earlier "end of 2012" fits.
- **Thus, perhaps there is no need for a mechanism that would yield enhanced $\mu = R$ values.**

  However, the fits above only reflect some average properties and it could be that individual channels ($e.g.$ the VBF$\to H \to \gamma\gamma$) will in the end turn out to be enhanced.

Multiple Higgs Models

- **Let us suppose that the "final" results for the Higgs signals cannot be fit by the SM Higgs.**

  At the moment there are many hints that this could be the case despite the fact that the average result is close to SM-like.

- **This would make it natural to consider models in which there is more than one Higgs boson. Some Higgs could dominate one kind of signal and other Higgs could dominate another kind of signal. Such models include:**

  1. **Two-Higgs Doublet Models (2HDM)**

     In this model the one-doublet complex Higgs field of the SM is replicated and each of the neutral components of the two doublet fields acquires a vacuum expectation value: we have v_1 and v_2.

An important parameter of such a model is is $\tan\beta = v_2/v_1$ — $v_1^2 + v_2^2 = v^2 = (246~\mathrm{GeV})^2$ is required to get the W, Z masses right.

2 complex doublets have 8 degrees of freedom, of which only 3 are absorbed or "eaten" in giving the $W^\pm, Z$ their masses. The remaining 5 d.o.f. become physical scalar particles:

$$\text{CP-even}: \; h, H\,, \quad \text{CP-odd}: \; A\,, \quad \text{charged pair}: \; H^\pm \tag{225}$$

2. **Minimal Supersymmetric Model (MSSM)**
   The Higgs sector is just a constrained version of the 2HDM model category. No additional Higgs bosons. However, the SUSY constraints are such that it hard to get a CP-even Higgs boson with SM-like properties without going to extremes.
3. **Adding additional doublets to the 2HDM or MSSM**

Makes a mess of gauge coupling unification in the MSSM case.

4. **Adding additional singlets to a 2HDM or the MSSM**

   Every additional complex singlet yields one more CP-even H and one more CP-odd A.

   No impact on gauge unification since it is a $singlet$ that is being added.

   – A particularly attractive version is the Next-to-Minimal Supersymmetric Model (NMSSM).

     Getting the lightest CP-even Higgs to be as heavy as $\mathbf{125}~\mathrm{GeV}$ does not require extremes. It is an altogether beautiful model.

Enhanced Higgs signals in the NMSSM

- NMSSM=MSSM+$\widehat{S}$.
- The extra complex S component of $\widehat{S}$ $\Rightarrow$ the NMSSM has h_1, h_2, h_2, a_1, a_2.
- The new NMSSM parameters of the superpotential (λ and κ) and scalar potential (A_λ and A_κ) appear as:

$$W \ni \lambda \widehat{S} \widehat{H}_u \widehat{H}_d + \frac{\kappa}{3} \widehat{S}^3 \,, \quad V_{\rm soft} \ni \lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3 \tag{226}$$

- $\langle S \rangle \neq 0$ is generated by SUSY breakng and solves μ problem: $\mu_{\rm eff} = \lambda \langle S \rangle$.
- First question: Can the NMSSM give a Higgs mass as large as $125~{\rm GeV}$?

  Answer: Yes, so long as parameters at the GUT scale are not fully unified. For our studies, we employed universal m_0, except for NUHM ($m^2_{H_u}$, $m^2_{H_d}$, m^2_S free), universal $A_t = A_b = A_\tau = A_0$ but allow A_λ and A_κ to vary freely. Of course, $\lambda > 0$ and κ are scanned demanding perturbativity up to the GUT scale.

- Can this model achieve rates in $\gamma\gamma$ and 4ℓ that are $>$SM?

  Answer: It depends on whether or not we require a good prediction for the muon anomalous magnetic moment, a_μ.

- The possible $R(\gamma\gamma) > 1$ mechanism (arXiv:1112.3548, Ellwanger) is to reduce the $b\overline{b}$ width of the mainly SM-like Higgs by giving it some singlet component. The gg and $\gamma\gamma$ couplings are less affected.

- Typically, this requires m_{h_1} and m_{h_2} to have similar masses (for singlet-doublet mixing) and large λ (to enhance Higgs mass).

  Large λ (by which we mean $\lambda > 0.1$) is only possible while retaining perturbativity up to $M_{\rm P}$ if $\tan\beta$ is modest in size.

In the semi-unified model we employ, enhanced rates and/or large λ cannot be made consistent with decent δa_μ. (J. F. Gunion, Y. Jiang and S. Kraml.arXiv:1201.0982 [hep-ph])

- **Some illustrative R_{gg} results from** (J. F. Gunion, Y. Jiang and S. Kraml. arXiv:1207.1545):

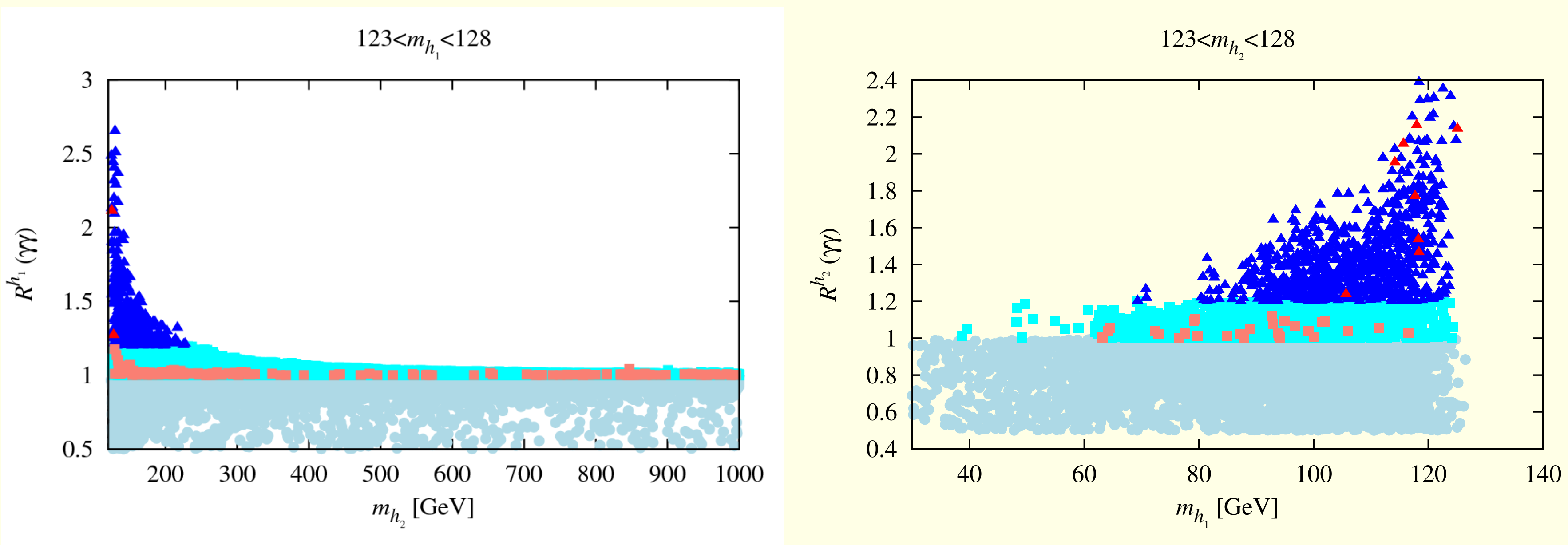


Figure 22: The plot shows $R_{gg}(\gamma\gamma)$ for the cases of $123 < m_{h_1} < 128$ GeV and $123 < m_{h_2} < 128$ GeV. Note: red triangle (orange square) is for WMAP window with $R_{gg}(\gamma\gamma) > 1.2$ ($R_{gg}(\gamma\gamma) = [1, 1.2]$).

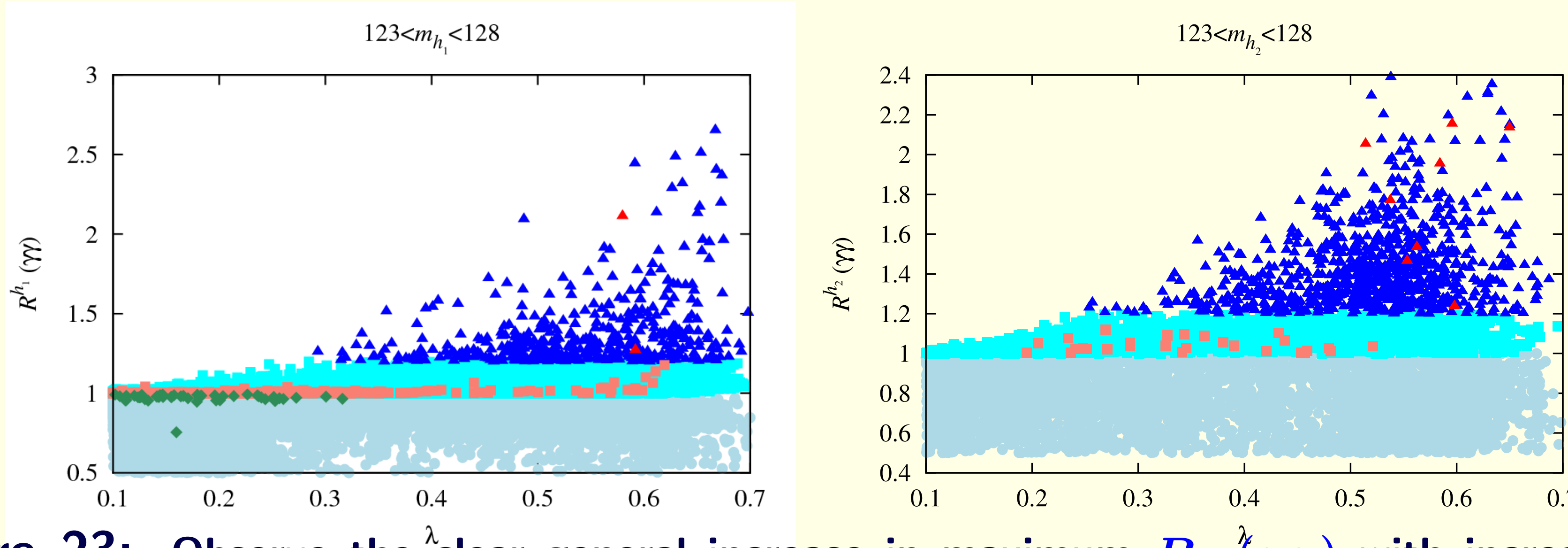


Figure 23: **Observe the clear general increase in maximum $R_{gg}(\gamma\gamma)$ with increasing λ. Green points have good δa_μ, $m_{h_2} > 1\ \mathrm{TeV}$ BUT $R_{gg}(\gamma\gamma) \sim 1$.**

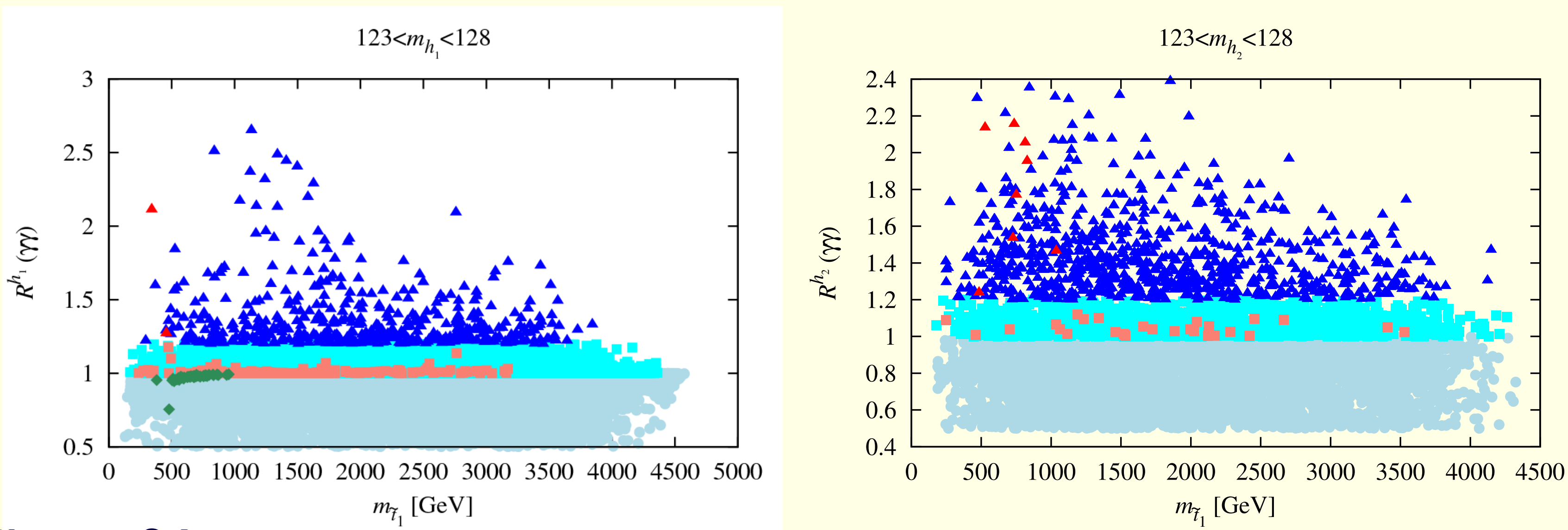


Figure 24: **The lightest stop has mass $\sim 300 - 700\ \mathrm{GeV}$ for red-triangle points.**

- If we ignore δa_μ, then $R_{gg}(\gamma\gamma) > 1.2$ (even > 2) is possible while satisfying all other constraints provided h_1 and h_2 are close in mass, especially in the case where $m_{h_2} \in [123, 128]~\mathrm{GeV}$ window.
- This raises the issue of scenarios in which *both* m_{h_1} and m_{h_2} are in the $[123, 128]~\mathrm{GeV}$ window where the experiments see the Higgs signal.

The ideas and issues related to degeneracy:

  - If h_1 and h_2 are sufficiently degenerate, the experimentalists might not have resolved the two distinct peaks, even in the $\gamma\gamma$ channel.
  - The rates for the h_1 and h_2 could then add together to give an enhanced $\gamma\gamma$ signal.
  - The apparent width or shape of the $\gamma\gamma$ mass distribution could be altered.
  - There is more room for an apparent mismatch between the

$\gamma\gamma$ **channel and other channels, such as** $b\overline{b}$ **or** 4ℓ**, than in non-degenerate situation.**

In particular, the h_1 **and** h_2 **will generally have different** gg **and** VV **production rates and branching ratios.**

Degenerate NMSSM Higgs Scenarios:

(arXiv:1207.1545, JFG, Jiang, Kraml)

- For the numerical analysis, we used NMSSMTools version 3.2.0, which has improved convergence of RGEs in the case of large Yukawa couplings.
- The precise constraints imposed are the following.

1. Basic constraints: proper RGE solution, no Landau pole, neutralino LSP, Higgs and SUSY mass limits as implemented in NMSSMTools-3.2.0.
2. B physics:
3. Dark Matter: $\Omega h^2 < 0.136$ — allows for scenarios in which the relic density arises in part from some other source. However, we single out points with $0.094 \leq \Omega h^2 \leq 0.136$, which is the 'WMAP window' defined in NMSSMTools-3.2.0.
4. 2011 XENON100: spin-independent LSP–proton scattering

cross section bounds implied by the neutralino-mass-dependent XENON100 bound. (2012 XENON100 has little additional impact.)

5. δa_μ ignored: impossible to satisfy for scenarios studied here.

- Compute the effective Higgs mass in given production and final decay channels Y and X, respectively, and R^h_{gg} as

$$m_h^Y(X) \equiv \frac{R_Y^{h_1}(X) m_{h_1} + R_Y^{h_2}(X) m_{h_2}}{R_Y^{h_1}(X) + R_Y^{h_2}(X)} \quad R_Y^h(X) = R_Y^{h_1}(X) + R_Y^{h_2}(X)\,. \quad (227)$$

- The extent to which it is appropriate to combine the rates from the h_1 and h_2 depends upon the degree of degeneracy and the experimental resolution. Very roughly, one should probably think of $\sigma_{\rm res} \sim 1.5~\mathrm{GeV}$ or larger. The widths of the h_1 and h_2 are very much smaller than this resolution.
- We only display points which pass constraints listed earlier, *and* have $123~\mathrm{GeV} < m_{h_1}, m_{h_2} < 128~\mathrm{GeV}$.

• **Many of the displayed points have $R^{h_1}_{gg}(\gamma\gamma) + R^{h_2}_{gg}(\gamma\gamma) > 1$.**

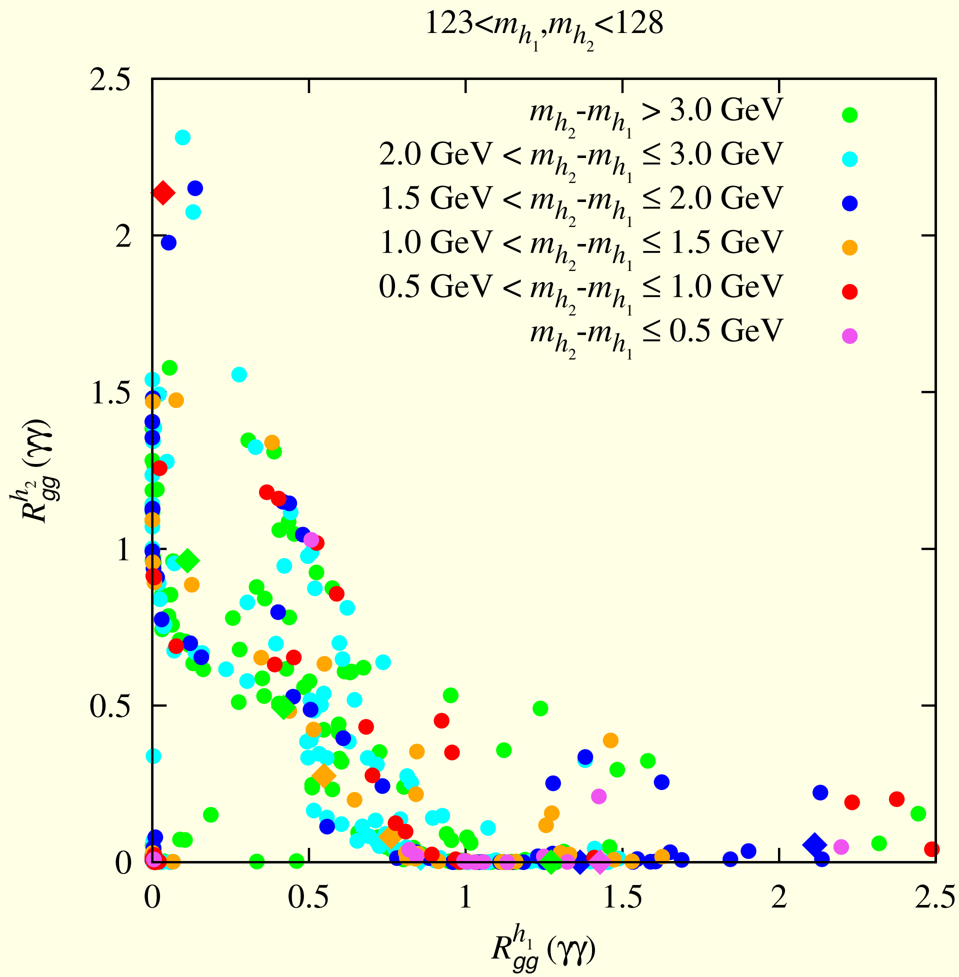


Figure 25: **Correlation of $gg \to (h_1, h_2) \to \gamma\gamma$ signal strengths when both h_1 and h_2 lie in the 123–128 GeV mass range. The circular points have $\Omega h^2 < 0.094$, while diamond points have $0.094 \leq \Omega h^2 \leq 0.136$. Points are color coded according to $m_{h_2} - m_{h_1}$. Probably green and cyan points can be resolved in mass.**

Now combine the h_1 and h_2 signals. Color code:

1. red for $m_{h_2} - m_{h_1} \leq 1~\mathrm{GeV}$;
2. blue for $1~\mathrm{GeV} < m_{h_2} - m_{h_1} \leq 2~\mathrm{GeV}$;
3. green for $2~\mathrm{GeV} < m_{h_2} - m_{h_1} \leq 3~\mathrm{GeV}$.

- For current statistics and $\sigma_{\mathrm{res}} \gtrsim 1.5~\mathrm{GeV}$ we estimate that the h_1 and h_2 signals will not be seen separately for $m_{h_2} - m_{h_1} \leq 2~\mathrm{GeV}$.
- In Fig. 26, we show results for $R^h_{gg}(X)$ for $X = \gamma\gamma, VV, b\overline{b}$. Enhanced $\gamma\gamma$ and VV rates from gluon fusion are very common.
- The bottom-right plot shows that enhancement in Vh with $h \to b\overline{b}$ rate is also natural, though not as large as the best fit value suggested by the new Tevatron analysis.
- Diamond points ($i.e.$ those in the WMAP window) are rare, but typically show enhanced rates.

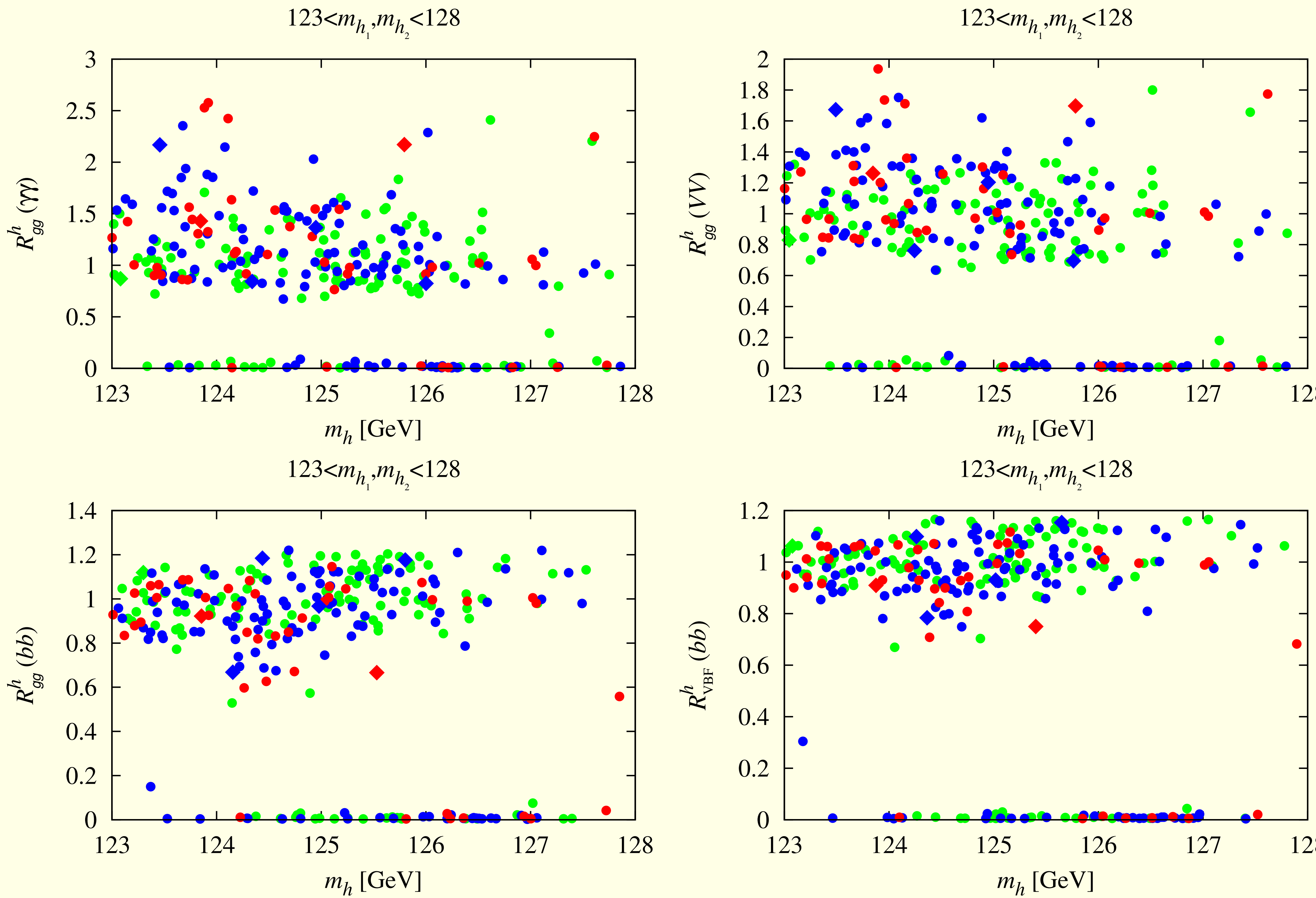


Figure 26: $R^h_{gg}(X)$ **for** $X = \gamma\gamma, VV, b\bar{b}$**, and** $R^h_{\mathrm{VBF}}(b\bar{b})$ **versus** m_h**. For application to the Tevatron, note that** $R^h_{\mathrm{VBF}}(b\bar{b}) = R^h_{V^*\to Vh}(b\bar{b})$**.**

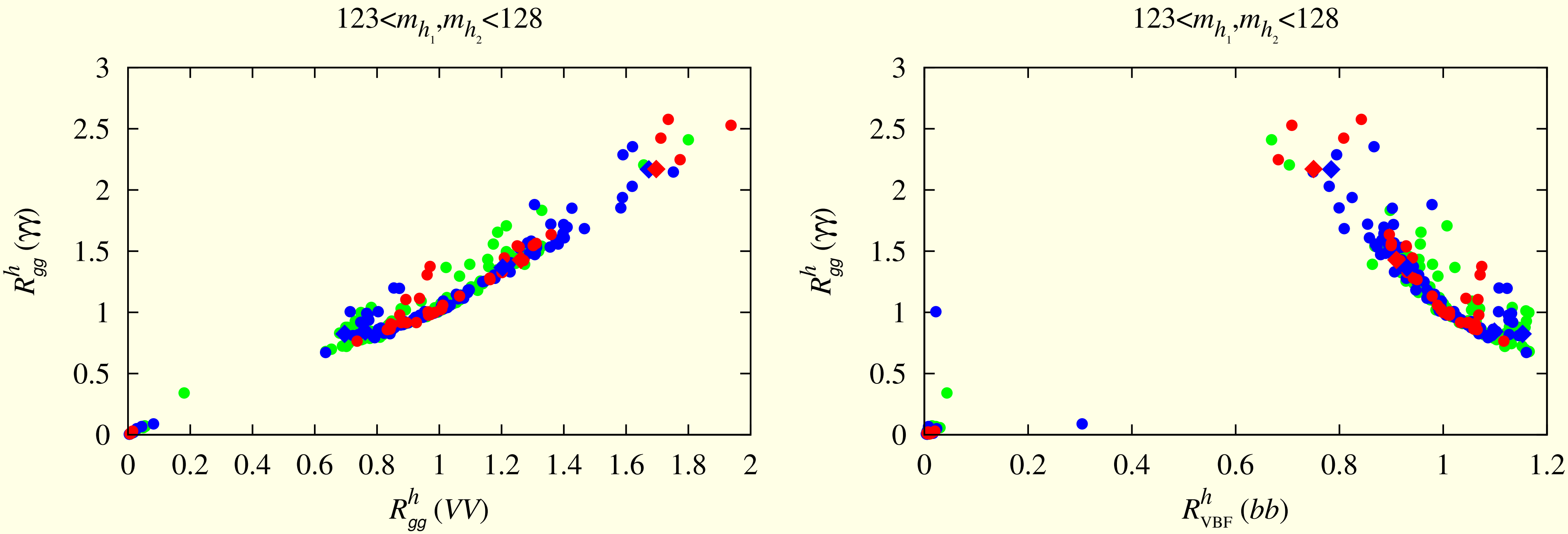
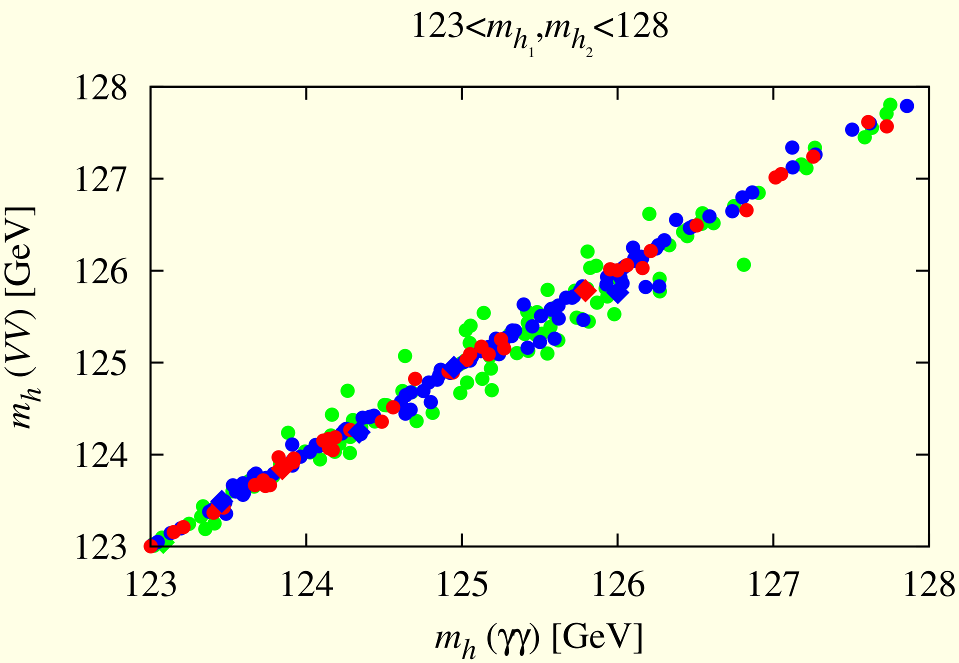


Figure 27: **Left: correlation between the gluon fusion induced $\gamma\gamma$ and VV rates relative to the SM. Right: correlation between the gluon fusion induced $\gamma\gamma$ rate and the VV fusion induced $b\overline{b}$ rates relative to the SM; the relative rate for $V^* \to Vh$ with $h \to b\overline{b}$ (relevant for the Tevatron) is equal to the latter.**

- **Comments on Fig. 27:**
  1. **Left-hand plot shows the strong correlation between $R^h_{gg}(\gamma\gamma)$ and $R^h_{gg}(VV)$.**

Note that if $R^h_{gg}(\gamma\gamma) \sim 1.3$ (as for ATLAS) then in this model $R^h_{gg}(VV) \geq 1$.

2. The right-hand plot shows the (anti) correlation between $R^h_{gg}(\gamma\gamma)$ and $R^h_{V^*\to Vh}(b\overline{b}) = R^h_{\rm VBF}(b\overline{b})$.
   In general, the larger $R^h_{gg}(\gamma\gamma)$ is, the smaller the value of $R^h_{V^*\to Vh}(b\overline{b})$.
3. It is often the case that one of the h_1 or h_2 dominates $R^h_{gg}(\gamma\gamma)$ while the other dominates $R^h_{V^*\to Vh}(b\overline{b})$.
   However, a significant number of the points are such that either the $\gamma\gamma$ or the $b\overline{b}$ signal receives substantial contributions from both the h_1 and the h_2.
   We did not find points where the $\gamma\gamma$ and $b\overline{b}$ final states $both$ receive substantial contributions from $both$ the h_1 and h_2.



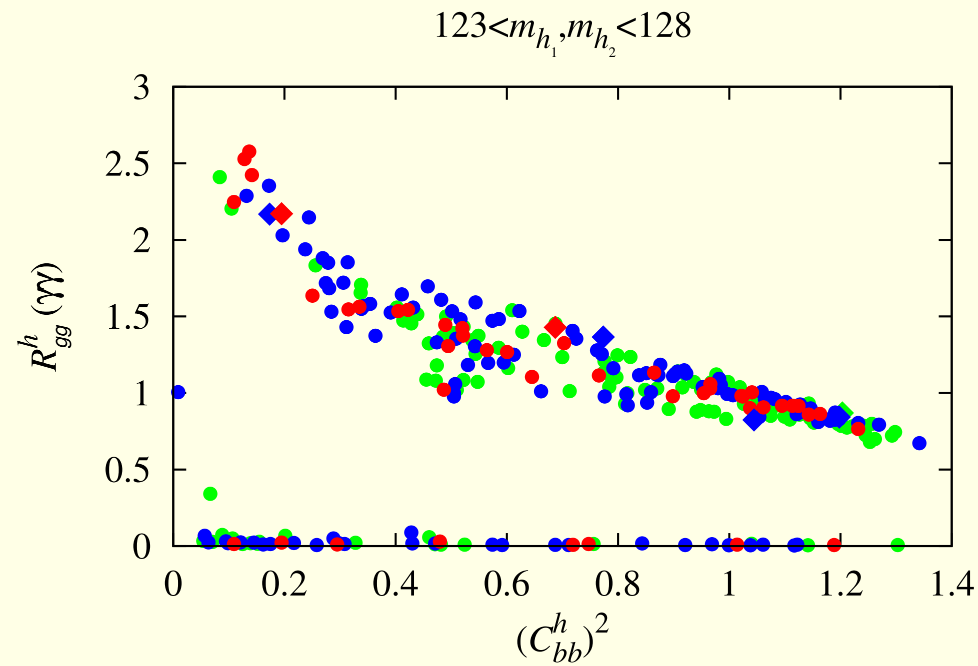


Figure 28: **Left: effective Higgs masses obtained from different channels: $m_h^{gg}(\gamma\gamma)$ versus $m_h^{gg}(VV)$. Right: $\gamma\gamma$ signal strength $R^h_{gg}(\gamma\gamma)$ versus effective coupling to $b\bar b$ quarks $(C^h_{b\bar b})^2$. Here, ${C^h_{b\bar b}}^2 \equiv \left[R^{h_1}_{gg}(\gamma\gamma){C^{h_1}_{b\bar b}}^2 + R^{h_2}_{gg}(\gamma\gamma){C^{h_2}_{b\bar b}}^2\right] / \left[R^{h_1}_{gg}(\gamma\gamma) + R^{h_2}_{gg}(\gamma\gamma)\right]$.**

Comments on Fig. 28

1. **The m_h values for the gluon fusion induced $\gamma\gamma$ and VV cases are also strongly correlated — in fact, they differ by**

no more than a fraction of a GeV and are most often much closer, see the left plot of Fig. 28.

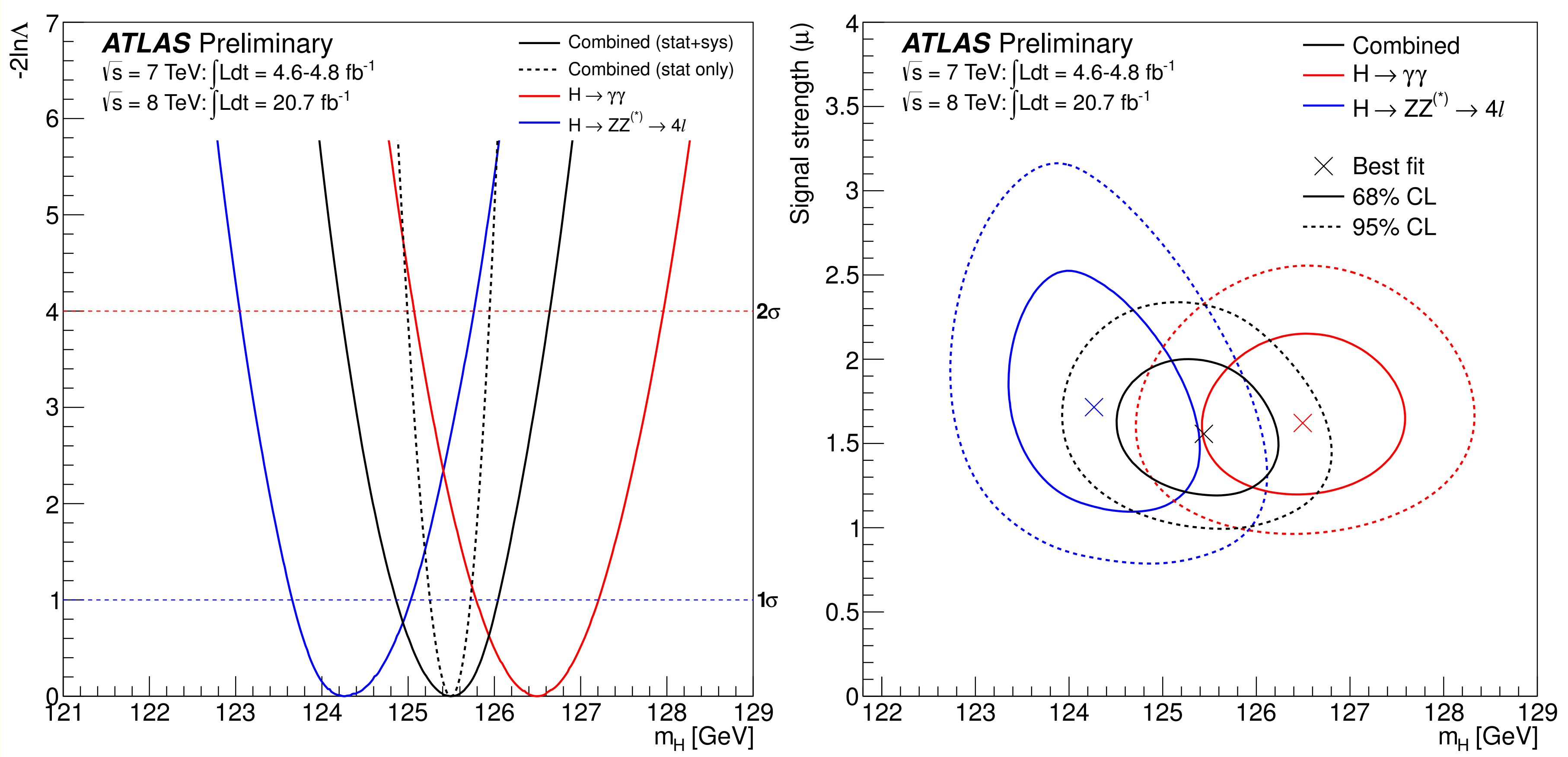
2. The right plot of Fig. 28 illustrates the mechanism behind enhanced rates, namely that large net $\gamma\gamma$ branching ratio is achieved by reducing the average total width by reducing the average $b\bar{b}$ coupling strength.

- Although we have emphasized that degeneracy can easily lead to enhanced signals, it is equally true that a pair of degenerate Higgs could easily yield a SM-like signal.

  For example, points with $R_{gg}(\gamma\gamma) \sim 1$ are easily found in Fig. 25. And, Fig. 26 shows that other rates will often be SM-like at the same time.

- Either way, an important question is: how can we check for underlying degeneracy? This will be discussed later.

Separate Mass Peaks for ZZ vs. $\gamma\gamma$



- h_1 should have $m_{h_1} \sim 124.2~\mathrm{GeV}$ and ZZ rate not too much smaller than SM-like rate, but suppressed $\gamma\gamma$ rate.
- h_2 should have $m_{h_2} \sim 126.5~\mathrm{GeV}$, enhanced $\gamma\gamma$ rate and somewhat suppressed ZZ rate.

- **The kind of extreme apparently seen by ATLAS is hard to arrange in the NMSSM.**

  This is because the mechanism for getting enhanced $\gamma\gamma$ (suppression of bb partial width through mixing) automatically also enhances ZZ. Recall the correlation plot given earlier

- **To assess a bit more quantitatively, we compute $m_h(VV)$ vs. $m_h(\gamma\gamma)$ using previous formula involving weighting by $R_{gg}^{h_1,h_2}(ZZ)$ and $R_{gg}^{h_1,h_2}(\gamma\gamma)$ and accepting points with $121~\mathrm{GeV} \leq m_{h_1}, m_{h_2} \leq 128~\mathrm{GeV}$.**

  Or, selecting points with $122~\mathrm{GeV} < m_{h_1} < 124~\mathrm{GeV}$ and $125~\mathrm{GeV} < m_{h_2} < 127~\mathrm{GeV}$.

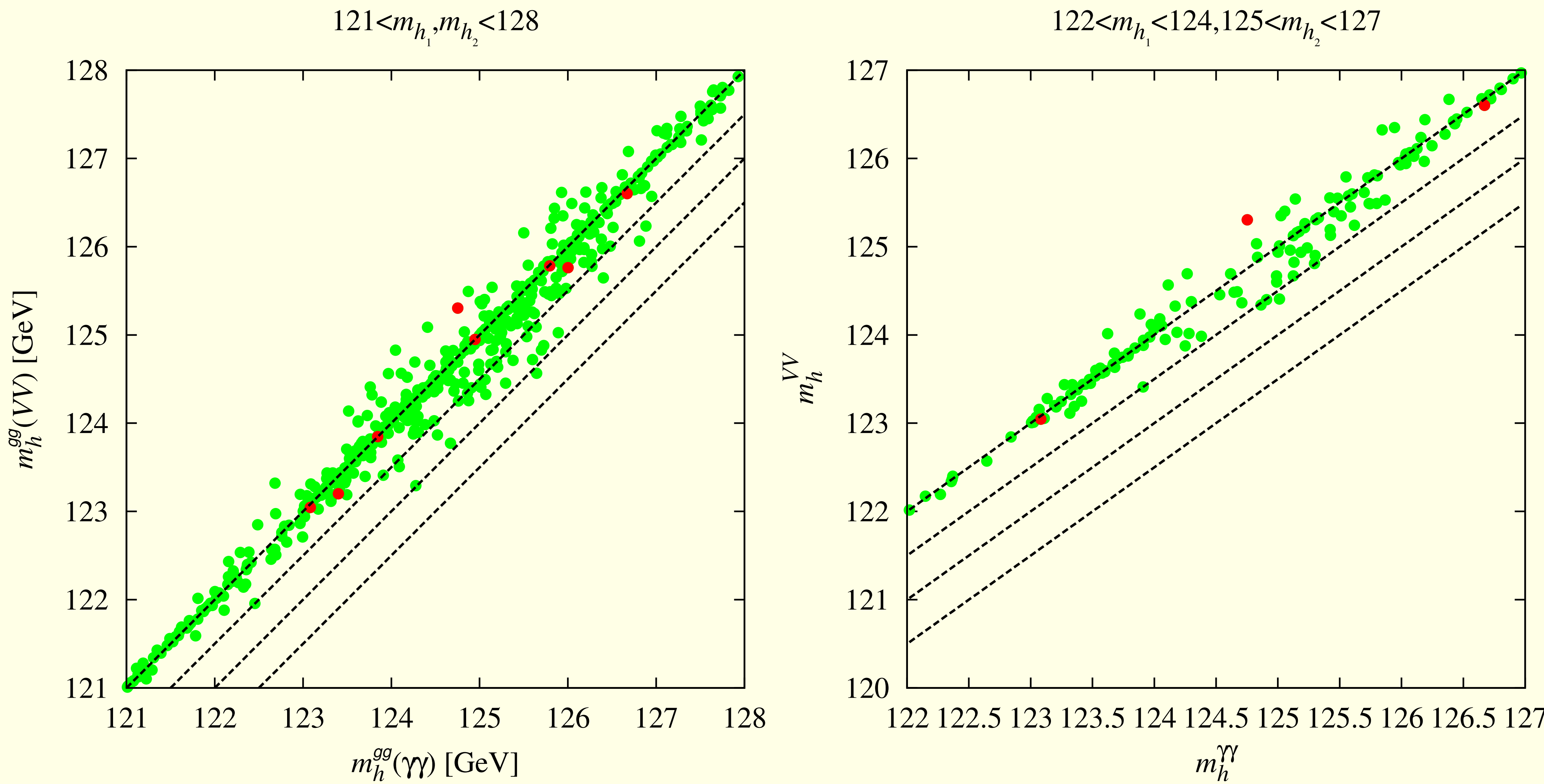


Figure 29: m_h obtained in ZZ vs. $\gamma\gamma$ final state when scanning and requiring: $121~\mathrm{GeV} \leq m_{h_1}, m_{h_2} \leq 128~\mathrm{GeV}$ (Left) or $122~\mathrm{GeV} < m_{h_1} < 124~\mathrm{GeV}$, $125~\mathrm{GeV} < m_{h_2} < 127~\mathrm{GeV}$ (Right). Dashed lines show $m_h(ZZ) = m_h(\gamma\gamma) - (0, 0.5, 1, 1.5)$. Hard to get a mass shift of more than $1~\mathrm{GeV}$.

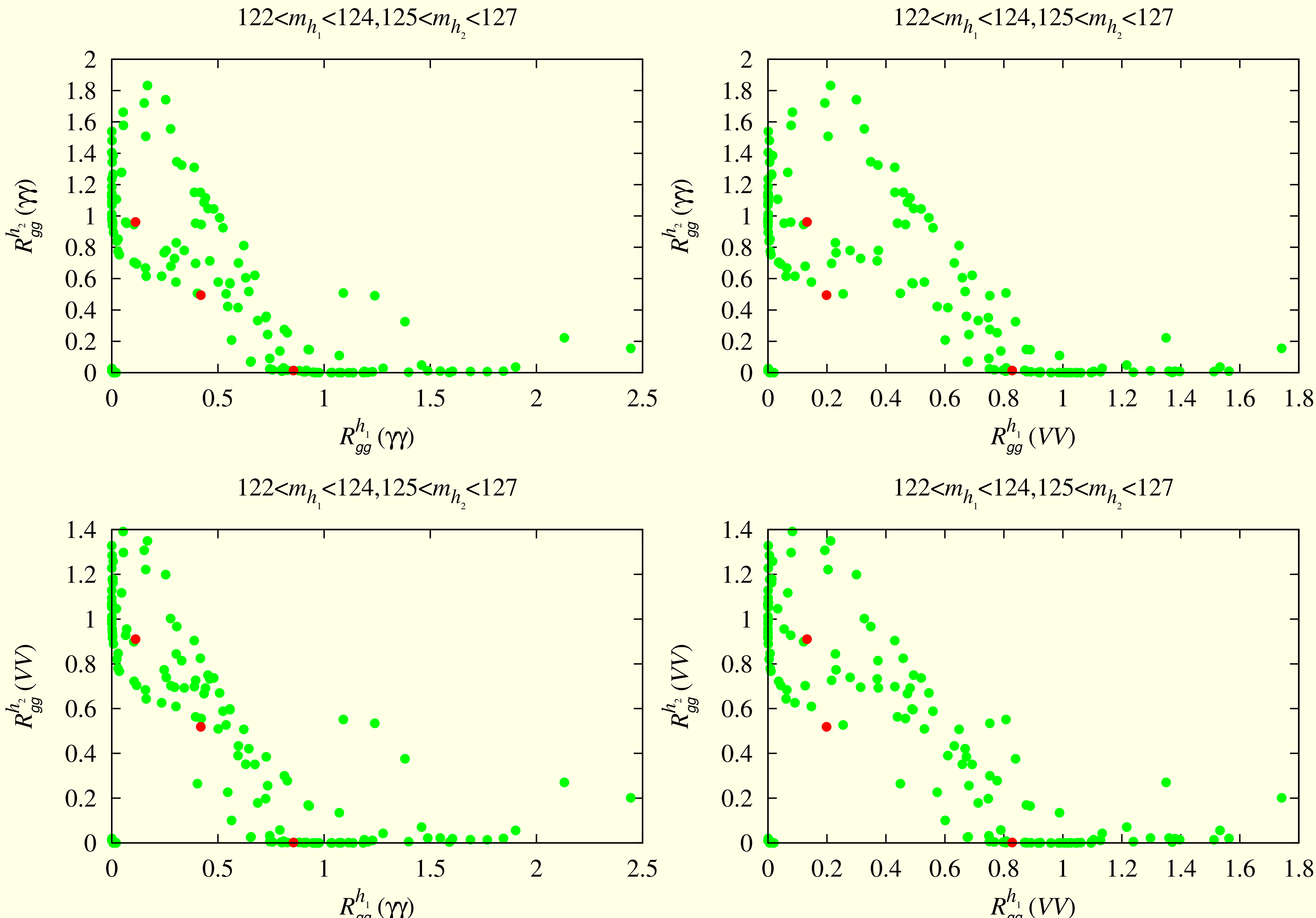


Figure 30: **Too much correlation between VV and $\gamma\gamma$ channels for the h_1 and h_2 separately.**

Diagnosing the presence of degenerate Higgses

(J. F. Gunion, Y. Jiang and S. Kraml. arXiv:1208.1817)

- Given that enhanced R^h_{gg} is very natural if there are degenerate Higgs mass eigenstates, how do we detect degeneracy if closely degenerate? Must look at correlations among different R^h's.
- In the context of any doublets plus singlets model not all the R^{h_i}'s are independent; a complete independent set of R^h's can be taken to be:

$$R^h_{gg}(VV), \quad R^h_{gg}(bb), \quad R^h_{gg}(\gamma\gamma), \quad R^h_{VBF}(VV), \quad R^h_{VBF}(bb), \quad R^h_{VBF}(\gamma\gamma)\,. \tag{228}$$

- Let us now look in more detail at a given $R^h_Y(X)$. It takes the form

$$R^h_Y(X) = \sum_{i=1,2} \frac{(C^{h_i}_Y)^2 (C^{h_i}_X)^2}{C^{h_i}_\Gamma} \tag{229}$$

where $C^{h_i}_X$ for $X = \gamma\gamma, WW, ZZ, \ldots$ is the ratio of the $h_i X$

to $h_{\rm SM} X$ coupling and $C_\Gamma^{h_i}$ is the ratio of the total width of the h_i to the SM Higgs total width.

- The diagnostic tools that can reveal the existence of a second, quasi-degenerate (but non-interfering in the small width approximation) Higgs state are the double ratios:

$$\text{I): } \frac{R^h_{VBF}(\gamma\gamma)/R^h_{gg}(\gamma\gamma)}{R^h_{VBF}(bb)/R^h_{gg}(bb)}, \quad \text{II): } \frac{R^h_{VBF}(\gamma\gamma)/R^h_{gg}(\gamma\gamma)}{R^h_{VBF}(VV)/R^h_{gg}(VV)}, \quad \text{III): } \frac{R^h_{VBF}(VV)/R^h_{gg}(VV)}{R^h_{VBF}(bb)/R^h_{gg}(bb)}, \tag{230}$$

each of which should be unity if only a single Higgs boson is present but, due to the non-factorizing nature of the sum in Eq. (229), are generally expected to deviate from 1 if two (or more) Higgs bosons are contributing to the net h signals.

- In a doublets+singlets model all other double ratios that are equal to unity for single Higgs exchange are not independent of the above three.

- Of course, the above three double ratios are not all independent. Which will be most useful depends upon the precision with which the R^h's for different initial/final states can be measured. E.g measurements of R^h for the bb final state may continue to be somewhat imprecise and it is then double ratio II) that might prove most discriminating.

Or, it could be that one of the double ratios deviates from unity by a much larger amount than the others, in which case it might be most discriminating even if the R^h's involved are not measured with great precision.

- In Fig. 31, we plot the numerator versus the denominator of the double ratios I) and II), [III) being very like I) due to the correlation between the $R^h_{gg}(\gamma\gamma)$ and $R^h_{gg}(VV)$ values discussed earlier].
- We observe that any one of these double ratios will often, but not always, deviate from unity (the diagonal dashed line in the figure).
- The probability of such deviation increases dramatically if we

require (as apparently preferred by ATLAS data) $R^h_{gg}(\gamma\gamma) > 1$, see the solid (vs. open) symbols of Fig. 31.

- This is further elucidated in Fig. 32 where we display the double ratios I) and II) as functions of $R^h_{gg}(\gamma\gamma)$ (left plots).

  For the NMSSM, it seems that the double ratio I) provides the greatest discrimination between degenerate vs. non-degenerate scenarios with values very substantially different from unity (the dashed line) for the majority of the degenerate NMSSM scenarios explored in the earlier section of this talk that have enhanced $\gamma\gamma$ rates.

  Note in particular that I), being sensitive to the $b\bar{b}$ final state, singles out degenerate Higgs scenarios even when one or the other of h_1 or h_2 dominates the $gg \to \gamma\gamma$ rate, see the top right plot of Fig. 32.

  In comparison, double ratio II) is most useful for scenarios with $R^h_{gg}(\gamma\gamma) \sim 1$, as illustrated by the bottom left plot of Fig. 32.

- **Thus, as illustrated by the bottom right plot of Fig. 32, the greatest discriminating power is clearly obtained by measuring both double ratios.**

In fact, a close examination reveals that there are no points for which $both$ double ratios are exactly 1!

Of course, experimental errors may lead to a region containing a certain number of points in which both double ratios are merely consistent with 1 within the errors.

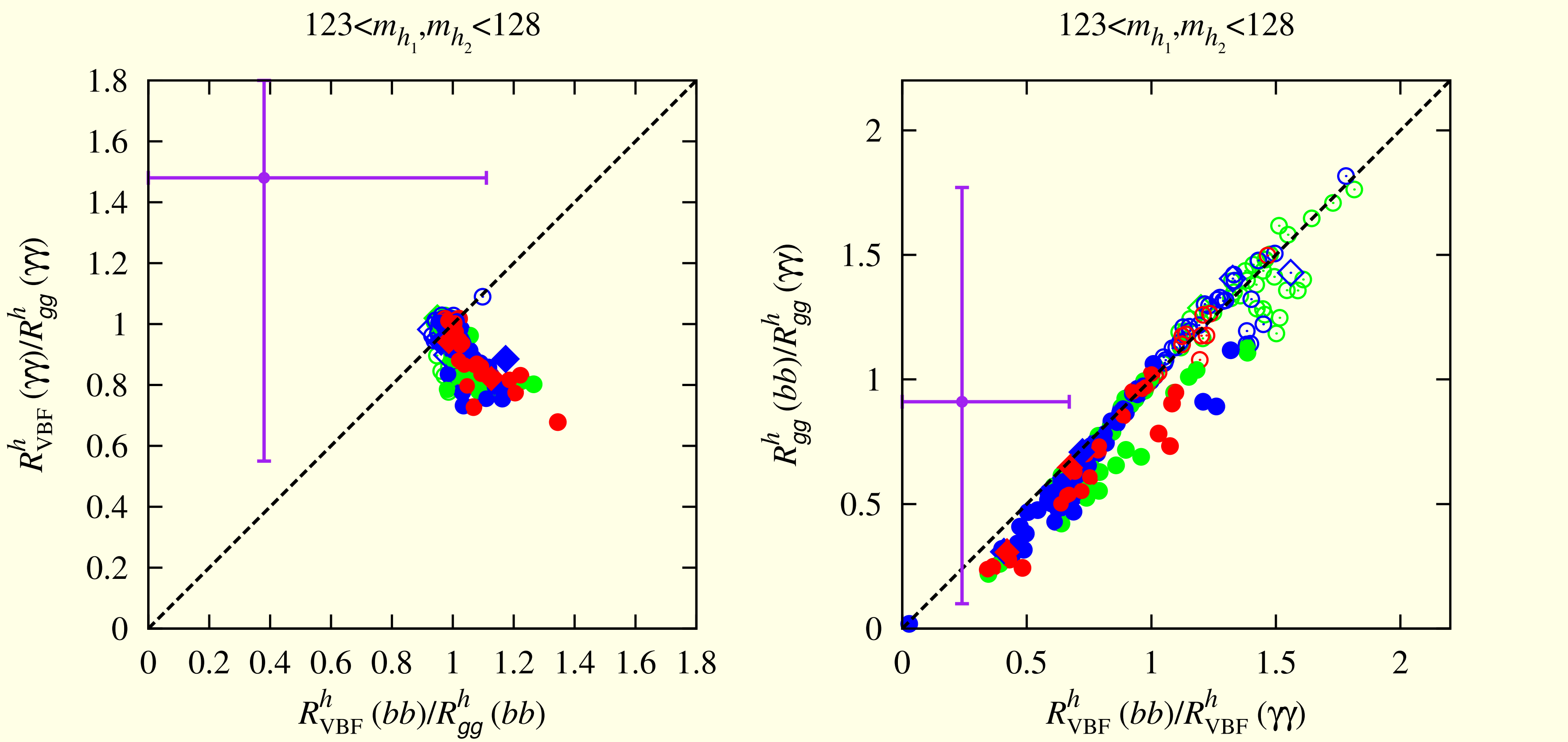


Figure 31: **Comparisons of pairs of event rate ratios that should be equal if only a single Higgs boson is present. The color code is green for points with $2~\mathrm{GeV} < m_{h_2} - m_{h_1} \leq 3~\mathrm{GeV}$, blue for $1~\mathrm{GeV} < m_{h_2} - m_{h_1} \leq 2~\mathrm{GeV}$, and red for $m_{h_2} - m_{h_1} \leq 1~\mathrm{GeV}$. Large diamond points have Ωh^2 in the WMAP window of $[0.094, 0.136]$, while circular points have $\Omega h^2 < 0.094$. Solid points are those with $R^h_{gg}(\gamma\gamma) > 1$ and open symbols have $R^h_{gg}(\gamma\gamma) \leq 1$. Current experimental values for the ratios from CMS data along with their 1σ error bars are also shown.**

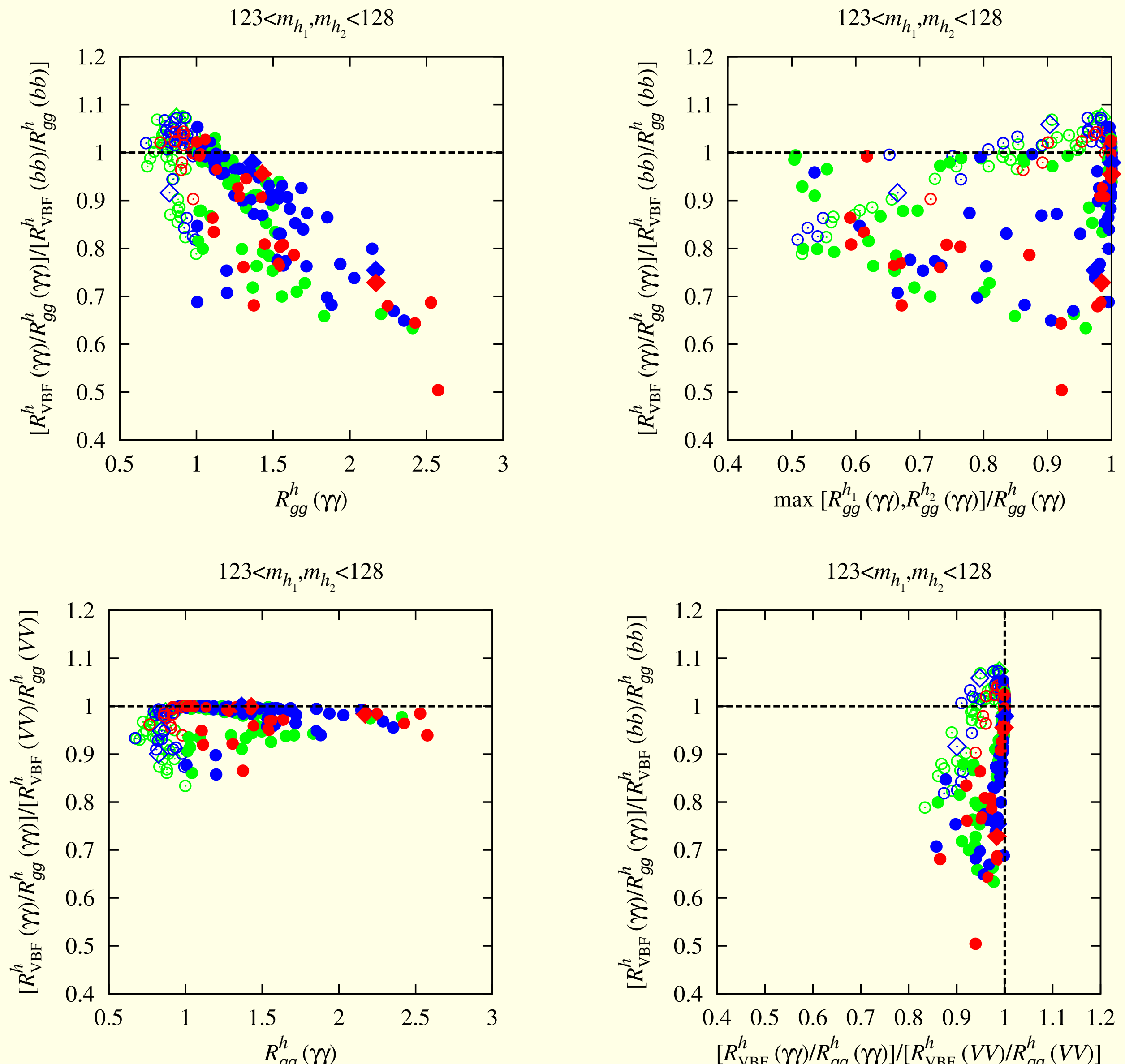


Figure 32: **Double ratios I) and II) of Eq. (230) as functions of $R^h_{gg}(\gamma\gamma)$ (on the left). On the right we show (top) double ratio I) vs. $\max\left[R^{h_1}_{gg}(\gamma\gamma), R^{h_2}_{gg}(\gamma\gamma)\right]/R^h_{gg}(\gamma\gamma)$ and (bottom) double ratio I) vs. double ratio II) for the points displayed in Fig. 31. Colors and symbols are the same as in Fig. 31.**

- **What does current LHC data say about these various double ratios?**

  The central values and 1σ error bars for the numerator and denominator of double ratios I) and II) obtained from CMS data (CMS-PAS-HIG-12-020) are also shown in Fig. 31.

  Obviously, current statistics are inadequate to discriminate whether or not the double ratios deviate from unity.

  About 100 times increased statistics will be needed. This will not be achieved until the $\sqrt{s} = 14~\mathrm{TeV}$ run with $\geq 100\mathrm{fb}^{-1}$ of accumulated luminosity.

  Nonetheless, it is clear that the double-ratio diagnostic tools will ultimately prove viable and perhaps crucial for determining if the $\sim 125~\mathrm{GeV}$ Higgs signal is really only due to a single Higgs-like resonance or if two resonances are contributing.

- **Degeneracy has significant probability in model contexts if enhanced $\gamma\gamma$ rates are indeed confirmed at higher statistics.**

Higgs-radion mixing model example

– **Much bigger deviations of double ratios from being equal, related to anomalous gg and $\gamma\gamma$ couplings of the radion. (Compare to first NMSSM plot of preceding section)**

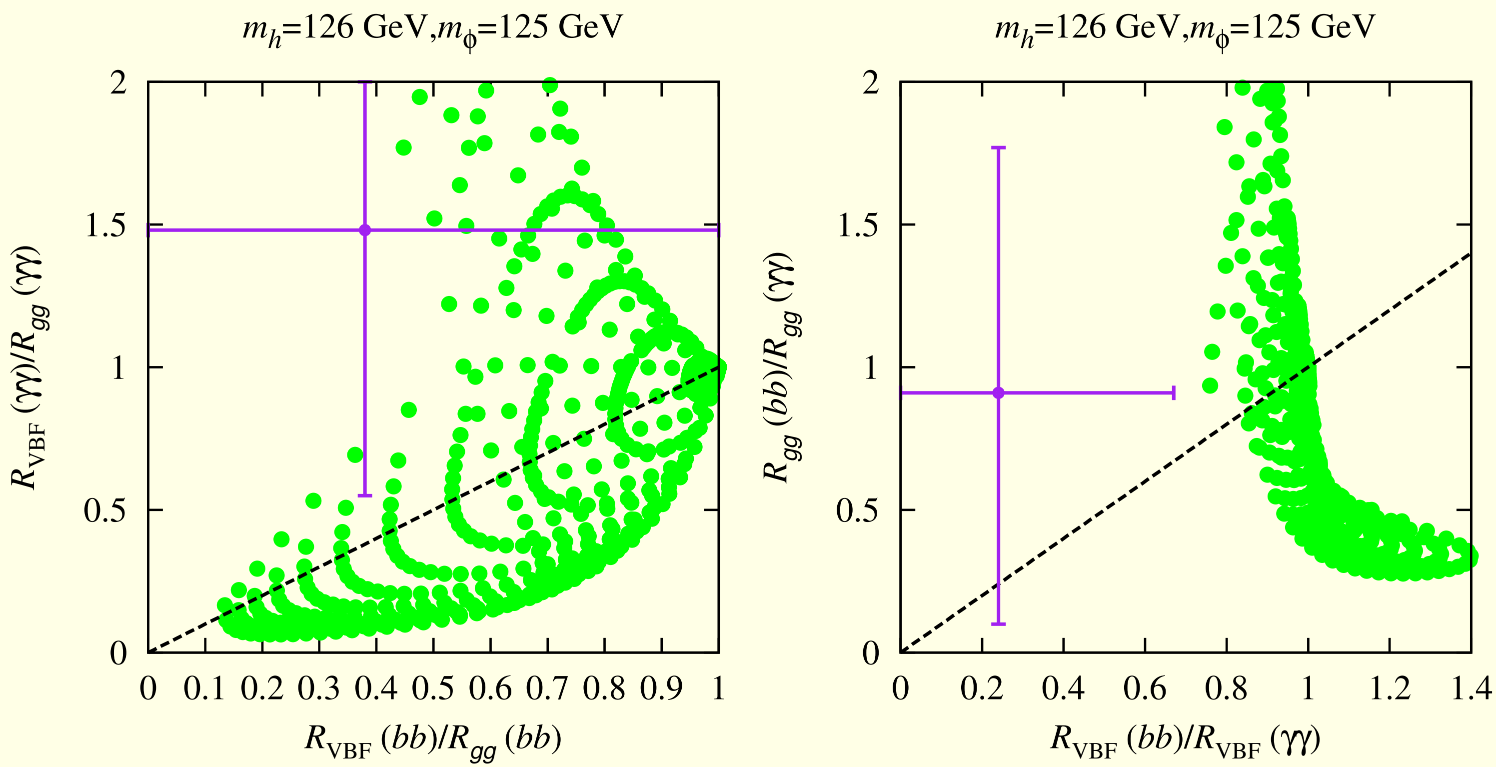


Figure 33: Figure shows only a small part of the full range of vertical axis.

The pure 2HDM

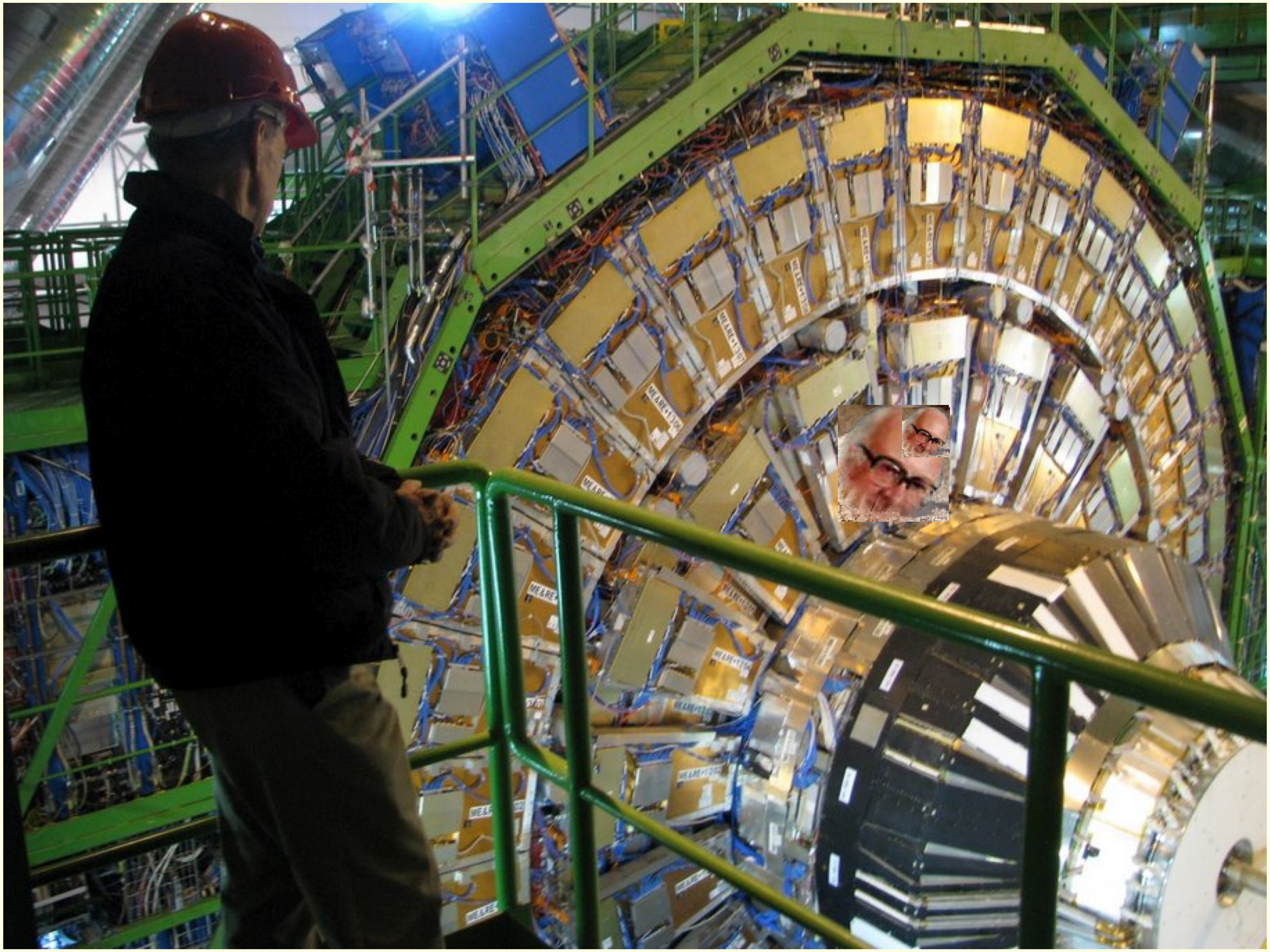
- *"Two-Higgs-Doublet Models and Enhanced Rates for a 125 GeV Higgs"* **A. Drozd, B. Grzadkowski, J. F. Gunion and Y. Jiang. arXiv:1211.3580 [hep-ph]**
- **see also,** *"Mass-degenerate Higgs bosons at 125 GeV in the Two-Higgs-Doublet Model"* **P. M. Ferreir H. E. Haber, R. Santos and J. P. Silva. arXiv:1211.3131 [hep-ph]**
- **There are some differences.**

NMSSM-like degeneracy can be explored in this context also, but no time to discuss.

Conclusions

- It seems likely that the Higgs responsible for EWSB has emerged.
- Perhaps, other Higgs-like objects are emerging.
- Survival of enhanced signals for one or more Higgs boson would be one of the most exciting outcomes of the current LHC run and would guarantee years of theoretical and experimental exploration of BSM models with elementary scalars.
- $>$SM signals would appear to guarantee the importance of a linear collider or LEP3 or muon collider in order to understand fully the responsible BSM physics.
- In any case, the current situation illusrates the fact that we must never assume we have uncovered all the Higgs.

Certainly, I will continue watching and waiting



Details regarding the 2HDM Models

Introduction

The minimal version of the Standard Model (SM) contains one complex Higgs doublet, resulting in one physical neutral CP-even Higgs boson, $h_{\rm SM}$, after electroweak symmetry breaking (EWSB).

However, the Standard Model is not likely to be the ultimate theoretical structure responsible for electroweak symmetry breaking.

Moreover, the Standard Model must be viewed as an effective field theory that is embedded in a more fundamental structure, characterized by an energy scale, Λ, which is larger than the scale of EWSB, $v = 246~{\rm GeV}$.

Although Λ may be as large as the Planck scale, there are strong theoretical arguments that suggest that Λ is significantly lower, perhaps of order 1 TeV [650].

For example, Λ could be the scale of supersymmetry breaking [651, 652, 653], the compositeness scale of new strong dynamics [654], or associated with the inverse size of extra dimensions [655].

In many of these approaches, there exists an effective low-energy theory with elementary scalars that comprise a non-minimal Higgs sector [656].

For example, the minimal supersymmetric extension of the Standard Model (MSSM) contains a scalar Higgs sector corresponding to that of a two-Higgs-doublet model (2HDM) [657, 658].

Models with Higgs doublets (and singlets) possess the important phenomenological property that $\rho = m_W/(m_Z \cos\theta_W) = 1$ up to *finite* radiative corrections.

Here, we focus on a general 2HDM. There are two possible cases.

1. In the first case, there is never an energy range in which the effective low-energy theory contains only one light Higgs boson.

2. In the second case, one CP-even neutral Higgs boson, h, is significantly lighter than a new scale, Λ_{2HDM}, which characterizes the masses of all the remaining 2HDM Higgs states.

   In this latter case, the scalar sector of the effective field theory below $\Lambda_{\rm 2HDM}$ is that of the SM Higgs sector.

   In particular, if $\Lambda_{\rm 2HDM} \gg v$, and all dimensionless Higgs self-coupling parameters $\lambda_i \lesssim \mathcal{O}(1)$ [see eq. (231)], then the couplings of h to gauge bosons and fermions and the h self-couplings approach the corresponding couplings of the $h_{\rm SM}$, with the deviations vanishing as some power of $v^2/\Lambda^2_{\rm 2HDM}$ [659].

   This limit is called the decoupling limit [660], and could be very relevant experimentally given the SM-like nature of the state observed at the LHC.

We now fully define and explore the decoupling limit of the

2HDM.[7]

We will explain the (often confusing) relations between different parameter sets ($e.g.$, Higgs masses and mixing angles $vs.$ Lagrangian tree-level couplings) and give a complete translation table in an Appendix.

We then make one simplifying assumption, namely that the Higgs sector is CP-conserving. (The conditions that guarantee that there is no explicit or spontaneous breaking of CP in the 2HDM are given in a 2nd Appendix. The more general CP-violating 2HDM is treated elsewhere [662, 663].)

In the CP-conserving 2HDM, there is still some freedom in the choice of Higgs-fermion couplings.

A number of different choices have been studied in the literature [664, 656]. Among these are:

- **type-I, in which only one Higgs doublet couples to the fermions;**

[7] Some of the topics of this paper have also been addressed recently in ref. [661].

- **and type-II, in which the neutral member of one Higgs doublet couples only to up-type quarks and the neutral member of the other Higgs doublet couples only to down-type quarks and leptons.**

For Higgs-fermion couplings of type-I or type-II, tree-level flavor-changing neutral currents (FCNC) mediated by Higgs bosons are automatically absent [665].

Type-I and type-II models can be implemented with an appropriately chosen discrete symmetry (which may be softly broken without dire phenomenologically consequences).

The type-II model Higgs sector also arises in the MSSM.

We can also allow for the most general Higgs-fermion Yukawa couplings (the so-called type-III model [666]). For type-III Higgs-fermion Yukawa couplings, tree-level Higgs-mediated FCNCs are present, and one must be careful to choose Higgs parameters which ensure that these FCNC effects are numerically small.

An important result is that in the approach to the decoupling limit, FCNC effects generated by tree-level Higgs exchanges are suppressed by a factor of $\mathcal{O}(v^2/\Lambda^2_{\rm 2HDM})$.

In the first section, we define the most general CP-conserving 2HDM and provide a number of useful relations among the parameters of the scalar Higgs potential and the Higgs masses in several Appendices.

In a 3rd Appendix, we note that certain combinations of the scalar potential parameters are invariant with respect to the choice of basis for the two scalar doublets. In particular, the Higgs masses and the physical Higgs interaction vertices can be written in terms of these invariant coupling parameters.

The decoupling limit of the 2HDM is defined in the following section, and its main properties are examined. In this limit, the properties of the lightest CP-even Higgs boson, h, precisely coincide with those of the SM Higgs boson.

We exhibit the tree-level Higgs couplings to vector bosons, fermions and Higgs bosons, and evaluate them in the decoupling limit.

The first non-trivial corrections to the Higgs couplings as one moves away from the decoupling limit are also given.

We also will show that parameter regimes exist outside the decoupling regime in which one of the CP-even Higgs bosons exhibits tree-level couplings that approximately coincide with those of the SM Higgs boson.

We discuss the origin of this behavior and show how one can distinguish this region of parameter space from that of true decoupling.

Still later, the two-Higgs-doublet sector of the MSSM is used to illustrate the features of the decoupling limit when $m_A \gg m_Z$.

In addition, we briefly describe the impact of radiative corrections, and show how these corrections satisfy the requirements of the decoupling limit.

We emphasize that the rate of approach to decoupling can be delayed at large $\tan\beta$, and we discuss the possibility of a SM-like Higgs boson in a parameter regime in which all Higgs masses are in a range $\lesssim \mathcal{O}(v)$.

The CP-Conserving Two-Higgs Doublet Model

We first review the general (non-supersymmetric) two-Higgs doublet extension of the Standard Model [656]. Let Φ_1 and Φ_2 denote two complex $Y = 1$, SU(2)$_L$ doublet scalar fields. The most general gauge invariant scalar potential is given by[8]

$$\begin{aligned} \mathcal{V} &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}] \\ &\quad +\tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &\quad + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right\} . \end{aligned} \tag{231}$$

[8] In refs. [656] and [658], the scalar potential is parameterized in terms of a different set of couplings, which are less useful for the decoupling analysis. In the 1st Appendix, we relate this alternative set of couplings to the parameters appearing in eq. (231).

In general, m_{12}^2, λ_5, λ_6 and λ_7 can be complex. In many discussions of two-Higgs-doublet models, the terms proportional to λ_6 and λ_7 are absent.

This can be achieved by imposing a discrete symmetry $\Phi_1 \to -\Phi_1$ on the model. Such a symmetry would also require $m_{12}^2 = 0$ unless we allow a soft violation of this discrete symmetry by dimension-two terms.[9] For the moment, we refrain from setting any of the coefficients in eq. (231) to zero.

We next derive the constraints on the parameters λ_i such that the scalar potential $\mathcal{V}$ is bounded from below. It is sufficient to examine the quartic terms of the scalar potential (which we denote by $\mathcal{V}_4$). We define $a \equiv \Phi_1^\dagger \Phi_1$, $b \equiv \Phi_2^\dagger \Phi_2$, $c \equiv \mathrm{Re}\, \Phi_1^\dagger \Phi_2$, $d \equiv \mathrm{Im}\, \Phi_1^\dagger \Phi_2$, and note that $ab \geq c^2 + d^2$. Then, one can rewrite the quartic terms of the scalar potential as follows:

[9]This discrete symmetry is also employed to restrict the Higgs-fermion couplings so that no tree-level Higgs-mediated FCNC's are present. If $\lambda_6 = \lambda_7 = 0$, but $m_{12}^2 \neq 0$, the soft breaking of the discrete symmetry generates *finite* Higgs-mediated FCNC's at one loop.

$$
\begin{aligned}
\mathcal{V}_4 &= \tfrac{1}{2}\left[\lambda_1^{1/2}a-\lambda_2^{1/2}b\right]^2+\left[\lambda_3+(\lambda_1\lambda_2)^{1/2}\right](ab-c^2-d^2)\\
&\quad+2[\lambda_3+\lambda_4+(\lambda_1\lambda_2)^{1/2}]\,c^2+[\mathrm{Re}\,\lambda_5-\lambda_3-\lambda_4-(\lambda_1\lambda_2)^{1/2}](c^2-d^2)\\
&\quad-2cd\,\mathrm{Im}\,\lambda_5+2a\,[c\,\mathrm{Re}\,\lambda_6-d\,\mathrm{Im}\,\lambda_6]+2b\,[c\,\mathrm{Re}\,\lambda_7-d\,\mathrm{Im}\,\lambda_7]\,. \qquad (232)
\end{aligned}
$$

We demand that no directions exist in field space in which $\mathcal{V}\to-\infty$. (We also require that no flat directions exist for $\mathcal{V}_4$.) Three conditions on the λ_i are easily obtained by examining asymptotically large values of a and/or b with $c=d=0$:

$$
\lambda_1>0\,,\qquad \lambda_2>0\,,\qquad \lambda_3>-(\lambda_1\lambda_2)^{1/2}\,. \qquad (233)
$$

A fourth condition arises by examining the direction in field space where $\lambda_1^{1/2}a=\lambda_2^{1/2}b$ and $ab=c^2+d^2$. Setting $c=\xi d$, and requiring that the potential is bounded from below for all ξ leads to a condition on a quartic polynomial in ξ, which must be satisfied for all ξ. There is no simple analytical constraint on the λ_i that can be derived from this condition. If $\lambda_6=\lambda_7=0$, the resulting polynomial is quadratic in ξ, and a constraint on the remaining nonzero λ_i is easily derived [667]

$$\lambda_3+\lambda_4-|\lambda_5| > -(\lambda_1\lambda_2)^{1/2} \qquad [\text{assuming } \lambda_6=\lambda_7=0]\,. \quad (234)$$

From now on, we shall ignore the possibility of explicit CP-violating effects in the Higgs potential by choosing all coefficients in eq. (231) to be real.[10]

The scalar fields will develop non-zero vacuum expectation values if the mass matrix m^2_{ij} has at least one negative eigenvalue.

We assume that the parameters of the scalar potential are chosen such that the minimum of the scalar potential respects the U(1)$_{\rm EM}$ gauge symmetry. Then, the scalar field vacuum

[10] The most general CP-violating 2HDM will be examined in ref. [663].

expectations values are of the form[11]

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \tag{235}$$

where the v_i are taken to be real, $i.e.$ we assume that spontaneous CP violation does not occur.[12] **The corresponding potential minimum conditions are:**

$$m_{11}^2 = m_{12}^2 \tilde{b} - \tfrac{1}{2} v^2 \left[\lambda_1 c_\beta^2 + \lambda_{345} s_\beta^2 + 3\lambda_6 s_\beta c_\beta + \lambda_7 s_\beta^2 \tilde{b} \right], \tag{236}$$

$$m_{22}^2 = m_{12}^2 \tilde{b}^{-1} - \tfrac{1}{2} v^2 \left[\lambda_2 s_\beta^2 + \lambda_{345} c_\beta^2 + \lambda_6 c_\beta^2 \tilde{b}^{-1} + 3\lambda_7 s_\beta c_\beta \right], \tag{237}$$

where we have defined:

$$\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5 , \qquad \tilde{b} \equiv \tan\beta \equiv \frac{v_2}{v_1}, \tag{238}$$

[11] Don't worry that we have temporarily chosen a convention in which both doublets have $Y = 1$ — this can be remapped to the case of one with $Y = 1$ and one with $Y = -1$, as for the MSSM.

[12] The conditions required for the absence of explicit and spontaneous CP-violation in the Higgs sector are elucidated in Appendix B.

and

$$v^2 \equiv v_1^2 + v_2^2 = \frac{4m_W^2}{g^2} = (246\ \mathrm{GeV})^2 \,. \tag{239}$$

It is always possible to choose the phases of the scalar doublet Higgs fields such that both v_1 and v_2 are positive; henceforth we take $0 \leq \beta \leq \pi/2$.

Of the original eight scalar degrees of freedom, three Goldstone bosons ($G^\pm$ and G) are absorbed ("eaten") by the $W^\pm$ and Z. The remaining five physical Higgs particles are: two CP-even scalars (h and H, with $m_h \leq m_H$), one CP-odd scalar (A) and a charged Higgs pair ($H^\pm$). The squared-mass parameters m_{11}^2 and m_{22}^2 can be eliminated by minimizing the scalar potential. The resulting squared-masses for the CP-odd and charged Higgs states are[13]

$$m_A^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \tfrac{1}{2} v^2 \left(2\lambda_5 + \lambda_6 \tilde{b}^{-1} + \lambda_7 \tilde{b} \right) , \tag{240}$$

$$m_{H^\pm}^2 = m_A^2 + \tfrac{1}{2} v^2 (\lambda_5 - \lambda_4) \,. \tag{241}$$

[13]Here and in the following, we use the shorthand notation $c_\beta \equiv \cos\beta$, $s_\beta \equiv \sin\beta$, $c_\alpha \equiv \cos\alpha$, $s_\alpha \equiv \sin\alpha$, $c_{2\alpha} \equiv \cos 2\alpha$, $s_{2\alpha} \equiv \cos 2\alpha$, $c_{\beta-\alpha} \equiv \cos(\beta-\alpha)$, $s_{\beta-\alpha} \equiv \sin(\beta-\alpha)$, etc.

The two CP-even Higgs states mix according to the following squared-mass matrix:

$$\mathcal{M}^2 \equiv m_A^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} + \mathcal{B}^2 \,, \tag{242}$$

where

$$\mathcal{B}^2 \equiv v^2 \begin{pmatrix} \lambda_1 c_\beta^2 + 2\lambda_6 s_\beta c_\beta + \lambda_5 s_\beta^2 & (\lambda_3+\lambda_4) s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 \\ (\lambda_3+\lambda_4) s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 & \lambda_2 s_\beta^2 + 2\lambda_7 s_\beta c_\beta + \lambda_5 c_\beta^2 \end{pmatrix} . \tag{243}$$

Defining the physical mass eigenstates

$$\begin{aligned} H &= (\sqrt{2}\mathrm{Re}\,\Phi_1^0 - v_1) c_\alpha + (\sqrt{2}\mathrm{Re}\,\Phi_2^0 - v_2) s_\alpha \,, \\ h &= -(\sqrt{2}\mathrm{Re}\,\Phi_1^0 - v_1) s_\alpha + (\sqrt{2}\mathrm{Re}\,\Phi_2^0 - v_2) c_\alpha \,, \end{aligned} \tag{244}$$

the masses and mixing angle α are found from the diagonalization process

$$\begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 \\ \mathcal{M}_{12}^2 & \mathcal{M}_{22}^2 \end{pmatrix} \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}$$

$$= \begin{pmatrix} \mathcal{M}_{11}^2 c_\alpha^2 + 2\mathcal{M}_{12}^2 c_\alpha s_\alpha + \mathcal{M}_{22}^2 s_\alpha^2 & \mathcal{M}_{12}^2 (c_\alpha^2 - s_\alpha^2) + (\mathcal{M}_{22}^2 - \mathcal{M}_{11}^2) s_\alpha c_\alpha \\ \mathcal{M}_{12}^2 (c_\alpha^2 - s_\alpha^2) + (\mathcal{M}_{22}^2 - \mathcal{M}_{11}^2) s_\alpha c_\alpha & \mathcal{M}_{11}^2 s_\alpha^2 - 2\mathcal{M}_{12}^2 c_\alpha s_\alpha + \mathcal{M}_{22}^2 c_\alpha^2 \end{pmatrix} \tag{2}$$

The mixing angle α is evaluated by setting the off-diagonal elements of the CP-even scalar squared-mass matrix [eq. (245)] to zero, and demanding that $m_H \geq m_h$. The end result is

$$m^2_{H,h} = \tfrac{1}{2}\left[\mathcal{M}^2_{11} + \mathcal{M}^2_{22} \pm \sqrt{(\mathcal{M}^2_{11} - \mathcal{M}^2_{22})^2 + 4(\mathcal{M}^2_{12})^2}\,\right]. \tag{246}$$

and the corresponding CP-even scalar mixing angle is fixed by

$$\begin{aligned} s_{2\alpha} &= \frac{2\mathcal{M}^2_{12}}{\sqrt{(\mathcal{M}^2_{11} - \mathcal{M}^2_{22})^2 + 4(\mathcal{M}^2_{12})^2}}\,, \\ c_{2\alpha} &= \frac{\mathcal{M}^2_{11} - \mathcal{M}^2_{22}}{\sqrt{(\mathcal{M}^2_{11} - \mathcal{M}^2_{22})^2 + 4(\mathcal{M}^2_{12})^2}}\,. \end{aligned} \tag{247}$$

We shall take $-\pi/2 \leq \alpha \leq \pi/2$.

It is convenient to define the following four combinations of parameters:

$$\begin{aligned} m^4_D &\equiv \mathcal{B}^2_{11}\mathcal{B}^2_{22} - [\mathcal{B}^2_{12}]^2\,, \\ m^2_L &\equiv \mathcal{B}^2_{11}\cos^2\beta + \mathcal{B}^2_{22}\sin^2\beta + \mathcal{B}^2_{12}\sin 2\beta\,, \\ m^2_T &\equiv \mathcal{B}^2_{11} + \mathcal{B}^2_{22}\,, \\ m^2_S &\equiv m^2_A + m^2_T\,, \end{aligned} \tag{248}$$

where the $\mathcal{B}^2_{ij}$ are the elements of the matrix defined in eq. (243). In terms of these quantities we have the exact relations

$$m^2_{H,h} = \tfrac{1}{2}\left[m^2_S \pm \sqrt{m^4_S - 4m^2_A m^2_L - 4m^4_D}\right]\,, \tag{249}$$

and

$$c^2_{\beta-\alpha} = \frac{m^2_L - m^2_h}{m^2_H - m^2_h}\,. \tag{250}$$

Eq. (250) is most easily derived by using $c^2_{\beta-\alpha} = \frac{1}{2}(1 + c_{2\beta}c_{2\alpha} + s_{2\beta}s_{2\alpha})$ and the results of eq. (247). Note that the case of $m_h = m_H$ is special and must be treated carefully. We do this in a separate Appendix, where we explicitly verify that $0 \leq c^2_{\beta-\alpha} \leq 1$.

Finally, for completeness we record the expressions for the original hypercharge-one scalar fields Φ_i in terms of the physical Higgs states and the Goldstone bosons:

$$\begin{aligned}
\Phi_1^\pm &= c_\beta G^\pm - s_\beta H^\pm\,, \\
\Phi_2^\pm &= s_\beta G^\pm + c_\beta H^\pm\,, \\
\Phi_1^0 &= \tfrac{1}{\sqrt{2}}\left[v_1 + c_\alpha H - s_\alpha h + i c_\beta G - i s_\beta A\right]\,, \\
\Phi_2^0 &= \tfrac{1}{\sqrt{2}}\left[v_2 + s_\alpha H + c_\alpha h + i s_\beta G + i c_\beta A\right]\,.
\end{aligned} \tag{251}$$

The Decoupling Limit

In effective field theory, we may examine the behavior of

the theory characterized by two disparate mass scales, $m_L \ll m_S$, by integrating out all particles with masses of order m_S, assuming that all the couplings of the "low-mass" effective theory comprising particles with masses of order m_L can be kept fixed.

In the 2HDM, the low-mass effective theory, if it exists, must correspond to the case where one of the Higgs doublets is integrated out. That is, the resulting effective low-mass theory is precisely equivalent to the one-scalar-doublet SM Higgs sector.

These conclusions follow from electroweak gauge invariance. Namely, there are two relevant scales—the electroweak scale characterized by the scale $v = 246$ GeV and a second scale $m_S \gg v$. The underlying electroweak symmetry requires that scalar mass splittings within doublets cannot be larger than $\mathcal{O}(v)$ [assuming that dimensionless couplings of the theory are no larger than $\mathcal{O}(1)$].

It follows that the $H^\pm$, A and H masses must be of $\mathcal{O}(m_S)$,

while $m_h \sim \mathcal{O}(v)$. Moreover, since the effective low-mass theory consists of a one-doublet Higgs sector, the properties of h must be indistinguishable from those of the SM Higgs boson.

We can illustrate these results more explicitly as follows.

Suppose that all the Higgs self-coupling constants λ_i are held fixed such that $|\lambda_i| \lesssim \mathcal{O}(1)$, while taking $m_A^2 \gg |\lambda_i| v^2$.

In particular, we constrain the $\alpha_i \equiv \lambda_i/(4\pi)$ so that the Higgs sector does not become strongly coupled, implying no violations of tree-unitarity [668, 669, 670, 671, 672].

Then, the $\mathcal{B}_{ij}^2 \sim \mathcal{O}(v^2)$, and it follows that:

$$m_h \simeq m_L = \mathcal{O}(v)\,, \tag{252}$$

$$m_H, m_A, m_{H^\pm} = m_S + \mathcal{O}\left(v^2/m_S\right)\,, \tag{253}$$

and

$$\cos^2(\beta-\alpha) \simeq \frac{m_L^2(m_T^2 - m_L^2) - m_D^4}{m_A^4}$$

$$= \quad \frac{\left[\frac{1}{2}(\mathcal{B}_{11}^2 - \mathcal{B}_{22}^2)s_{2\beta} - \mathcal{B}_{12}^2 c_{2\beta}\right]^2}{m_A^4} = \mathcal{O}\left(\frac{v^4}{m_S^4}\right). \tag{254}$$

We shall establish the above results in more detail below.

The limit $m_A^2 \gg |\lambda_i| v^2$ (subject to $|\alpha_i| \lesssim 1$) is called the *decoupling limit* of the model.[14]

Note that eq. (254) implies that in the decoupling limit, $c_{\beta-\alpha} = \mathcal{O}(v^2/m_A^2)$. We will demonstrate that this implies that the couplings of h in the decoupling limit approach values that correspond precisely to those of the SM Higgs boson.

We will also obtain explicit expressions for the squared-mass differences between the heavy Higgs bosons (as a function of the λ_i couplings in the Higgs potential) in the decoupling limit.

One can give an alternative condition for the decoupling limit.

As above, we assume that all $|\alpha_i| \lesssim 1$.

[14] Later [see eq. (282) and surrounding discussion], we shall refine this definition slightly, and also require that $m_A^2 \gg |\lambda_6| v^2 \cot\beta$ and $m_A^2 \gg |\lambda_7| v^2 \tan\beta$, in order to guarantee that at large $\cot\beta$ [$\tan\beta$] the couplings of h to up-type [down-type] fermions approach the corresponding SM Higgs-fermion couplings.

First consider the following special cases.

- If neither $\tan\beta$ nor $\cot\beta$ is close to 0, then $m_{12}^2 \gg |\lambda_i| v^2$ [see eq. (240)] in the decoupling limit.
- On the other hand, if $m_{12}^2 \sim \mathcal{O}(v^2)$ and $\tan\beta \gg 1$ [$\cot\beta \gg 1$], then it follows from eqs. (236) and (237) that $m_{11}^2 \gg \mathcal{O}(v^2)$ if $\lambda_7 < 0$ [$m_{22}^2 \gg \mathcal{O}(v^2)$ if $\lambda_6 < 0$] in the decoupling limit.

All such conditions depend on the original choice of scalar field basis Φ_1 and Φ_2. For example, we can diagonalize the squared-mass terms of the scalar potential [eq. (231)] thereby setting $m_{12} = 0$.

In the decoupling limit in the new basis, one is simply driven to the second case above.

A basis-independent characterization of the decoupling limit is simple to formulate.

Starting from the scalar potential in an arbitrary basis, form the matrix m_{ij}^2 [made up of the coefficients of the quadratic terms in the potential, see eq. (231)].

Denote the eigenvalues of this matrix by m_a^2 and m_b^2 respectively; note that the eigenvalues are real but can be of either sign. By convention, we can take $|m_a^2| \leq |m_b^2|$. Then, the decoupling limit corresponds to $m_a^2 < 0$, $m_b^2 > 0$ such that $m_b^2 \gg |m_a^2|, v^2$ (with $|\alpha_i| \lesssim 1$).

For some choices of the scalar potential, no decoupling limit exists.

Consider the case of $m_{12}^2 = \lambda_6 = \lambda_7 = 0$ (and all other $|\alpha_i| \lesssim 1$). Then, the potential minimum conditions [eqs. (236) and (237)] do not permit either m_{11}^2 or m_{22}^2 to become large; m_{11}^2, $m_{22}^2 \sim \mathcal{O}(v^2)$, and clearly all Higgs masses are of $\mathcal{O}(v)$. Thus, in this case no decoupling limit exists.[15]

The case of $m_{12}^2 = \lambda_6 = \lambda_7 = 0$ corresponds to the existence

[15] However, it may be difficult to distinguish between the non-decoupling effects of the SM with a heavy Higgs boson and those of the 2HDM where all Higgs bosons are heavy [673].

of a discrete symmetry in which the potential is invariant under the change of sign of one of the Higgs doublet fields.

Although the latter statement is basis-dependent, one can check that the following stronger condition holds: no decoupling limit exists if and only if $\lambda_6 = \lambda_7 = 0$ in the basis where $m_{12}^2 = 0$.

Thus, the absence of a decoupling limit implies the existence of some discrete symmetry under which the scalar potential is invariant (although the precise form of this symmetry is most evident for the special choice of basis).

We now return to the results for the Higgs masses and the CP-even Higgs mixing angle in the decoupling limit.

For fixed values of λ_6, λ_7, α and β, there are two equivalent parameter sets: (i) λ_1, λ_2, λ_3, λ_4 and λ_5; (ii) m_h^2, m_H^2, m_{12}^2, $m_{H^\pm}^2$ and m_A^2.

The relations between these two parameter sets are given in

an Appendix.

Using the results eqs. (313)–(318) we can give explicit expressions in the decoupling limit for the Higgs masses in terms of the potential parameters and the mixing angles.

First, it is convenient to define the following four linear combinations of the λ_i:[16]

$$\lambda \equiv \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}(\lambda_6 c_\beta^2 + \lambda_7 s_\beta^2)\,, \tag{255}$$

$$\widehat{\lambda} \equiv \tfrac{1}{2} s_{2\beta}\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta}\right] - \lambda_6 c_\beta c_{3\beta} - \lambda_7 s_\beta s_{3\beta}\,, \tag{256}$$

$$\lambda_A \equiv c_{2\beta}(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2) + \lambda_{345} s_{2\beta}^2 - \lambda_5 + 2\lambda_6 c_\beta s_{3\beta} - 2\lambda_7 s_\beta c_{3\beta}\,, \tag{257}$$

$$\lambda_F \equiv \lambda_5 - \lambda_4\,, \tag{258}$$

where λ_{345} is defined in eq. (238). The significance of these coupling combinations is discussed in Appendix . We consider the limit $c_{\beta-\alpha} \to 0$, corresponding to the decoupling limit,

[16] We make use of the triple-angle identities: $c_{3\beta} = c_\beta(c_\beta^2 - 3s_\beta^2)$ and $s_{3\beta} = s_\beta(3c_\beta^2 - s_\beta^2)$.

$m_A^2 \gg |\lambda_i| v^2$. In nearly all of the parameter space, $\mathcal{M}_{12}^2 < 0$ [see eq. (242)], and it follows from eq. (247) that $-\pi/2 \leq \alpha \leq 0$ (which implies that $c_{\beta-\alpha} \to 0$ is equivalent to $\beta - \alpha \to \pi/2$ given that $0 \leq \beta \leq \pi/2$). However, in the small regions of parameter space in which β is near zero [or $\pi/2$], roughly corresponding to $m_A^2 \tan\beta < \lambda_6 v^2$ [or $m_A^2 \cot\beta < \lambda_7 v^2$], one finds $\mathcal{M}_{12}^2 > 0$ (and consequently $0 < \alpha < \pi/2$). In these last two cases, the decoupling limit is achieved for $\alpha = \pi/2 - \beta$ and $\cot\beta \gg 1$ [$\tan\beta \gg 1$]. That is, $\cos(\beta - \alpha) = \sin 2\beta \ll 1$ and $\sin(\beta - \alpha) \simeq -1$ [$+1$]. [17] In practice, since $\tan\beta$ is fixed and cannot be arbitrarily large (or arbitrarily close to zero), one can always find a value of m_A large enough such that $\mathcal{M}_{12}^2 < 0$. This is equivalent to employing the refined version of the decoupling limit mentioned in footnote 14. In this case, the decoupling

[17] We have chosen a convention in which $-\pi/2 \leq \alpha \leq \pi/2$. An equally good alternative is to choose $\sin(\beta - \alpha) \geq 0$. If negative, one may simply change the sign of $\sin(\beta - \alpha)$ by taking $\alpha \to \alpha \pm \pi$, which is equivalent to the field redefinitions $h \to -h$, $H \to -H$.

limit simply corresponds to $\beta - \alpha \to \pi/2$ [*i.e.*, $\sin(\beta-\alpha) = 1$] independently of the value of β.

In the approach to the decoupling limit where $\alpha \simeq \beta - \pi/2$ (that is, $|c_{\beta-\alpha}| \ll 1$ and $s_{\beta-\alpha} \simeq 1 - \frac{1}{2}c^2_{\beta-\alpha}$), we may use eqs. (320)–(323) and eq. (241) to obtain:[18]

$$m_A^2 \simeq v^2\left[\frac{\widehat{\lambda}}{c_{\beta-\alpha}} + \lambda_A - \tfrac{3}{2}\widehat{\lambda}\, c_{\beta-\alpha}\right], \tag{259}$$

$$m_h^2 \simeq v^2(\lambda - \widehat{\lambda}\, c_{\beta-\alpha}), \tag{260}$$

$$m_H^2 \simeq v^2\left[\frac{\widehat{\lambda}}{c_{\beta-\alpha}} + \lambda - \tfrac{1}{2}\widehat{\lambda}\, c_{\beta-\alpha}\right] \simeq m_A^2 + (\lambda - \lambda_A + \widehat{\lambda}\, c_{\beta-\alpha})v^2, \tag{261}$$

$$m_{H^\pm}^2 \simeq v^2\left[\frac{\widehat{\lambda}}{c_{\beta-\alpha}} + \lambda_A + \tfrac{1}{2}\lambda_F - \tfrac{3}{2}\widehat{\lambda}\, c_{\beta-\alpha}\right] = m_A^2 + \tfrac{1}{2}\lambda_F v^2. \tag{262}$$

The condition $m_H > m_h$ implies the inequality (valid to first

[18] In obtaining eqs. (259), (261) and (262) we divided both sides of each equation by $c_{\beta-\alpha}$, so these equations need to be treated with care if $c_{\beta-\alpha} = 0$ exactly. In this latter case, it suffices to note that $\widehat{\lambda}/c_{\beta-\alpha}$ has a finite limit whose value depends on m_A and λ_A [see eq. (266)].

order in $c_{\beta-\alpha}$):

$$m_A^2 > v^2(\lambda_A - 2\widehat{\lambda} c_{\beta-\alpha})\,, \tag{263}$$

[*cf.* eq. (343)]. The positivity of m_h^2 also imposes a useful constraint on the Higgs potential parameters. For example, $m_h^2 > 0$ requires that $\lambda > 0$.

In the decoupling limit (where $m_A^2 \gg |\lambda_i| v^2$), eqs. (259)–(262) provide the first nontrivial corrections to eqs. (252) and (253). Finally, we employ eq. (240) to obtain

$$m_{12}^2 \simeq v^2 s_\beta c_\beta \left[\frac{\widehat{\lambda}}{c_{\beta-\alpha}} + \lambda_A + \lambda_5 + \tfrac{1}{2}\lambda_6 \tilde{b}^{-1} + \tfrac{1}{2}\lambda_7 \tilde{b} - \tfrac{3}{2}\widehat{\lambda} c_{\beta-\alpha}\right]. \tag{264}$$

This result confirms our previous observation that $m_{12}^2 \gg |\lambda_i| v^2$ in the decoupling limit as long as β is not close to 0 or $\pi/2$. However, m_{12}^2 can be of $\mathcal{O}(v^2)$ in the decoupling limit [$c_{\beta-\alpha} \to 0$] if either $t_\beta \gg 1$ [and $c_\beta / c_{\beta-\alpha} \sim \mathcal{O}(1)$] or $\tilde{b}^{-1} \gg 1$ [and $s_\beta / c_{\beta-\alpha} \sim \mathcal{O}(1)$].

The significance of eq. (260) is easily understood by noting that the decoupling limit corresponds to integrating out the second heavy Higgs doublet. The resulting low-mass effective theory is simply the one-Higgs-doublet model with corresponding scalar potential $V = m^2(\Phi^\dagger\Phi) + \frac{1}{2}\lambda(\Phi^\dagger\Phi)^2$, where λ is given by eq. (255) and

$$m^2 \equiv m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - 2m_{12}^2 s_\beta c_\beta \,. \tag{265}$$

Imposing the potential minimum conditions [eqs. (236) and (237)], we see that $v^2 = -2m^2/\lambda$ [where $\langle\Phi^0\rangle \equiv v/\sqrt{2}$] as expected. Moreover, the Higgs mass is given by $m_h^2 = \lambda v^2$, in agreement with the $c_{\beta-\alpha} \to 0$ limit of eq. (260).

We can rewrite eq. (259) in another form [or equivalently use eqs. (341) and (342) to obtain]:

$$\cos(\beta-\alpha) \simeq \frac{\widehat{\lambda} v^2}{m_A^2 - \lambda_A v^2} \simeq \frac{\widehat{\lambda} v^2}{m_H^2 - m_h^2} . \tag{266}$$

This yields an $\mathcal{O}(v^2/m_A^2)$ correction to eq. (254). Note that eq. (266) also implies that in the approach to the decoupling limit, the sign of $\cos(\beta-\alpha)$ is given by the sign of $\widehat{\lambda}$.

Two-Higgs Doublet Model Couplings in the Decoupling Limit

The phenomenology of the two-Higgs doublet model depends in detail on the various couplings of the Higgs bosons to gauge bosons, Higgs bosons and fermions [656]. The Higgs couplings to gauge bosons follow from gauge invariance and are thus model independent:

$$g_{hVV} = g_V m_V s_{\beta-\alpha} \,, \quad g_{HVV} = g_V m_V c_{\beta-\alpha} \,, \tag{267}$$

where $g_V \equiv 2m_V/v$ for $V = W$ or Z. There are no tree-level couplings of A or $H^{\pm}$ to VV. In the decoupling limit where $c_{\beta-\alpha} = 0$, we see that $g_{hVV} = g_{h_{\rm SM}VV}$, whereas the HVV coupling vanishes. Gauge invariance also determines the strength of the trilinear couplings of one gauge boson to two Higgs bosons:

$$g_{hAZ} = \frac{g c_{\beta-\alpha}}{2\cos\theta_W}, \quad g_{HAZ} = \frac{-g s_{\beta-\alpha}}{2\cos\theta_W}. \tag{268}$$

In the decoupling limit, the hAZ coupling vanishes, while the HAZ coupling attains its maximal value. This pattern is repeated in all the three-point and four-point couplings of h and H to VV, $V\phi$, and $VV\phi$ final states (where V is a vector boson and ϕ is one of the Higgs scalars). These results can be summarized as follows: the coupling of h and H to vector boson pairs or vector–scalar boson final states is proportional to either $\sin(\beta-\alpha)$ or

$\cos(\beta-\alpha)$ as indicated below [656, 658].

$$
\begin{array}{ll}
\underline{\cos(\beta-\alpha)} & \underline{\sin(\beta-\alpha)} \\
HW^+W^- & hW^+W^- \\
HZZ & hZZ \\
ZAh & ZAH \\
W^\pm H^\mp h & W^\pm H^\mp H \\
ZW^\pm H^\mp h & ZW^\pm H^\mp H \\
\gamma W^\pm H^\mp h & \gamma W^\pm H^\mp H
\end{array}
\tag{269}
$$

Note in particular that *all* vertices in the theory that contain at least one vector boson and *exactly one* of the non-minimal Higgs boson states (H, A or $H^\pm$) are proportional to $\cos(\beta-\alpha)$ and hence vanish in the decoupling limit.

The Higgs couplings to fermions are model dependent. The most general structure for the Higgs-fermion Yukawa couplings, often referred to as the type-III model [666], is given by:

$$
-\mathcal{L}_Y = \overline{Q}^0_L \widetilde{\Phi}_1 \eta_1^{U,0} U^0_R + \overline{Q}^0_L \Phi_1 \eta_1^{D,0} D^0_R + \overline{Q}^0_L \widetilde{\Phi}_2 \eta_2^{U,0} U^0_R + \overline{Q}^0_L \Phi_2 \eta_2^{D,0} D^0_R + \text{h.c.}\,, \tag{270}
$$

where $\Phi_{1,2}$ are the Higgs doublets, $\widetilde{\Phi}_i \equiv i\sigma_2 \Phi_i^*$, Q_L^0 is the weak isospin quark doublet, and U_R^0, D_R^0 are weak isospin quark singlets. [The right and left-handed fermion fields are defined as usual: $\psi_{R,L} \equiv P_{R,L}\psi$, where $P_{R,L} \equiv \frac{1}{2}(1 \pm \gamma_5)$.] Here, Q_L^0, U_R^0, D_R^0 denote the interaction basis states, which are vectors in flavor space, whereas $\eta_1^{U,0}, \eta_2^{U,0}, \eta_1^{D,0}, \eta_2^{D,0}$ are matrices in flavor space. We have omitted the leptonic couplings in eq. (270); these follow the same pattern as the down-type quark couplings.

We next shift the scalar fields according to their vacuum expectation values, and then re-express the scalars in terms of the physical Higgs states and Goldstone bosons [see eq. (251)]. In addition, we diagonalize the quark mass matrices and define the quark mass eigenstates. The resulting Higgs-fermion Lagrangian can be written in several ways [674]. We choose to display the form that makes the type-II model limit of the general type-III couplings apparent. The type-II model (where $\eta_1^{U,0} =$

$\eta_2^{D,0} = 0$) automatically has no tree-level flavor-changing neutral Higgs couplings, whereas these are generally present for type-III couplings. The fermion mass eigenstates are related to the interaction eigenstates by biunitary transformations:

$$P_L U = V_L^U P_L U^0 \,, \qquad P_R U = V_R^U P_R U^0 \,,$$
$$P_L D = V_L^D P_L D^0 \,, \qquad P_R D = V_R^D P_R D^0 \,, \qquad (271)$$

and the Cabibbo-Kobayashi-Maskawa matrix is defined as $K \equiv V_L^U V_L^{D\,\dagger}$. It is also convenient to define "rotated" coupling matrices:

$$\eta_i^U \equiv V_L^U \eta_i^{U,0} V_R^{U\,\dagger} \,, \qquad \eta_i^D \equiv V_L^D \eta_i^{D,0} V_R^{D\,\dagger} \,. \qquad (272)$$

The diagonal quark mass matrices are obtained by replacing the

scalar fields with their vacuum expectation values:

$$M_D = \frac{1}{\sqrt{2}}(v_1\eta_1^D + v_2\eta_2^D)\,, \quad M_U = \frac{1}{\sqrt{2}}(v_1\eta_1^U + v_2\eta_2^U)\,. \tag{273}$$

After eliminating η_2^U and η_1^D, the resulting Yukawa couplings are:

$$\begin{aligned}
\mathcal{L}_Y &= \frac{1}{v}\overline{D}M_D D\left(\frac{s_\alpha}{c_\beta}h - \frac{c_\alpha}{c_\beta}H\right) + \frac{i}{v}\overline{D}M_D\gamma_5 D(\tilde{b}A - G) \\
&-\frac{1}{\sqrt{2}c_\beta}\overline{D}(\eta_2^D P_R + {\eta_2^D}^\dagger P_L)D(c_{\beta-\alpha}h - s_{\beta-\alpha}H) \\
&-\frac{i}{\sqrt{2}c_\beta}\overline{D}(\eta_2^D P_R - {\eta_2^D}^\dagger P_L)D\,A \\
&-\frac{1}{v}\overline{U}M_U U\left(\frac{c_\alpha}{s_\beta}h + \frac{s_\alpha}{s_\beta}H\right) + \frac{i}{v}\overline{U}M_U\gamma_5 U(t_\beta^{-1}A + G) \\
&+\frac{1}{\sqrt{2}s_\beta}\overline{U}(\eta_1^U P_R + {\eta_1^U}^\dagger P_L)U(c_{\beta-\alpha}h - s_{\beta-\alpha}H) \\
&-\frac{i}{\sqrt{2}s_\beta}\overline{U}(\eta_1^U P_R - {\eta_1^U}^\dagger P_L)U\,A
\end{aligned} \tag{274}$$

$$+\frac{\sqrt{2}}{v}\Big[\overline{U}KM_DP_RD(\tilde{b}H^+ - G^+) + \overline{U}M_UKP_LD(t_\beta^{-1}H^+ + G^+) + \mathrm{h.c.}\Big]$$
$$- \left[\frac{1}{s_\beta}\overline{U}\eta_1^{U\dagger}KP_LD\,H^+ + \frac{1}{c_\beta}\overline{U}K\eta_2^DP_RD\,H^+ + \mathrm{h.c.}\right] . \qquad (275)$$

In general, η_1^U and η_2^D are complex non-diagonal matrices. Thus, the Yukawa Lagrangian displayed in eq. (275) exhibits both flavor-nondiagonal and CP-violating couplings between the neutral Higgs bosons and the quarks.

In the decoupling limit (where $c_{\beta-\alpha} \to 0$), the Yukawa Lagrangian displays a number of interesting features. First, the flavor non-diagonal and the CP-violating couplings of h vanish (although the corresponding couplings to H and A persist). Moreover, in this limit, the h coupling to fermions reduces precisely to its Standard Model value, $\mathcal{L}_Y^{\mathrm{SM}} = -(m_f/v)\bar{f}fh$. To better see the behavior of couplings in the decoupling limit, the following trigonometric identities are particularly useful:

$$h\overline{D}D \quad : \qquad -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)\,, \qquad (276)$$

$$h\overline{U}U \quad : \qquad \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)\,, \tag{277}$$

$$H\overline{D}D \quad : \qquad \frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha)\,, \tag{278}$$

$$H\overline{U}U \quad : \qquad \frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha)\,, \tag{279}$$

where we have indicated the type of Higgs-fermion coupling with which a particular trigonometric expression arises. It is now easy to read off the corresponding Higgs-fermion couplings in the decoupling limit and one verifies that the h-fermion couplings reduce to their Standard Model values. Working to $\mathcal{O}(c_{\beta-\alpha})$, the Yukawa couplings of h are given by

$$\begin{aligned}\mathcal{L}_{hQQ} = \quad & -\overline{D} \quad \left[\frac{1}{v}M_D - \tan\beta\left[\frac{1}{v}M_D - \frac{1}{\sqrt{2}s_\beta}(S_D + iP_D\gamma_5)\right]c_{\beta-\alpha}\right]D\,h \\ & -\overline{U} \quad \left[\frac{1}{v}M_U + \cot\beta\left[\frac{1}{v}M_U - \frac{1}{\sqrt{2}c_\beta}(S_U + iP_U\gamma_5)\right]c_{\beta-\alpha}\right]U\,h\,,\end{aligned} \tag{280}$$

where

$$S_D \equiv \frac{1}{2}\left(\eta_2^D + \eta_2^{D\,\dagger}\right)\,, \qquad P_D \equiv -\frac{i}{2}\left(\eta_2^D - \eta_2^{D\,\dagger}\right)\,, \tag{281}$$

are 3×3 hermitian matrices and S_U and P_U are defined similarly by making the replacements $D \to U$ and $2 \to 1$.

Note that both h-mediated FCNC interactions (implicit in the off-diagonal matrix elements of S and P) and CP-violating interactions proportional to P are suppressed by a factor of $c_{\beta-\alpha}$ in the decoupling limit. Moreover, FCNCs and CP-violating effects mediated by A and H are suppressed by the square of the heavy Higgs masses (relative to v), due to the propagator suppression.

Since $m_h \ll m_H$, m_A and $c_{\beta-\alpha} \simeq \mathcal{O}(v^2/m_A^2)$ near the decoupling limit, we see that the flavor-violating processes and CP-violating processes mediated by h, H and A are all suppressed by the same factor.

Thus, for $m_A \gtrsim \mathcal{O}(1~\mathrm{TeV})$, the decoupling limit provides a viable mechanism for suppressed Higgs-mediated FCNCs and suppressed Higgs-mediated CP-violating effects in the most general 2HDM.

Note that the approach to decoupling can be delayed if either $\tan\beta \gg 1$ or $\cot\beta \gg 1$, as is evident from eq. (280). For example, decoupling at large $\tan\beta$ or $\cot\beta$ occurs when $|c_{\beta-\alpha}\tan\beta| \ll 1$ or $|c_{\beta-\alpha}\cot\beta| \ll 1$, respectively. Using eqs. (266) and (256), these conditions are respectively equivalent to

$$m_A^2 \gg |\lambda_6| v^2 \cot\beta \qquad \text{and} \qquad m_A^2 \gg |\lambda_7| v^2 \tan\beta\,, \tag{282}$$

which supplement the usual requirement of $m_A^2 \gg \lambda_i v^2$.

That is, there are two possible ranges of the CP-odd Higgs squared-mass, $\lambda_i v^2 \ll m_A^2 \lesssim |\lambda_7| v^2 \tan\beta$ [or $\lambda_i v^2 \ll m_A^2 \lesssim |\lambda_6| v^2 \cot\beta$] when $\tan\beta \gg 1$ [or $\cot\beta \gg 1$], where the h

couplings to VV, hh and hhh are nearly indistinguishable from the corresponding $h_{\rm SM}$ couplings, whereas one of the $hf\bar{f}$ couplings can deviate significantly from the corresponding $h_{\rm SM}f\bar{f}$ couplings.

The cubic and quartic Higgs self-couplings depend on the parameters of the 2HDM potential [eq. (231)], and can easily be worked out.

In the decoupling limit (DL) of $\alpha \to \beta - \pi/2$, we denote the terms of the scalar potential corresponding to the cubic Higgs couplings by $\mathcal{V}^{(3)}_{\rm DL}$ and the terms corresponding to the quartic Higgs couplings by $\mathcal{V}^{(4)}_{\rm DL}$.

The coefficients of the quartic terms in the scalar Higgs potential can be written more simply in terms of the linear combinations of couplings defined earlier [eqs. (255)–(258)] and three additional combinations:

$$\lambda_T = \tfrac{1}{4}s^2_{2\beta}(\lambda_1+\lambda_2)+\lambda_{345}(s^4_\beta+c^4_\beta)-2\lambda_5-s_{2\beta}c_{2\beta}(\lambda_6-\lambda_7)\,, \tag{283}$$

$$\lambda_U = \tfrac{1}{2}s_{2\beta}(s^2_\beta\lambda_1-c^2_\beta\lambda_2+c_{2\beta}\lambda_{345})-\lambda_6 s_\beta s_{3\beta}-\lambda_7 c_\beta c_{3\beta}\,. \tag{284}$$

$$\lambda_V = \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2 - 2 s_{2\beta}(\lambda_6 s_\beta^2 + \lambda_7 c_\beta^2)\,. \tag{285}$$

The resulting expressions for $\mathcal{V}_{\rm DL}^{(3)}$ and $\mathcal{V}_{\rm DL}^{(4)}$ are

$$\begin{aligned}
\mathcal{V}_{\rm DL}^{(3)} = & \tfrac{1}{2}\lambda v(h^3 + hG^2 + 2hG^+G^-) + (\lambda_T + \lambda_F) v h H^+ H^- \\
& + \tfrac{1}{2}\widehat{\lambda} v \left[3Hh^2 + HG^2 + 2HG^+G^- - 2h(AG + H^+G^- + H^-G^+)\right] \\
& + \tfrac{1}{2}\lambda_U v(H^3 + HA^2 + 2HH^+H^-) \\
& + \left[\lambda_A - \lambda + \tfrac{1}{2}\lambda_F\right] vH(H^+G^- + H^-G^+) \\
& + (\lambda_A - \lambda) vHAG + \tfrac{1}{2}\lambda_T v h A^2 + (\lambda - \lambda_A + \tfrac{1}{2}\lambda_T) v h H^2 \\
& + \tfrac{i}{2}\lambda_F v A(H^+G^- - H^-G^+)\,,
\end{aligned} \tag{286}$$

and

$$\begin{aligned}
\mathcal{V}_{\rm DL}^{(4)} & = \tfrac{1}{8}\lambda({g_{\mu\nu}}^2 + 2G^+G^- + h^2)^2 \\
& + \widehat{\lambda}(h^3 H - h^2 A g_{\mu\nu} - h^2 H^+G^- - h^2 H^-G^+ + hH{g_{\mu\nu}}^2 + 2hHG^+G^- - Ag_{\mu\nu}{}^3 \\
& \quad -2Ag_{\mu\nu}G^+G^- - {g_{\mu\nu}}^2 H^-G^+ - {g_{\mu\nu}}^2 H^+G^- - 2H^+G^-G^+G^- - 2H^-G^+G^- \\
& + \tfrac{1}{2}(\lambda_T + \lambda_F)(h^2 H^+H^- + H^2 G^+G^- + A^2 G^+G^- + {g_{\mu\nu}}^2 H^+H^-)
\end{aligned}$$

$$+\lambda_U(hH^3 + hHA^2 + 2hHH^+H^- - H^2Ag_{\mu\nu} - H^2H^+G^- - H^2H^-G^+ - A^3$$
$$-A^2H^+G^- - A^2H^-G^+ - 2Ag_{\mu\nu}H^+H^- - 2H^+H^-H^+G^- - 2H^-H^+H^-G$$

$$+[2(\lambda_A - \lambda) + \lambda_F](hHH^+G^- + hHH^-G^+ - Ag_{\mu\nu}H^+G^- - Ag_{\mu\nu}H^-G^+)$$

$$+\tfrac{1}{4}\lambda_V(H^4 + 2H^2A^2 + A^4 + 4H^2H^+H^- + 4A^2H^+H^- + 4H^+H^-H^+H^-)$$

$$+\tfrac{1}{2}(\lambda - \lambda_A)(H^+H^+G^-G^- + H^-H^-G^+G^+ - 2hHAg_{\mu\nu}) + \tfrac{1}{4}\lambda_T(h^2A^2 + H^2g_{\mu}$$

$$+\tfrac{1}{4}[2(\lambda - \lambda_A) + \lambda_T](h^2H^2 + A^2{g_{\mu\nu}}^2) + (\lambda - \lambda_A + \lambda_T)H^+H^-G^+G^-$$

$$+\tfrac{i}{2}\lambda_F(hAH^+G^- - hAH^-G^+ + Hg_{\mu\nu}H^+G^- - Hg_{\mu\nu}H^-G^+)\,, \qquad ($$

where G and $G^\pm$ are the Goldstone bosons (eaten by the Z and $W^\pm$, respectively). Moreover, for $c_{\beta-\alpha} = 0$, we have $m_h^2 = \lambda v^2$ and $m_H^2 - m_A^2 = (\lambda - \lambda_A)v^2$, whereas $m_{H^\pm}^2 - m_A^2 = \frac{1}{2}\lambda_F v^2$ is exact at tree-level. As expected, in the decoupling limit, the low-energy effective scalar theory (which includes h and the three Goldstone bosons) is precisely the same as the corresponding SM

Higgs theory, with λ proportional to the Higgs quartic coupling.

An alternative parameterization of the 2HDM scalar potential

In this Appendix, we give the translation of the parameters of eq. (231) employed in this paper to the parameters employed in the Higgs Hunter's Guide (HHG) [656]. While the HHG parameterization was useful for some purposes ($e.g.$, the scalar potential minimum is explicitly exhibited), it obscures the decoupling limit.

In the HHG parameterization, the most general 2HDM scalar potential, subject to a discrete symmetry $\Phi_1 \to -\Phi_1$ that is only softly violated by dimension-two terms, is given by[19]

$$\mathcal{V} = \Lambda_1 \left(\Phi_1^\dagger \Phi_1 - V_1^2\right)^2 + \Lambda_2 \left(\Phi_2^\dagger \Phi_2 - V_2^2\right)^2 + \Lambda_3 \left[\left(\Phi_1^\dagger \Phi_1 - V_1^2\right)\right.$$

[19] In the HHG, the V_i and Λ_i are denoted by v_i and λ_i, respectively. In eq. (288), we employ the former notation in order to distinguish between the HHG parameterization and the notation of eqs. (231) and (235).

$$
\begin{aligned}
&+ \;\; \Lambda_4 \left[(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - (\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\right] + \Lambda_5 \left[\mathrm{Re}(\Phi_1^\dagger\Phi_2) - \mathrm{V}_1\right. \\
&+ \;\; \Lambda_6 \left[\mathrm{Im}(\Phi_1^\dagger\Phi_2) - \mathrm{V}_1\mathrm{V}_2 \sin\xi\right]^2 \\
&+ \;\; \Lambda_7 \left[\mathrm{Re}(\Phi_1^\dagger\Phi_2) - \mathrm{V}_1\mathrm{V}_2 \cos\xi\right] \left[\mathrm{Im}(\Phi_1^\dagger\Phi_2) - \mathrm{V}_1\mathrm{V}_2 \sin\xi\right] ,
\end{aligned}
$$

where the Λ_i are real parameters.[20] **The $V_{1,2}$ are related to the $v_{1,2}$ of eq. (235) by $V_{1,2} = v_{1,2}/\sqrt{2}$. The conversion from these Λ_i to the λ_i and m_{ij}^2 of eq. (231) is:**

$$
\begin{aligned}
\lambda_1 &= 2(\Lambda_1 + \Lambda_3) , \\
\lambda_2 &= 2(\Lambda_2 + \Lambda_3) , \\
\lambda_3 &= 2\Lambda_3 + \Lambda_4 , \\
\lambda_4 &= -\Lambda_4 + \tfrac{1}{2}(\Lambda_5 + \Lambda_6) ,
\end{aligned}
$$

[20] In eq. (288) we include the Λ_7 term that was left out in the hardcover edition of the HHG. See the erratum that has been included in the paperback edition of the HHG (Perseus Publishing, Cambridge, MA, 2000).

$$\lambda_5 = \tfrac{1}{2}(\Lambda_5 - \Lambda_6 - i\Lambda_7)\,,$$
$$\lambda_6 = \lambda_7 = 0$$
$$m_{11}^2 = -2V_1^2\Lambda_1 - 2(V_1^2 + V_2^2)\Lambda_3\,,$$
$$m_{22}^2 = -2V_2^2\Lambda_2 - 2(V_1^2 + V_2^2)\Lambda_3\,,$$
$$m_{12}^2 = V_1V_2(\Lambda_5\cos\xi - i\Lambda_6\sin\xi - \tfrac{i}{2}e^{i\xi}\Lambda_7)\,. \quad (289)$$

Excluding λ_6 and λ_7, the scalar potential [eqs. (231) and (288)] are fixed by ten real parameters. The CP-conserving limit of eq. (288) is most easily obtained by setting $\xi = 0$ and $\Lambda_7 = 0$. In the CP-conserving limit, it is easy to invert eq. (289) and solve for the Λ_i ($i = 1, \ldots, 6$). The result is:

$$\Lambda_1 = \tfrac{1}{2}\left[\lambda_1 - \lambda_{345} + 2m_{12}^2/(v^2 s_\beta c_\beta)\right]\,,$$
$$\Lambda_2 = \tfrac{1}{2}\left[\lambda_2 - \lambda_{345} + 2m_{12}^2/(v^2 s_\beta c_\beta)\right]\,,$$
$$\Lambda_3 = \tfrac{1}{2}\left[\lambda_{345} - 2m_{12}^2/(v^2 s_\beta c_\beta)\right]\,,$$

$$\Lambda_4 = 2m_{12}^2/(v^2 s_\beta c_\beta) - \lambda_4 - \lambda_5\,,$$
$$\Lambda_5 = 2m_{12}^2/(v^2 s_\beta c_\beta)\,,$$
$$\Lambda_6 = 2m_{12}^2/(v^2 s_\beta c_\beta) - 2\lambda_5\,, \qquad (290)$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$ and $v^2 s_\beta c_\beta = 2V_1V_2$.

Conditions for CP conservation in the two-Higgs doublet model

First, we derive the conditions such that the Higgs sector does not exhibit explicit CP violation.[21] It is convenient to adopt a convention in which one of the vacuum expectation values, say v_1 is real and positive.[22] This still leaves one additional phase redefinition for the Higgs doublet fields. If there is no explicit CP violation, it should be possible to choose the phases of the Higgs fields so that there are no explicit phases in the Higgs potential parameters of eq. (231). If we consider $\Phi_1^\dagger\Phi_2 \to e^{-i\eta}\Phi_1^\dagger\Phi_2$, then

[21] For another approach, in which invariants are employed to identify basis-independent conditions for CP violation in the Higgs sector, see refs. [692] and [693].

[22] Due to the U(1)-hypercharge symmetry of the theory, it is always possible to make a phase rotation on the scalar fields such that $v_1 > 0$.

the η-dependent terms in $\mathcal{V}$ are given by

$$\mathcal{V} \ni -m_{12}^2 e^{-i\eta}\Phi_1^\dagger\Phi_2 + \tfrac{1}{2}\lambda_5 e^{-2i\eta}\left(\Phi_1^\dagger\Phi_2\right)^2$$
$$+\lambda_6 e^{-i\eta}\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_1^\dagger\Phi_2\right) + \lambda_7 e^{-i\eta}\left(\Phi_2^\dagger\Phi_2\right)\left(\Phi_1^\dagger\Phi_2\right) + \mathrm{h.c.} \qquad (291)$$

Let us write

$$m_{12}^2 = |m_{12}^2| e^{i\theta_m}\,, \qquad \lambda_{5,6,7} = |\lambda_{5,6,7}| e^{i\theta_{5,6,7}}\,. \qquad (292)$$

Then, all explicit parameter phases are removed if

$$\theta_m - \eta = n_m\pi\,, \quad \theta_5 - 2\eta = n_5\pi\,, \quad \theta_{6,7} - \eta = n_{6,7}\pi\,, \quad (293)$$

where $n_{m,5,6,7}$ are integers. Writing $\eta = \theta_m - n_m\pi$ from the first condition of eq. (293), and substituting into the other conditions, gives

$$\theta_5 - 2\theta_m = (n_5 - 2n_m)\pi \quad \Rightarrow \mathrm{Im}[(m_{12}^2)^2\lambda_5^*] = 0\,, \quad (294)$$
$$\theta_6 - \theta_m = (n_6 - n_m)\pi \quad \Rightarrow \mathrm{Im}[m_{12}^2\lambda_6^*] = 0\,, \quad (295)$$

$$\theta_7 - \theta_m = (n_7 - n_m)\pi \quad \Rightarrow \mathrm{Im}[m_{12}^2 \lambda_7^*] = 0 \,. \qquad (296)$$

Eqs. (294)–(296) constitute the conditions for the absence of explicit CP violation in the (tree-level) Higgs sector. A useful convention is one in which m_{12}^2 is real (by a suitable choice of the phase η). It then follows that λ_5, λ_6 and λ_7 are also real. Henceforth, we shall assume that all parameters in the scalar potential are real.

Let us consider now the conditions for the absence of spontaneous CP violation.[23] Let us write $\langle \Phi_1^\dagger \Phi_2 \rangle = \frac{1}{2} v_1 v_2 e^{i\xi}$ with v_1 and v_2 real and positive and $0 \le \xi \le \pi$. The ξ-dependent terms in $\mathcal{V}$ are given by

$$\mathcal{V} \ni -m_{12}^2 v_1 v_2 \cos\xi + \tfrac{1}{4}\lambda_5 v_1^2 v_2^2 \cos 2\xi + \tfrac{1}{2}\lambda_6 v_1^3 v_2 \cos\xi + \tfrac{1}{2}\lambda_7 v_2^3 v_1 \cos\xi \,, \qquad (297)$$

which yields

$$\frac{\partial \mathcal{V}}{\partial \cos\xi} = -m_{12}^2 v_1 v_2 + \lambda_5 v_1^2 v_2^2 \cos\xi + \tfrac{1}{2}\lambda_6 v_1^3 v_2 + \tfrac{1}{2}\lambda_7 v_2^3 v_1 \qquad (298)$$

[23] Similar considerations can be found in refs. [693, 694, 695] and [662].

and

$$\frac{\partial^2 \mathcal{V}}{\partial(\cos\xi)^2} = \lambda_5 v_1^2 v_2^2 \,. \tag{299}$$

Spontaneous CP violation occurs when $\xi \neq 0, \pi/2$ or π at the potential minimum. That is, $\lambda_5 > 0$ and there exists a CP-violating solution to

$$\cos\xi = \frac{m_{12}^2 - \frac{1}{2}\lambda_6 v_1^2 - \frac{1}{2}\lambda_7 v_2^2}{\lambda_5 v_1 v_2} \,. \tag{300}$$

Thus, we conclude that the criterion for spontaneous CP violation (in a convention where all parameters of the scalar potential are real) is

$$0 \neq \left| m_{12}^2 - \tfrac{1}{2}\lambda_6 v_1^2 - \tfrac{1}{2}\lambda_7 v_2^2 \right| < \lambda_5 v_1 v_2 \qquad \text{and } \lambda_5 > 0 \,. \tag{301}$$

Otherwise, the minimum of the potential occurs either at $\xi = 0$, $\pi/2$ or π and CP is conserved.[24] The case of $\xi = \pi/2$ is

[24] The CP-conserving minimum corresponding to $\xi = 0$ or $\xi = \pi$ does not in general correspond to an extremum in

singular and arises when $m_{12}^2 = \frac{1}{2}\lambda_6 v_1^2 + \frac{1}{2}\lambda_7 v_2^2$ and $\lambda_5 > 0$.[25] It is convenient to choose a convention where $\langle\Phi_1^0\rangle$ is real and $\langle\Phi_2^0\rangle$ is pure imaginary. One must then re-evaluate the Higgs mass eigenstates. As shown in ref. [696], the neutral Goldstone boson is now a linear combination of $\mathrm{Im}\ \Phi_1^0$ and $\mathrm{Re}\ \Phi_2^0$, while the physical CP-odd scalar, A corresponds to the orthogonal combination. The two CP-even Higgs scalars are orthogonal linear combinations of $\mathrm{Re}\ \Phi_1^0$ and $\mathrm{Im}\ \Phi_2^0$. Most of the results of this paper do not apply for this case without substantial revision. Nevertheless, it is clear that the decoupling limit ($m_A^2 \gg \lambda_i v^2$) does not exist due to the condition on m_{12}^2.

We shall not consider the $\xi = \pi/2$ model further in this paper. Then, if the parameters of the scalar potential are real and if

$V(\cos\xi)$. Specifically, for $\lambda_5 < 0$, the extremum corresponds to a maximum in $\mathcal{V}$, while for $\lambda_5 > 0$, the extremum corresponding to a minimum of $\mathcal{V}(\cos\xi)$ arises for $|\cos\xi| > 1$. In both cases, when restricted to the physical region corresponding to $|\cos\xi| \leq 1$, the minimum of $\mathcal{V}(\cos\xi)$ is attained on the boundary, $|\cos\xi| = 1$.

[25]Note that the case of $\xi = \pi/2$ arises automatically in the case of the discrete symmetry discussed in Section , $m_{12}^2 = \lambda_6 = \lambda_7 = 0$, when $\lambda_5 > 0$.

there is no spontaneous CP-violation, then it is always possible to choose the phase η in eq. (291) so that the potential minimum corresponds to $\xi = 0$.[26] In this convention,

$$m_{12}^2 - \tfrac{1}{2}\lambda_6 v_1^2 - \tfrac{1}{2}\lambda_7 v_2^2 \geq \lambda_5 v_1 v_2 \qquad \text{for } \lambda_5 > 0\,, \tag{302}$$

$$m_{12}^2 - \tfrac{1}{2}\lambda_6 v_1^2 - \tfrac{1}{2}\lambda_7 v_2^2 \geq 0 \qquad \text{for } \lambda_5 \leq 0\,, \tag{303}$$

where eq. (302) follows from eq. (301), and eq. (303) is a consequence of the requirement that $\mathcal{V}(\xi = 0) \leq \mathcal{V}(\xi = \pi)$. Since $\xi = 0$ and both v_1 and v_2 are real and positive, this convention corresponds to the one chosen below eq. (239). Note that if we rewrite eq. (240) as [27]

$$m_A^2 = \frac{v^2}{v_1 v_2}\left[m_{12}^2 - \lambda_5 v_1 v_2 - \tfrac{1}{2}\lambda_6 v_1^2 - \tfrac{1}{2}\lambda_7 v_2^2\right]\,, \tag{304}$$

[26] In particular, if $\xi = \pi$, simply choose $\eta = \pi$, which corresponds to changing the overall sign of $\Phi_1^\dagger \Phi_2$. This is equivalent to redefining the parameters $m_{12}^2 \to -m_{12}^2$, $\lambda_6 \to -\lambda_6$ and $\lambda_7 \to -\lambda_7$.

[27] Under the assumption that v_1 and v_2 are positive, eq. (240) implicitly employs the convention in which $\xi = 0$.

it follows that if $\lambda_5 > 0$, then the condition $m_A^2 \geq 0$ is equivalent to eq. (302). However, if $\lambda_5 \leq 0$, then eq. (303) implies that $m_A^2 \geq |\lambda_5| v^2$.

A singular limit: $m_h = m_H$

By definition, $m_h \leq m_H$. The limiting case of $m_h = m_H$ is special and requires careful treatment in some cases. For example, despite the appearance of $m_H^2 - m_h^2$ in the denominator of eq. (250), one can show that $0 \leq c_{\beta-\alpha}^2 \leq 1$. To prove this, we first write

$$c_{\beta-\alpha}^2 = \frac{1}{2}\left[1 - \frac{m_S^2 - 2m_L^2}{\sqrt{m_S^4 - 4m_A^2 m_L^2 - 4m_D^4}}\right] . \tag{305}$$

Next, we use eq. (248) to explicitly compute:

$$m_S^4 - 4m_A^2 m_L^2 - 4m_D^4 = m_A^4 - 2m_A^2\left[(\mathcal{B}_{22}^2 - \mathcal{B}_{11}^2)c_{2\beta} + 2\mathcal{B}_{12}^2 s_{2\beta}\right] + (\mathcal{B}_{11}^2 - \mathcal{B}_{22}^2)^2 + 4[\mathcal{B}_{12}^2]^2 , \tag{306}$$

and

$$(m_S^2 - 2m_L^2)^2 = m_S^4 - 4m_A^2 m_L^2 - 4m_D^4 - \left[(\mathcal{B}_{11}^2 - \mathcal{B}_{22}^2)s_{2\beta} - 2\mathcal{B}_{12}^2 c_{2\beta}\right]^2 . \tag{307}$$

Note that eq. (306), viewed as a quadratic function of m_A^2 (of the form $Am_A^4 + Bm_A^2 + C$), is non-negative if $B^2 - 4AC = \left[(\mathcal{B}_{11}^2 - \mathcal{B}_{22}^2)s_{2\beta} - 2\mathcal{B}_{12}^2 c_{2\beta}\right]^2 \geq 0$. It then follows from eq. (305) that $0 \leq c_{\beta-\alpha}^2 \leq 1$ if

$$(m_S^2 - 2m_L^2)^2 \leq m_S^4 - 4m_A^2 m_L^2 - 4m_D^4 , \tag{308}$$

a result which is manifestly true [see eq. (307)].

We now turn to the case of $m_h = m_H$. This can arise if and only if the CP-even Higgs squared-mass matrix (in any basis) is proportional to the unit matrix. From eq. (242), it then follows that:

$$\mathcal{B}_{11}^2 - \mathcal{B}_{22}^2 = m_A^2 c_{2\beta} , \qquad 2\mathcal{B}_{12}^2 = m_A^2 s_{2\beta} . \tag{309}$$

where $m_h^2 = m_H^2 = \mathcal{B}_{11}^2 + m_A^2 s_\beta^2 = \mathcal{B}_{22}^2 + m_A^2 c_\beta^2$. Alternatively, from eq. (249), the condition for $m_h = m_H$ is given by $m_S^4 - 4m_A^2 m_L^2 - 4m_D^4 \equiv Am_A^4 + Bm_A^2 + C = 0$. However, one must check that this quadratic equation possesses a positive (real) solution for m_A^2. Noting the discussion above eq. (308), such a solution can exist if and only if $B^2 - 4AC = 0$, which is indeed consistent with eq. (309). Of course, the results of eq. (309) are not compatible with the decoupling limit, since it is not possible to have $m_h = m_H$ and $m_A^2 \gg |\lambda_i| v^2$.

If we take $B^2 - 4AC = 0$ but keep m_A arbitrary, then eq. (305) yields

$$c_{\beta-\alpha}^2 = \begin{cases} 0\,, & \text{if} \quad m_L^2 < \frac{1}{2} m_S^2\,, \\ 1\,, & \text{if} \quad m_L^2 > \frac{1}{2} m_S^2\,. \end{cases} \tag{310}$$

For $m_L^2 = \frac{1}{2} m_S^2$, we have $m_h^2 = m_H^2 = \frac{1}{2} m_S^2$, and the angle α is not well-defined. In this case, one cannot distinguish between h and H in either production or decays, and the corresponding squared-amplitudes should be (incoherently) added in all processes. It is easy to check that the undetermined angle α that appears in the relevant Higgs couplings would then drop out in any such sum of squared-amplitudes. The singular point of parameter space corresponding to $m_h = m_H$ will not be considered further in this paper.

Relations among Higgs potential parameters and masses

It is useful to express the physical Higgs masses in terms of the parameters of the scalar potential [eq. (231)]. First, inserting eqs. (242) and (243) into eq. (245) and examining the diagonal elements yields the CP-even Higgs boson squared-masses:

$$m_h^2 = m_A^2 c_{\beta-\alpha}^2 + v^2\Big[\lambda_1 c_\beta^2 s_\alpha^2 + \lambda_2 s_\beta^2 c_\alpha^2 - 2\lambda_{345} c_\alpha c_\beta s_\alpha s_\beta + \lambda_5 c_{\beta-\alpha}^2 - 2\lambda_6 c_\beta s_\alpha c_{\beta+\alpha} + 2\lambda_7 s_\beta c_\alpha c_{\beta+\alpha}\Big], \quad (311)$$

$$m_H^2 = m_A^2 s_{\beta-\alpha}^2 + v^2\Big[\lambda_1 c_\beta^2 c_\alpha^2 + \lambda_2 s_\beta^2 s_\alpha^2 + 2\lambda_{345} c_\alpha c_\beta s_\alpha s_\beta + \lambda_5 s_{\beta-\alpha}^2 + 2\lambda_6 c_\beta c_\alpha s_{\beta+\alpha} + 2\lambda_7 s_\beta s_\alpha s_{\beta+\alpha}\Big], \quad (312)$$

while the requirement that the off-diagonal entries in eq. (245) are zero yields

$$\begin{aligned} m_A^2\, s_{\beta-\alpha}c_{\beta-\alpha} &= \tfrac{1}{2}v^2\Big[s_{2\alpha}(-\lambda_1 c_\beta^2+\lambda_2 s_\beta^2)+\lambda_{345}s_{2\beta}c_{2\alpha}-2\lambda_5 s_{\beta-\alpha}c_{\beta-\alpha} \\ &\qquad +2\lambda_6 c_\beta c_{\beta+2\alpha}+2\lambda_7 s_\beta s_{\beta+2\alpha}\Big]\,, \end{aligned} \tag{313}$$

where $\lambda_{345}\equiv\lambda_3+\lambda_4+\lambda_5$. We can now eliminate m_A^2 from eqs. (311) and (312) and eqs. (240) and (241) using the result of eq. (313). This yields equations for the other three physical Higgs boson squared-masses and the scalar potential mass parameter m_{12}^2 in terms of the Higgs scalar quartic couplings

$$\begin{aligned} \frac{m_h^2}{v^2}s_{\beta-\alpha} &= -\lambda_1 c_\beta^3 s_\alpha+\lambda_2 s_\beta^3 c_\alpha+\tfrac{1}{2}\lambda_{345}c_{\beta+\alpha}s_{2\beta} \\ &\quad +\lambda_6 c_\beta^2(c_\beta c_\alpha-3s_\beta s_\alpha)+\lambda_7 s_\beta^2(3c_\beta c_\alpha-s_\beta s_\alpha)\,, \end{aligned} \tag{314}$$

$$\begin{aligned} \frac{m_H^2}{v^2}c_{\beta-\alpha} &= \lambda_1 c_\beta^3 c_\alpha+\lambda_2 s_\beta^3 s_\alpha+\tfrac{1}{2}\lambda_{345}s_{\beta+\alpha}s_{2\beta} \\ &\quad +\lambda_6 c_\beta^2(3s_\beta c_\alpha+c_\beta s_\alpha)+\lambda_7 s_\beta^2(s_\beta c_\alpha+3c_\beta s_\alpha)\,, \end{aligned} \tag{315}$$

$$\frac{2m^2_{H^\pm}}{v^2} s_{\beta-\alpha}c_{\beta-\alpha} = -s_{2\alpha}(\lambda_1 c^2_\beta - \lambda_2 s^2_\beta) + \lambda_{345} s_{2\beta}c_{2\alpha} - (\lambda_4+\lambda_5)s_{\beta-\alpha}c_{\beta-\alpha} + 2\lambda_6 c_\beta c_{\beta+2\alpha} + 2\lambda_7 s_\beta s_{\beta+2\alpha}\,, \tag{316}$$

$$\frac{2m^2_{12}}{v^2} s_{\beta-\alpha}c_{\beta-\alpha} = -\tfrac{1}{2}s_{2\beta}s_{2\alpha}(\lambda_1 c^2_\beta - \lambda_2 s^2_\beta) + \tfrac{1}{2}\lambda_{345}s^2_{2\beta}c_{2\alpha} + \lambda_6 c^2_\beta\left[3c_\beta s_\beta c_{2\alpha} - c_\alpha s_\alpha(1+2s^2_\beta)\right] \tag{317}$$

$$+\lambda_7 s^2_\beta\left[3s_\beta c_\beta c_{2\alpha} + c_\alpha s_\alpha(1+2c^2_\beta)\right]\,. \tag{318}$$

Note that eq. (316) is easily derived by inserting eq. (313) into eq. (241). A related useful result is easily derived from eqs. (313) and (315):

$$\frac{(m^2_A - m^2_H)}{v^2} s_{\beta-\alpha} = \tfrac{1}{2}s_{2\beta}\left(-\lambda_1 c_\alpha c_\beta + \lambda_2 s_\alpha s_\beta + \lambda_{345}c_{\beta+\alpha}\right) - \lambda_5 s_{\beta-\alpha} + \lambda_6 c_\beta\left[c_\beta c_{\beta+\alpha} - 2s^2_\beta c_\alpha\right] + \lambda_7 s_\beta\left[s_\beta c_{\beta+\alpha} + 2c^2_\beta s_\alpha\right]. \tag{319}$$

It is remarkable that the left hand side of eq. (319) is proportional only to $s_{\beta-\alpha}$ (*i.e.*, the factor of $c_{\beta-\alpha}$ has canceled). As a result, in the decoupling limit where $c_{\beta-\alpha} \to 0$, we see that $m_A^2 - m_H^2 = \mathcal{O}(v^2)$.

The expressions given in eqs. (313)–(316) are quite complicated. These results simplify considerably when expressed in terms of λ, $\widehat{\lambda}$ and λ_A [eqs. (255)–(257)]:

$$m_A^2 = v^2\left[\lambda_A + \widehat{\lambda}\left(\frac{s_{\beta-\alpha}}{c_{\beta-\alpha}} - \frac{c_{\beta-\alpha}}{s_{\beta-\alpha}}\right)\right], \tag{320}$$

$$m_h^2 = v^2\left[\lambda - \frac{\widehat{\lambda}\, c_{\beta-\alpha}}{s_{\beta-\alpha}}\right], \tag{321}$$

$$m_H^2 = v^2\left[\lambda + \frac{\widehat{\lambda}\, s_{\beta-\alpha}}{c_{\beta-\alpha}}\right]. \tag{322}$$

One can then rewrite eq. (319) as

$$m_H^2 - m_A^2 = v^2\left[\lambda - \lambda_A + \frac{\widehat{\lambda} c_{\beta-\alpha}}{s_{\beta-\alpha}}\right]. \tag{323}$$

We can invert eqs. (313)–(318) and solve for any five of the scalar potential parameters in terms of the physical Higgs masses and the remaining three undetermined variables [661, 697, 698]. It is convenient to solve for $\lambda_1, \ldots, \lambda_5$ in terms of λ_6, λ_7, m_{12}^2 and the Higgs masses. We obtain:

$$\lambda_1 = \frac{m_H^2 c_\alpha^2 + m_h^2 s_\alpha^2 - m_{12}^2 \tilde{b}}{v^2 c_\beta^2} - \tfrac{3}{2}\lambda_6 \tilde{b} + \tfrac{1}{2}\lambda_7 \tilde{b}^3 \,, \tag{324}$$

$$\lambda_2 = \frac{m_H^2 s_\alpha^2 + m_h^2 c_\alpha^2 - m_{12}^2 \tilde{b}^{-1}}{v^2 s_\beta^2} + \tfrac{1}{2}\lambda_6 \tilde{b}^{-3} - \tfrac{3}{2}\lambda_7 \tilde{b}^{-1} \,, \tag{325}$$

$$\lambda_3 = \frac{(m_H^2 - m_h^2) c_\alpha s_\alpha + 2 m_{H^\pm}^2 s_\beta c_\beta - m_{12}^2}{v^2 s_\beta c_\beta} - \tfrac{1}{2}\lambda_6 \tilde{b}^{-1} - \tfrac{1}{2}\lambda_7 \tilde{b} \tag{326}$$

$$\lambda_4 = \frac{(m_A^2 - 2 m_{H^\pm}^2) s_\beta c_\beta + m_{12}^2}{v^2 s_\beta c_\beta} - \tfrac{1}{2}\lambda_6 \tilde{b}^{-1} - \tfrac{1}{2}\lambda_7 \tilde{b} \,, \tag{327}$$

$$\lambda_5 = \frac{m_{12}^2 - m_A^2 s_\beta c_\beta}{v^2 s_\beta c_\beta} - \tfrac{1}{2}\lambda_6 \tilde{b}^{-1} - \tfrac{1}{2}\lambda_7 \tilde{b} \,. \tag{328}$$

In addition, the minimization conditions of eqs. (236) and (237) reduce to:

$$m_{11}^2 = -\frac{1}{2 c_\beta}\left(m_H^2 c_\alpha c_{\beta-\alpha} - m_h^2 s_\alpha s_{\beta-\alpha}\right) + m_{12}^2 \tilde{b} \,, \tag{329}$$

$$m_{22}^2 = -\frac{1}{2s_\beta}\left(m_h^2 c_\alpha s_{\beta-\alpha} + m_H^2 s_\alpha c_{\beta-\alpha}\right) + m_{12}^2 \tilde{b}^{-1}\,. \quad (330)$$

Note that λ_6 and λ_7 do not appear when m_{11}^2 and m_{22}^2 are expressed entirely in terms of m_{12}^2 and physical Higgs masses.

In some cases, it proves more convenient to eliminate m_{12}^2 in favor of λ_5 using eq. (328). The end result is:

$$\lambda_1 = \frac{m_H^2 c_\alpha^2 + m_h^2 s_\alpha^2 - m_A^2 s_\beta^2}{v^2 c_\beta^2} - \lambda_5 \tilde{b}^2 - 2\lambda_6 \tilde{b}\,, \quad (331)$$

$$\lambda_2 = \frac{m_H^2 s_\alpha^2 + m_h^2 c_\alpha^2 - m_A^2 c_\beta^2}{v^2 s_\beta^2} - \lambda_5 \tilde{b}^{-2} - 2\lambda_7 \tilde{b}^{-1}\,, \quad (332)$$

$$\lambda_3 = \frac{(m_H^2 - m_h^2) s_\alpha c_\alpha + (2m_{H^\pm}^2 - m_A^2) s_\beta c_\beta}{v^2 s_\beta c_\beta} - \lambda_5 - \lambda_6 \tilde{b}^{-1} - \lambda_7 \tilde{b}\,, \quad (333)$$

$$\lambda_4 = \frac{2(m_A^2 - m_{H^\pm}^2)}{v^2} + \lambda_5\,, \quad (334)$$

and

$$m_{11}^2 = -\frac{1}{2c_\beta}\left(m_H^2 c_\alpha c_{\beta-\alpha} - m_h^2 s_\alpha s_{\beta-\alpha}\right) + (m_A^2 + \lambda_5 v^2) s_\beta^2 + \tfrac{1}{2} v^2 (\lambda_6 s_\beta c_\beta + \lambda_7 s_\beta^2 \tilde{b})\,,$$

(33

$$m_{22}^2 = -\frac{1}{2s_\beta}\left(m_h^2 c_\alpha s_{\beta-\alpha} + m_H^2 s_\alpha c_{\beta-\alpha}\right) + (m_A^2 + \lambda_5 v^2)c_\beta^2 + \tfrac{1}{2}v^2(\lambda_6 c_\beta^2 \tilde{b}^{-1} + \lambda_7 s_\beta c_\beta$$

(33

Using eqs. (320)–(322), one may obtain simple expressions for λ, $\widehat{\lambda}$ and λ_A [eqs. (255)–(257)] in terms of the neutral Higgs squared-masses:

$$\lambda v^2 = m_h^2 s_{\beta-\alpha}^2 + m_H^2 c_{\beta-\alpha}^2 , \tag{337}$$

$$\widehat{\lambda} v^2 = (m_H^2 - m_h^2) s_{\beta-\alpha} c_{\beta-\alpha} , \tag{338}$$

$$\lambda_A v^2 = m_A^2 + (m_H^2 - m_h^2)(c_{\beta-\alpha}^2 - s_{\beta-\alpha}^2) , \tag{339}$$

$$\lambda_F v^2 = 2(m_{H^\pm}^2 - m_A^2) , \tag{340}$$

where we have also included an expression for $\lambda_F \equiv \lambda_5 - \lambda_4$ in terms of the Higgs squared-masses [see eq. (241)]. Thus, four of the the invariant coupling parameters can be expressed in terms

of the physical Higgs masses and the basis-independent quantity $\beta - \alpha$ (see Appendix).

Finally, we note that eqs. (338) and (339) also yield a simple expression for $\beta - \alpha$, which plays such a central role in the decoupling limit. We find two forms that are noteworthy:

$$\tan\left[2(\beta-\alpha)\right] = \frac{-2\widehat{\lambda}v^2}{m_A^2 - \lambda_A v^2}\,, \tag{341}$$

and

$$\sin\left[2(\beta-\alpha)\right] = \frac{2\widehat{\lambda}v^2}{m_H^2 - m_h^2}\,. \tag{342}$$

Indeed, if $\widehat{\lambda} = 0$ then either $c_{\beta-\alpha} = 0$ or $s_{\beta-\alpha} = 0$ as discussed in Section 5. For $\widehat{\lambda} \neq 0$, the condition $m_H > m_h$ implies that $\widehat{\lambda} s_{\beta-\alpha} c_{\beta-\alpha} > 0$. This inequality, when applied to eq. (320), imposes the following constraint on m_A

$$v^2\left[\lambda_A-\frac{2\widehat{\lambda}\,c_{\beta-\alpha}}{s_{\beta-\alpha}}\right]<m_A^2<v^2\left[\lambda_A+\frac{2\widehat{\lambda}\,s_{\beta-\alpha}}{c_{\beta-\alpha}}\right]. \qquad (343)$$

In addition, we require that $m_A^2 \geq 0$.

The expressions for the Higgs masses [eqs. (320)–(322)] and $\beta-\alpha$ [eq. (341) or (342)] are especially useful when considering the approach to the decoupling limit, where $|c_{\beta-\alpha}| \ll 1$. For example, eqs. (320)–(322) reduce in this limit to the results of eqs. (259)–(261). Moreover, $\sin[2(\beta-\alpha)] \simeq -\tan[2(\beta-\alpha)] \simeq 2c_{\beta-\alpha}$, and eqs. (341) and (342) reduce to the results given by eq. (266). The corresponding results in limiting case of $|s_{\beta-\alpha}| \ll 1$ treated in Section 5 are also similarly obtained.

Invariant combinations of the Higgs scalar potential parameters

In the most general 2HDM model, there is no distinction between the two $Y = 1$ complex doublets, Φ_1 and Φ_2. In principle, one could choose any two orthogonal linear combinations of Φ_1 and Φ_2 ($i.e.$, choose a new basis for the scalar doublets), and construct the scalar sector Lagrangian with respect to the new basis. Clearly, the parameters of eq. (231), m_{ij}^2 and the λ_i, would all be modified, along with α and β. However, there exists seven invariant combinations of the λ_i that are independent of basis choice [699]. These are: λ, $\widehat{\lambda}$, λ_A, λ_F defined in eqs. (255)–(258), and λ_T, λ_U and λ_V defined in eqs. (283)–(285). In addition, the combination $\beta - \alpha$ is clearly basis independent. Thus, all physical Higgs masses and Higgs self-couplings can be expressed in terms of the above invariant coupling parameters and $\beta - \alpha$. Earlier, we have already shown how to express the Higgs masses in terms of the invariant parameters. One can also write the three-Higgs and four-Higgs couplings in terms of the invariant parameters.[28]

To obtain expressions for the Higgs self-couplings in terms of invariant parameters, one must invert the relations between the λ_i and the invariant coupling parameters. The end result is:

$$
\begin{aligned}
\lambda_1 &= c_\beta^2(1+3s_\beta^2)\lambda + 2s_{2\beta}(c_\beta^2\,\widehat{\lambda} + s_\beta^2\,\lambda_U) - \tfrac{1}{2}s_{2\beta}^2(2\lambda_A - \lambda_T) + s_\beta^4\lambda_V\,, \\
\lambda_2 &= s_\beta^2(1+3c_\beta^2)\lambda - 2s_{2\beta}(s_\beta^2\,\widehat{\lambda} + c_\beta^2\,\lambda_U) - \tfrac{1}{2}s_{2\beta}^2(2\lambda_A - \lambda_T) + c_\beta^4\lambda_V\,, \\
\lambda_{345} &= (2c_{2\beta}^2 - c_\beta^2 s_\beta^2)\lambda - 3s_{2\beta}c_{2\beta}(\widehat{\lambda} - \lambda_U) - (c_{2\beta}^2 - 2c_\beta^2 s_\beta^2)(2\lambda_A - \lambda_T) + \tfrac{3}{4}s_{2\beta}^2\lambda_V\,, \\
\lambda_5 &= (c_{2\beta}^2 + c_\beta^2 s_\beta^2)\lambda - s_{2\beta}c_{2\beta}(\widehat{\lambda} - \lambda_U) - c_{2\beta}^2\lambda_A + \tfrac{1}{4}s_{2\beta}^2(\lambda_V - 2\lambda_T)\,, \\
\lambda_6 &= \tfrac{1}{2}s_{2\beta}(3s_\beta^2 - 1)\lambda - c_\beta c_{3\beta}\widehat{\lambda} - s_\beta s_{3\beta}\lambda_U + \tfrac{1}{2}s_{2\beta}c_{2\beta}(2\lambda_A - \lambda_T) - \tfrac{1}{2}s_\beta^2 s_{2\beta}\lambda_V\,, \\
\lambda_7 &= \tfrac{1}{2}s_{2\beta}(3c_\beta^2 - 1)\lambda - s_\beta s_{3\beta}\widehat{\lambda} - c_\beta c_{3\beta}\lambda_U - \tfrac{1}{2}s_{2\beta}c_{2\beta}(2\lambda_A - \lambda_T) - \tfrac{1}{2}c_\beta^2 s_{2\beta}\lambda_V\,, \qquad (344)
\end{aligned}
$$

and $\lambda_4 = \lambda_5 - \lambda_F$.

The significance of the invariant coupling parameters is most evident in the so-called Higgs basis of ref. [693], in which only

[28] The Higgs couplings to vector bosons depend only on $\beta - \alpha$ [see eqs. (267)–(269)].

The Higgs couplings to fermions in the Type-III model (in which both up and down-type fermions couple to both Higgs doublets) can also be written in terms of invariant parameters. However, one would then have to identify the appropriate invariant combinations of the Higgs-fermion Yukawa coupling parameters [699], η_i^U and η_i^D [see eq. (272)].

the neutral component of one of the two Higgs doublets (say, the first one) possesses a vacuum expectation value. Let us denote the two Higgs doublets in this basis by Φ_a and Φ_b. Then, after a rotation from the Φ_1–Φ_2 basis by an angle β,

$$\begin{aligned} \Phi_a &= \Phi_1 \cos\beta + \Phi_2 \sin\beta\,, \\ \Phi_b &= -\Phi_1 \sin\beta + \Phi_2 \cos\beta\,, \end{aligned} \tag{345}$$

one obtains

$$\Phi_a = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + \varphi_a^0 + iG^0\right) \end{pmatrix}, \qquad \Phi_b = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(\varphi_b^0 + iA\right) \end{pmatrix}, \tag{346}$$

where φ_a^0 and φ_b^0 are related in the CP-conserving model to the CP-even neutral Higgs bosons by:

$$H = \varphi_a^0 \cos(\beta-\alpha) - \varphi_b^0 \sin(\beta-\alpha)\,, \tag{347}$$

$$h = \varphi_a^0 \sin(\beta-\alpha) + \varphi_b^0 \cos(\beta-\alpha)\,. \tag{348}$$

Here, we see that $\beta-\alpha$ is the invariant angle that characterizes the direction of the CP-even mass eigenstates (in the two-

dimensional Higgs "flavor" space) relative to that of the vacuum expectation value.

In the Higgs basis, the corresponding values of $\lambda_1, \cdots, \lambda_7$ are easily evaluated by putting $\beta = 0$ in eq. (344). Thus, the scalar potential takes the following form:

$$\begin{aligned}
\mathcal{V} &= m_{aa}^2 \Phi_a^\dagger \Phi_a + m_{bb}^2 \Phi_b^\dagger \Phi_b - [m_{ab}^2 \Phi_a^\dagger \Phi_b + \mathrm{h.c.}] \\
&\quad + \tfrac{1}{2}\lambda (\Phi_a^\dagger \Phi_a)^2 + \tfrac{1}{2}\lambda_V (\Phi_b^\dagger \Phi_b)^2 + (\lambda_T + \lambda_F)(\Phi_a^\dagger \Phi_a)(\Phi_b^\dagger \Phi_b) \qquad (349) \\
&\quad + (\lambda - \lambda_A - \lambda_F)(\Phi_a^\dagger \Phi_b)(\Phi_b^\dagger \Phi_a) \\
&\quad + \left\{ \tfrac{1}{2}(\lambda - \lambda_A)(\Phi_a^\dagger \Phi_b)^2 - \left[\widehat{\lambda}\, (\Phi_a^\dagger \Phi_a) + \lambda_U (\Phi_b^\dagger \Phi_b) \right] \Phi_a^\dagger \Phi_b + \mathrm{h.c.} \right\}, \qquad (350)
\end{aligned}$$

where three new invariant quantities are revealed:

$$m_{aa}^2 = m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - [m_{12}^2 + (m_{12}^*)^2] s_\beta c_\beta \,, \qquad (351)$$

$$m_{bb}^2 = m_{11}^2 s_\beta^2 + m_{22}^2 c_\beta^2 + [m_{12}^2 + (m_{12}^*)^2] s_\beta c_\beta \,, \qquad (352)$$

$$m_{ab}^2 = (m_{11}^2 - m_{22}^2) s_\beta c_\beta + m_{12}^2 c_\beta^2 - (m_{12}^*)^2 s_\beta^2 \,. \qquad (353)$$

In the CP-conserving theory where m_{12}^2 is real, the corresponding

potential minimum conditions [eqs. (236)–(237)] simplify to:

$$m_{aa}^2 = -\tfrac{1}{2}v^2\lambda\,, \qquad m_{ab}^2 = -\tfrac{1}{2}v^2\widehat{\lambda}\,, \tag{354}$$

with no constraint on m_{bb}^2. In fact, m_{bb}^2 is related to m_A^2:

$$\begin{aligned} m_A^2 &= \mathrm{Tr}\, m^2 + \tfrac{1}{2}v^2(\lambda + \lambda_T) \\ &= m_{bb}^2 + \tfrac{1}{2}v^2\lambda_T\,, \end{aligned} \tag{355}$$

after imposing the potential minimum condition [eq. (354)]. It is convenient to trade the free parameter m_{bb}^2 for $\beta - \alpha$. Using the results of eqs. (341) and (342), it follows that

$$\tan[2(\beta-\alpha)] = \frac{2\widehat{\lambda}}{\lambda_A - \tfrac{1}{2}\lambda_T - m_{bb}^2/v^2}\,, \tag{356}$$

where the sign of $\sin[2(\beta-\alpha)]$ is equal to the sign of $\widehat{\lambda}$.

It is now straightforward to obtain the three-Higgs and four-

Higgs couplings in terms of the invariant coupling parameters and $\beta - \alpha$, by inserting eqs. (346)–(348) into eq. (350).

Some more on the MSSM Higgs sector: theoretical basis

Note: There are equation numbers appearing as ?? marks in what follows. If you want to learn more about the referenced equations, you can go look for these equation numbers in the .pdf file for the darkmatter-susy course linked to my home page. The discussion below is taken from the notes for that course.

- **Electroweak symmetry breaking and the Higgs bosons**

  In the MSSM, the description of electroweak symmetry breaking is slightly complicated by the fact that there are two complex Higgs doublets $H_u = (H_u^+,\ H_u^0)$ and $H_d = (H_d^0,\ H_d^-)$ rather than just one as in the ordinary Standard Model.
  The classical scalar potential for the Higgs scalar fields in the MSSM by

$$V = (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2)$$

$$
\begin{aligned}
&+ [b\,(H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.}] \\
&+\frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 \\
&+\tfrac{1}{2} g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2.
\end{aligned}
\tag{357}
$$

We note the following:

– The terms proportional to $|\mu|^2$ come from F-terms [see Eq. (??)].

– The terms proportional to g^2 and g'^2 are the D-term contributions, obtained from the general formula Eq. (??) after some rearranging.

– Finally, the terms proportional to $m_{H_u}^2$, $m_{H_d}^2$ and b are just a rewriting of the last three terms of Eq. (??) using the identity

$$
|H_u^{i\,*} H_d^i|^2 + |\epsilon_{ij} H_u^i H_d^j|^2 = (H_u^{i\,*} H_u^i)(H_d^{j\,*} H_d^j) \tag{358}
$$

The full scalar potential of the theory also includes many terms involving the squark and slepton fields that we can ignore here,

since they do not get VEVs because they have large positive squared masses.

We now have to demand that the minimum of this potential should break electroweak symmetry down to electromagnetism $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$, in accord with experiment.

We can use the freedom to make gauge transformations to simplify this analysis.

- **First, the freedom to make $SU(2)_L$ gauge transformations allows us to rotate away a possible VEV for one of the weak isospin components of one of the scalar fields, so without loss of generality we can take $H_u^+ = 0$ at the minimum of the potential.**
- **Then, we can examine the condition for a minimum of the**

potential satisfying

$$\left.\frac{\partial V}{\partial H_u^+}\right|_{H_u^+=0} = bH_d^- + \tfrac{1}{2}g^2 H_d^{0*} H_d^- H_u^{0*} = 0\,. \qquad (359)$$

For generic parameter choices this will not vanish unless $H_d^- = 0$.
This is good, because it means that at the minimum of the potential electromagnetism is necessarily unbroken, due to the fact that the charged components of the Higgs scalars cannot get VEVs.

– **After setting $H_u^+ = H_d^- = 0$, we are left to consider the scalar potential involving only the neutral Higgs fields:**

$$\begin{aligned} V &= (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (b\, H_u^0 H_d^0 + \mathrm{c.c.}) \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2. \end{aligned} \qquad (360)$$

- The only term in this potential that depends on the phases of the fields is the b-term.
  Therefore, a redefinition of the phase of H_u or H_d can absorb any phase in b, so we can take b to be real and positive.
- Then it is clear that a minimum of the potential V requires that $H_u^0 H_d^0$ is also real and positive, so $\langle H_u^0 \rangle$ and $\langle H_d^0 \rangle$ must have cancelling phases.
- We can therefore use a $U(1)_Y$ gauge transformation to make them both be real and positive without loss of generality, since H_u and H_d have opposite weak hypercharges ($\pm 1/2$).
- It follows that CP cannot be spontaneously broken by the Higgs scalar potential, since the VEVs and b can be simultaneously chosen real, as a convention.
- This means that the Higgs scalar mass eigenstates can be assigned well-defined eigenvalues of CP, at least at tree-level. (CP-violating phases in other couplings can induce

loop-suppressed CP violation in the Higgs sector, but do not change the fact that b, $\langle H_u^0\rangle$, and $\langle H_d\rangle$ can always be chosen real and positive.)

In order for the MSSM scalar potential to be viable, we must first make sure that the potential is bounded from below for arbitrarily large values of the scalar fields, so that V will really have a minimum. (Recall that scalar potentials in purely supersymmetric theories are automatically non-negative and so clearly bounded from below. But, now that we have introduced supersymmetry breaking, we must be careful.)

The scalar quartic interactions in V will stabilize the potential for almost all arbitrarily large values of H_u^0 and H_d^0.

However, for the special directions in field space $|H_u^0| = |H_d^0|$, the quartic contributions to V [the second line in Eq. (363)] are identically zero.

Such directions in field space are called D-flat directions, because along them the part of the scalar potential coming from D-terms vanishes.

In order for the potential to be bounded from below, we need the quadratic part of the scalar potential to be positive along the D-flat directions. This requirement amounts to

$$2b < 2|\mu|^2 + m^2_{H_u} + m^2_{H_d}. \tag{361}$$

Note that the b-term always favors electroweak symmetry breaking.

Requiring that one linear combination of H_u^0 and H_d^0 has a negative squared mass near $H_u^0 = H_d^0 = 0$ ($i.e.$ requiring that the determinant of the mass-squared matrix be negative) gives

$$b^2 > (|\mu|^2 + m^2_{H_u})(|\mu|^2 + m^2_{H_d}). \tag{362}$$

If this inequality is not satisfied, then $H_u^0 = H_d^0 = 0$ will be a stable minimum of the potential (or there will be no stable minimum at all), and electroweak symmetry breaking will not occur.

Interestingly, if $m^2_{H_u} = m^2_{H_d}$ then the constraints Eqs. (361) and (362) cannot both be satisfied.

In models derived from the minimal supergravity or gauge-mediated boundary conditions, $m^2_{H_u} = m^2_{H_d}$ is supposed to hold at tree level at the input scale, but the contribution to the RG equation for $m^2_{H_u}$ proportional to the square of the large top-quark Yukawa coupling y_t naturally pushes $m^2_{H_u}$ to negative or small values $m^2_{H_u} < m^2_{H_d}$ at the electroweak scale.

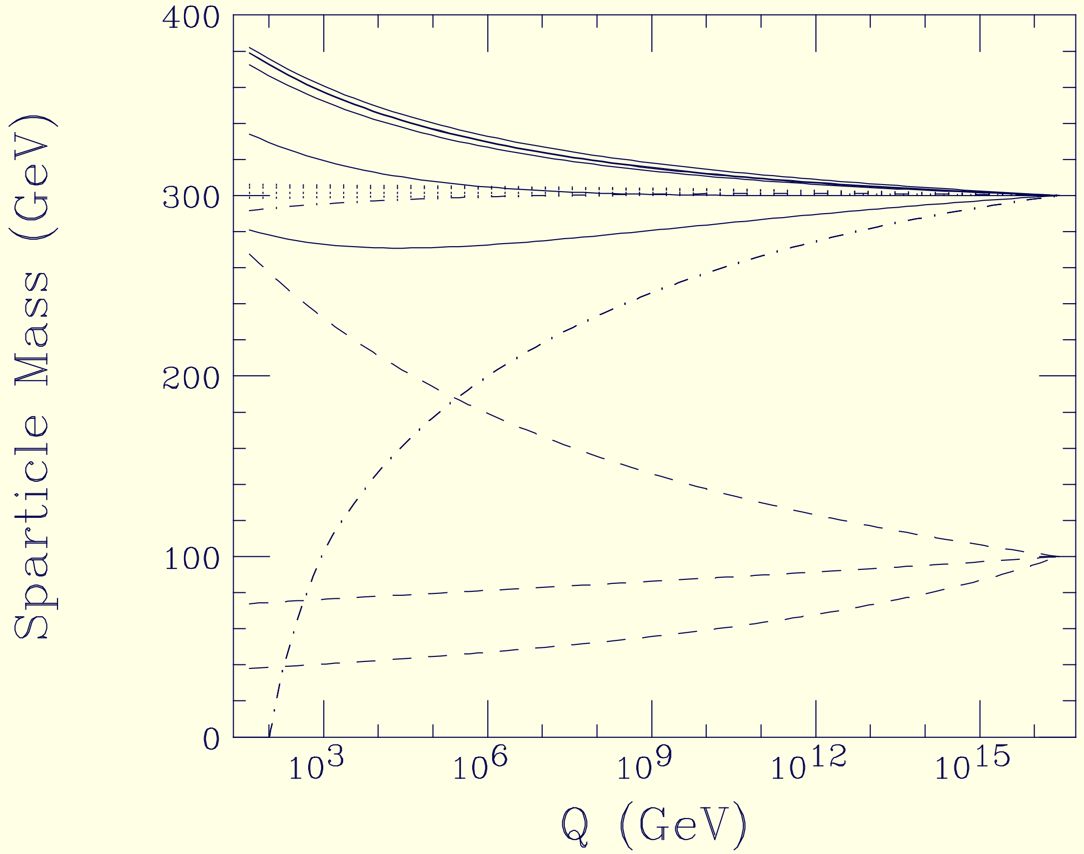


Figure 34: **Illustration of RG evolution of soft parameters showing how m^2_{Hu} is driven negative in evolving from GUT scale to m_Z scale. Some other things to note: gaugino masses can unify if $M_3 \sim 3M_2 \sim 6M_1$ at scale m_Z; squark masses increase as scale decreases, but slepton masses don't change a lot.**

Unless this effect is significant, the parameter space in which the electroweak symmetry is broken would be quite small. **So, in these models electroweak symmetry breaking is actually driven by quantum corrections; this mechanism is therefore known as** *radiative electroweak symmetry breaking*.

Note that although a negative value for $|\mu|^2 + m^2_{H_u}$ will help Eq. (362) to be satisfied, it is not strictly necessary.

Furthermore, even if $m^2_{H_u} < 0$, there may be no electroweak symmetry breaking if $|\mu|$ is too large or if b is too small.

Still, the large negative contributions to $m^2_{H_u}$ from the RG equation are an important factor in ensuring that electroweak symmetry breaking can occur in models with simple GUT-scale boundary conditions for the soft terms.

The realization that this works most naturally with a large top-quark Yukawa coupling provides additional motivation for these models.

Having established the conditions necessary for H_u^0 and H_d^0 to get non-zero VEVs, we can now require that they are compatible with the observed phenomenology of electroweak symmetry breaking, $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$. Returning to the scalar potential involving only the neutral Higgs fields:

$$\begin{aligned} V &= (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (b\, H_u^0 H_d^0 + \text{c.c.}) \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2, \end{aligned} \tag{363}$$

let us write

$$v_u = \langle H_u^0 \rangle, \qquad\qquad v_d = \langle H_d^0 \rangle. \tag{364}$$

These VEVs are related to the known mass of the Z boson and the electroweak gauge couplings (I have changed my convention so that $v = v_{previous}/\sqrt{2}$.):

$$v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \approx (174\,\text{GeV})^2. \tag{365}$$

The ratio of the VEVs is traditionally written as

$$\tan\beta \equiv v_u/v_d. \tag{366}$$

The value of $\tan\beta$ is not fixed by present experiments, but it depends on the Lagrangian parameters of the MSSM in a calculable way.

Since $v_u = v\sin\beta$ and $v_d = v\cos\beta$ were taken to be real and positive by convention, we have $0 < \beta < \pi/2$, a requirement that will be sharpened below.

Now one can write down the conditions $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$ under which the potential Eq. (363) will have a minimum satisfying Eqs. (365) and (366):

$$m_{H_u}^2 + |\mu|^2 - b\cot\beta - (m_Z^2/2)\cos(2\beta) \;=\; 0, \tag{367}$$

$$m_{H_d}^2 + |\mu|^2 - b\tan\beta + (m_Z^2/2)\cos(2\beta) \;=\; 0. \tag{368}$$

It is easy to check that these equations indeed satisfy the necessary conditions Eqs. (361) and (362). They allow us to eliminate two of the Lagrangian parameters b and $|\mu|$ in favor of $\tan\beta$, but do not determine the phase of μ.

Taking $|\mu|^2$, b, $m^2_{H_u}$ and $m^2_{H_d}$ as input parameters, and m^2_Z and $\tan\beta$ as output parameters obtained by solving these two equations, one obtains:

$$\sin(2\beta) = \frac{2b}{m^2_{H_u} + m^2_{H_d} + 2|\mu|^2}, \tag{369}$$

$$m^2_Z = \frac{|m^2_{H_d} - m^2_{H_u}|}{\sqrt{1 - \sin^2(2\beta)}} - m^2_{H_u} - m^2_{H_d} - 2|\mu|^2. \tag{370}$$

(Note that $\sin(2\beta)$ is always positive. If $m^2_{H_u} < m^2_{H_d}$, as is usually assumed, then $\cos(2\beta)$ is negative; otherwise it is positive.)

As an aside, Eqs. (369) and (370) highlight the "μ problem" already mentioned earlier.

- **Without miraculous cancellations, all of the input parameters ought to be within an order of magnitude or two of m_Z^2.**
- **However, in the MSSM, μ is a supersymmetry-respecting parameter appearing in the superpotential, while b, $m_{H_u}^2$, $m_{H_d}^2$ are supersymmetry-breaking parameters.**
- **This has lead to a widespread belief that the MSSM must be extended at very high energies to include a mechanism that relates the effective value of μ to the supersymmetry-breaking mechanism in some way.**
- **Even if the value of μ is set by soft supersymmetry breaking, the cancellation needed by Eq. (370) is often very substantial ($\Rightarrow$ finetuning) when evaluated in specific model frameworks, after constraints from direct searches for the Higgs bosons and superpartners are taken into account.**

– For example, expanding for large $\tan\beta$, Eq. (370) becomes

$$m_Z^2 = -2(m_{H_u}^2 + |\mu|^2) + \frac{2}{\tan^2\beta}(m_{H_d}^2 - m_{H_u}^2) + \mathcal{O}(1/\tan^4\beta). \tag{371}$$

Typical viable solutions for the MSSM have $-m_{H_u}^2$ and $|\mu|^2$ each much larger than m_Z^2, so that significant cancellation is needed.

– In particular, large top squark squared masses, needed to avoid having the Higgs boson mass turn out too small [see Eq. (385) below] compared to the 126 GeV observed mass, will feed into (via the RGEs) $m_{H_u}^2$ and make it normally quite large in magnitude and negative.

The cancellation needed in the minimal model is typically of order a fraction of a per cent level. It is impossible to objectively characterize whether this should be considered worrisome, but it could be taken as a weak hint in favor of non-minimal models such as the NMSSM.

Radiative corrections to the Higgs masses

The discussion above is based on the tree-level potential, and involves running renormalized Lagrangian parameters, which depend on the choice of renormalization scale.

In practice, one must include radiative corrections at one-loop order, at least, in order to get numerically stable results.

To do this, one can compute the loop corrections ΔV to the effective potential $V_{\rm eff}(v_u, v_d) = V + \Delta V$ as a function of the VEVs. The impact of this is that the equations governing the VEVs of the full effective potential are obtained by simply replacing

$$m_{H_u}^2 \to m_{H_u}^2 + \frac{1}{2v_u}\frac{\partial(\Delta V)}{\partial v_u}, \qquad m_{H_d}^2 \to m_{H_d}^2 + \frac{1}{2v_d}\frac{\partial(\Delta V)}{\partial v_d} \tag{372}$$

in Eqs. (367)-(370), treating v_u and v_d as real variables in the differentiation.

The above replacements come from the simple identity that (focusing on v_u as an example)

$$\begin{aligned} \frac{\partial}{\partial v_u}(V+\Delta V) &\ni \frac{\partial}{\partial v_u}[m_{H_u}^2 v_u^2 + \Delta V) \\ &= (m_{H_u}^2 2v_u + \frac{\partial \Delta V}{\partial v_u}) \\ &= (m_{H_u}^2 + \frac{1}{2v_u}\frac{\partial \Delta V}{\partial v_u})2v_u\,. \end{aligned} \tag{373}$$

The result for ΔV has now been obtained through two-loop order in the MSSM.

The most important corrections come from the one-loop diagrams involving the top squarks and top quark, and experience shows that the validity of the tree-level approximation and the convergence of perturbation theory are therefore improved by choosing a renormalization scale roughly of order the average of the top squark masses.

Mass eigenstates

The Higgs scalar fields in the MSSM consist of two complex $SU(2)_L$-doublet, or eight real, scalar degrees of freedom.

When the electroweak symmetry is broken, three of them are the would-be Nambu-Goldstone bosons G^0, $G^\pm$, which become the longitudinal modes of the Z and $W^\pm$ massive vector bosons.

The remaining five Higgs scalar mass eigenstates consist of two CP-even neutral scalars h and H, one CP-odd neutral scalar A, and a charge $+1$ scalar H^+ and its conjugate charge -1 scalar H^-. (Here we define $G^- = G^{+*}$ and $H^- = H^{+*}$. Also, by convention, h is lighter than H.)

The gauge-eigenstate fields can be expressed in terms of the mass eigenstate fields as:

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix} h \\ H \end{pmatrix} + \frac{i}{\sqrt{2}} R_{\beta_0} \begin{pmatrix} G^0 \\ A \end{pmatrix} \qquad (374)$$

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = R_{\beta_\pm} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \qquad (375)$$

where the orthogonal rotation matrices

$$R_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}, \qquad (37$$

$$R_{\beta_0} = \begin{pmatrix} \sin\beta_0 & \cos\beta_0 \\ -\cos\beta_0 & \sin\beta_0 \end{pmatrix}, \qquad R_{\beta_\pm} = \begin{pmatrix} \sin\beta_\pm & \cos\beta_\pm \\ -\cos\beta_\pm & \sin\beta_\pm \end{pmatrix}, \qquad (37$$

are chosen so that the quadratic part of the potential has diagonal squared-masses:

$$\begin{aligned} V &= \tfrac{1}{2} m_h^2 (h)^2 + \tfrac{1}{2} m_H^2 (H)^2 + \tfrac{1}{2} m_{G^0}^2 (G^0)^2 + \tfrac{1}{2} m_A^2 (A^0)^2 \\ &\quad + m_{G^\pm}^2 |G^+|^2 + m_{H^\pm}^2 |H^+|^2 + \ldots, h \end{aligned} \qquad (378)$$

Then, provided that v_u, v_d minimize the tree-level potential,[29] **one finds that $\beta_0 = \beta_\pm = \beta$, and $m^2_{G^0} = m^2_{G^\pm} = 0$, and**

$$m^2_A \;=\; 2b/\sin(2\beta) \;=\; 2|\mu|^2 + m^2_{H_u} + m^2_{H_d} \tag{379}$$

$$m^2_{h,H} \;=\; \tfrac{1}{2}\Big(m^2_A + m^2_Z \mp \sqrt{(m^2_A - m^2_Z)^2 + 4m^2_Z m^2_A \sin^2(2\beta)}\Big), \tag{380}$$

$$m^2_{H^\pm} \;=\; m^2_A + m^2_W. \tag{381}$$

The mixing angle α is determined, at tree-level, by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\left(\frac{m^2_H + m^2_h}{m^2_H - m^2_h}\right), \qquad \frac{\tan 2\alpha}{\tan 2\beta} = \left(\frac{m^2_A + m^2_Z}{m^2_A - m^2_Z}\right), \tag{382}$$

and is traditionally chosen to be negative; it follows that $-\pi/2 < \alpha < 0$ (provided $m_A > m_Z$). The Feynman rules for couplings of the mass eigenstate Higgs scalars to the Standard Model quarks and leptons and the electroweak vector bosons,

[29] It is often more useful to expand around VEVs v_u, v_d that do not minimize the tree-level potential, for example to minimize the loop-corrected effective potential instead. In that case, β, β_0, and $\beta_\pm$ are all slightly different.

as well as to the various sparticles, have been worked out in detail (Gunion-Haber, and HHG).

The masses of A, H and $H^{\pm}$ can in principle be arbitrarily large since they all grow with $b/\sin(2\beta)$. In contrast, the mass of h is bounded above. From Eq. (380), one finds at tree-level:

$$m_h < m_Z|\cos(2\beta)| \tag{383}$$

This corresponds to a shallow direction in the scalar potential, along the direction $(H_u^0 - v_u, H_d^0 - v_d) \propto (\cos\alpha, -\sin\alpha)$.

The existence of this shallow direction can be traced to the fact that the quartic Higgs couplings are given by the square of the electroweak gauge couplings, via the D-term.

A contour map of the potential, for a typical case with $\tan\beta \approx -\cot\alpha \approx 10$, is shown in Figure 35.

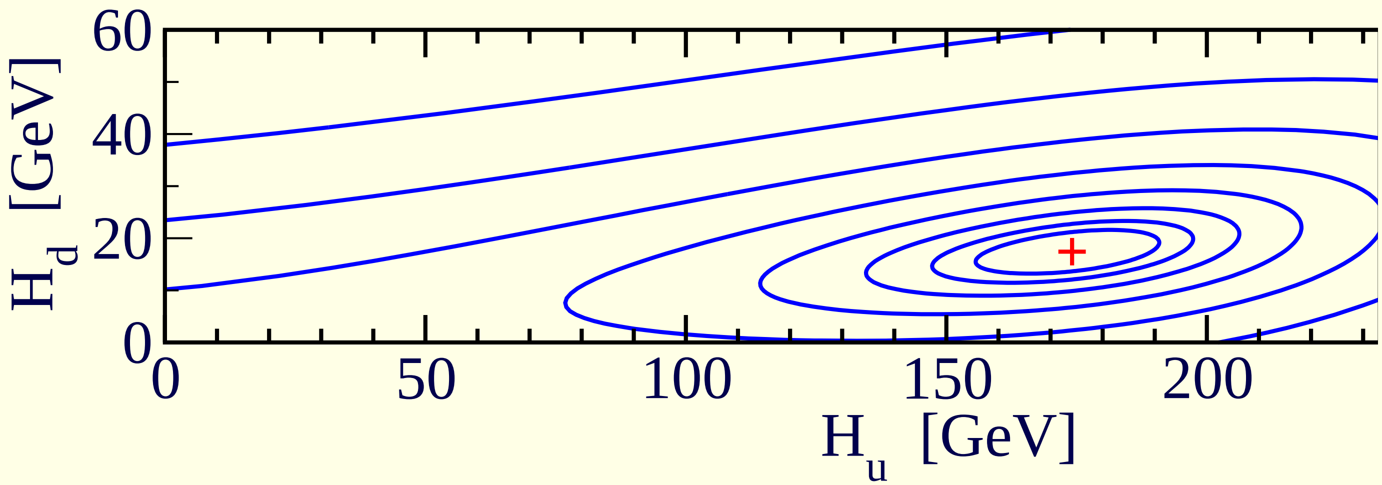


Figure 35: A contour map of the Higgs potential, for a typical case with $\tan\beta \approx -\cot\alpha \approx 10$. The minimum of the potential is marked by $+$, and the contours are equally spaced equipotentials. Oscillations along the shallow direction, with $H_u/H_d^0 \approx 10$, correspond to the mass eigenstate h, while the orthogonal steeper direction corresponds to the mass eigenstate H.

If the tree-level inequality (383) were robust, the lightest Higgs boson of the MSSM would have been discovered at LEP2. However, the tree-level formula for the squared mass of h is subject to quantum corrections that are relatively drastic.

The largest such contributions typically come from top and stop loops, as shown[30] in Fig. 36.

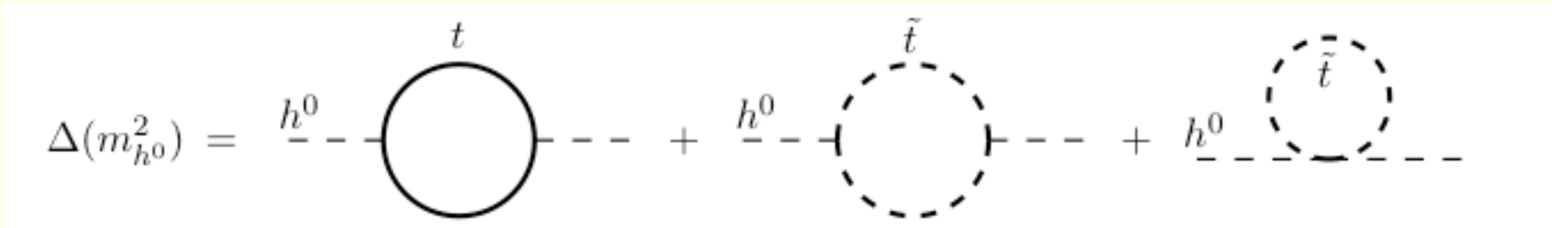


Figure 36: Contributions to the MSSM lightest Higgs mass from top-quark and top-squark one-loop diagrams. Incomplete cancellation, due to soft supersymmetry breaking, leads to a large positive correction to m_h^2 in the limit of heavy top squarks.

In the simple limit of top squarks that have a small mixing in the gauge eigenstate basis and with masses $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ much greater than the top quark mass m_t, one finds a large positive

[30] In general, one-loop 1-particle-reducible tadpole diagrams should also be included. However, they just cancel against tree-level tadpoles, and so both can be omitted, if the VEVs v_u and v_d are taken at the minimum of the loop-corrected effective potential (see previous footnote).

one-loop radiative correction to Eq. (380):

$$\Delta(m_h^2) = \frac{3}{4\pi^2}\cos^2\alpha\ y_t^2 m_t^2 \ln\left(m_{\widetilde{t}_1} m_{\widetilde{t}_2}/m_t^2\right). \qquad (384)$$

This shows that m_h can at least approach the observed 126 GeV mass.

An alternative way to understand the size of the radiative correction to the h mass is to consider an effective theory in which the heavy top squarks and top quark have been integrated out.

The quartic Higgs couplings in the low-energy effective theory get large positive contributions from the the one-loop diagrams of Fig. 37. This increases the steepness of the Higgs potential, and can be used to obtain the same result for the enhanced h mass.

Figure 37: Integrating out the top quark and top squarks yields large positive contributions to the quartic Higgs coupling in the low-energy effective theory, especially from these one-loop diagrams.

An interesting case, often referred to as the "decoupling limit", occurs when $m_A \gg m_Z$.

– Then m_h can saturate the upper bounds just mentioned, with $m_h^2 \approx m_Z^2 \cos^2(2\beta) +$ loop corrections.

– The particles A, H, and $H^\pm$ will be much heavier and nearly degenerate, forming an isospin doublet that decouples from sufficiently low-energy experiments.

– The angle α is very nearly $\beta - \pi/2$, and h has the same couplings to quarks and leptons and electroweak gauge bosons as would the physical Higgs boson of the ordinary

Standard Model without supersymmetry.

– Indeed, model-building experiences have shown that it is not uncommon for h to behave in a way nearly indistinguishable from a Standard Model-like Higgs boson, even if m_A is not too huge.

– However, it should be kept in mind that the couplings of h might turn out to deviate significantly from those of a Standard Model Higgs boson.

Top-squark mixing (which we may discuss later) can result in a further large positive contribution to m_h^2. At one-loop order, and working in the decoupling limit for simplicity, Eq. (384) generalizes to:

$$
\begin{aligned}
m_h^2 \quad = \quad & m_Z^2 \cos^2(2\beta) \\
& + \frac{3}{4\pi^2} \sin^2\beta \, y_t^2 \Big[m_t^2 \ln\left(m_{\tilde{t}_1} m_{\tilde{t}_2} / m_t^2 \right) + c_{\tilde{t}}^2 s_{\tilde{t}}^2 (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2) \ln(m_{\tilde{t}_2}^2 / m_{\tilde{t}_1}^2) \\
& + c_{\tilde{t}}^4 s_{\tilde{t}}^4 \Big\{ (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)^2 - \tfrac{1}{2} (m_{\tilde{t}_2}^4 - m_{\tilde{t}_1}^4) \ln(m_{\tilde{t}_2}^2 / m_{\tilde{t}_1}^2) \Big\} / m_t^2 \Big]. \qquad (385)
\end{aligned}
$$

Here $c_{\tilde{t}}$ and $s_{\tilde{t}}$ are the cosine and sine of a top squark mixing

angle $\theta_{\tilde{t}}$, defined more specifically later on when we discuss the squark sector.

For fixed top-squark masses, the maximum possible h mass occurs for rather large top squark mixing, $c_{\tilde{t}}^2 s_{\tilde{t}}^2 = m_t^2/[m_{\tilde{t}_2}^2 + m_{\tilde{t}_1}^2 - 2(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)/\ln(m_{\tilde{t}_2}^2/m_{\tilde{t}_1}^2)]$ or $1/4$, whichever is less.

It follows that the quantity in square brackets in Eq. (385) is always less than $m_t^2[\ln(m_{\tilde{t}_2}^2/m_t^2) + 3]$.

The observed $126~\mathrm{GeV}$ mass makes large top-squark mixing mandatory unless the top-squark masses are themselves extremely large.

Including these and other important corrections one can obtain only a weaker, but still very interesting, bound

$$m_h \lesssim 135~\mathrm{GeV} \tag{386}$$

in the MSSM. This assumes that all of the sparticles that can contribute to m_h^2 in loops have masses that do not exceed 1

TeV.

Thus, before the LHC discovery it was a fairly robust prediction of supersymmetry at the electroweak scale that at least one of the Higgs scalar bosons had to be light.

The measured $126~\mathrm{GeV}$ mass does not require extreme measures, but does lead to large fine-tuning in order to have consistency with the Z mass, as described earlier.

Let us now recall that the top, charm and up quark mass matrix is proportional to $v_u = v\sin\beta$ and the bottom, strange, and down quarks and the charge leptons get masses proportional to $v_d = v\cos\beta$. At tree-level,

$$m_t = y_t v\sin\beta, \qquad m_b = y_b v\cos\beta, \qquad m_\tau = y_\tau v\cos\beta. \tag{387}$$

These relations hold for the running masses rather than the physical pole masses, which are significantly larger for t, b.

Considering y_b and y_τ, we see that at tree level $y_b/y_t =$

$(m_b/m_t)\tan\beta$ and $y_\tau/y_t = (m_\tau/m_t)\tan\beta$, so that y_b and y_τ are large if $\tan\beta$ is much larger than 1.

In fact, there are good theoretical motivations for considering models with large $\tan\beta$. For example, models based on the GUT gauge group $SO(10)$ can unify the running top, bottom and tau Yukawa couplings at the unification scale; this requires $\tan\beta$ to be very roughly of order m_t/m_b.

Further notes:

– If one tries to make $\sin\beta$ too small, y_t will be nonperturbatively large.
  Requiring that y_t does not blow up above the electroweak scale, one finds that $\tan\beta \gtrsim 1.2$ or so, depending on the mass of the top quark, the QCD coupling, and other details.

– In principle, there is also a constraint on $\cos\beta$ if one requires that y_b and y_τ do not become nonperturbatively large.
  This gives a rough upper bound of $\tan\beta \lesssim 65$. However,

this is complicated somewhat by the fact that the bottom quark mass gets significant one-loop non-QCD corrections in the large $\tan\beta$ limit.

- **One can obtain a stronger upper bound on $\tan\beta$ in some models where $m_{H_u}^2 = m_{H_d}^2$ at the GUT or other high energy input scale, by requiring that y_b does not significantly exceed y_t.**[31]
- **The parameter $\tan\beta$ also directly impacts the masses and mixings of the MSSM sparticles,.**

[31] If y_b were substantially larger than y_t, then the RG evolution equations for the soft-SUSY-breaking masses $m_{H_u}^2, m_{H_d}^2$ that we did not discuss, would imply $m_{H_d}^2 < m_{H_u}^2$ at the electroweak scale. In this case, the minimum of the potential would have $\langle H_d^0\rangle > \langle H_u^0\rangle$, which would be a contradiction with the supposition that $\tan\beta$ is large.

The RNS approach to the Little Hierarchy Problem

Outline

- **Models of natural supersymmetry seek to solve the little hierarchy problem by positing a spectrum of light higgsinos $\lesssim 200-300$ GeV and light top squarks $\lesssim 600$ GeV along with very heavy squarks and TeV-scale gluinos.**

  Such models have low electroweak fine-tuning and satisfy the LHC constraints.

- **However, in the context of the MSSM, they predict too low a value of m_h, are frequently in conflict with the measured $b \to s\gamma$ branching fraction and the relic density of thermally produced higgsino-like WIMPs falls well below dark matter (DM) measurements.**

- **RNS posits a framework dubbed** *radiative natural SUSY* **(RNS) which can be realized within the MSSM (avoiding the addition of extra exotic matter) and which maintains features such as gauge coupling unification and radiative electroweak symmetry breaking.**
- **The RNS model can be generated from SUSY GUT type models with non-universal Higgs masses (NUHM). Allowing for high scale soft SUSY breaking Higgs mass $m_{H_u} > m_0$ leads to** *automatic cancellations* **during renormalization group (RG) running, and to** *radiatively-induced low fine-tuning* **at the electroweak scale.**
- **Coupled with large mixing in the top squark sector, RNS allows for fine-tuning at the 3-10% level with TeV-scale top squarks and a 125 GeV light Higgs scalar h.**
- **The model allows for at least a partial solution to the SUSY flavor, CP and gravitino problems since first/second generation**

scalars (and the gravitino) may exist in the 10-30 TeV regime.

- There are some interesting possible signatures for RNS at the LHC such as the appearance of low invariant mass opposite-sign isolated dileptons from gluino cascade decays.
- The smoking gun signature for RNS is the appearance of light higgsinos at a linear e^+e^- collider.
- If the strong CP problem is solved by the Peccei-Quinn mechanism, then RNS naturally accommodates mixed axion-higgsino cold dark matter, where the light higgsino-like WIMPS – which in this case make up only a fraction of the measured relic abundance – should be detectable at upcoming WIMP detectors.

Introduction

- The recent discovery by Atlas and CMS of a Higgs-like resonance at the CERN LHC adds credence to supersymmetric

models (SUSY) of particle physics in that the mass value $m_h \simeq 125$ GeV falls squarely within the narrow window predicted by the Minimal Supersymmetric Standard Model (MSSM): $m_h \sim 115 - 135$ GeV.

- **At the same time, the lack of a SUSY signal at LHC7 and LHC8 implies $m_{\tilde{g}} \gtrsim 1.4$ TeV (for $m_{\tilde{g}} \sim m_{\tilde{q}}$) and $m_{\tilde{g}} \gtrsim 0.9$ TeV (for $m_{\tilde{g}} \ll m_{\tilde{q}}$).**

- **While weak scale SUSY provides a solution to the gauge hierarchy problem via the cancellation of quadratic divergences, the apparently multi-TeV sparticle masses required by LHC searches seemingly exacerbates** *the little hierarchy problem* **(LHP):**
  - **how do multi-TeV values of SUSY model parameters conspire to yield a Z-boson (Higgs boson) mass of just 91.2 (125) GeV?**

- **Models of** *natural supersymmetry* **address the LHP by positing a spectrum of light higgsinos $\lesssim 200$ GeV and light top- and** bottom-squarks with $m_{\tilde{t}_{1,2},\tilde{b}_1} \lesssim 600$ GeV along with very heavy first/second generation squarks and TeV-scale gluinos.

Such a spectrum allows for low electroweak fine-tuning (EWFT) while at the same time keeping sparticles safely beyond LHC search limits.

Because third generation scalars are in the few hundred GeV range, the radiative corrections to m_h, which increase only logarithmically with $m_{\tilde{t}_i}^2$, are never very large and these models have great difficulty in accommodating a light SUSY Higgs scalar with mass $m_h \sim 125$ GeV.

Thus, we are faced with a new conundrum:

– how does one reconcile low EWFT with such a large value of m_h?

A second problem occurs in that

– the predicted branching fraction for $b \to s\gamma$ decay is frequently at odds with the measured value due to the very light third generation squarks.

A third issue appears in that

– the light higgsino-like WIMP particles predicted by models of natural SUSY lead to a thermally-generated relic density which is typically a factor 10-15 below the WMAP measured value of $\Omega_{CDM} h^2 \simeq 0.11$.

- One solution to the fine-tuning/Higgs problem is to add extra fields to the theory, thus moving beyond the MSSM.

  For example, adding an extra singlet as in the NMSSM permits a new quartic coupling in the Higgs potential thus allowing for an increased value of m_h.

- Alternatively, one may add extra vector-like matter to increase m_h while maintaining light top squarks.

- In the former case of the NMSSM, adding extra gauge singlets may lead to re-introduction of destabilizing divergences into the theory.

In the latter case, one might wonder about the ad-hoc introduction of extra weak scale matter multiplets and how they might have avoided detection.

- **A third possibility, which is presented below, is to re-examine EWFT and to ascertain if there do indeed exist sparticle spectra** *within the MSSM* **that lead to $m_h \sim 125$ GeV while maintaining modest levels of electroweak fine-tuning.**

Electroweak fine-tuning

- **One way to evaluate EWFT in SUSY models is to examine the minimization condition from the Higgs sector scalar potential which determines the Z boson mass. (Alternatively, one may examine the mass formula for m_h and arrive at similar conclusions.)**

  Minimization of the one-loop effective potential $V_{\rm tree} + \Delta V$,

leads to

$$\frac{M_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \,, \tag{388}$$

where Σ_u^u and Σ_d^d are radiative corrections that arise from the derivatives of ΔV evaluated at the minimum.

Eq. (388) reduces to the familiar tree-level expression for M_Z^2 when radiative correction terms are ignored.

As you know, Σ_u^u and Σ_d^d include contributions from various particles and sparticles with sizeable Yukawa and/or gauge couplings to the Higgs sector.

- To obtain a *natural* value of M_Z on the left-hand-side, one would like each term C_i (with $i = H_d$, H_u, μ as well as $\Sigma_u^u(k)$, $\Sigma_d^d(k)$, where k denotes the various contributions to the Σs that we just mentioned) on the right-hand-side to have an absolute value of order $M_Z^2/2$.

Traditionally, EWFT has been quantified using the Barbieri-Giudice measure[710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724]

$$\Delta_{BG} \equiv max_i \left| \frac{\partial \ln m_Z^2}{\partial \ln a_i} \right| \tag{389}$$

where a_i represents various fundamental parameters of the theory, usually taken to be some set of soft SUSY breaking parameters defined at some high energy scale Λ_{HS} below which the theory in question is posited to be the correct effective field theory description of nature. $1/\Delta$ is the % of fine tuning. The value of Δ_{BG} then answers the question: how stable is the fractional Z-boson mass against fractional variation of high scale model parameters?

Depending on which parameters are included in the set a_i, very different answers emerge[724].

In addition, theories which are defined at very different values of Λ_{HS}, but which nonetheless lead to exactly the same weak scale sparticle mass spectra, give rise to very different values of Δ_{BG}.

- To understand how the underlying framework for superpartner masses may be relevant, consider a model with input parameters defined at some high scale $\Lambda \gg m_{SUSY}$, where m_{SUSY} is the SUSY breaking scale ~ 1 TeV and Λ may be as high as $M_{\rm GUT}$ or even the reduced Planck mass M_P. Then

$$m^2_{H_u}(m_{SUSY}) = m^2_{H_u}(\Lambda) + \delta m^2_{H_u} \tag{390}$$

where

$$\delta m^2_{H_u} \simeq -\frac{3f_t^2}{8\pi^2}\left(m^2_{Q_3} + m^2_{U_3} + A_t^2\right)\log\left(\frac{\Lambda}{m_{SUSY}}\right). \tag{391}$$

- Requiring $\delta m_{H_u}^2 \leq \Delta \times \frac{m_h^2}{2}$ then leads for $m_h = 125$ GeV to,

$$\sqrt{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2} \lesssim 600 \text{ GeV} \frac{\sin\beta}{\sqrt{1+R_t^2}} \left(\frac{\log\frac{\Lambda}{\text{TeV}}}{3}\right)^{-1/2} \left(\frac{\Delta}{5}\right)^{1/2}, \tag{392}$$

where $R_t = A_t/\sqrt{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}$. Taking $\Delta = 10$ and Λ as low as 20 TeV corresponds to

– $|\mu| \lesssim 200$ GeV,

– $m_{\tilde{t}_i}$, $m_{\tilde{b}_1} \lesssim 600$ GeV,

– $m_{\tilde{g}} \lesssim 1.5 - 2$ TeV.

The last of these conditions arises because the squark radiative corrections $\delta m_{\tilde{t}_i}^2 \sim (2g_s^2/3\pi^2) m_{\tilde{g}}^2 \times \log\Lambda$. Setting the $\log$ to unity and requiring $\delta m_{\tilde{t}_i}^2 < m_{\tilde{t}_i}^2$ then implies $m_{\tilde{g}} \lesssim 3 m_{\tilde{t}_i}$, or $m_{\tilde{g}} \lesssim 1.5 - 2$ GeV for $\Delta \lesssim 10$.

- **Taking Λ as high as M_{GUT} leads to even tighter constraints: $m_{\tilde t_{1,2}}, m_{\tilde b_1} \lesssim 200$ GeV and $m_{\tilde g} \lesssim 600$ GeV, almost certainly in violation of LHC sparticle search constraints.**
- **Since (degenerate) first/second generation squarks and sleptons enter into (388) only at the two loop level, these can be much heavier: beyond LHC reach and also possibly heavy enough to provide a (partial) decoupling solution to the SUSY flavor and CP problems.**
- **In gravity mediation where $m_{\tilde q} \sim m_{3/2}$, then one also solves the cosmological gravitino problem and in GUTs one also suppresses proton decay. Then we may also have**
  - **$m_{\tilde q,\tilde \ell} \sim 10-50$ TeV.**
- **The generic natural SUSY (NS) solution reconciles lack of a SUSY signal at LHC with allowing for electroweak naturalness. It also predicts that the $\tilde t_{1,2}$ and $\tilde b_1$ may soon be accessible to LHC searches. New limits from direct top- and bottom-**

squark pair production searches, interpreted within the context of simplified models, have begun to bite into the NS parameter space. Of course, if $m_{\tilde{t}_{1,2}}$, $m_{\tilde{b}_1} \simeq m_{\widetilde{Z}_1}$, then the visible decay products from stop and sbottom production will be soft and difficult to see at the LHC.

- A more worrisome problem comes from the newly discovered value of the Higgs mass $m_h \simeq 125$ GeV. In the MSSM, one obtains (assuming that the t-squarks are not very split),

$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + \frac{3g^2}{8\pi^2}\frac{m_t^4}{m_W^2}\left[\ln\frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2}\left(1 - \frac{X_t^2}{12 m_{\tilde{t}}^2}\right)\right] \tag{393}$$

where $X_t = A_t - \mu\cot\beta$ and $m_{\tilde{t}}^2 \simeq m_{Q_3} m_{U_3}$. For a given $m_{\tilde{t}}^2$, this expression is maximal for large mixing in the top-squark sector with $X_t^{max} = \sqrt{6} m_{\tilde{t}}$.

With top-squark masses below about 500 GeV, the radiative corrections to m_h are not large enough to yield $m_h \simeq 125$ GeV even with maximal mixing.

- This situation has been used to argue that additional multiplets beyond those of the MSSM must be present in order to raise up m_h while maintaining very light third generation squarks.

  Added to these are the two issues mentioned earlier:

  1. the very light third generation squarks endemic to NS lead to a predicted branching fraction for $b \to s\gamma$ decay which is frequently much lower than the measured value, and
  2. that the relic abundance of higgsino-like WIMPs inherent in NS, calculated in the standard MSSM-only cosmology, is typically a factor 10-15 below measured values.

- These issues have led to increasing skepticism of weak scale SUSY as realized in the natural SUSY incarnation described above.

- A possible resolution to the above issues associated with a NS spectrum is to simply invoke a SUSY particle spectrum at the weak scale (or some other nearby scale), as in the pMSSM model so that large logarithms associated with a high value of Λ are absent.

In this case, $\Lambda \sim m_{SUSY}$ and $\delta m_{H_u}^2$ is not enhanced by large logarithms and we may select parameters $m_{H_u}^2 \sim \mu^2 \sim M_Z^2 \sim m_h^2$. Of course, heavy top squarks are needed to obtain the observed value of m_h.

While a logical possibility, this solution loses several attractive features of models which are valid up to scales as high as $\Lambda \sim M_{\rm GUT}$, such as gauge coupling unification and radiative electroweak symmetry breaking driven by a large top quark mass.

- Another alternative is to use Δ_{EW} discussed below as a fine-tuning measure even for models defined at the high scale.

  Small Δ_{EW} will be simply the statement that we only use the weak scale parameters appearing in Eq. (388) to define the fine-tuning criterion. This approach is a weaker condition since it allows for possible cancellations in (390).

Indeed this is precisely what happens in what is known as the hyperbolic branch or focus point region (HB/FP) where

$$m_{H_u}^2(\Lambda) + \delta m_{H_u}^2 \sim m_{H_u}^2(m_{SUSY}) \sim \mu^2 \sim M_Z^2. \quad (394)$$

The HB/FP region of mSUGRA occurs, however, only for small values of A_0/m_0 and yields $m_h < 120$ GeV, well below the Atlas/CMS measured value of $m_h \simeq 125$ GeV.

Scans over parameter space show that the HB/FP region is nearly excluded if one requires both low $|\mu|$ and $m_h \sim 123 - 127$ GeV.

- To obtain a viable high scale model we see that we clearly need to go beyond mSUGRA.

$\boxed{\Delta_{EW} \text{ and } \Delta_{HS}}$

Let us now define more precisely the two different measures of EWFT– Δ_{EW} and Δ_{HS}– that have been proposed to answer the following question:

how is it possible that m_Z has a value of just 91.2 GeV while gluino and squark masses exist at TeV or even far beyond values?

As we have discussed, the answer should be: those independent contributions which enter the scalar potential and conspire to build up the Z-boson mass should all be comparable to m_Z.

Minimization of the scalar potential in the MSSM[704] leads to the well-known relation that

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \,, \tag{395}$$

where $m^2_{H_u}$ and $m^2_{H_d}$ are soft SUSY breaking (not physical) Higgs mass terms, μ is the superpotential Higgsino mass term, $\tan\beta \equiv v_u/v_d$ is the ratio of Higgs field vevs and Σ^u_u and Σ^d_d include a variety of independent radiative corrections[725].

$\boxed{\Delta_{EW}}$

Noting that all entries in Eq. 395 are defined at the weak scale, the electroweak fine-tuning parameter

$$\Delta_{EW} \equiv max_i \left|C_i\right| / (m^2_Z/2) \ , \tag{396}$$

may be constructed, where $C_{H_d} = m^2_{H_d}/(\tan^2\beta - 1)$, $C_{H_u} = -m^2_{H_u}\tan^2\beta/(\tan^2\beta - 1)$ and $C_\mu = -\mu^2$. Also, $C_{\Sigma^u_u(k)} = \Sigma^u_u(k)/(m^2_Z/2)$ and $C_{\Sigma^d_d(k)} = \Sigma^d_d(k)/(m^2_Z/2)$, where k labels the various loop contributions included in Eq. 395.

A low value of Δ_{EW} means less fine-tuning, *e.g.* $\Delta_{EW} = 20$ corresponds to $\Delta^{-1}_{EW} = 5\%$ finetuning amongst terms contributing to $m_Z^2/2$. Since C_{H_d} and $C_{\Sigma_d^d(k)}$ terms are suppressed by $\tan^2\beta - 1$, for even moderate $\tan\beta$ values the expression Eq. 395 reduces approximately to

$$\frac{m_Z^2}{2} \simeq -(m_{H_u}^2 + \Sigma_u^u) - \mu^2 \,. \tag{397}$$

In order to achieve low Δ_{EW}, it is necessary that $-m_{H_u}^2$, $-\mu^2$ and each contribution to $-\Sigma_u^u$ all be nearby to $m_Z^2/2$ to within a factor of a few.

We note that Δ_{EW} coincides with Δ_{BG} when M_Z^2 depends linearly on input parameters (such as μ^2, $m_{H_u}^2$ or $m_{H_d}^2$ using electroweak scale parameters) but differs when the parameter dependence is non-linear.

For electroweak scale parameters, the non-linear dependence only occurs in the radiative correction terms Σ_u^u and Σ_d^d and in $\tan\beta$.

A scan over mSUGRA/CMSSM parameter space, requiring that LHC sparticle mass constraints and $m_h = 125 \pm 2$ GeV be obeyed, finds a minimal value of $\Delta_{EW} \sim 10^2$, with more common values being $\Delta_{EW} \sim 10^3 - 10^4$. Thus, one may conclude that the Z mass is rather highly finetuned in this paradigm model.

In the case of mSUGRA, the value C_μ becomes low only in the hyperbolic branch/focus point[716, 718] (HB/FP) region. In this region, however, m_0 and consequently $m_{\tilde{t}_{1,2}}$ are very large, so that $\Sigma_u^u(\tilde{t}_{1,2})$ are each large, and the model remains finetuned.

Alternatively, if one moves to the two-parameter non-universal Higgs model (NUHM2)[726], with free parameters

$$m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ \mu,\ m_A \tag{398}$$

then

1. μ can be chosen in the $100 - 300$ GeV range since it is now a free input parameter,
2. a value of $m^2_{H_u}(m_{GUT}) \sim (1.3 - 2.5)m_0$ may be chosen so that $m^2_{H_u}$ is driven only slightly negative at the weak scale, leading to $m^2_{H_u}(weak) \sim -m^2_Z/2$, and
3. with large stop mixing from $A_0 \sim \pm 1.6 m_0$, the top-squark radiative corrections are softened while m_h is raised to the ~ 125 GeV level[725].

In the NUHM2 model, Δ_{EW} as low as $5 - 10$ can be generated. For such cases, the Little Hierarchy Problem seems to disappear. The low Δ_{EW} models are typified by the presence of light higgsinos $m_{\widetilde{W}_1}^{\pm}$, $m_{\widetilde{Z}_{1,2}} \sim 100 - 300$ GeV which should be accessible to a linear e^+e^- collider operating with $\sqrt{s} \gtrsim 2|\mu|$. Also, $m_{\tilde{g}} \sim 1 - 5$ TeV while $m_{\tilde{t}_1} \sim 1 - 2$

TeV and $m_{\tilde{t}_2} \sim 2 - 4$ TeV.

$\boxed{\Delta_{HS}}$

To include explicit dependence on the high scale Λ at which the SUSY theory may be defined, we may write the *weak scale* parameters $m^2_{H_{u,d}}$ and μ^2 in Eq. (395) as

$$m^2_{H_{u,d}} = m^2_{H_{u,d}}(\Lambda) + \delta m^2_{H_{u,d}}; \qquad \mu^2 = \mu^2(\Lambda) + \delta\mu^2 \ , \tag{399}$$

where $m^2_{H_{u,d}}(\Lambda)$ and $\mu^2(\Lambda)$ are the corresponding parameters renormalized at the high scale Λ. It is the $\delta m^2_{H_{u,d}}$ terms that will contain the $\log\Lambda$ dependence emphasized in constructs of natural SUSY models[727, 728, 729]. In this way, we write

$$\frac{m_Z^2}{2} = \frac{(m^2_{H_d}(\Lambda) + \delta m^2_{H_d} + \Sigma^d_d) - (m^2_{H_u}(\Lambda) + \delta m^2_{H_u} + \Sigma^u_u)\tan^2\beta}{\tan^2\beta - 1} - (\mu^2(\Lambda) + \delta\mu^2) \ . \tag{400}$$

In the same spirit used to construct Δ_{EW}, we can now define a fine-tuning measure that encodes the information about the high scale origin of the parameters by requiring that each of the terms on the right-hand-side of Eq. (400) (normalized again to $m_Z^2/2$) be smaller than a value $\Delta_{\rm HS}$. The high scale fine-tuning measure $\Delta_{\rm HS}$ is thus defined to be

$$\Delta_{\rm HS} \equiv max_i|B_i|/(m_Z^2/2) \, , \tag{401}$$

with $B_{H_d} \equiv m_{H_d}^2(\Lambda)/(\tan^2\beta - 1)$ etc., defined analogously to the set C_i.

As discussed above, in models such as mSUGRA whose domain of validity extends to very high scales, because of the large logarithms one would expect that (barring seemingly accidental cancellations) the $B_{\delta H_u}$ contributions to $\Delta_{\rm HS}$ would be much larger than any contributions to $\Delta_{\rm EW}$ because the term $m_{H_u}^2$ evolves from large m_0^2 through zero to negative values in order to radiatively break electroweak symmetry. Thus, Δ_{HS} is numerically very similar to the EWFT measure advocated by

Kitano-Nomura[727] where $\Delta_{KN} = \delta m^2_{H_u}/(m^2_h/2)$

Scans of the mSUGRA/CMSSM model in Ref. [730] found $\Delta_{HS} \gtrsim 10^3$. In Ref. [725], scans over NUHM2 model similarly found $\Delta_{HS} \gtrsim 10^3$. Thus, both the mSUGRA and NUHM2 models would qualify as highly EW finetuned under Δ_{HS}.

- A perhaps surprising result is that Δ_{HS} values far below the NUHM2/mSUGRA minimal value of 10^3 can now be found. In fact, the lowest Δ_{HS} point from a broad scan has a value of 32, or 3.1% EWFT, even including the effect of high scale logarithms.

  There are several features of the input parameters which lead to low Δ_{HS}.

  First, the GUT scale value of $m^2_{H_u} = (314 \text{ GeV})^2$, so our high scale starting point for m_{H_u} is not too far from m_Z.

  Second, the GUT scale gaugino masses M_1 and M_2 are $\sim$

$3M_3 \sim 3$ **TeV. The RG running of** $m^2_{H_u}$ **is governed by**

$$\frac{dm^2_{H_u}}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 + \frac{3}{10}g_1^2S + 3f_t^2X_t\right) \tag{402}$$

where $t = \log(Q^2/\mu^2)$,

$$S = m^2_{H_u} - m^2_{H_d} + Tr\left[\mathrm{m}^2_Q - \mathrm{m}^2_L - 2\mathrm{m}^2_U + \mathrm{m}^2_D + \mathrm{m}^2_E\right]$$

and

$$X_t = m^2_Q(3) + m^2_U(3) + m^2_{H_u} + A^2_t.$$

At $Q = m_{GUT}$**, the large gaugino masses provide a large negative slope (green curve of Fig. 38) for** $m^2_{H_u}$**, causing its value to increase while running towards lower mass scales.**

As the parameters evolve, X_t **increases due to the increasing squark soft terms so that the Yukawa coupling term grows (red curve from Fig. 38) and ultimately dominates; then** $m^2_{H_u}$ **is driven towards negative values, so that electroweak symmetry**

is finally broken.

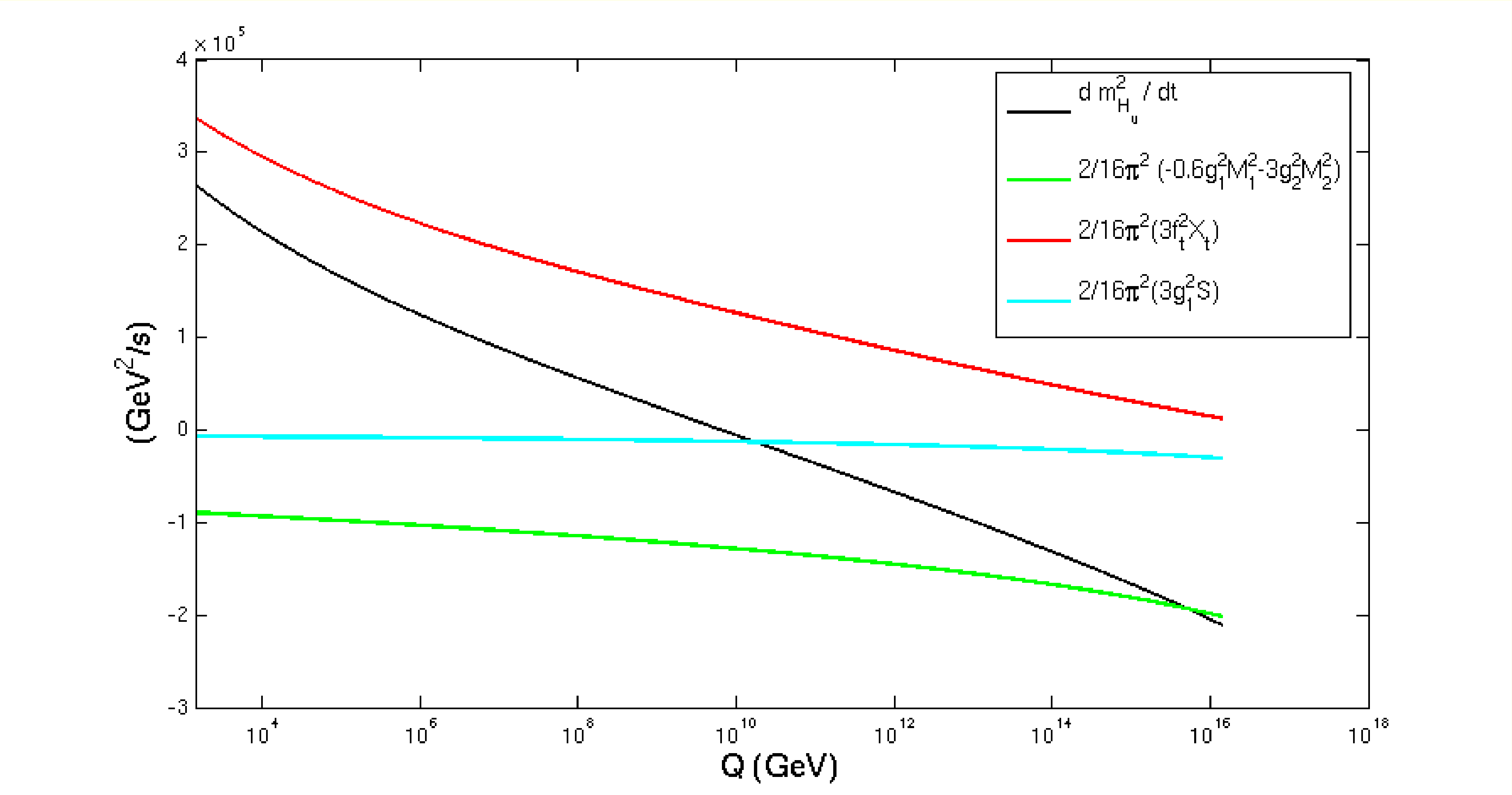


Figure 38: Plot of a) slope $dm^2_{H_u}/dt$ vs. Q from model HS1 with $\Delta_{HS} = 32$.

The total slope (black curve) passes through zero around

$Q \sim 10^{10}$ **GeV, indicating large cancellations in the RG running of $m^2_{H_u}$.**

Ultimately, the value of $m^2_{H_u}(m_{weak}) \sim -(185\ \mathrm{GeV})^2$ so that both the starting and ending points of $m^2_{H_u}$ remain not too far from m^2_Z, and hence $\delta m^2_{H_u}$ is not too far from m^2_Z, thus fulfilling the most important condition required by low Δ_{HS}.

The RG running of gaugino masses and selected soft scalar masses for HS1 are shown in Fig. 39.

We see that indeed M_1 and M_2 start at ~ 3 TeV values and decrease, whilst M_3 starts small at $Q = m_{GUT}$ and sharply increases.

The gaugino mass boundary conditions then influence the running of the soft scalar masses shown in Fig. 40.

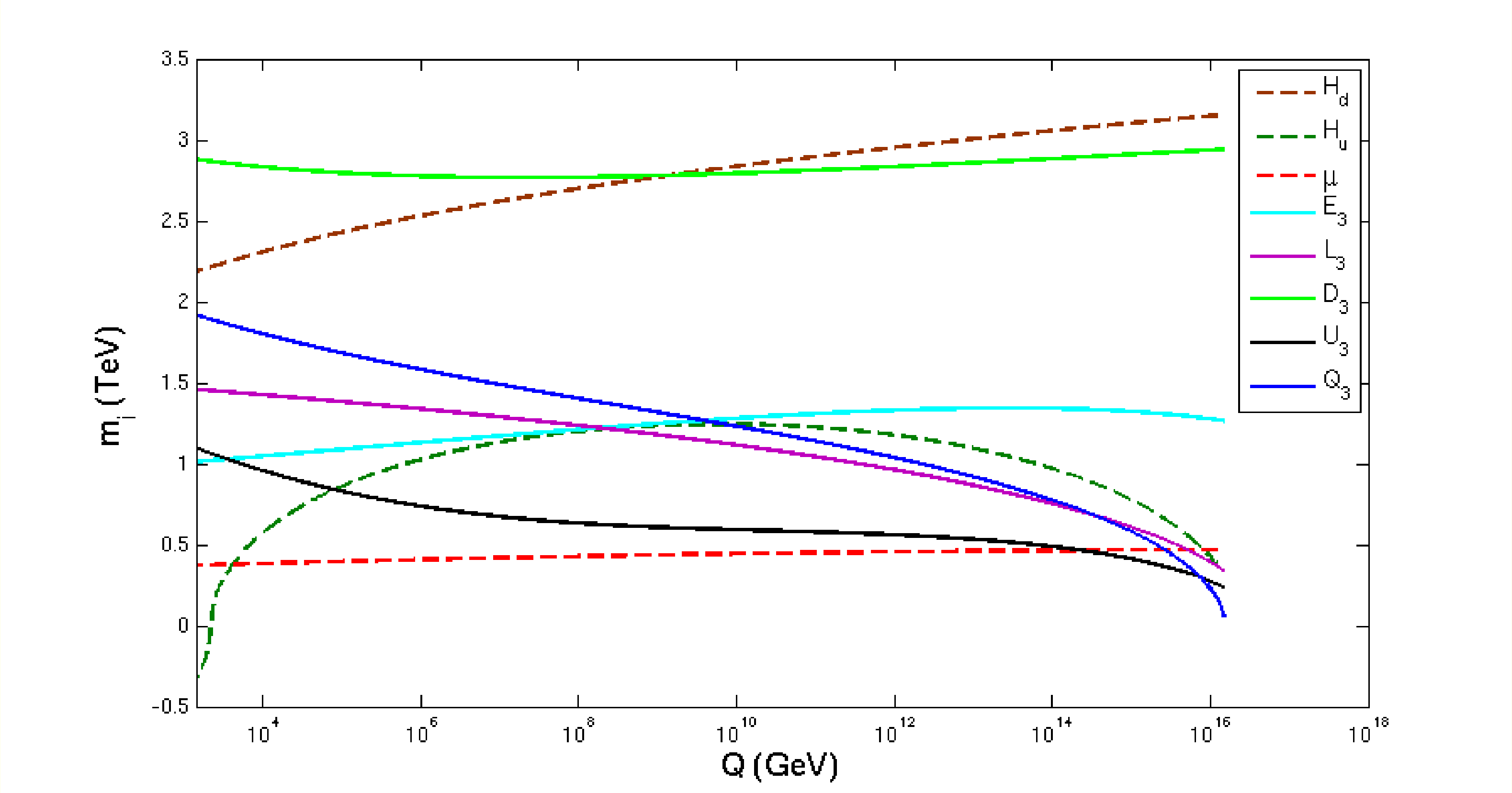


Figure 39: **Plot of running gaugino masses vs. Q from model HS1 with $\Delta_{HS} = 32$.**

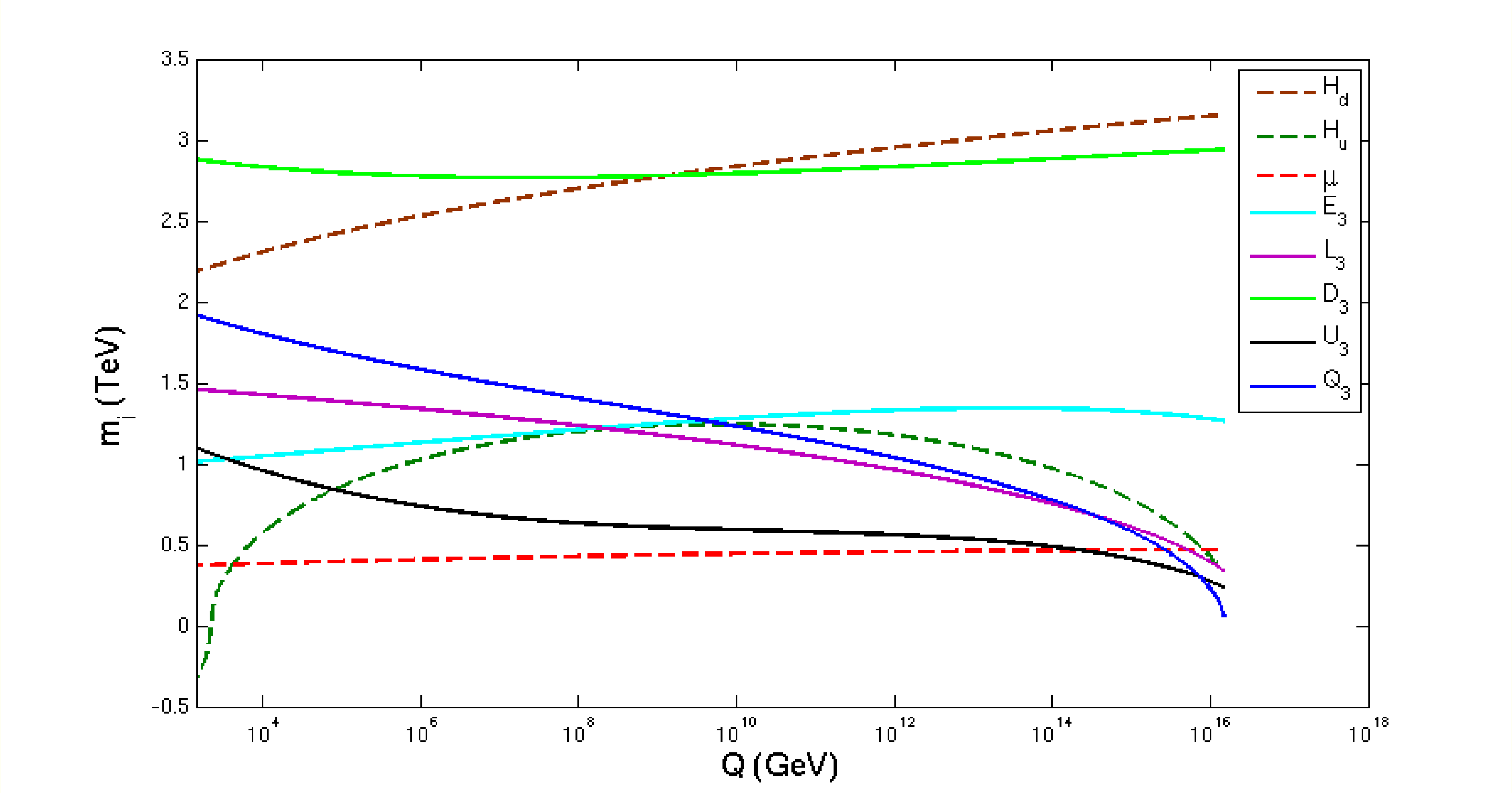


Figure 40: Plot of running scalar masses vs. Q from model HS1 with $\Delta_{HS} = 32$.

Most important is the running of $m^2_{H_u}$, as shown in Fig. 40, which starts near m^2_Z at m_{GUT}, runs up to about the TeV scale at $Q \sim 10^{10}$ GeV, and then is pushed to small negative values by $Q \sim m_{weak}$.

Also, $m_U(3)$ and $m_Q(3)$ start small, which aides the high Q gaugino dominance in the running of $m^2_{H_u}$.

By $Q \sim m_{weak}$, these third generation squark soft terms have been pushed to the TeV scale. Thus, top squarks are not so heavy and the radiative corrections $\Sigma^u_u(\tilde{t}_{1,2})$ are under control.

- To contrast the above to how a low value of $m^2_{H_u}(m_{SUSY})$ is obtained in the previously discussed RNS models, in Fig. 41 we show the running of various SUSY parameters versus the renormalization scale Q for the RNS2 benchmark point.

  The RNS2 point has parameters $m_0 = 7025$ GeV, $m_{1/2} = 568.3$ GeV, $A_0 = -11426.6$ GeV, $\tan\beta = 8.55$ with $\mu = 150$ GeV and $m_A = 1000$ GeV.

  The gaugino and matter scalar mass parameters evolve from $m_{1/2}$ and m_0 to their weak scale values, resulting in a pattern of masses very similar to that in mSUGRA. The parameters μ and $m^2_{H_d}$ hardly evolve.

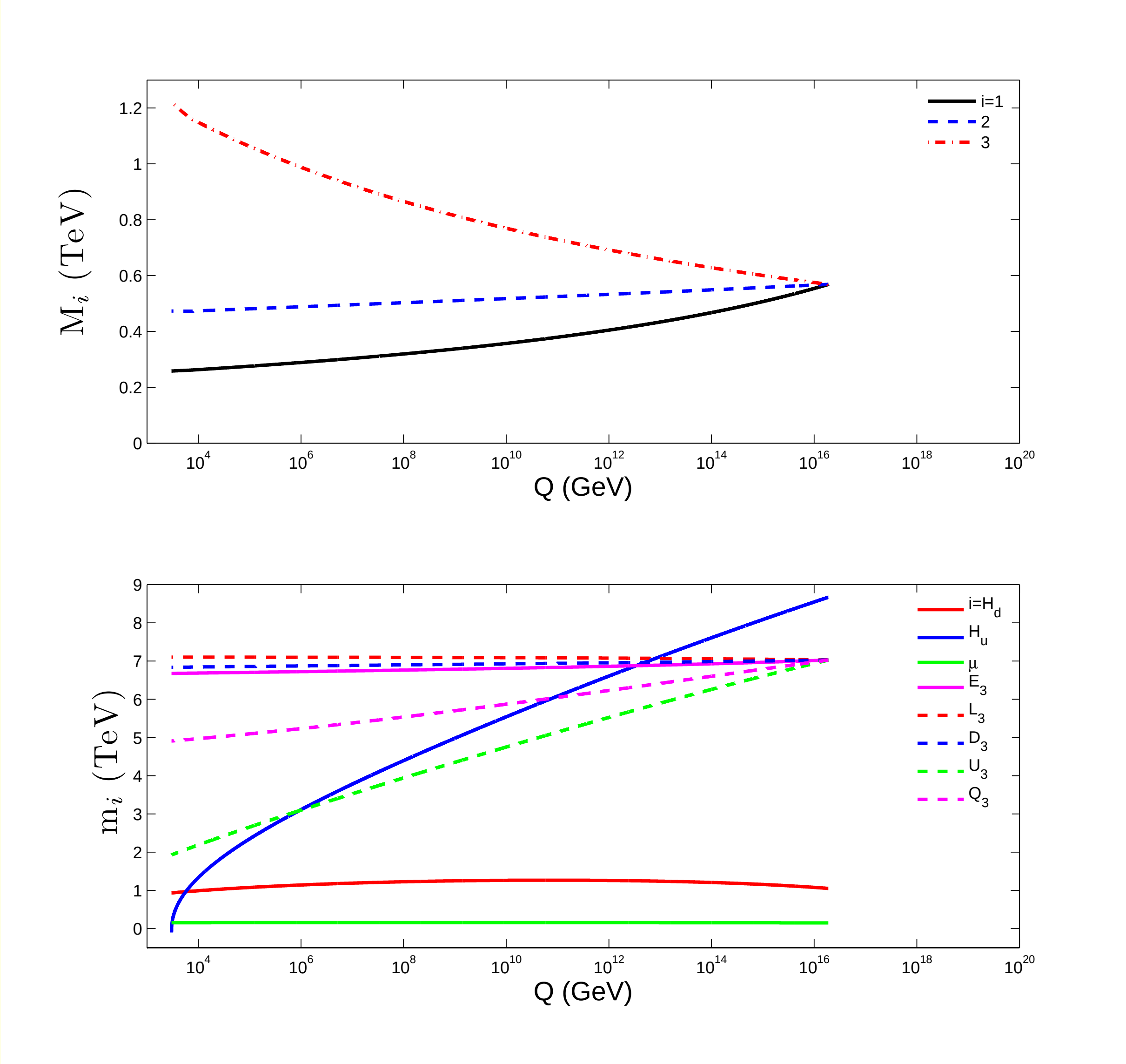


Figure 41: Evolution of SSB parameters from $M_{\rm GUT}$ to M_{weak} for the RNS2 benchmark point taken from in Ref. [?] whose parameters are given in the text. The graph extends to values below $Q^2 = m_{\tilde t_1} m_{\tilde t_2}$ where the Higgs mass parameters are extracted.

Of most interest to us here is the RG evolution of $m^2_{H_u}$. As is well known, the SUSY breaking parameters $m^2_{Q_3}$, $m^2_{U_3}$ and $m^2_{H_u}$ of the scalar fields that couple via the large top quark Yukawa coupling are driven down with reducing values of the scale Q.

The reduction is the greatest for $m^2_{H_u}$ which, in fact, is driven negative, triggering the radiative breakdown of electroweak symmetry. We see from the figure that the weak scale value of $-m^2_{H_u}$ has a magnitude $\sim M_Z^2$, and is much smaller than the weak scale value of other mass parameters.

In this way, small Δ_{EW} becomes possible if μ is also of order M_Z.

In this kind of RNS model, small Δ_{EW} is thus not an accident because *the NUHM2 model provides us the flexibility to adjust the GUT scale value of $m^2_{H_u}$ so that it barely runs to negative values at the weak scale*. **Since $m^2_{H_u}$ is driven radiatively to $\sim -M_Z^2$ at**

the weak scale, this was why this scenario was dubbed $Radiative$ $Natural\ SUSY$**, or RNS for short.**

Some Conclusions

- **In previous studies, the radiative natural SUSY model had emerged as a way to reconcile low EWFT with lack of SUSY signals at LHC8 and the presence of a light Higgs scalar with mass** $m_h \sim 125$ **GeV.**

  The RNS model cannot be realized within the restrictive mSUGRA/CMSSM framework, but can be realized within the context of NUHM2 models (which depend on 6 input parameters) and where μ **can be a free input value.**

  In RNS models, Δ_{EW} **as low as** ~ 10 **can be generated while** Δ_{HS} **as low as** 10^3 **can be found.**

  However, if one does scans over the most general minimal flavor- and minimal CP**-violating GUT scale SUSY model – SUGRA19, which is like the pMSSM, but defined at high scale)**

— the conclusions change.

- The first question is if the additional freedom of 13 extra parameters allows for much lower Δ_{EW} solutions. In previous work– by proceeding from mSUGRA to NUHM2 models– a reduction in the minimum of Δ_{EW} of at least a factor of 10 was found[730, 725].

  In fact, one does not find any substantial reduction in the minimal Δ_{EW} value by proceeding from the NUHM2 model to SUGRA19. The parameter freedom of NUHM2 appears sufficient to minimize Δ_{EW} to its lowest values of $\sim 5 - 10$.
- The second question is whether the additional parameter SUGRA19 freedom can improve on the high scale EWFT parameter Δ_{HS}.

  In this regard, one does find improvements by factors ranging up to ~ 150!

  In order to generate low values of Δ_{HS}, one must generate

$\mu \sim 100 - 300$ GeV as usual, but also one must start with $m_{H_u}^2 \sim m_Z^2$ at the GUT scale, and then generate relatively little change $\delta m_{H_u}^2$ during evolution from m_{GUT} to m_{weak}.

Small values of $\delta m_{H_u}^2$ can be found if one begins with electroweak gaugino masses $M_{1,2} \sim 3M_3$ at the GUT scale so that gaugino-induced RG evolution dominates at high $Q \sim m_{GUT}$.

Then at lower Q values approaching the weak scale, top-Yukawa terms dominate the running of $m_{H_u}^2$, leading to broken electroweak symmetry, but also to not much net change in $m_{H_u}^2$ during its evolution from m_{GUT} to m_{weak}.

The solutions with low Δ_{HS} are characterized by the presence of four light higgsinos $\widetilde{W}_1^{\pm}$ and $\widetilde{Z}_{1,2}$ similar to RNS models.

However, in contrast to RNS models, the third generation squarks tend to be lighter (although not as light as generic natural SUSY which favors $m_{\tilde{t}_{1,2}} \lesssim 500$ GeV).

The lighter third generation squarks lead to significant SUSY contributions to the decay $b \to s\gamma$, and seem to be disfavored by the measured value of this branching fraction.

In the case of low Δ_{HS} models, the lightest neutralino is more higgsino-like than in RNS models, leading to even lower values of predicted relic density and low direct detection rates.

The remaining CDM abundance may be augmented by scalar field or axino/saxion production and decay in the early universe, and in the latter case, the additional presence of axions is expected.

Fine Tuning in the NMSSM

There are a number of recent relevant papers.

1. arXiv:1212.5243 (Gherghetta et. al.) shows that you can get small fine-tuning if you use an upper cutoff of order $\Lambda \sim 20~\mathrm{TeV}$.

   Well, I regard this as no big deal since this was already true in the MSSM
2. There is generalized version of the NMSSM which gets you to smaller fine-tuning using GUT scale boundary conditions, but it is kind of messy: see arXiv:1205.1509 (Graham Ross, et. al.).
3. I claim that my work with Kraml and Yun (arXiv:1201.0982) is actually more relevant to this question when GUT scale boundary conditions are employed.

Let us recall some defining equations for the NMSSM.

The parameters in the Higgs sector are chosen as follows:

a) Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

$$\lambda\ \widehat{S}\widehat{H_u}\widehat{H_d} + \frac{\kappa}{3}\ \widehat{S}^3 \tag{403}$$

depending on two dimensionless couplings λ, κ beyond the MSSM. (Hatted capital letters denote superfields, and unhatted capital letters will denote their scalar components).

b) The associated trilinear soft terms are

$$\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3 \,. \tag{404}$$

c) The final two input parameters are

$$\tan\beta = \langle H_u\rangle / \langle H_d\rangle \ , \ \mu_{\rm eff} = \lambda \langle S\rangle \ . \tag{405}$$

These, along with M_Z, can be viewed as determining the three SUSY breaking masses squared for H_u, H_d and S through the three minimization equations of the scalar potential.

Thus, as compared to two independent parameters in the Higgs sector of the MSSM (often chosen as $\tan\beta$ and M_A), the Higgs sector of the NMSSM is described by the six parameters

$$\lambda\ ,\ \kappa\ ,\ A_\lambda\ ,\ A_\kappa,\ \tan\beta\ ,\ \mu_{\mathrm{eff}}\ . \tag{406}$$

We will choose sign conventions for the fields such that λ and $\tan\beta$ are positive, while κ, A_λ, A_κ and μ_{eff} should be allowed to have either sign. For any choice of these parameters – as well as of the values for the gaugino masses and of the soft terms related to the squarks and sleptons that contribute to the radiative corrections in the Higgs sector

The three models we considered are defined in terms of grand-unification (GUT) scale parameters as follows:

I) a version of the constrained NMSSM (CNMSSM) in which we adopt universal m_0, $m_{1/2}$, $A_0 = A_{t,b,\tau}$ values but require $A_\lambda = A_\kappa = 0$, as motivated by the $U(1)_R$ symmetry limit of the NMSSM;

II) the non-universal Higgs mass (NUHM) relaxation of model I in which m_{H_u} and m_{H_d} are chosen independently of m_0, but still with $A_\lambda = A_\kappa = 0$; and

III) universal m_0, $m_{1/2}$, A_0 with NUHM relaxation and general A_λ and A_κ.

We used NMSSMTools-3.0.2 [126][162][?] for the numerical analysis, performing extensive scans over the parameter spaces of the models considered.

The precise constraints imposed were the following.

1. Our 'basic constraints' were to require that an NMSSM parameter choice be such as to give a proper RGE solution, have no Landau pole, have a neutralino LSP and obey Higgs

and SUSY mass limits as implemented in NMSSMTools-3.0.2 (Higgs mass limits are from LEP, TEVATRON, and early LHC data; SUSY mass limits are essentially from LEP).

2. Regarding B physics, the constraints considered were those on $\mathrm{BR}(B_s \to X_s\gamma)$, ΔM_s, ΔM_d, $\mathrm{BR}(B_s \to \mu^+\mu^-)$, $\mathrm{BR}(B^+ \to \tau^+\nu_\tau)$ and $\mathrm{BR}(B \to X_s\mu^+\mu^-)$ at 2σ as encoded in NMSSMTools-3.0.2, except that we updated the bound on the radiative B_s decay to $3.04 < \mathrm{BR}(B_s \to X_s\gamma) \times 10^4 < 4.06$; theoretical uncertainties in B-physics observables are taken into account as implemented in NMSSMTools-3.0.2. These combined constraints we term the 'B-physics contraints'.

3. Regarding a_μ, we required that the extra NMSSM contribution, δa_μ, falls into the window defined in NMSSMTools of $8.77 \times 10^{-10} < \delta a_\mu < 4.61 \times 10^{-9}$ expanded to $5.77 \times 10^{-10} < \delta a_\mu < 4.91 \times 10^{-9}$ after allowing for a 1σ theoretical error in the NMSSM calculation of $\pm 3 \times 10^{-10}$.

In fact, points that fail to fall into the above δa_μ window always do so by virtue of δa_μ being too small.

4. For Ωh^2, we declare that the relic density is consistent with WMAP data provided $0.094 < \Omega h^2 < 0.136$, which is the 'WMAP window' defined in NMSSMTools-3.0.2 after including theoretical and experimental systematic uncertainties.

   We also considered the implications of relaxing this constraint to simply $\Omega h^2 < 0.136$ so as to allow for scenarios in which the relic density arises at least in part from some other source.

5. A "perfect" point is one for which all constraints are satisfied including requiring that δa_μ is in the above defined window and Ωh^2 is in the WMAP window.

Results

- We find that only in models II and III is it possible for a "perfect" point to have a light scalar Higgs in the mass range

$123 - 128$ GeV as consistent with the hints from the recent LHC Higgs searches. The largest m_{h_1} achieved for perfect points is about 125 GeV.

- However, relaxing the a_μ constraint vastly increases the number of accepted points and it is possible to have $m_{h_1} \gtrsim 126$ GeV in both models II and III even if δa_μ is just slightly outside (below) the allowed window.

  Comparing with [?], the tension between obtaining an ideal or nearly ideal δa_μ while predicting a SM-like light Higgs near 125 GeV appears to be somewhat less in NUHM variants of the NMSSM than in those of the MSSM.

In the plots shown in the following, the coding for the plotted points is as follows:

- grey squares pass the 'basic' constraints but fail B-physics constraints (such points are rare);
- green squares pass the basic constraints *and* satisfy B-physics

constraints;

- blue plusses ($+$) observe B-physics constraints as above and in addition have $\Omega h^2 < 0.136$, thereby allowing for other contributions to the dark matter density (a fraction of order 20% of these points have $0.094 < \Omega h^2 < 0.136$) but they do not necessarily have acceptable δa_μ;
- magenta crosses ($\times$) have satisfactory δa_μ as well as satisfying B-physics constraints, but arbitrary Ωh^2;
- golden triangle points pass all the same constraints as the magenta points and in addition have $\Omega h^2 < 0.136$;
- open black/grey[32] triangles are perfect, completely allowed points in the sense that they pass all the constraints listed earlier, including $5.77 \times 10^{-10} < \delta a_\mu < 4.91 \times 10^{-9}$ and $0.094 < \Omega h^2 < 0.136$;
- open white diamonds are points with $m_{h_1} \geq 123~\mathrm{GeV}$ that pass

[32] For perfect points, we will use black triangles if $m_{h_1} \geq 123~\mathrm{GeV}$ and grey triangles if $m_{h_1} < 123~\mathrm{GeV}$ in plots where m_{h_1} does not label the x axis.

basic constraints, B-physics constraints and predict $0.094 < \Omega h^2 < 0.136$ but have $4.27 \times 10^{-10} < \delta a_\mu < 5.77 \times 10^{-10}$, that is we allow an excursion of half the 1σ theoretical systematic uncertainty below the earlier defined window. We will call these "almost perfect" points.

- We begin by presenting the crucial plots of Fig. 42 in which we show $R^{h_1}(\gamma\gamma)$ as a function of m_{h_1} for cases I, II and III.
- Only in cases II and III do we find points that pass all constraints (the open black triangles) with $m_{h_1} \sim 124 - 125~\mathrm{GeV}$.

  These typically have $R^{h_1}(\gamma\gamma)$ of order 0.98.
- Somewhat surprisingly, such points were more easily found by our scanning procedure in case II than in case III.
- Many additional points with $m_{h_1} \sim 125~\mathrm{GeV}$ emerge if we relax only slightly the δa_μ constraint.

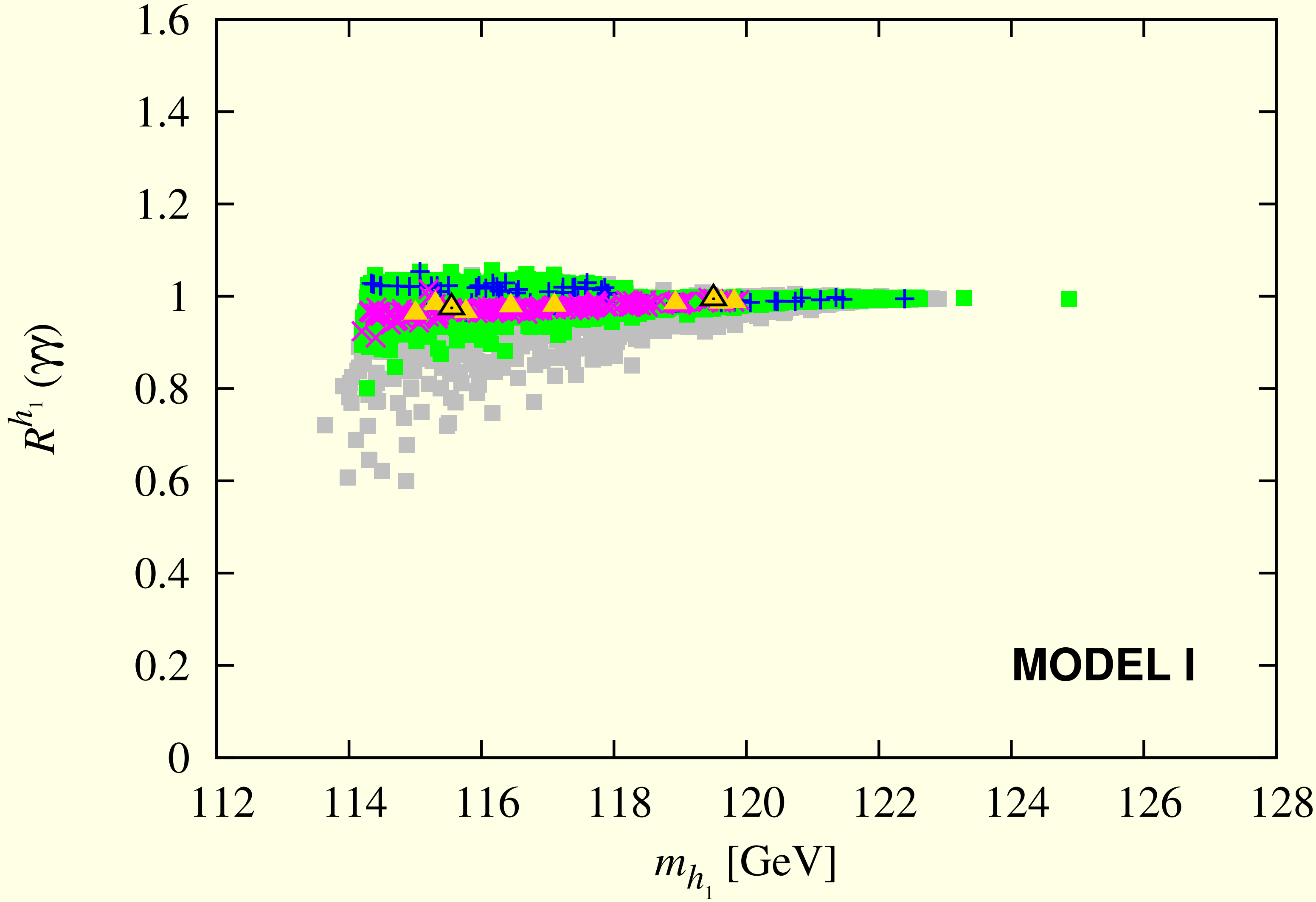


Figure 42: Scatter plots of $R^{h_1}(\gamma\gamma)$ versus m_{h_1} for boundary condition case I. See text for symbol/color notations.

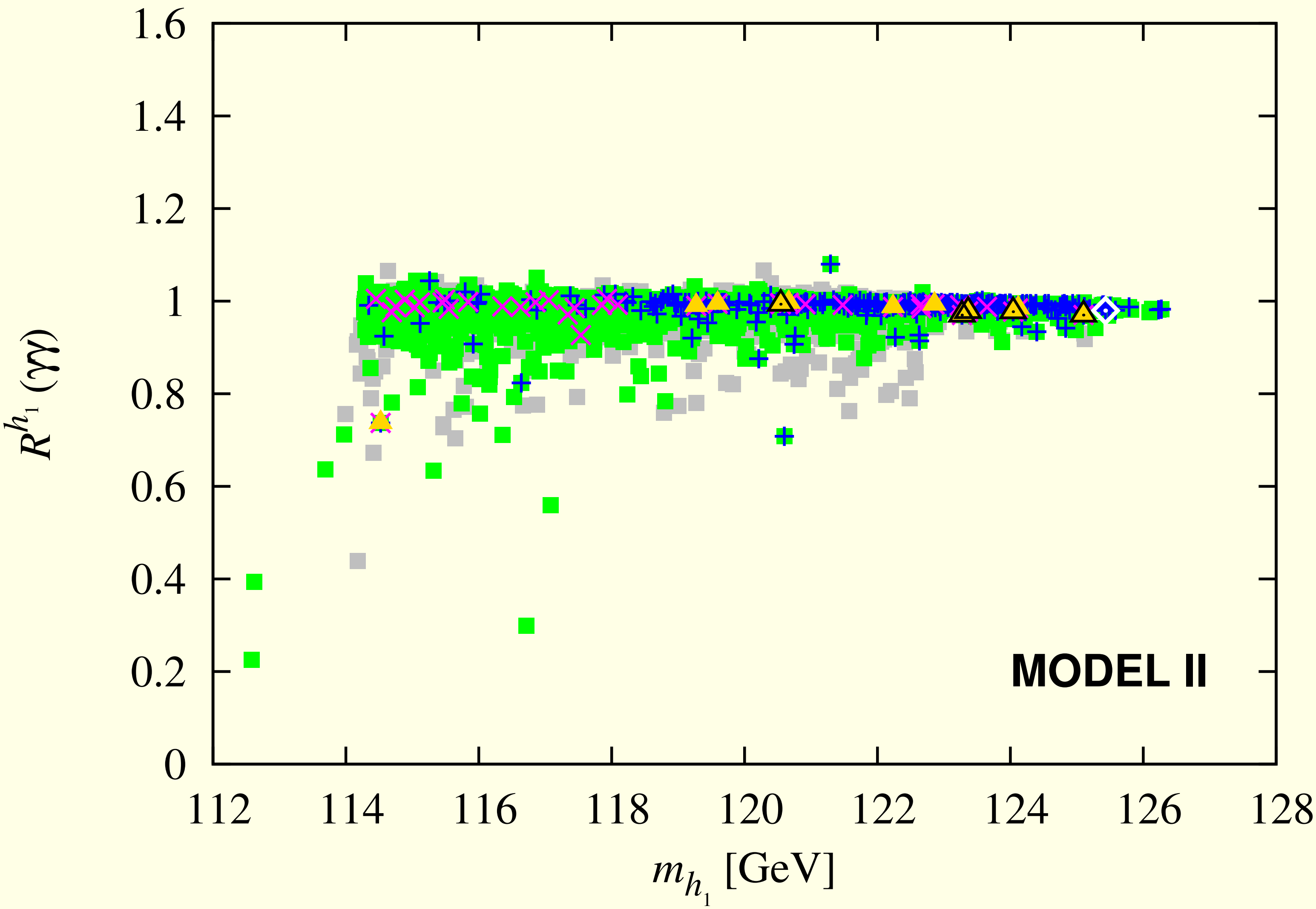


Figure 43: Scatter plots of $R^{h_1}(\gamma\gamma)$ versus m_{h_1} for boundary condition case II. See text for symbol/color notations.

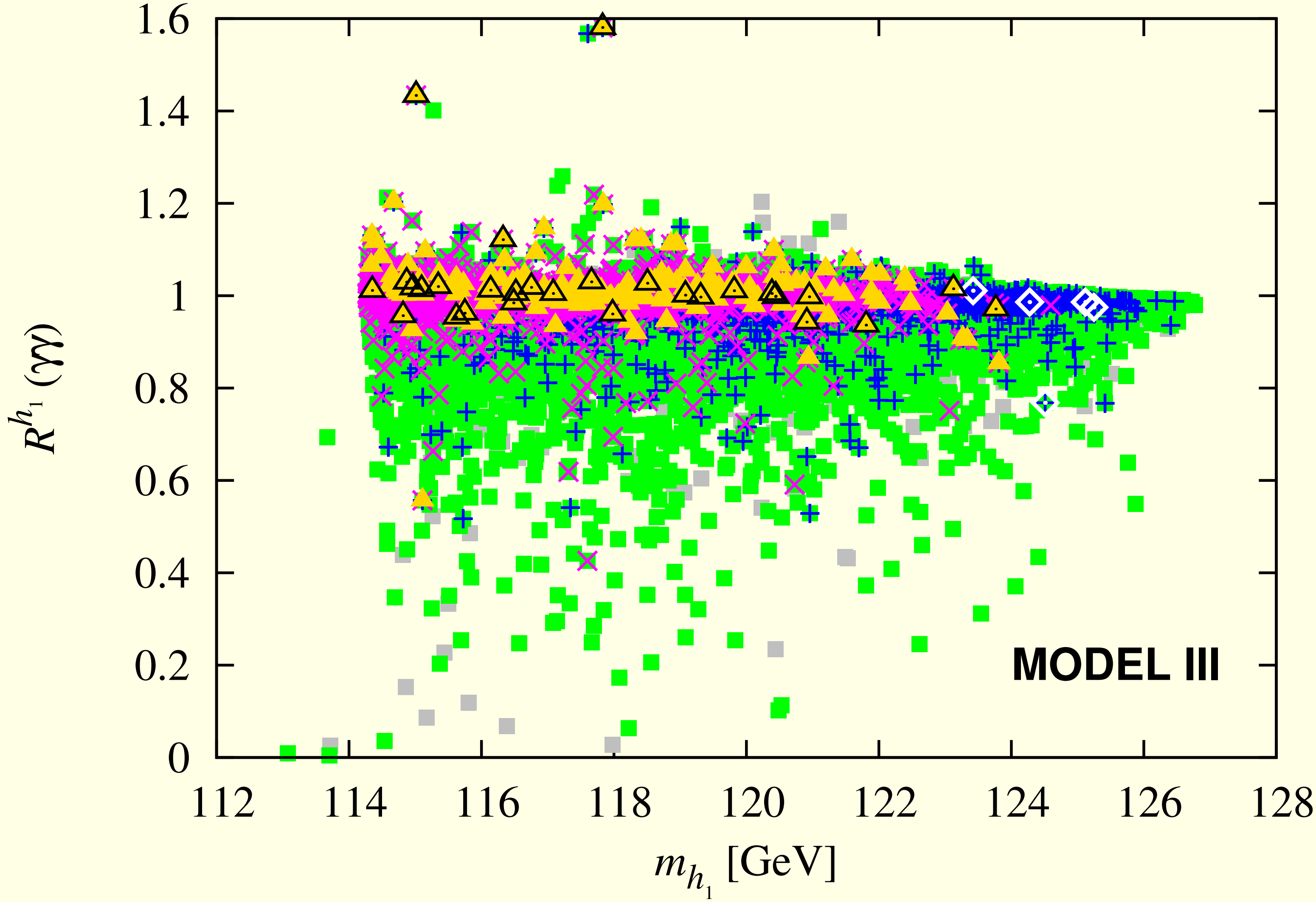


Figure 44: Scatter plots of $R^{h_1}(\gamma\gamma)$ versus m_{h_1} for boundary condition case III. See text for symbol/color notations.

- **The white diamonds show points for cases for which $\mathbf{4.27 \times 10^{-10} < \delta a_\mu < 5.77 \times 10^{-10}}$ having $\boldsymbol{m_{h_1} \geq 123}$ GeV.**

- **The parameter choices that give the largest $\boldsymbol{m_{h_1}}$ values are ones for which the $\boldsymbol{h_1}$ is really very SM-like in terms of its couplings and branching ratios.**

- **These scans did not find parameter choices for which $\boldsymbol{R^{h_1}(\gamma\gamma)}$ was significantly larger than 1 for $\boldsymbol{m_{h_1} = 123 - 128}$ GeV, as hinted at by the ATLAS data.**

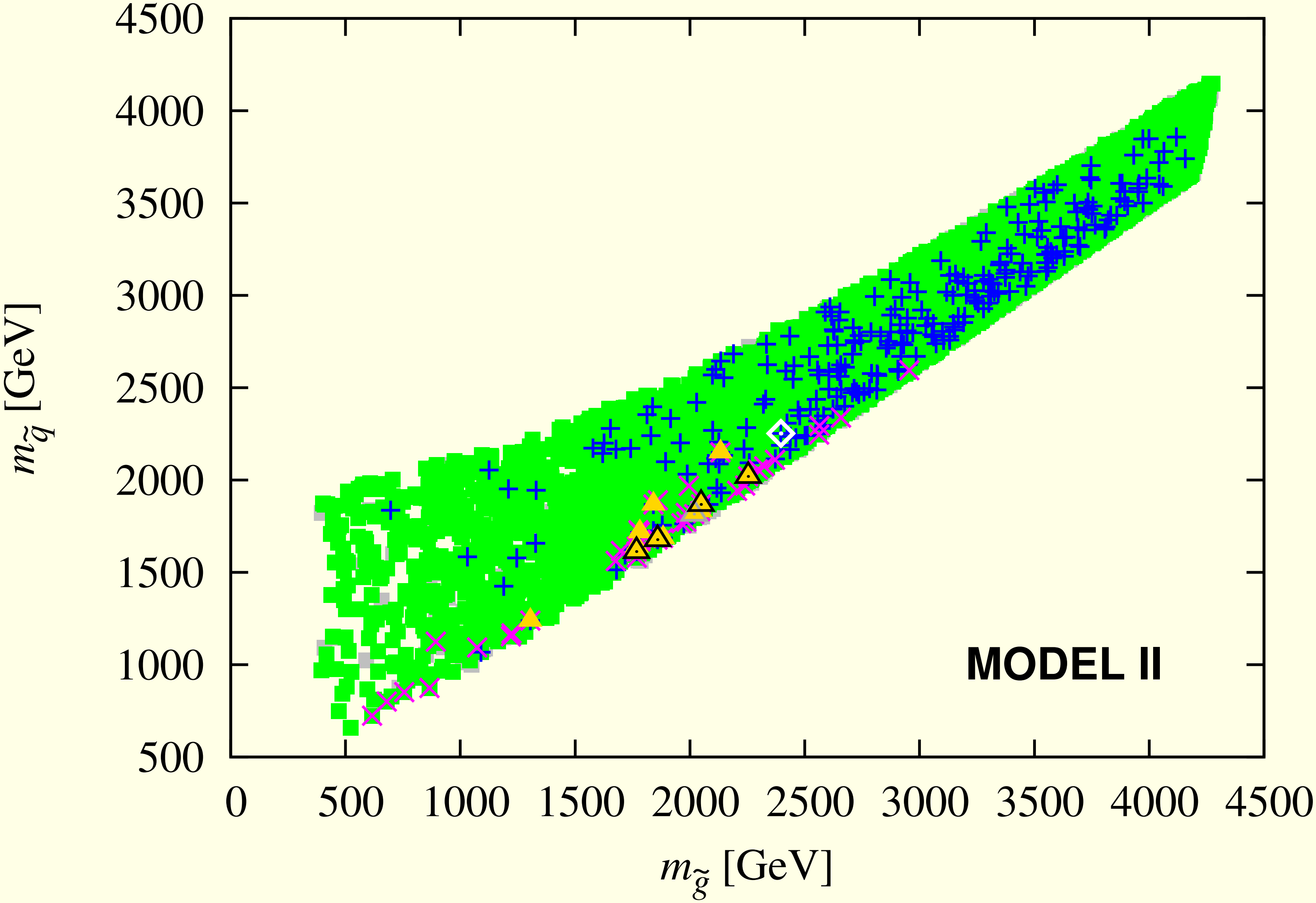


Figure 45: **Scatter plot of squark versus gluino masses for model II. Here we use black (grey) open triangles for perfect points with $m_{h_1} \geq 123$ GeV ($m_{h_1} < 123$ GeV). See text for remaining symbol/color notations.**

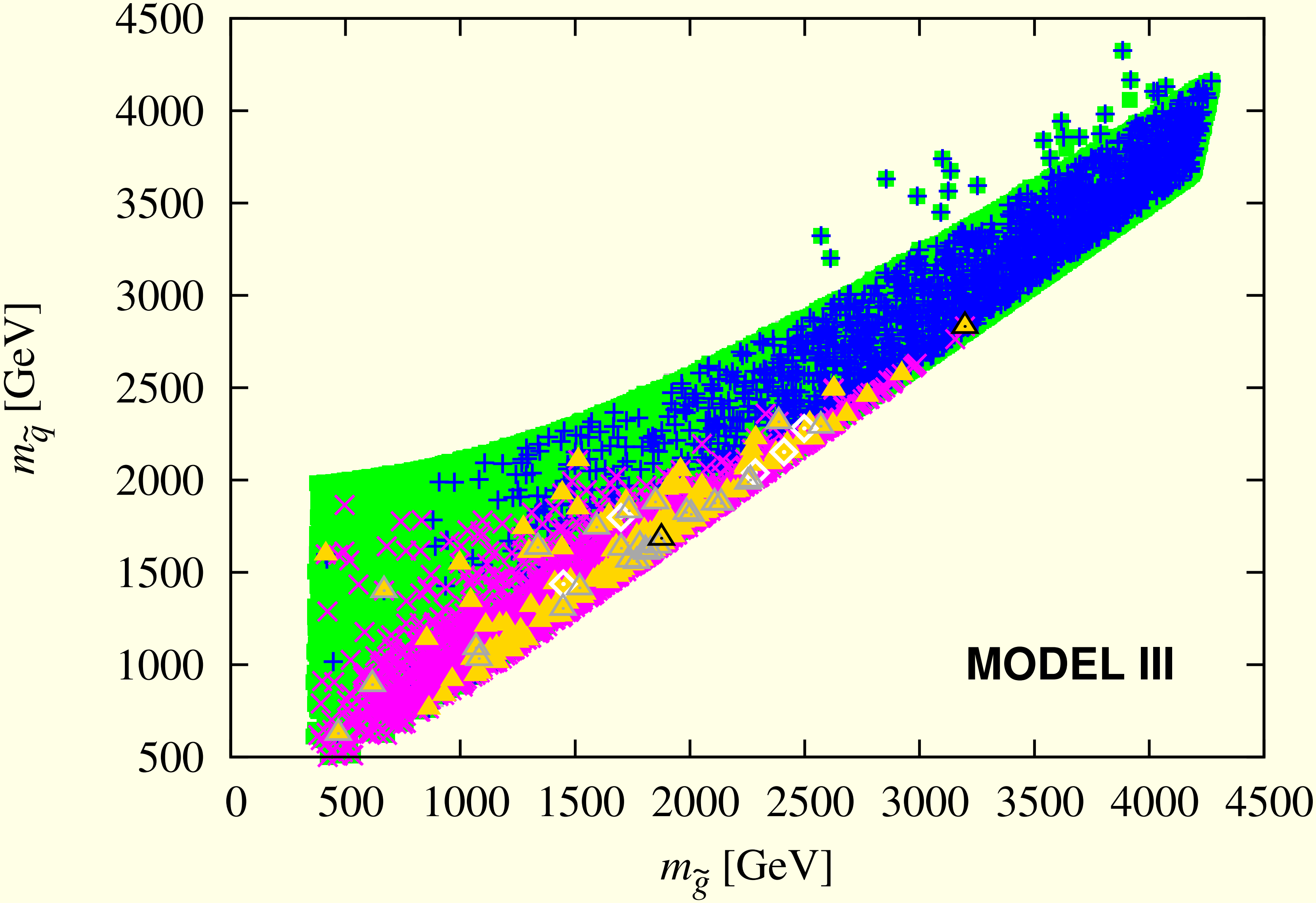


Figure 46: **As above, but for model III.**

Given that the LHC data is consistent with a rather SM-like Higgs in the vicinity of $m_{h_1} \sim 125~\mathrm{GeV}$ (rather than one with an enhanced $\gamma\gamma$ rate), it is of interest to know the nature of the parameter choices that yield the perfect, black triangle and almost perfect white diamond points with $m_{h_1} \sim 125~\mathrm{GeV}$ and what the other experimental signatures of these points are. We therefore present a brief summary of the most interesting features.

First, one must ask if such points are consistent with current LHC limits on SUSY particles, in particular squarks and gluinos. To this end, Fig. 45 shows the distribution of squark and gluino masses for the various kinds of points for models II and III. Interestingly, all the perfect, black triangle and almost perfect, white diamond points with $m_{h_1} \gtrsim 123~\mathrm{GeV}$ have squark and gluino masses above $1~\mathrm{TeV}$ and thus have not yet been probed by current LHC results. (Note that since we are considering models with universal m_0 and $m_{1/2}$ for squarks and gauginos, analyses in the context of the CMSSM apply.) It is quite intriguing that the regions of parameter space that are consistent with a Higgs of mass close to $125~\mathrm{GeV}$ automatically evade the current limits from LHC SUSY searches.

In order to further detail the parameters and some relevant features of perfect and almost perfect points we present in Tables 2–5 seven exemplary points with $m_{h_1} \gtrsim 124~\mathrm{GeV}$ from models II and III. Some useful observations include the following:

- Because of the way we initiated our model III MCMC scans, restricting $|A_{\lambda,\kappa}| \leq 1$ TeV, most of the tabulated model III points have quite modest A_λ and A_κ. However, a completely random scan finds almost perfect points with quite large A_λ and A_κ values as exemplified by tabulated point #7. The fact that the general scan over A_λ and A_κ did not find any perfect points with $m_{h_1} \gtrsim 124~\mathrm{GeV}$, whereas such points were fairly quickly found using the MCMC technique, suggests that such points are quite fine-tuned in the general scan sense. See Table 2 for specifics.

- In Table 3, we display various details regarding the Higgs bosons for each of our exemplary points. As already noted, for the perfect and almost perfect points the h_1 is very SM-like when $m_{h_1} \gtrsim 123~\mathrm{GeV}$. To quantify how well the LHC Higgs data is described for each of our exemplary points, we use a chi-squared approach. In practice, only the ATLAS collaboration has presented the best fit values for $R^h(\gamma\gamma, ZZ \to 4\ell, WW \to \ell\nu\ell\nu)$ along with 1σ upper and lower errors as a function of m_h. Identifying h with the NMSSM h_1, we have employed Fig. 8 of [?] to compute a $\chi^2(\mathrm{ATLAS})$ for each point in the NMSSM parameter space (but this was not included in the global likelihood used for our MCMC scans). From Table 3 we see that the smallest $\chi^2(\mathrm{ATLAS})$ values (of order 0.6 to 0.7) are obtained for $m_{h_1} \sim 124~\mathrm{GeV}$. This is simply because at this mass the ATLAS fits to $R^h(\gamma\gamma)$ and $R^h(4\ell)$ are very close to one, the natural prediction in the NMSSM context. For $m_h \sim 125~\mathrm{GeV}$, the R^h's for the ATLAS data are somewhat larger than 1 leading to a discrepancy with the NMSSM SM-like prediction and a roughly doubling of $\chi^2(\mathrm{ATLAS})$ to values of order 1.3 to 1.6 for our exemplary points. In this context, we should note that at a Higgs mass of $125~\mathrm{GeV}$ the CMS data is best fit if the Higgs signals are not enhanced and, indeed, are very close to SM values.

- The mass of the neutralino LSP, $\widetilde\chi_1^0$, is rather similar, $m_{\widetilde\chi_1^0} \approx 300 - 450~\mathrm{GeV}$, for the different perfect and almost perfect points with $m_{h_1} \gtrsim 124~\mathrm{GeV}$. For all but pt. #5, the $\widetilde\chi_1^0$ is approximately an equal mixture of higgsino and bino. There is some variation in the primary annihilation mechanism, with $\widetilde\tau_1\widetilde\tau_1$ and $\widetilde\chi_1^0\widetilde\chi_1^0$ annihilation being the dominant channels except for pt. #2 for which $\widetilde{\nu}_\tau \widetilde{\nu}_\tau$ and $\widetilde{\nu}_\tau \overline{\widetilde{\nu}}_\tau$ annihilations are dominant. In the case of dominant $\widetilde{\tau}_1 \widetilde{\tau}_1$ annihilation, the bulk of the $\widetilde{\chi}_1^0$'s come from those $\widetilde{\tau}$'s that have not annihilated against one another or co-annihilated with a $\widetilde{\chi}_1^0$.

- All the tabulated points yield a spin-independent direct detection cross section of order $(3.5-6)\times 10^{-8}~\mathrm{pb}$. For the above $m_{\widetilde{\chi}_1^0}$ values, current limits on $\sigma_{\rm SI}$ are not that far above this mark and upcoming probes of $\sigma_{\rm SI}$ will definitely reach this level.
- The 7 points all have $m_{\tilde g}$ and $m_{\tilde q}$ above $1.5~\mathrm{TeV}$ and in some cases above $2~\mathrm{TeV}$. Detection of the superparticles may have to await the LHC upgrade to $14~\mathrm{TeV}$.
- Only the $\tilde t_1$ is seen to have a mass distinctly below $1~\mathrm{TeV}$ for the tabulated points. Still, for all the points $m_{\tilde t_1}$ is substantial, ranging from $\sim 500~\mathrm{GeV}$ to above $1~\mathrm{TeV}$. For such masses, detection of the $\tilde t_1$ as an entity separate from the other squarks and the gluino will be quite difficult and again may require the

$\mathbf{14}$ TeV **LHC upgrade.**

- **The effective superpotential μ-term, $\mu_{\rm eff}$, is small for all the exemplary points. This is interesting regarding the question of electroweak fine-tuning.**

	Model II			Model III			
Pt. #	1*	2*	3	4*	5	6	7
$\tan\beta(m_Z)$	17.9	17.8	21.4	15.1	26.2	17.9	24.2
λ	0.078	0.0096	0.023	0.084	0.028	0.027	0.064
κ	0.079	0.011	0.037	0.158	-0.045	0.020	0.343
$m_{1/2}$	923	1026	1087	842	738	1104	1143
m_0	447	297	809	244	1038	252	582
A_0	-1948	-2236	-2399	-1755	-2447	-2403	-2306
A_λ	0	0	0	-251	-385	-86.8	-2910
A_κ	0	0	0	-920	883	-199	-5292
$m^2_{H_d}$	$(2942)^2$	$(3365)^2$	$(4361)^2$	$(2481)^2$	$(935)^2$	$(3202)^2$	$(3253)^2$
$m^2_{H_u}$	$(1774)^2$	$(1922)^2$	$(2089)^2$	$(1612)^2$	$(1998)^2$	$(2073)^2$	$(2127)^2$

Table 2: Input parameters for the exemplary points. We give $\tan\beta(m_Z)$ and GUT scale parameters, with masses in GeV and masses-squared in GeV^2. Starred points are the perfect points satisfying all constraints, including $\delta a_\mu > 5.77\times10^{-10}$ and $0.094 < \Omega h^2 < 0.136$. Unstarred points are the almost perfect points that have $4.27\times10^{-10} < \delta a_\mu < 5.77\times10^{-10}$ and $0.094 < \Omega h^2 < 0.136$.

	Model II			Model III			
Pt. #	1*	2*	3	4*	5	6	7
m_{h_1}	124.0	125.1	125.4	123.8	124.5	125.2	125.1
m_{h_2}	797	1011	1514	1089	430	663	302
m_{a_1}	66.5	9.83	3.07	1317	430	352	302
C_u	0.999	0.999	0.999	0.999	0.999	0.999	0.999
C_d	1.002	1.002	1.001	1.003	1.139	1.002	1.002
C_V	0.999	0.999	0.999	0.999	0.999	0.999	0.999
$C_{\gamma\gamma}$	1.003	1.004	1.004	1.004	1.012	1.003	1.001
C_{gg}	0.987	0.982	0.988	0.984	0.950	0.986	0.994
$\Gamma_{\rm tot}(h_1)$ [GeV]	0.0037	0.0039	0.0039	0.0037	0.0046	0.0039	0.0039
$\mathrm{BR}(h_1 \to \gamma\gamma)$	0.0024	0.0024	0.0024	0.0024	0.002	0.0024	0.0024
$\mathrm{BR}(h_1 \to gg)$	0.056	0.055	0.056	0.056	0.043	0.055	0.056
$\mathrm{BR}(h_1 \to b\bar{b})$	0.638	0.622	0.616	0.643	0.680	0.619	0.621
$\mathrm{BR}(h_1 \to WW)$	0.184	0.201	0.207	0.180	0.159	0.203	0.201
$\mathrm{BR}(h_1 \to ZZ)$	0.0195	0.022	0.023	0.019	0.017	0.022	0.022
$R^{h_1}(\gamma\gamma)$	0.977	0.970	0.980	0.980	0.971	0.768	0.975
$R^{h_1}(ZZ, WW)$	0.971	0.962	0.974	0.974	0.964	0.750	0.969
$\chi^2_{\rm ATLAS}$	0.59	1.27	1.47	0.72	1.57	1.34	1.20

Table 3: Upper section: Higgs masses. Middle section: reduced h_1 couplings to up- and down-type quarks, $V = W, Z$ bosons, photons, and gluons. Bottom section: total width in GeV, decay branching ratios, $R^{h_1}(\gamma\gamma)$, $R^{h_1}(VV)$ and $\chi^2_{\rm ATLAS}$ of the lightest CP-even Higgs for the seven exemplary points.

	Model II			Model III			
Pt. #	1*	2*	3	4*	5	6	7
$\mu_{\rm eff}$	400	447	472	368	421	472	477
$m_{\tilde g}$	2048	2253	2397	1876	1699	2410	2497
$m_{\tilde q}$	1867	2020	2252	1685	1797	2151	2280
$m_{\tilde b_1}$	1462	1563	1715	1335	1217	1664	1754
$m_{\tilde t_1}$	727	691	775	658	498	784	1018
$m_{\tilde e_L}$	648	581	878	520	1716	653	856
$m_{\tilde e_R}$	771	785	1244	581	997	727	905
$m_{\tilde\tau_1}$	535	416	642	433	784	443	458
$m_{\widetilde\chi_1^\pm}$	398	446	472	364	408	471	478
$m_{\widetilde\chi_1^0}$	363	410	438	328	307	440	452
$f_{\tilde B}$	0.506	0.534	0.511	0.529	0.914	0.464	0.370
$f_{\tilde W}$	0.011	0.009	0.008	0.012	0.002	0.009	0.009
$f_{\tilde H}$	0.483	0.457	0.482	0.459	0.083	0.528	0.622
$f_{\tilde S}$	10^{-4}	10^{-6}	10^{-6}	10^{-4}	10^{-6}	10^{-4}	10^{-6}

Table 4: Top section: $\mu_{\rm eff}$ and sparticle masses at the SUSY scale in GeV. Bottom section: LSP decomposition. $m_{\tilde q}$ is the average squark mass of the first two generations. The LSP bino, wino, higgsino and singlino fractions are $f_{\tilde B} = N_{11}^2$, $f_{\tilde W} = N_{12}^2$, $f_{\tilde H} = N_{13}^2 + N_{14}^2$ and $f_{\tilde S} = N_{15}^2$, respectively, with N the neutralino mixing matrix.

Pt. #	δa_μ	Ωh^2	Prim. Ann. Channels	$\sigma_{\rm SI}$ [pb]
1*	6.01	0.094	$\widetilde\chi_1^0\widetilde\chi_1^0 \to W^+W^-(31.5\%), ZZ(21.1\%)$	4.3×10^{-8}
2*	5.85	0.099	$\widetilde\nu_\tau\widetilde\nu_\tau \to \nu_\tau\nu_\tau(11.4\%), \widetilde\nu_\tau\overline{\widetilde\nu}_\tau \to W^+W^-(8.8\%)$	3.8×10^{-8}
3	4.48	0.114	$\widetilde\chi_1^0\widetilde\chi_1^0 \to W^+W^-(23.9\%), ZZ(17.1\%)$	3.7×10^{-8}
4*	6.87	0.097	$\widetilde\chi_1^0\widetilde\chi_1^0 \to W^+W^-(36.9\%), ZZ(23.5\%)$	4.5×10^{-8}
5	5.31	0.135	$\widetilde\chi_1^0\widetilde\chi_1^0 \to b\bar b(39.5\%), h_1a_1(20.3\%)$	5.8×10^{-8}
6	4.89	0.128	$\widetilde\tau_1\widetilde\tau_1 \to \tau\tau(17.4\%), \widetilde\chi_1^0\widetilde\chi_1^0 \to W^+W^-(14.8\%)$	4.0×10^{-8}
7	4.96	0.101	$\widetilde\chi_1^0\widetilde\chi_1^0 \to W^+W^-(17.7\%), ZZ(12.9\%)$	4.0×10^{-8}

Table 5: δa_μ in units of 10^{-10}, LSP relic abundance, primary annihilation channels and spin-independent LSP scattering cross section off protons.

Our second study

- NMSSM=MSSM+$\widehat S$.
- The extra complex S component of $\widehat S$ $\Rightarrow$ the NMSSM has h_1, h_2, h_2, a_1, a_2.
- The new NMSSM parameters of the superpotential (λ and κ) and scalar potential (A_λ and A_κ) appear as:

$$W \ni \lambda \widehat S \widehat H_u \widehat H_d + \frac{\kappa}{3}\widehat S^3\,, \quad V_{\rm soft} \ni \lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3 \tag{407}$$

- $\langle S\rangle \neq 0$ is generated by SUSY breakng and solves μ problem: $\mu_{\rm eff} = \lambda\langle S\rangle$.
- First question: Can the NMSSM give a Higgs mass as large as $125~{\rm GeV}$?
  Answer: Yes, so long as it is not a highly unified model.
  For our studies, we employed universal m_0, except for NUHM ($m^2_{H_u}$, $m^2_{H_d}$, m^2_S free), universal $A_t = A_b = A_\tau = A_0$ but allow A_λ and A_κ to vary freely. Of course, $\lambda > 0$ and κ are scanned demanding perturbativity up to the GUT scale.
- Can this model achieve rates in $\gamma\gamma$ and 4ℓ that are $>$SM?
  Answer: it depends on whether or not we insist on getting good a_μ.
- The possible mechanism (arXiv:1112.3548, Ellwanger) is to reduce the $b\overline{b}$ width of the mainly SM-like Higgs by giving it some singlet component. The gg and $\gamma\gamma$ couplings are less affected.
  In the semi-unified model we employ, enhanced rates and/or large λ cannot be made consistent with decent δa_μ. (J. F. Gunion, Y. Jiang and S. Kraml.arXiv:1201.0982 [hep-ph])
  In this lecture we want to focus on the green points that have SM-like rates and good δa_μ — the LHC data may be headed in the direction of a quite SM-like Higgs.

Some illustrative R_{gg} results from (J. F. Gunion, Y. Jiang and S. Kraml. arXiv:1207.1545):

Figure Legend

	LEP/Teva	B-physics	$\Omega h^2 > 0$	$\delta a_\mu(\times 10^{10})$	XENON100	$R^{h_1/h_2}(\gamma\gamma)$
●	$\surd$	$\surd$	$0 - 0.136$	$\times$	$\surd$	$[0.5, 1]$
■	$\surd$	$\surd$	$0 - 0.094$	$\times$	$\surd$	$(1, 1.2]$
▲	$\surd$	$\surd$	$0 - 0.094$	$\times$	$\surd$	> 1.2
■	$\surd$	$\surd$	0.094-0.136	$\times$	$\surd$	$(1, 1.2]$
▲	$\surd$	$\surd$	0.094-0.136	$\times$	$\surd$	> 1.2
◆	$\surd$	$\surd$	$0.094 - 0.136$	4.27-49.1	$\surd$	~ 1

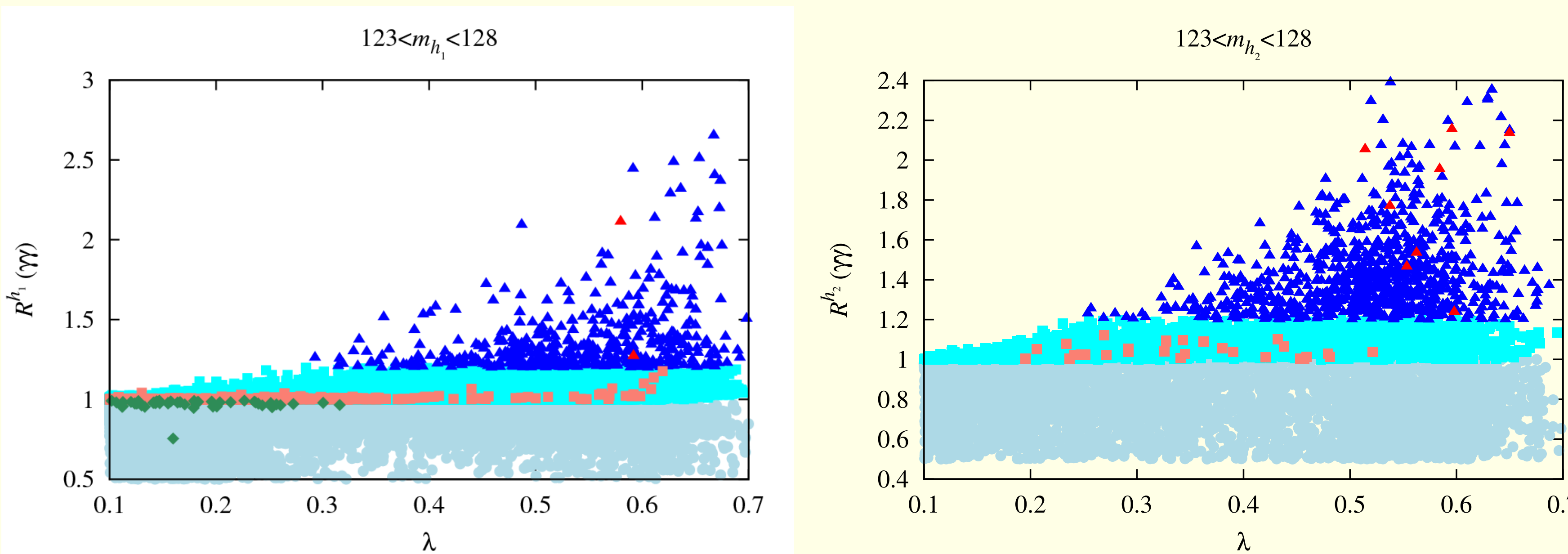
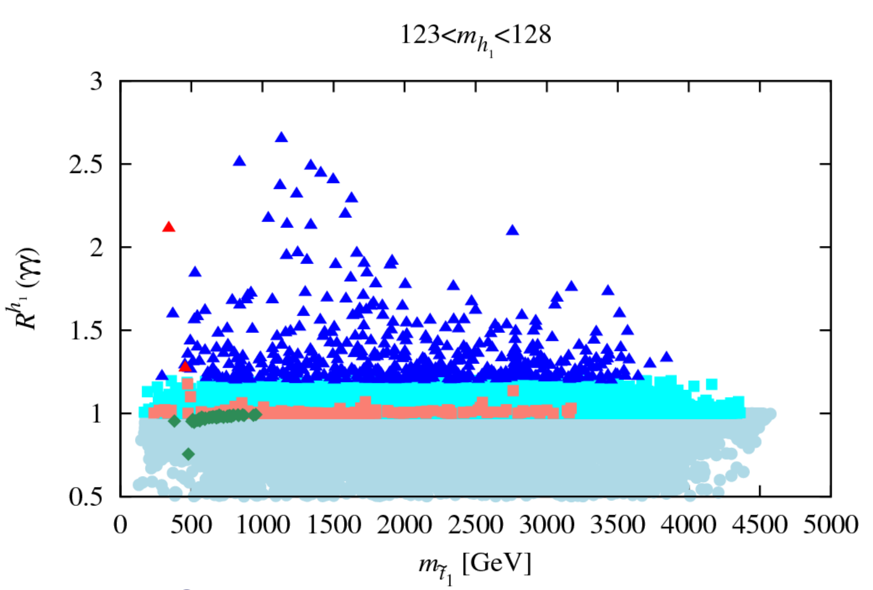


Figure 47: **Observe the clear general increase in maximum $R_{gg}(\gamma\gamma)$ with increasing λ. Green points have good δa_μ, $m_{h_2} > 1\ \mathrm{TeV}$ BUT $R_{gg}(\gamma\gamma) \sim 1$.**



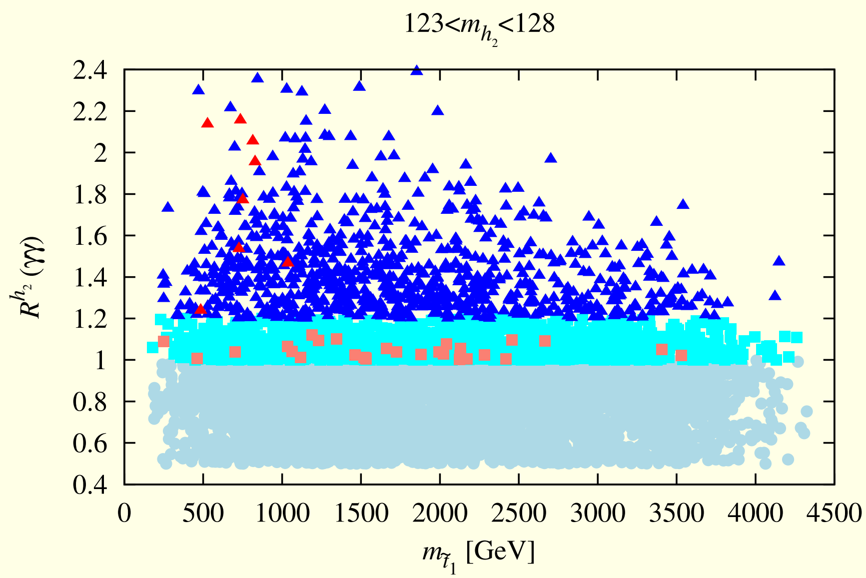


Figure 48: **The lightest stop has mass $\sim 300 - 900$ GeV for green "perfect" points.**

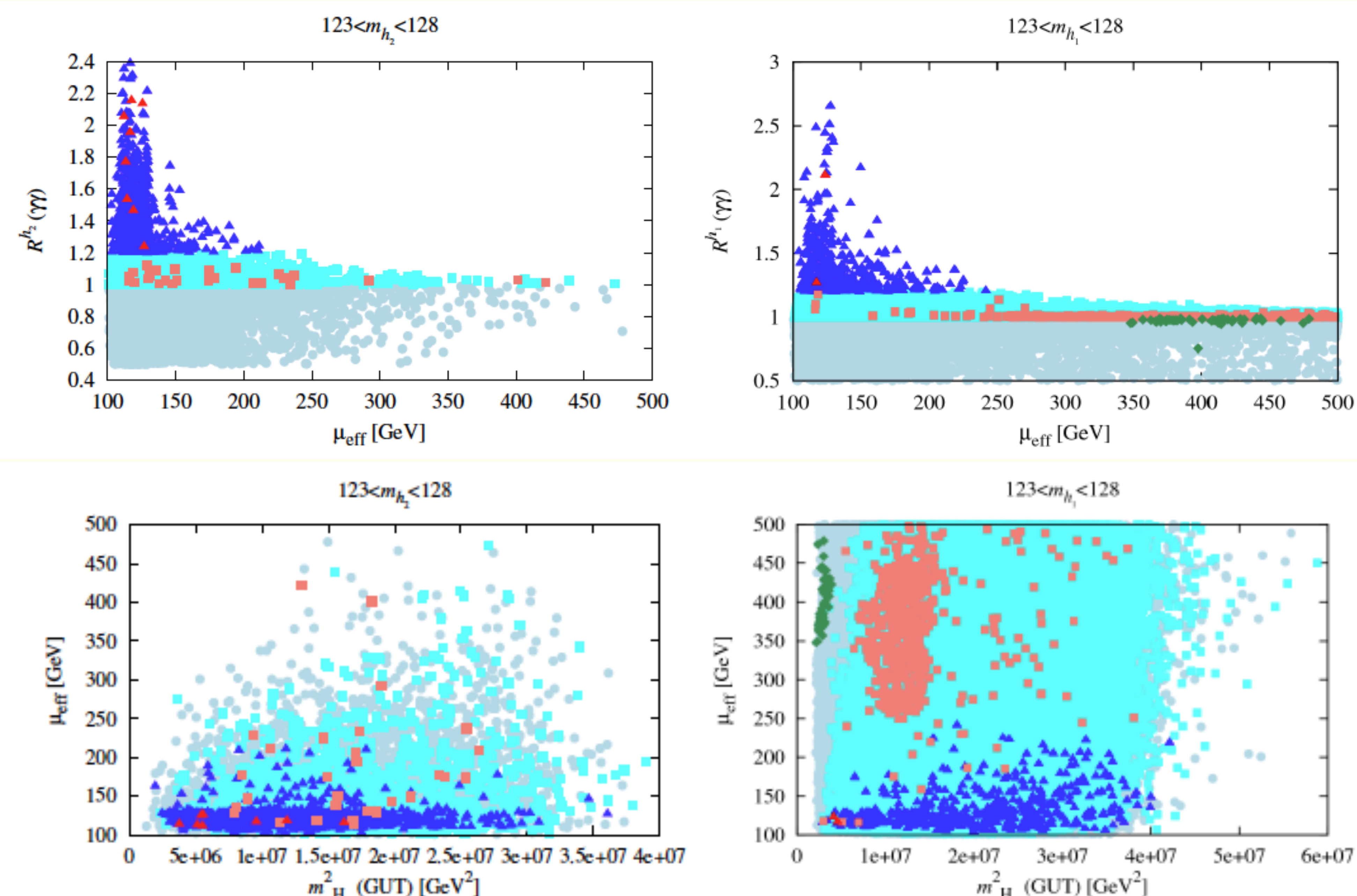


Figure 49: Plots involving $\mu_{\rm eft}$ and $m^2_{H_u}(GUT)$. For the green "perfect" points, $\mu_{\rm eff}$ ranges from $350 - 470~\mathrm{GeV}$ while $m_{H_u}(GUT)$ is of order $\lesssim 1.5~\mathrm{TeV}$.

- **Well, of course we need $m_{H_u}(m_{SUSY})$ but this will require reprocessing the data files.**

  But given modest $\mu_{\rm eff}$, and the usual formulate for $m_Z^2/2$, $m_{H_u}(m_{SUSY})$ cannot be that different.

- **So, from our work, I would claim that the simple NMSSM with rather unified boundary conditions comparable to NUHM MSSM boundary conditions will have very modest Δ_{EW}.**

  If we just use $\frac{\mu_{\rm eff}^2}{(m_Z^2/2)}$, $i.e.$ the C_μ estimator part of Δ_{EW}, this would give $\Delta_{EW} \sim 55$, which is quite comparable to the best values obtained in the more complicated MSSM scenarios and to the best values obtained in the lower Λ NMSSM or generalized NMSSM scenarios.

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