Higgs bosons in the Standard Model, the MSSM and beyond

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Nearing the 40th anniversary of the Higgs particle idea.

Detailed references can be found in two recent reviews: one by Haber and Carena and the 2nd (posted today?) by Gunion, Haber and Van Kooten. See also the recent paper on decoupling by Gunion and Haber.

Outline

- The Standard Model Higgs boson
  - Constraints
  - Basic Phenomenology
  - Problems
- The MSSM Higgs sector
  - The naturalness issue
  - Coupling unification
  - Basic tree-level results
  - Radiative corrections to tree-level results
- The NMSSM
- Still more singlets?
- Left-right supersymmetric models

The SM Higgs boson

- **The SM employs just a single doublet (under SU(2)$_L$) complex scalar field to give masses to all particles.** *Given the mass* $m_{h_{\rm SM}} = \frac{1}{2}v^2\lambda$ **(where** λ **is the quartic self-coupling strength) all couplings of the** $h_{\rm SM}$ **are determined.**

$$g_{h_{\rm SM} f\overline{f}} = \frac{m_f}{v}, \quad g_{h_{\rm SM} VV} = \frac{2m_V^2}{v}, \quad g_{h_{\rm SM} h_{\rm SM} VV} = \frac{2m_V^2}{v^2},$$

$$g_{h_{\rm SM} h_{\rm SM} h_{\rm SM}} = \frac{3}{2}\lambda v = \frac{3m_{h_{\rm SM}}^2}{v}, \quad g_{h_{\rm SM} h_{\rm SM} h_{\rm SM} h_{\rm SM}} = \frac{3}{2}\lambda = \frac{3m_{h_{\rm SM}}^2}{v^2}.$$

where $V = W$ **or** Z **and** $v = 2m_W/g = 246~{\rm GeV}$**.**

- **The couplings determine the branching ratios and total width.**

  The Higgs is very narrow until $m_{h_{\rm SM}} > 2m_W$**, at which point the** VV **decay modes start to take over and the width increases rapidly, reaching a unitarity, etc. bound for** $m_{h_{\rm SM}} \sim 700~{\rm GeV}$**.**

Note: that $B(h_{SM} \to \gamma\gamma)$ is substantial for $m_{h_{SM}} \sim 120$ GeV is important for LHC discovery mode for light Higgs.

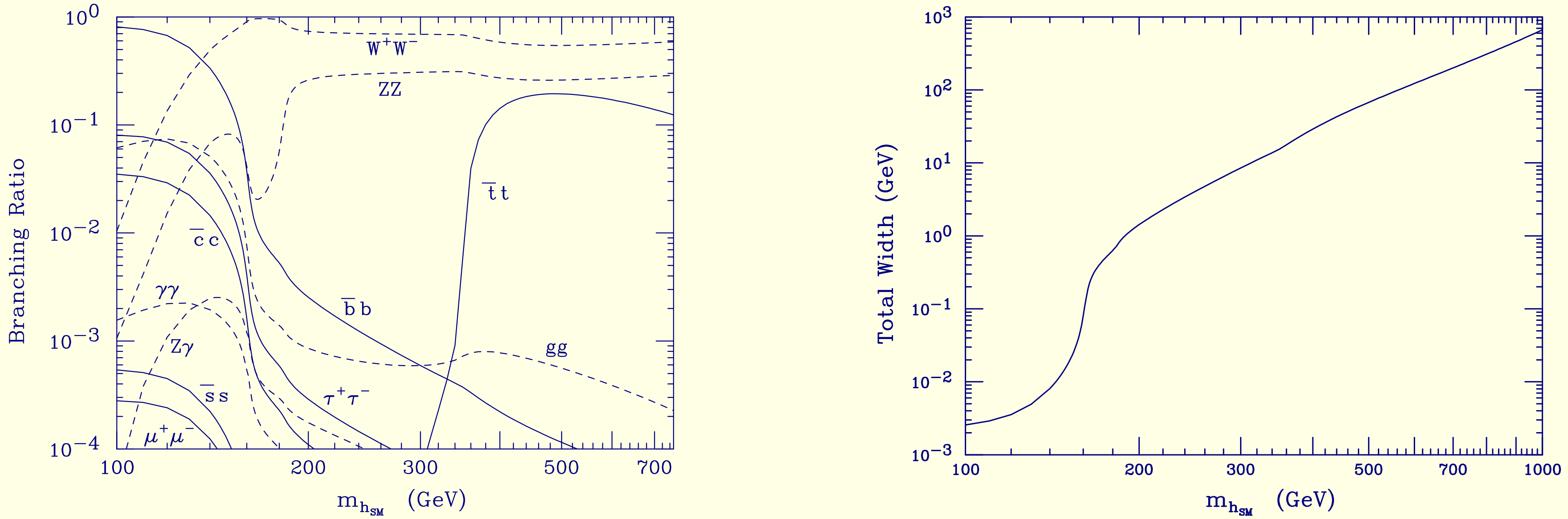


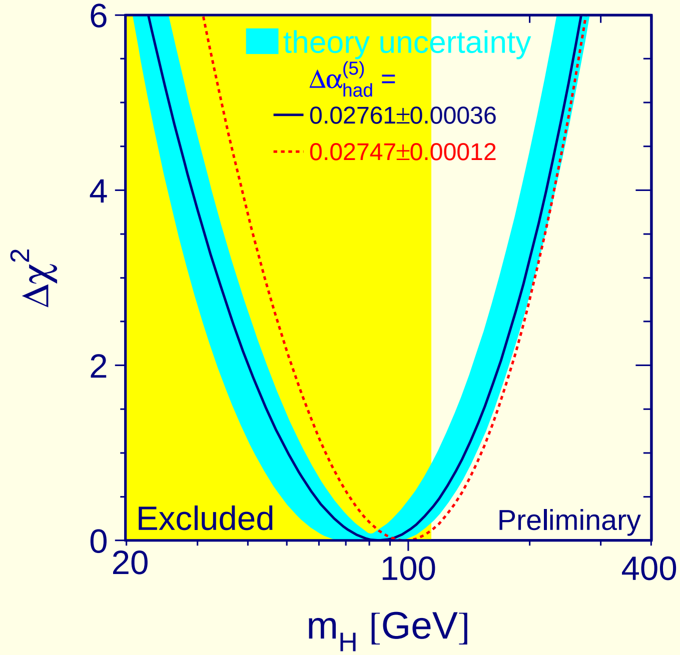
Figure 1: The SM Higgs branching ratios and total width.

- **The most immediate goal of present and future colliders will be to discover the SM Higgs (or a SM-like Higgs) if it exists and then to measure its branching ratios, total width, self-coupling, spin, parity and CP. This will not be possible without having both the LHC and a future LC.**

- **But, what is $m_{h_{\rm SM}}$?**

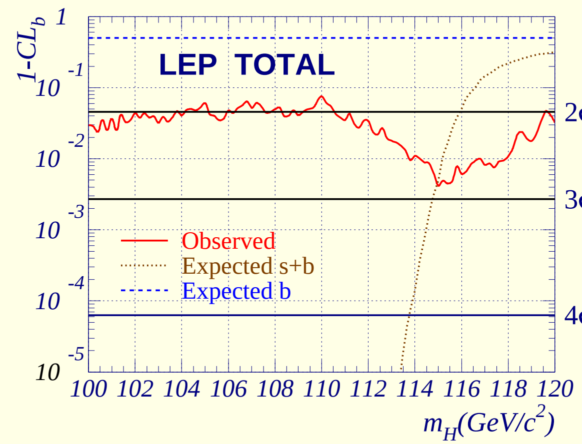
Hints from Current Data?

Precision electroweak constraints give $m_{h_{\rm SM}} < 196~{\rm GeV}$ at 95% CL, with a preferred central value of $m_{h_{\rm SM}} = 81~^{+52}_{-33}~{\rm GeV}$, below the LEP bound of $m_{h_{\rm SM}} < 114.4~{\rm GeV}$.



Background Compatability

The definition of $-2\ln Q$ changes with Higgs mass,
so we have a background confidence level curve,
instead of just a single value.



Minimum in $1 - CL_b = 4.2 \times 10^{-3}$ at 115 GeV/c^2

This is equivalent to a 2.9σ excess
over the background expectation.

Peter McNamara Standard Model Higgs Results from LEP January 27, 2001

There is possibility for spread-out Higgs weight (at $<$ SM strength) throughout the interval plotted.

There are also the "weak" signals: $m_h \sim 115\,\mathrm{GeV}$ and $m_h = 97\,\mathrm{GeV}$ in hZ production and $m_h + m_{A^0} = 187\,\mathrm{GeV}$ in hA^0 production.

All are consistent with a more complicated Higgs sector with multiple Higgs sharing the ZZ coupling.

- The influence of new physics on Higgs constraints?

  Two basic theoretical constraints are:

  - the Higgs self coupling does not blow up below scale Λ; $\Rightarrow$ upper bound on $m_{h_{\mathrm{SM}}}$ as function of Λ.
  - the Higgs potential does not develop a new minimum at large values of the scalar field of order Λ; $\Rightarrow$ lower bound on $m_{h_{\mathrm{SM}}}$ as function of Λ.

  These two constraints imply that the SM can be valid all the way up to

M_{Pl} if $130 \lesssim m_{h_{\rm SM}} \lesssim 180~{\rm GeV}$.

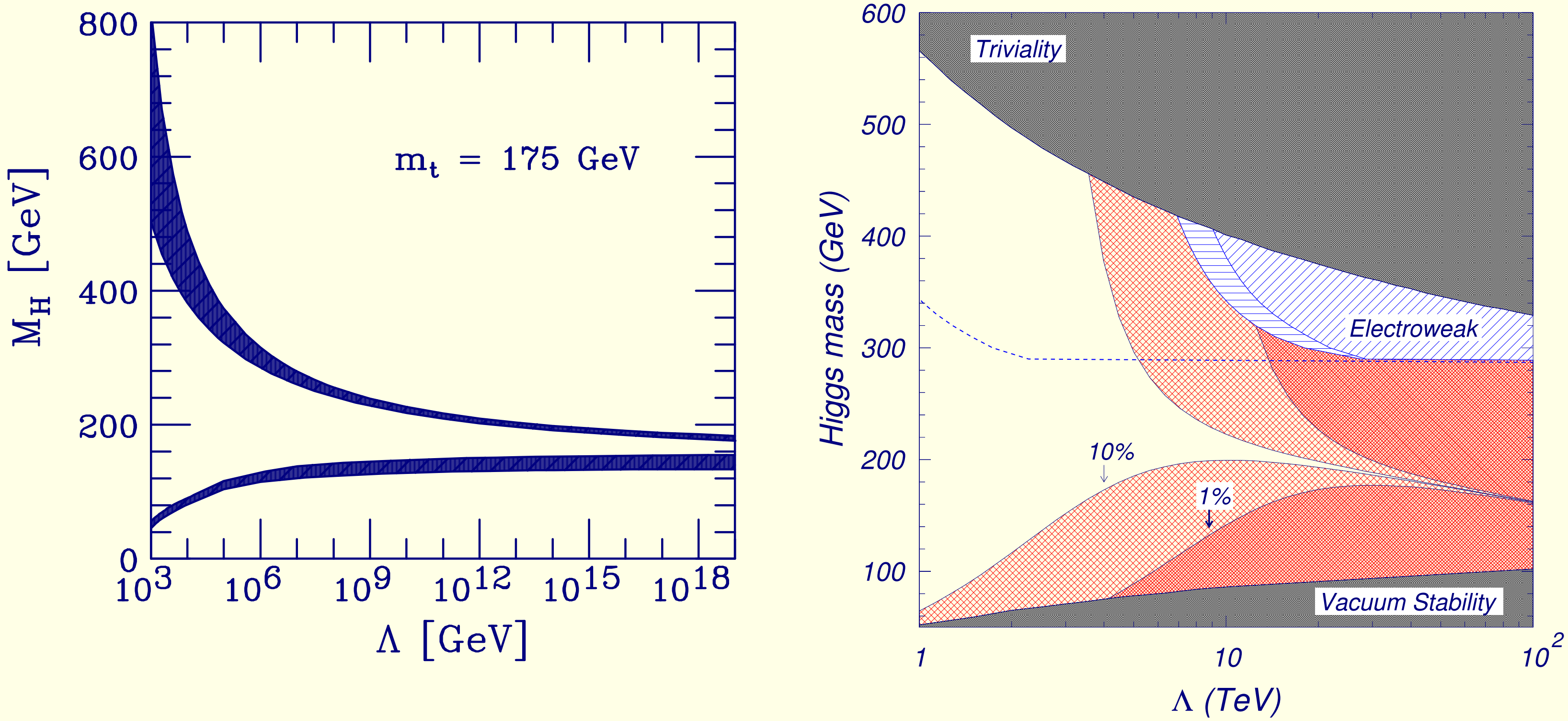


Figure 2: Left: triviality and global minimum constraints on $m_{h_{\rm SM}}$ vs. Λ. Right: fine-tuning constraints on Λ.

The precision electroweak constraints can also be somewhat modified if we allow for new physics operators (there are two important ones — one contributing to the S parameter and the other to the T parameter) characterized by some scale Λ. If the coefficients of these two operators are tree-level in size, then the above upper bound on $m_{h_{\rm SM}}$ can be considerably weakened. (See 2nd graph above.)

However, the survival of the SM as an effective theory all the way up to M_{Pl} is unlikely due to the problem of "naturalness" and the associated "fine-tuning" issue. We should impose the additional condition that:

– $m_{h_{\rm SM}} \sim m_Z$ is not a consequence of extreme fine-tuning.

Recall that after including the one loop corrections we have

$$m_{h_{\rm SM}}^2 = \mu^2 + \frac{3\Lambda^2}{32\pi^2 v^2}(2m_W^2 + m_Z^2 + m_{h_{\rm SM}}^2 - 4m_t^2) \qquad (1)$$

where $\mu^2 = -2\lambda v^2 \sim \mathcal{O}(m_Z^2)$ is a fundamental parameter of the theory.

These two terms have entirely different sources, and so a value of $m_{h_{\rm SM}} \sim m_Z$ should not arise by fine-tuned cancellation between the two terms. There are then two possible solutions:

1. Λ should be restricted to values $\lesssim 1~\mathrm{TeV}$
2. $m_{h_{\rm SM}}$ should obey the "Veltman" condition

$$m_{h_{\rm SM}}^2 = 4m_t^2 - 2m_W^2 - m_Z^2 \sim (317~\mathrm{GeV})^2 . \qquad (2)$$

In fact, this latter is a bit too simple and is somewhat modified in a

Λ-dependent way by going to the next order in the loop calculations. $\Rightarrow$ a $m_{h_{SM}}(\Lambda)$ solution to the no-fine-tuning "Veltman" condition.
Of course, just as we do not want to have a fine-tuned cancellation of the two terms in Eq. (1), we also do not want to insist on too fine-tuned a choice for $m_{h_{SM}}$ (in the SM there is no symmetry or theory that can predict this value), $\Rightarrow$ cannot continue the game to too high a Λ.

In practice, it is also appropriate to allow a certain percentage (e.g. 1% or 10%) amount of fine-tuning in the cancellation between μ^2 and the loop contributions or in the choice of $m_{h_{SM}}(\Lambda)$.

- The 2HDM is an example of new physics that could weaken precision EW bound, but not cure naturalness without additional new physics above a TeV

  Consider CP-conserving case: h^0, H^0, A^0 and $H^\pm$.

  It is possible to have all Higgs bosons heavy ($\sim 1~\mathrm{TeV}$) other than the A^0, with $m_{A^0} < 500~\mathrm{GeV}$ (perhaps very light).

  – Heavy h_{SM}-like Higgs $\Rightarrow$ large $\Delta S > 0$ and large $\Delta T < 0$.
  – Compensate by large $\Delta T > 0$ from small mass non-degeneracy (weak isospin breaking) of heavier Higgs. Light A^0 + heavy SM-like h^0 $\Rightarrow$

$$\Delta\rho = \frac{\alpha}{16\pi m_W^2 c_W^2}\left\{\frac{c_W^2}{s_W^2}\frac{m_{H^\pm}^2 - m_{H^0}^2}{2} - 3m_W^2\left[\log\frac{m_{h^0}^2}{m_W^2} + \frac{1}{6} + \frac{1}{s_W^2}\log\frac{m_W^2}{m_Z^2}\right]\right. \quad (3)$$

Can adjust $m_{H^\pm} - m_{H^0} \sim \mathrm{few\ GeV}$ (both heavy) so that the S, T prediction is OK.

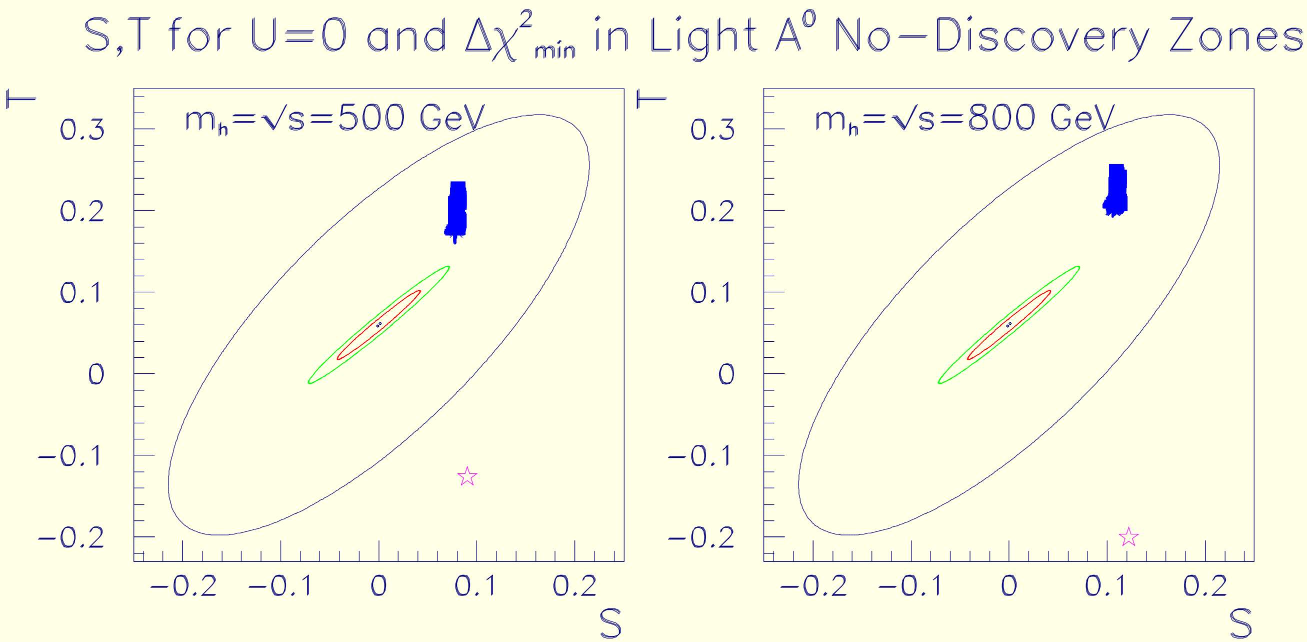


Figure 3: Outer ellipses = current 90% CL region for $U = 0$ and $m_{h_{\rm SM}} = 115$ GeV. Blobs = S, T predictions for Yukawa-wedge 2HDM models with minimum relative $\Delta\chi^2$. Innermost (middle) ellipse = 90% (99.9%) CL region for $m_{h_{\rm SM}} = 115$ GeV after Giga-Z *and* a $\Delta m_W \lesssim 6$ MeV threshold scan measurement. Stars = SM S, T prediction if $m_{h_{\rm SM}} = 500$ or 800 GeV.

Small m_{A^0} and large $\tan\beta \Rightarrow$ (part of) explanation of a_μ deviation relative to SM.

- Stil, if we want a consistent effective theory all the way up to M_{Pl} *without fine-tuning*, we must have some new physics at a scale $\Lambda \sim 1 - 10~\mathrm{TeV}$. The prime candidate is Supersymmetry.

  Λ would be identified with the scale of SUSY breaking, suggesting low energy SUSY with new particles at a mass scale of order $1~\mathrm{TeV}$. This also gives coupling constant unification in the MSSM context.

  In the decoupling limit, the light h^0 of the MSSM is SM-like.

  In general, it is clear that there will be many scenarios in which the SM is the effective theory up to some scale $\Lambda \gtrsim 1~\mathrm{TeV}$ and that we will wish to assess our ability to discovery the h_{SM} or a SM-like Higgs in the mass range from $114.4~\mathrm{GeV}$ up to $\sim 700~\mathrm{GeV}$ or so.

  We now turn to a review of the prospects for h_{SM} discovery and precision measurements of its properties.

- **Production/detection modes at hadron colliders**

$$gg \to h_{\rm SM} \to \gamma\gamma\,,$$
$$gg \to h_{\rm SM} \to VV^{(*)}\,,$$
$$q\bar q \to V^{(*)} \to h_{\rm SM}V\,,\ \text{with } h_{\rm SM} \to b\bar b, VV^{(*)}\,,$$
$$qq \to qqV^{(*)}V^{(*)} \to qqh_{\rm SM}\,,\ \text{with } h_{\rm SM} \to \gamma\gamma, \tau^+\tau^-, VV^{(*)}$$
$$qq, gg \to t\bar t h_{\rm SM}\,,\ \text{with } h_{\rm SM} \to b\bar b, \gamma\gamma, VV^{(*)}\,.$$

Some NLO and higher corrections for these production processes have been computed. Generally, the "K" factors are > 1 but not always ($K(t\bar t h_{\rm SM}) < 1$ at the Tevatron).

Remember that the Tevatron will accumulate no more than $15{\rm fb}^{-1}$ (probably more like $5{\rm fb}^{-1}$) before the LHC is in full swing.

$\Rightarrow$ $h_{\rm SM}$ discovery at the Tevatron is on the edge except at low masses.

The LHC will accumulate of order $100{\rm fb}^{-1}$ to $300{\rm fb}^{-1}$ for sure and $h_{\rm SM}$ detection is guaranteed regardless of $m_{h_{\rm SM}}$.

In fact, some moderately precise checking of Higgs properties (ratios of branching ratios, some partial widths) will be possible at the LHC.

However, really precise measurements must await the Linear Collider (LC).

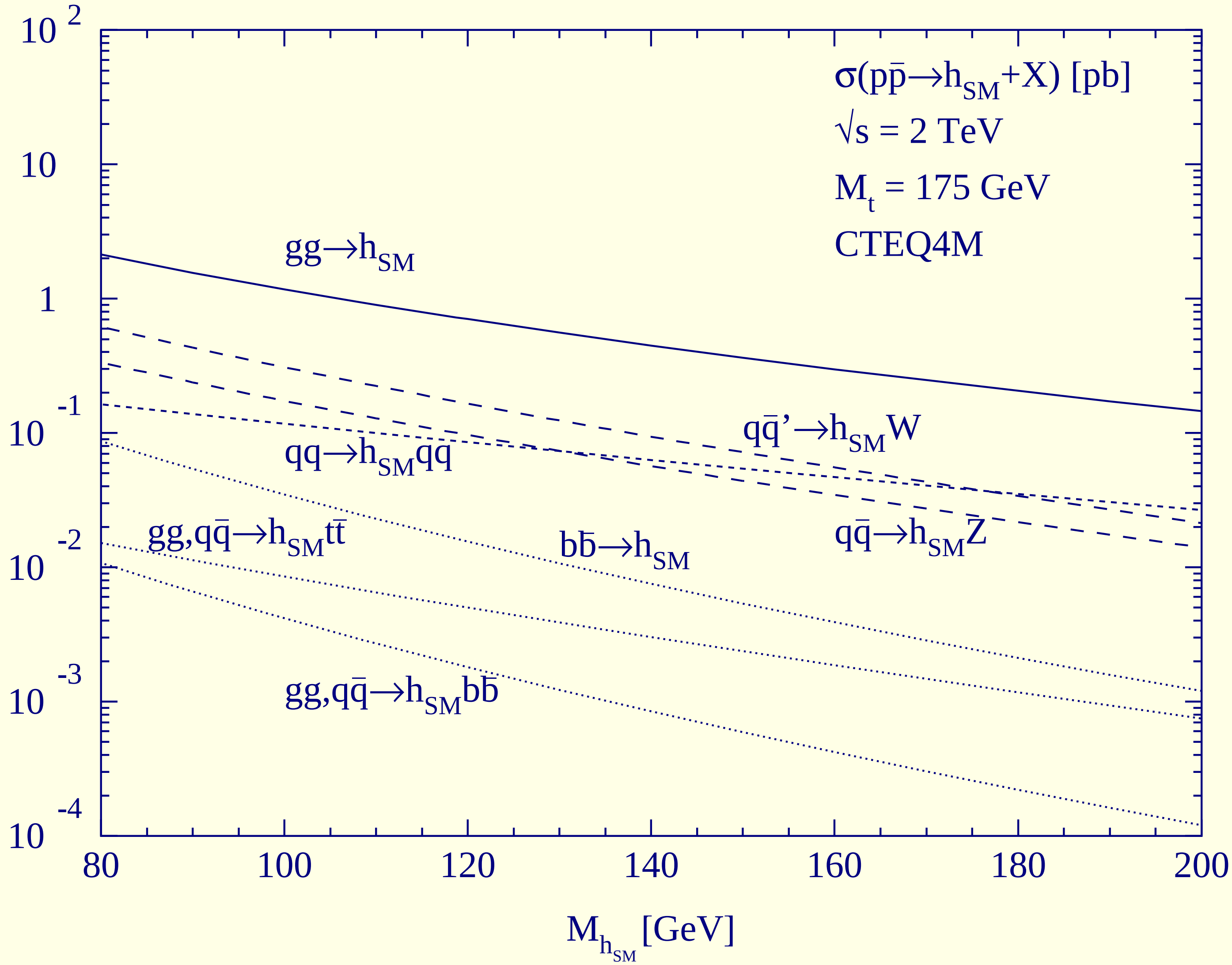


Figure 4: Tevatron cross sections for the $h_{\rm SM}$.

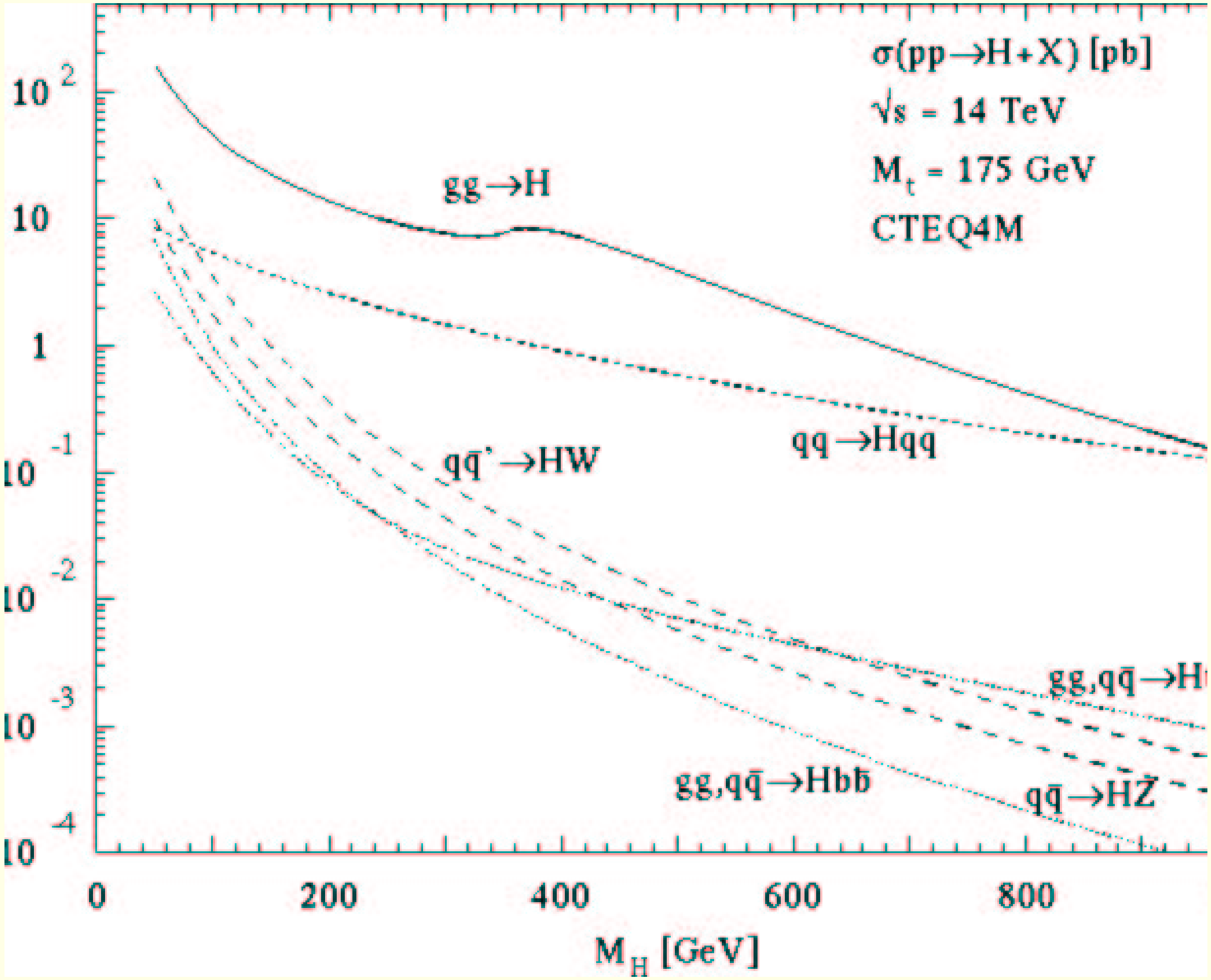


Figure 5: LHC cross sections for the $h_{\rm SM}$.

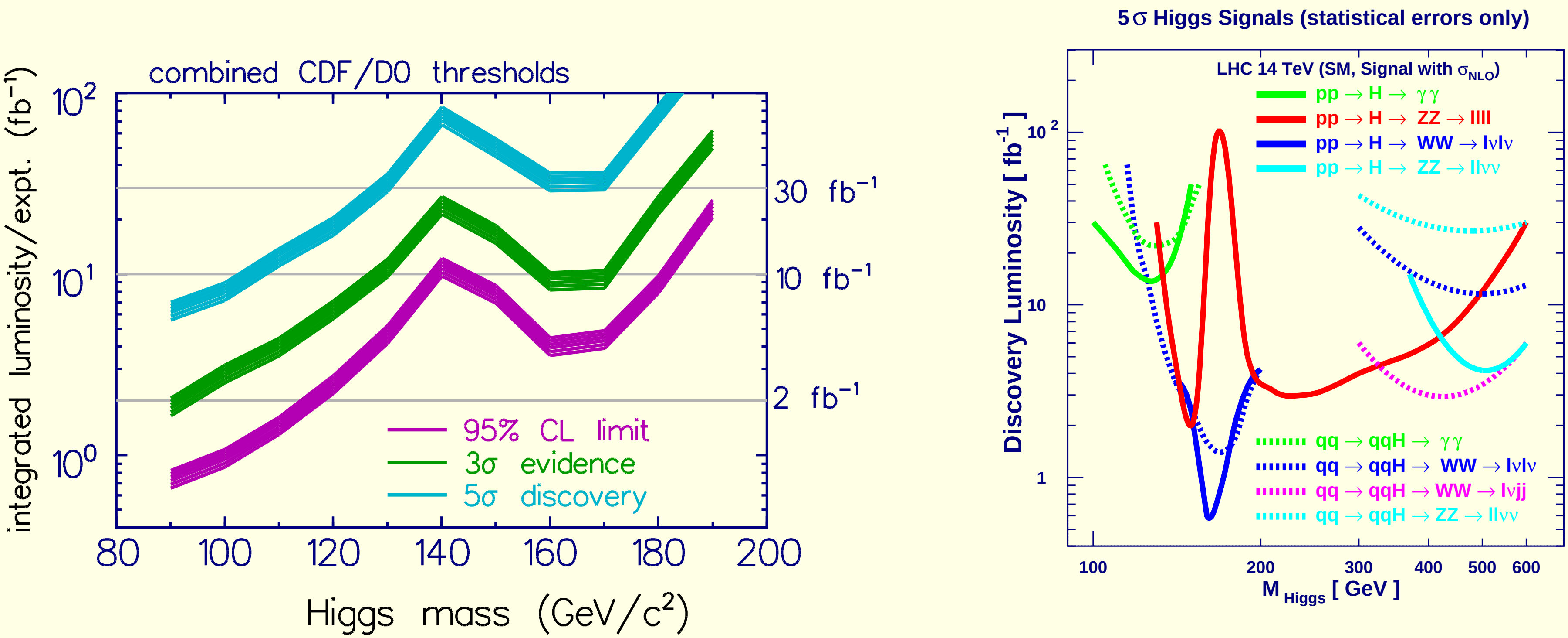


Figure 6: SM Higgs discovery at Tevatron and LHC. $h_{\rm SM}$ detection is guaranteed at the LHC.

- **Precision measurements at the LC**

The primary production modes are:

$$e^+e^- \to Z^* \to Zh_{\rm SM}\,, \quad e^+e^- \to \nu\bar\nu W^*W^* \to \nu\bar\nu h_{\rm SM}\,, \quad e^+e^- \to t\bar t h_{\rm SM}$$

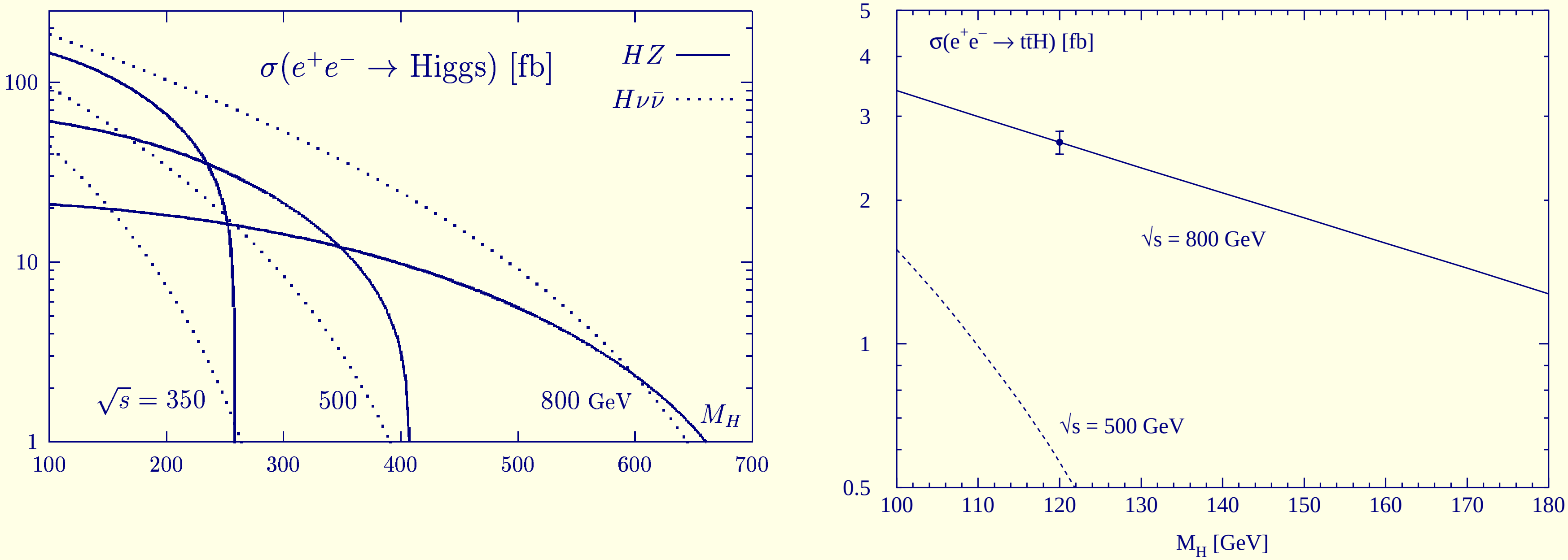


Figure 7: SM Higgs σ's at the LC. Recall: $L = 200 - 300{\rm fb}^{-1}$ per year.

The $Zh_{\rm SM}$ mode is very! important as it allows one to observe the $h_{\rm SM}$ as a bump in the M_X spectrum of the $e^+e^- \to ZX$ final state, *independent of how the* $h_{\rm SM}$ *decays*.

This provides a model-independent determination of $g^2_{ZZh_{\rm SM}}$, using which

all $B(h_{\rm SM} \to F)$ can be extracted:

$$B(h_{\rm SM} \to F) = \frac{\sigma(e^+e^- \to Zh_{\rm SM} \to ZF)}{\sigma(e^+e^- \to Zh_{\rm SM})}\,. \qquad (4)$$

A determination of $\Gamma^{\rm tot}_{h_{\rm SM}}$ is needed to compute $\Gamma(h_{\rm SM} \to F) = B(h_{\rm SM} \to F)\Gamma^{\rm tot}_{h_{\rm SM}}$. One technique employs the W-fusion cross section.

$$\Gamma(h_{\rm SM} \to WW) \propto \frac{\sigma(e^+e^- \to h_{\rm SM} \to WW)}{B(h_{\rm SM} \to WW)_{Zh_{\rm SM}}}\,, \quad \Gamma^{\rm tot}_{h_{\rm SM}} = \frac{\Gamma(h_{\rm SM} \to WW)}{B(h_{\rm SM} \to WW)} \qquad (5)$$

A rough determination of $g_{h_{\rm SM}h_{\rm SM}h_{\rm SM}}$ is possible using sensitivity of $e^+e^- \to Zh_{\rm SM}h_{\rm SM}$ coming from the sub-graph described by $e^+e^- \to Zh^*_{\rm SM}$ with $h^*_{\rm SM} \to h_{\rm SM}h_{\rm SM}$. The background is all the other graphs contributing to the same $Zh_{\rm SM}h_{\rm SM}$ final state.

The spin-0 nature of the $h_{\rm SM}$ can be checked by looking at the threshold rise of the $Zh_{\rm SM}$ cross section, which is much more rapid for $J=0$ than for $J=1$ or $J=2$.

Table 1: Measurement precisions for the properties of a SM-like Higgs boson, $h_{\rm SM}$, for a range of Higgs boson masses. Unless otherwise noted (see footnotes below the table), we assume $\sqrt{s}=500$ GeV and $L=500$ fb^{-1}.

$\Delta m_{h_{\rm SM}}$	$\simeq 120$ MeV (recoil against leptons from Z) $\simeq 50$ MeV (direct reconstruction)				
$m_{h_{\rm SM}}$ (GeV)	120	140	160	200	400–500
$\sqrt{s}$ (GeV)	500				800
$\Delta\sigma(Zh_{\rm SM})/\sigma(Zh_{\rm SM})$	4.7%	6.5%	6%	7%	10%
$\Delta\sigma(\nu\bar\nu h_{\rm SM})B(b\bar b)/\sigma B$	3.5%	6%	17%	–	–
$\delta g_{h_{\rm SM}xx}/g_{h_{\rm SM}xx}$ (from B's)					
$t\bar t$	$6-21\%$ †	–	–	–	10%
$b\bar b$	1.5%	2%	3.5%	12.5%	–
$c\bar c$	20%	22.5%	–	–	–
$\tau^+\tau^-$	4%	5%	–	–	–
$\mu^+\mu^-$	15% ‡	–	–	–	–
$WW^{(*)}$	4.5%	2%	1.5%	3.5%	8.5%
$ZZ^{(*)}$	–	–	8.5%	4%	10%
gg	10%	12.5%	–	–	–
$\gamma\gamma$	7%	10%	–	–	–
$g_{h_{\rm SM}h_{\rm SM}h_{\rm SM}}$	20% §	–	–	–	–
$\Gamma^{\rm tot}_{h_{\rm SM}}$ ††	10.1%	8.2%	12.9%	10.6%	22.3%

† The $h_{\rm SM}t\bar t$ coupling errors are from $e^+e^-\to t\bar t h_{\rm SM}$, with $\sqrt{s}=500-800$ GeV and 1 ab^{-1} of data.

‡ based on $\sqrt{s}=800$ GeV and 1 ab^{-1} of data.

§ based on $\sqrt{s}=500$ GeV and 1 ab^{-1} of data.

†† indirect determination from $\Gamma(VV^*)/B(VV^*)$, $V=W,Z$.

Determination of the CP of the $h_{\rm SM}$?

- **Checking that $CP=+$ for the $h_{\rm SM}$ using the $\gamma\gamma$ collider option at the LC**

  Why the $\gamma\gamma$ collider?

  – Angular leptonic distributions in $Zh_{\rm SM}\to \ell^+\ell^- h_{\rm SM}$ production and/or $h_{\rm SM}\to Z^*Z^*\to 4\ell$ only check that the $h_{\rm SM}$ has a substantial $CP=+$ component – since any $CP=-$ component couples only at one loop, one could have up to 80% CP-odd without seeing it in the angular distribution.
  The $Zh_{\rm SM}$ cross section would be smaller than anticipated, but such a reduction could arise from other sources than CP-mixing.

  – One can employ $e^+e^-\to Zh_{\rm SM}$ with $h_{\rm SM}\to\tau^+\tau^-$ and use the self-analyzing decays $\tau^+\to\rho,\pi+\nu$, but this is quite hard and the accuracy of the CP determination is not wonderful.

  – At the $\gamma\gamma$ collider, one transversely polarizes the laser photons (yielding partially transversely polarized back scattered photons) and then uses the facts that:
  a) the CP-even and CP-odd components of a Higgs boson both couple strongly to $\gamma\gamma$ (via the top-quark loop for the CP-odd part) and
  b) the CP-even part couples to transversely polarized photons as $\vec\epsilon\cdot\vec\epsilon'$

while the CP-odd part couples as $\vec{\epsilon} \times \vec{\epsilon}'$ $\Rightarrow$ easy to isolate one from the other by comparing rates for parallel vs. perpendicular transverse polarizations.

Net result: can check $CP = +$ with accuracy of $\sim 11\%$.

Beyond the SM Higgs boson

There are many possible directions:

- **Simply extend the SM to include extra Higgs representations, e.g. by adding singlet Higgs, one or more extra double Higgs representations (general 2HDM), one or more triplet representations (left-right symmetric model),. . .**

  All have some motivation: e.g.

  Two-doublets plus one $Y = 0$ triplet yields coupling unification at $M_U = 1.7 \times 10^{14}$, which is ok if there is no gauge unification (as in some string models).

  $Y = 2$ triplets are good for see-saw mechanism and can also give coupling unification (at low M_U).

  But, all have the naturalness / fine-tuning problem.

- **Could go to technicolor, top assisted technicolor, little higgses.**

  But these all tend to have difficulties with precision electroweak data.

- **Could avoid the fine-tuning and naturalness issues if there are large extra-dimensions.**

  Coupling unification can survive but is not very motivated.

- **Supersymmetry with exactly two Higgs doublets (the MSSM) is the best motivated.**

  a) naturalness and fine-tuning are resolved for $m_{\rm SUSY} \sim 1~{\rm TeV} - 10~{\rm TeV}$.

  b) coupling unification is excellent for $m_{\rm SUSY} \sim 1~{\rm TeV} - 10~{\rm TeV}$

  c) electroweak symmetry breaking starting from universal scalar masses at M_U is "automatic" as a result of the H_u scalar mass squared being driven negative under rge evolution by the large top-quark Yukawa coupling.

The Higgs bosons of the MSSM

- **Minimal SUSY model contains exactly two Higgs doublets, one with $Y = +1$ (Φ_u) and one with $Y = -1$ (Φ_d). Why?**

  a) Φ_u (Φ_d) is required for giving masses to up quarks (down quarks and leptons).

  b) Need the opposite Y doublets for anomaly cancellation.

  Associated very nice features:

  1. **The MSSM yields excellent coupling unification at $M_U \sim few \times 10^{16}~\mathrm{GeV}$; for more doublets, this fails badly.**
  2. **The MSSM yields "automatic" EWSB.**

- **The MSSM Higgs sector is CP-conserving (CPC) at tree-level (although radiative corrections involving complex soft-SUSY-breaking parameters can introduce CP-mixing at the one-loop level).**

For CPC, the Higgs mass eigenstates are: the CP-even h^0, H^0; the CP-odd A^0; and the charged Higgs pair $H^\pm$.

Tree-level Higgs masses and diagonalization

- At tree-level, all Higgs masses and couplings are determined by just two parameters: $\tan\beta = \frac{v_u}{v_d}$ (where $v_u = \sqrt{2}\langle\Phi_u^0\rangle, v_d = \sqrt{2}\langle\Phi_d^0\rangle$) and m_{A^0}.

  The CP-even eigenstates are obtained by diagonalizing a 2×2 matrix using a rotation angle α:

$$\begin{aligned} h^0 &= -(\sqrt{2}\,\mathrm{Re}\,\Phi_d^0 - v_d)\sin\alpha + (\sqrt{2}\,\mathrm{Re}\,\Phi_u^0 - v_u)\cos\alpha\,, \\ H^0 &= (\sqrt{2}\,\mathrm{Re}\,\Phi_d^0 - v_d)\cos\alpha + (\sqrt{2}\,\mathrm{Re}\,\Phi_u^0 - v_u)\sin\alpha\,, \end{aligned} \tag{6}$$

  A particularly useful formula is:

$$\cos^2(\beta-\alpha) = \frac{m_{h^0}^2(m_Z^2 - m_{h^0}^2)}{m_{A^0}^2(m_{H^0}^2 - m_{h^0}^2)}\,. \tag{7}$$

  The decoupling phenomenon is already apparent from this equation which shows $\cos^2(\beta-\alpha) \to 0$ for $m_{A^0} \gg m_Z$. In this limit, we will see that the h^0 is SM-like.

At tree-level $m_{h^0} \leq m_Z|\cos 2\beta| \leq m_Z$, due to the fact that all Higgs self-coupling parameters of the MSSM are related to the squares of the electroweak gauge couplings.

Tree-level Couplings

- Three-point Higgs boson-vector boson couplings are conveniently summarized by listing the couplings that are proportional to either $\sin(\beta-\alpha)$ or $\cos(\beta-\alpha)$, and the couplings that are independent of α and β:

$\cos(\beta-\alpha)$	$\sin(\beta-\alpha)$	angle-independent
$H^0W^+W^-$	$h^0W^+W^-$	$--$
H^0ZZ	h^0ZZ	$--$
ZA^0h^0	ZA^0H^0	ZH^+H^-, γH^+H^-
$W^\pm H^\mp h^0$	$W^\pm H^\mp H^0$	$W^\pm H^\mp A^0$

(8)

All vertices that contain at least one vector boson and *exactly one* non-minimal Higgs boson state (H^0, A^0 or $H^\pm$) are proportional to $\cos(\beta-\alpha)$.

The couplings of the neutral Higgs bosons to $f\bar{f}$ relative to the Standard Model value, $gm_f/2m_W$, are given by

$$h^0 b\bar{b} \quad (\text{or } h^0\tau^+\tau^-) \quad : \quad -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)\,, \tag{9}$$

$$h^0 t\bar{t} \quad : \quad \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)\,, \tag{10}$$

$$H^0 b\bar{b} \quad (\text{or } H^0\tau^+\tau^-) \quad : \quad \frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha)\,, \tag{11}$$

$$H^0 t\bar{t} \quad : \quad \frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha)\,, \tag{12}$$

$$A^0 b\bar{b} \quad (\text{or } A^0\tau^+\tau^-) \quad : \quad \gamma_5\tan\beta\,, \tag{13}$$

$$A^0 t\bar{t} \quad : \quad \gamma_5\cot\beta\,, \tag{14}$$

(the γ_5 indicates a pseudoscalar coupling), and the charged Higgs boson couplings to fermion pairs, with all particles pointing into the vertex, are given by

$$g_{H^-t\bar{b}} = \frac{g}{\sqrt{2}m_W}\left[m_t\cot\beta\,P_R + m_b\tan\beta\,P_L\right], \quad (15)$$

$$g_{H^-\tau^+\nu} = \frac{g}{\sqrt{2}m_W}\left[m_\tau\tan\beta\,P_L\right]. \quad (16)$$

The decoupling limit at tree-level

- It is the $\sin(\beta-\alpha)$ terms that survive in the decoupling limit of $m_{A^0} \gg m_Z$. In this limit we have

$$m_{h^0}^2 \simeq m_Z^2\cos^2 2\beta\,, \quad (17)$$

$$m_{H^0}^2 \simeq m_{A^0}^2 + m_Z^2\sin^2 2\beta\,, \quad (18)$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2\,, \quad (19)$$

$$\cos^2(\beta-\alpha) \simeq \frac{m_Z^4\sin^2 4\beta}{4m_{A^0}^4}. \quad (20)$$

Thus, $m_{A^0} \sim m_{H^0} \sim m_{H^\pm}$ up to terms of order m_Z^2/m_{A^0}, and $\cos(\beta - \alpha) = 0$ up to corrections of order $m_Z^2/m_{A^0}^2$. Further, the h^0 couplings are all SM-like. This means that the effective low-energy theory below scales of order m_{A^0} is the SM.

But, note that at large $\tan\beta$, the $h^0 b\bar{b}$ could have significant deviations from the SM value if $\tan\beta\cos(\beta-\alpha)$ is not small. This is sometimes called "delayed decoupling".

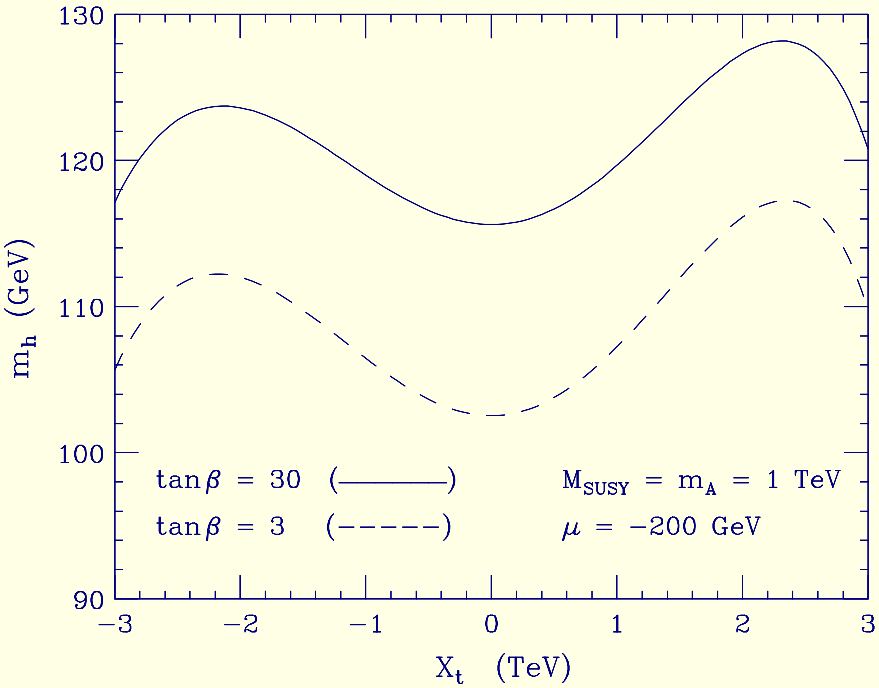
The couplings of the heavy Higgs bosons include $H^0 A^0 Z$ and $W^\pm H^\mp Z$ at maximal strength and $H^0 t\bar{t}, A^0 t\bar{t} \propto \cot\beta$ and $H^0 b\bar{b}, A^0 b\bar{b} \propto \tan\beta$.

Radiative Corrections to m_{h^0}

- There are top and stop loop contributions to the mass-matrix. These do not cancel completely since SUSY is broken. The crucial parameters are the average of the two top-squark squared-masses, $M_S \equiv \frac{1}{2}(m_{\widetilde{t}_1}^2 + m_{\widetilde{t}_2}^2)$ .and the parameter $X_t \equiv A_t - \mu\cot\beta$ that enters into stop-mixing. (A_t describes trilinear soft-SUSY-breaking and μ appears in the $\mu\widehat{H}_u\widehat{H}_d$ term of the superpotential.) The upper bound on the lightest CP-even Higgs mass is approximately given by

$$m_{h^0}^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right]. \qquad (21)$$

This reaches a maximum for $X_t \sim \sqrt{6}M_S$.



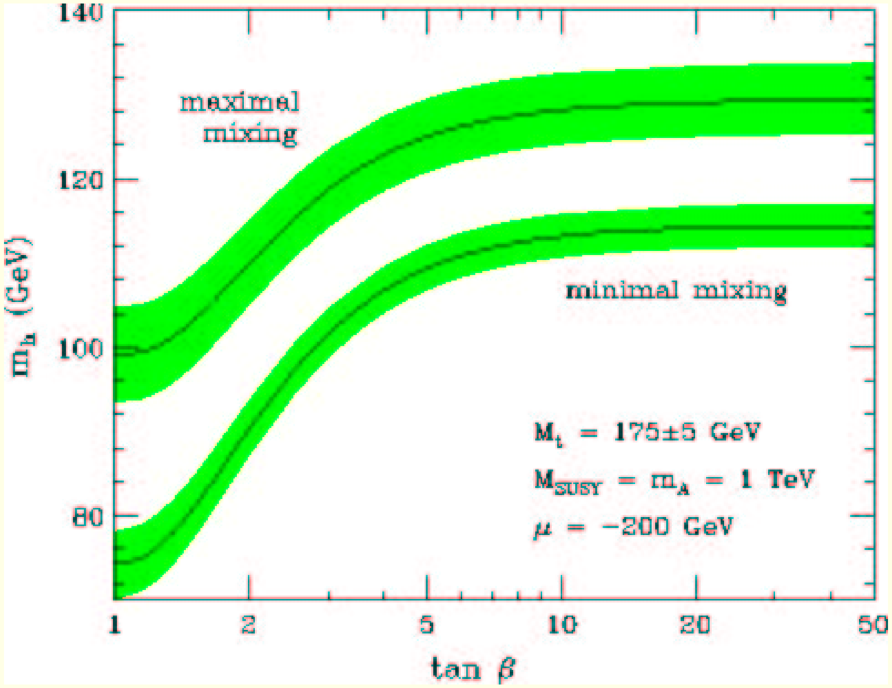


Figure 8: Left: m_{h^0} as function of X_t. Right: m_{h^0} as a function of $\tan\beta$ for $X_t = 0$ (minimal mixing) and $X_t = \sqrt{6}M_S$ (maximal mixing). $m_{\rm SUSY} = M_Q = M_U = M_D$.

There is only slow dependence on $m_{\rm SUSY}$ once $m_{\rm SUSY} \gtrsim 1$ TeV.

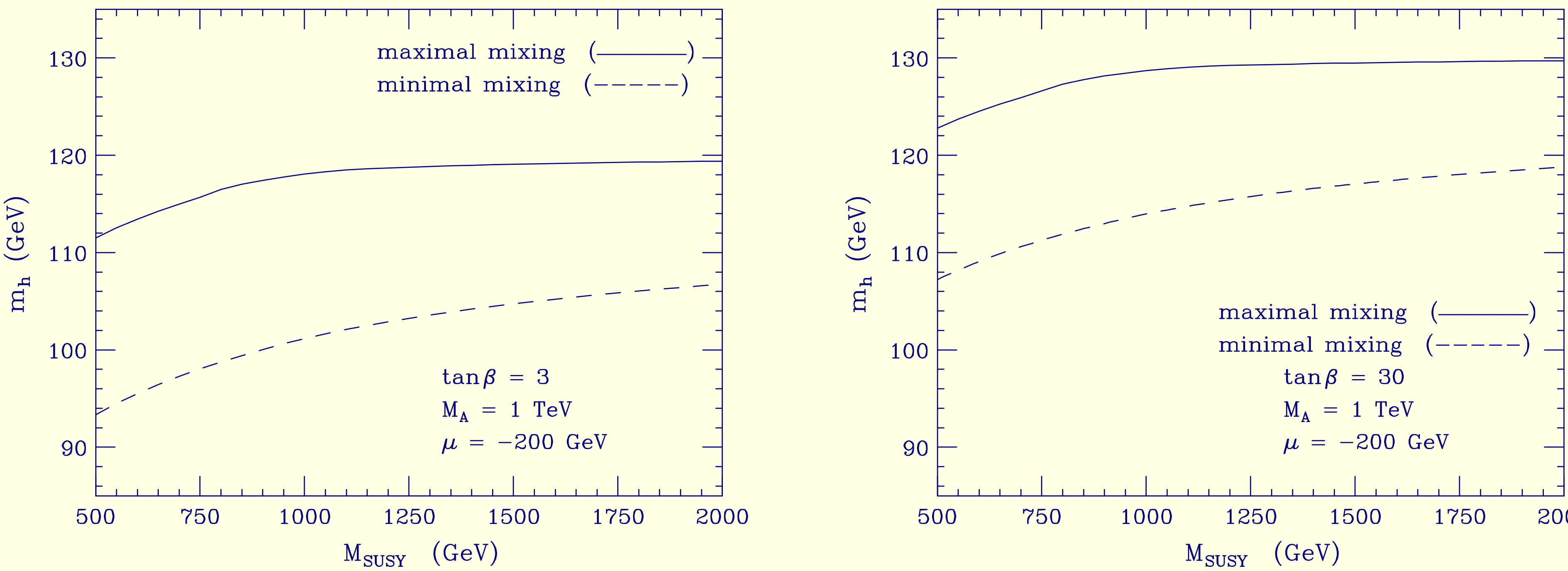


Figure 9: Minimal and maximal mixing results for m_{h^0} as a function of $m_{\rm SUSY} = M_Q = M_U = M_D$.

- **A final summary plot including other Higgs bosons is below.**

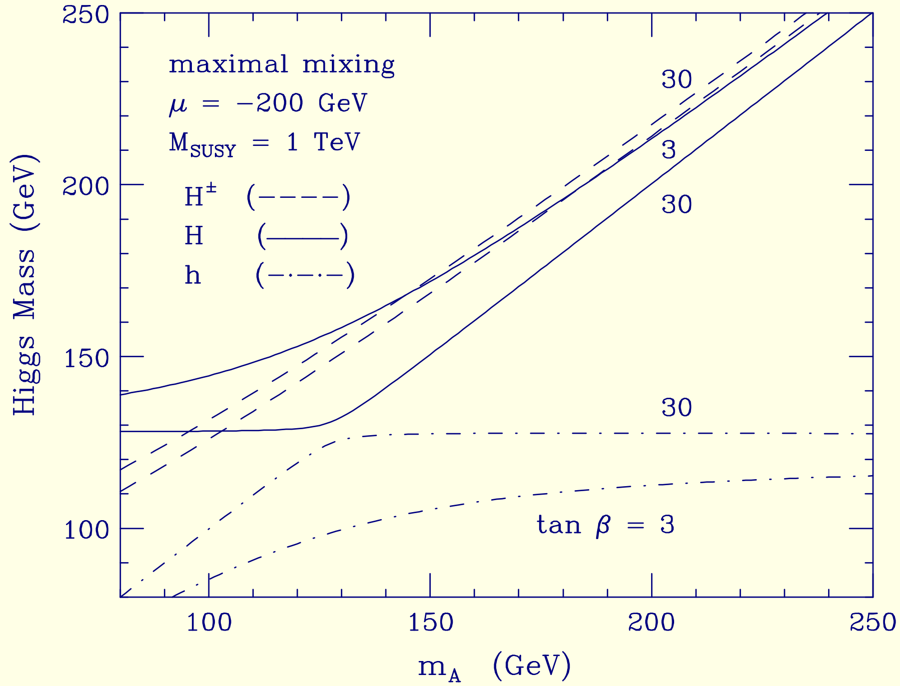


Figure 10: Higgs masses as a function of m_{A^0} for maximal mixing with $m_{\rm SUSY} = M_Q = M_U = M_D = 1~{\rm TeV}$.

Radiative corrections to couplings

- **The dominant corrections for Higgs couplings to vector bosons arise from radiative corrections to $\cos(\beta-\alpha)$ (which we shall shortly discuss).**

- **For Yukawa couplings there are additional (non-decoupling) vertex corrections.**

$$-\mathcal{L}_{\rm eff} = \epsilon_{ij}\left[(h_b+\delta h_b)\bar{b}_R\Phi_{\rm d}^i Q_L^j + (h_t+\delta h_t)\bar{t}_R Q_L^i \Phi_{\rm u}^j\right] + \Delta h_t \bar{t}_R Q_L^k \Phi_{\rm d}^{k*} + \Delta h_b \bar{b}_R Q_L^k \Phi_{\rm u}^{k*} + {\rm h.c.}\,, \tag{22}$$

implying a modification of the tree-level relations between h_t, h_b and m_t, m_b as follows:

$$m_b = \frac{h_b v}{\sqrt{2}}\cos\beta\left(1+\frac{\delta h_b}{h_b}+\frac{\Delta h_b\tan\beta}{h_b}\right) \equiv \frac{h_b v}{\sqrt{2}}\cos\beta(1+\Delta_b)\,, \tag{23}$$

$$m_t = \frac{h_t v}{\sqrt{2}}\sin\beta\left(1+\frac{\delta h_t}{h_t}+\frac{\Delta h_t\cot\beta}{h_t}\right) \equiv \frac{h_t v}{\sqrt{2}}\sin\beta(1+\Delta_t)\,. \tag{24}$$

The dominant contributions to Δ_b are $\tan\beta$-enhanced, with $\Delta_b \simeq (\Delta h_b/h_b)\tan\beta$; for $\tan\beta \gg 1$, $\delta h_b/h_b$ provides a small correction to Δ_b. In the same limit, $\Delta_t \simeq \delta h_t/h_t$, with the additional contribution of $(\Delta h_t/h_t)\cot\beta$ providing a small correction

$$\Delta_b \simeq \left[\frac{2\alpha_s}{3\pi}\mu M_{\tilde{g}}\, I(M^2_{\tilde{b}_1}, M^2_{\tilde{b}_2}, M^2_{\tilde{g}}) + \frac{h_t^2}{16\pi^2}\mu A_t\, I(M^2_{\tilde{t}_1}, M^2_{\tilde{t}_2}, \mu^2)\right]\tan\beta\,, \tag{25}$$

$$\Delta_t \simeq -\frac{2\alpha_s}{3\pi}A_t M_{\tilde{g}} I(M^2_{\tilde{t}_1}, M^2_{\tilde{t}_2}, M^2_{\tilde{g}}) - \frac{h_b^2}{16\pi^2}\mu^2 I(M^2_{\tilde{b}_1}, M^2_{\tilde{b}_2}, \mu^2)\,, \tag{26}$$

where $\alpha_s \equiv g_3^2/4\pi$, $M_{\tilde{g}}$ is the gluino mass, $M_{\tilde{b}_{1,2}}$ are the bottom squark

masses, and smaller electroweak corrections have been ignored.

$$I(a,b,c) = \frac{a^2b^2\ln(a^2/b^2) + b^2c^2\ln(b^2/c^2) + c^2a^2\ln(c^2/a^2)}{(a^2-b^2)(b^2-c^2)(a^2-c^2)}, \tag{27}$$

is of order $1/\max(a^2,b^2,c^2)$ when at least one of its arguments is large compared to m_Z^2.

Note Δ_b does not decouple (i.e. it does not $\to 0$) in the limit of large values of the supersymmetry breaking masses.

$\Delta_b \sim \pm 1$ is possible for large $\tan\beta$.

Similarly

$$m_\tau = \frac{h_\tau v_d}{\sqrt{2}}(1+\Delta_\tau). \tag{28}$$

The correction Δ_τ contains a contribution from a tau slepton–neutralino loop (depending on the two tau-slepton masses $M_{\tilde{\tau}_1}$ and $M_{\tilde{\tau}_2}$ and the mass parameter of the $\widetilde{B}$ component of the neutralino, M_1) and a tau sneutrino–chargino loop (depending on the tau sneutrino mass $M_{\tilde{\nu}_\tau}$, the mass parameter of the $\widetilde{W}^\pm$ component of the chargino, M_2, and μ). It is given by:

$$\Delta_\tau = \left[\frac{\alpha_1}{4\pi} M_1 \mu\, I(M_{\tilde\tau_1}, M_{\tilde\tau_2}, M_1) - \frac{\alpha_2}{4\pi} M_2 \mu\, I(M_{\tilde\nu_\tau}, M_2, \mu)\right] \tan\beta\,, \tag{29}$$

where $\alpha_2 \equiv g^2/4\pi$ and $\alpha_1 \equiv g'^{\,2}/4\pi$ are the electroweak gauge couplings.

$\Delta_\tau \ll \Delta_b$ because Δ_b knows about α_s and h_t while Δ_τ is proportional to only the weak gauge couplings.

Radiative Corrections to $\cos(\beta - \alpha)$

- In terms of the radiative corrections $\delta\mathcal{M}^2_{11}, \delta\mathcal{M}^2_{22}, \mathcal{M}^2_{12}$ to the 2×2 CP-even mass matrix, we obtain a correction to our earlier computation of $\cos(\beta - \alpha)$. One finds:

$$\cos(\beta - \alpha) = c\left[\frac{m_Z^2 \sin 4\beta}{2m_A^2} + \mathcal{O}\left(\frac{m_Z^4}{m_A^4}\right)\right]\,, \tag{30}$$

in the limit of $m_{A^0} \gg m_Z$, where

$$c \equiv 1 + \frac{\delta\mathcal{M}^2_{11} - \delta\mathcal{M}^2_{22}}{2m_Z^2 \cos 2\beta} - \frac{\delta\mathcal{M}^2_{12}}{m_Z^2 \sin 2\beta}. \quad (31)$$

Eq. (30) exhibits the expected decoupling behavior for $m_A \gg m_Z$. In the generic $c \neq 0$ cases, we get rapid decoupling just as at tree-level.

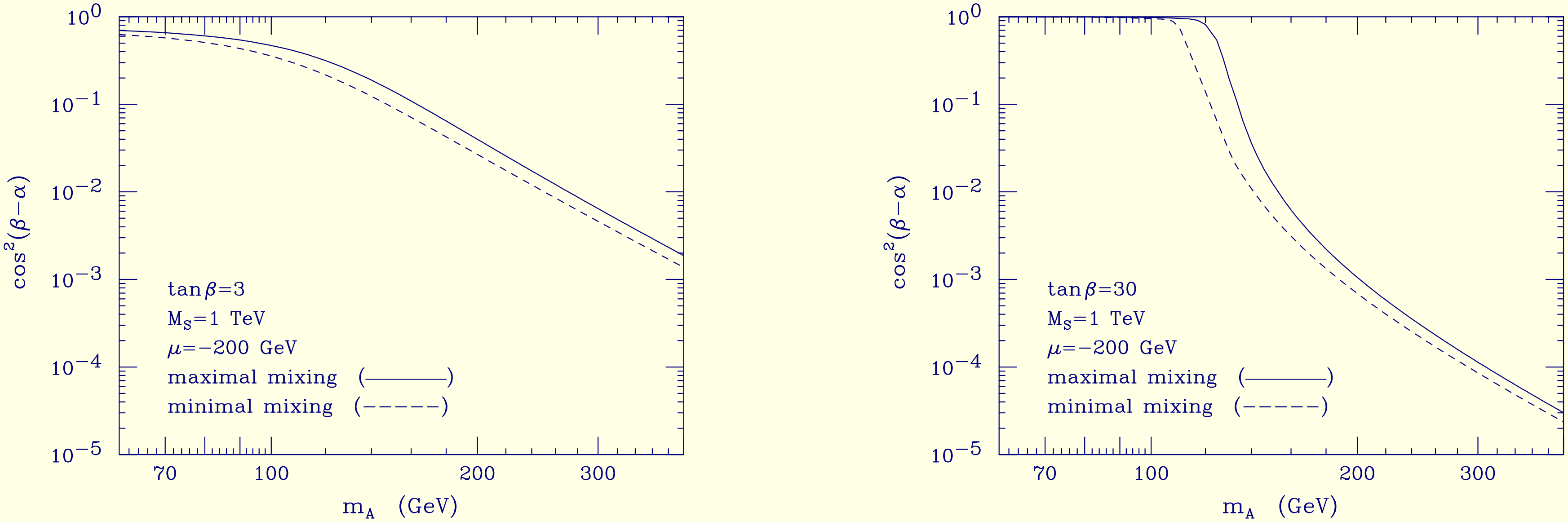


Figure 11: Minimal and maximal mixing results for approach to decoupling.

m_{A^0}-independent decoupling

However, $\cos(\beta-\alpha)=0$ can be achieved also by choosing the MSSM parameters (that govern the Higgs mass radiative corrections) such that $c=0$. That is,

$$2m_Z^2\sin 2\beta = 2\,\delta\mathcal{M}_{12}^2 - \tan 2\beta\left(\delta\mathcal{M}_{11}^2-\delta\mathcal{M}_{22}^2\right)\,. \tag{32}$$

Note that Eq. (32) is independent of the value of m_A. For a typical choice of MSSM parameters, Eq. (32) yields a solution at large $\tan\beta$. That is, by approximating $\tan 2\beta\simeq -\sin 2\beta\simeq -2/\tan\beta$, one can determine the value of β at which the decoupling occurs:

$$\tan\beta\simeq\frac{2m_Z^2-\delta\mathcal{M}_{11}^2+\delta\mathcal{M}_{22}^2}{\delta\mathcal{M}_{12}^2}\,. \tag{33}$$

We conclude that for the value of $\tan\beta$ specified in Eq. (33), $\cos(\beta-\alpha)=0$ independently of the value of m_A.

We shall refer to this phenomenon as m_{A^0}-independent decoupling.

Explicit solutions to Eq. (32) depend on ratios of MSSM parameters and are insensitive to the overall supersymmetric mass scale, modulo a mild logarithmic dependence on M_S/m_t.

Combining the loop corrections

- **The summary is**

$$g_{h^0 b\bar{b}} = -\frac{m_b}{v}\frac{\sin\alpha}{\cos\beta}\left[1+\frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b}-\Delta_b\right)(1+\cot\alpha\cot\beta)\right], \quad (34)$$

$$g_{H^0 b\bar{b}} = \frac{m_b}{v}\frac{\cos\alpha}{\cos\beta}\left[1+\frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b}-\Delta_b\right)(1-\tan\alpha\cot\beta)\right], \quad (35)$$

$$g_{A^0 b\bar{b}} = \frac{m_b}{v}\tan\beta\left[1+\frac{1}{(1+\Delta_b)\sin^2\beta}\left(\frac{\delta h_b}{h_b}-\Delta_b\right)\right], \quad (36)$$

$$g_{h^0 t\bar{t}} = \frac{m_t}{v}\frac{\cos\alpha}{\sin\beta}\left[1-\frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta+\tan\alpha)\right], \quad (37)$$

$$g_{H^0 t\bar{t}} = \frac{m_t}{v}\frac{\sin\alpha}{\sin\beta}\left[1-\frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta-\cot\alpha)\right], \quad (38)$$

$$g_{A^0 t\bar{t}} = \frac{m_t}{v}\cot\beta\left[1-\frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}(\cot\beta+\tan\beta)\right], \quad (39)$$

The τ couplings are obtained from the above equations by replacing m_b,

$\mathbf{\Delta}_b$ **and** δh_b **with** m_τ**,** $\mathbf{\Delta}_\tau$ **and** δh_τ**, respectively.**

One must employ the renormalized value of α **in the above formulae to incorporate the radiative corrections just discussed. In writing out the Higgs-top quark couplings above, we found it convenient to express the results in terms of** $\mathbf{\Delta}_t$ **and** $\mathbf{\Delta} h_t/h_t$**, since** $\mathbf{\Delta}_t \simeq \delta h_t/h_t$ **and the corresponding contribution of** $\mathbf{\Delta} h_t/h_t$ **is** $\tan\beta$ **suppressed.**

Once again, we reemphasize that $\mathbf{\Delta}_b \sim \alpha_s f(M_S)$**, where** $f(M_S)$ **is a dimensionless function of the ratios of SUSY particle masses.**

Back to the decoupling limit

- It is useful to work to first order in $\cos(\beta-\alpha)$, for which

$$\tan\alpha\tan\beta \sim -1 + (\cot\beta + \tan\beta)\cos(\beta-\alpha) + \mathcal{O}(\cos^2(\beta-\alpha)) \quad (40)$$

Using this expansion, one finds

$$\begin{aligned} g_{h^0bb} &\simeq g_{h_{\rm SM}bb}\left[1 + (\tan\beta + \cot\beta)\cos(\beta-\alpha)\left(\cos^2\beta - \frac{1+\delta h_b/h_b}{1+\Delta_b}\right)\right], \\ g_{h^0tt} &\simeq g_{h_{\rm SM}tt}\left[1 + \cos(\beta-\alpha)\left(\cot\beta - \frac{1}{1+\Delta_t}\frac{\Delta h_t}{h_t}\frac{1}{\sin^2\beta}\right)\right]. \end{aligned} \quad (41)$$

Note that Eq. (30) implies that $(\tan\beta+\cot\beta)\cos(\beta-\alpha) \simeq \mathcal{O}(m_Z^2/m_{A^0}^2)$, even if $\tan\beta$ is very large (or small). Thus, at large m_{A^0} the deviation of the $h^0b\bar{b}$ coupling from its SM value vanishes as $m_Z^2/m_{A^0}^2$ for all values of $\tan\beta$.

Thus, if we keep only the leading $\tan\beta$-enhanced radiative corrections, then

$$\frac{g^2_{hVV}}{g^2_{h_{\rm SM}VV}} \simeq 1 - \frac{c^2 m_Z^4 \sin^2 4\beta}{4 m_A^4}, \qquad \frac{g^2_{htt}}{g^2_{h_{\rm SM}tt}} \simeq 1 + \frac{c m_Z^2 \sin 4\beta \cot\beta}{m_A^2},$$

$$\frac{g^2_{hbb}}{g^2_{h_{\rm SM}bb}} \simeq 1 - \frac{4 c m_Z^2 \cos 2\beta}{m_A^2}\left[\sin^2\beta - \frac{\Delta_b}{1+\Delta_b}\right]. \qquad (42)$$

The approach to decoupling is fastest for the h^0 couplings to vector bosons and slowest for the couplings to down-type quarks.

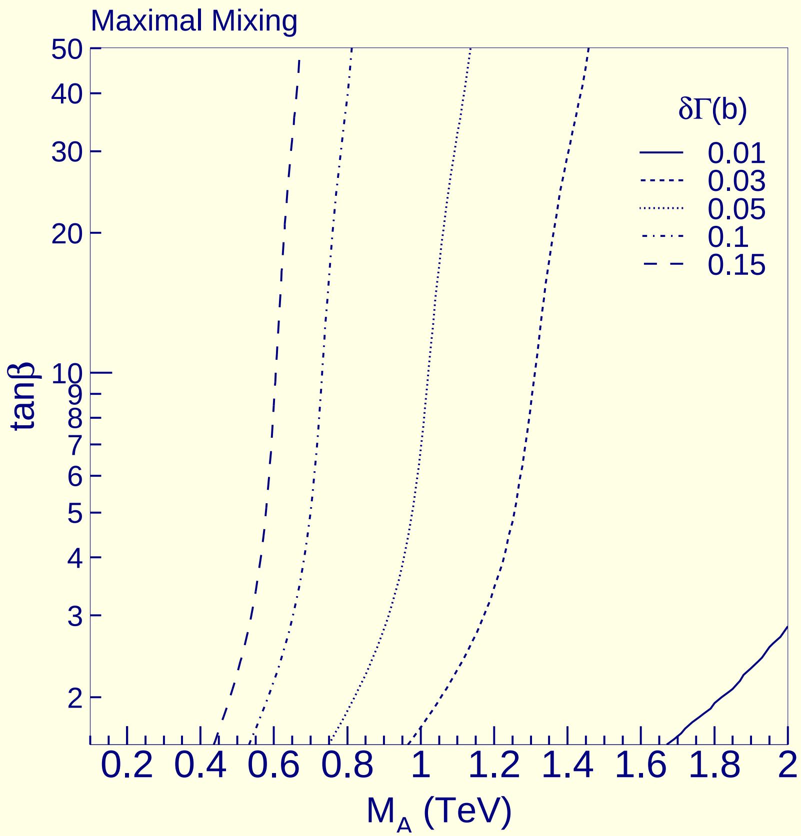
If $c = 0$, as possible for large $\tan\beta$, then we have m_{A^0}-independent decoupling.

- For loop induced decays/couplings such as ggh^0 or $\gamma\gamma h^0$ there are really two decoupling issues.

1. Is $m_{A^0} \gg m_Z$?
2. Is $m_{\rm SUSY} \gg m_Z$?

If only the first holds, then SUSY loops (of colored or charged particles, respectively) can still yield deviations with respect to SM expectations.

- **Some plots**



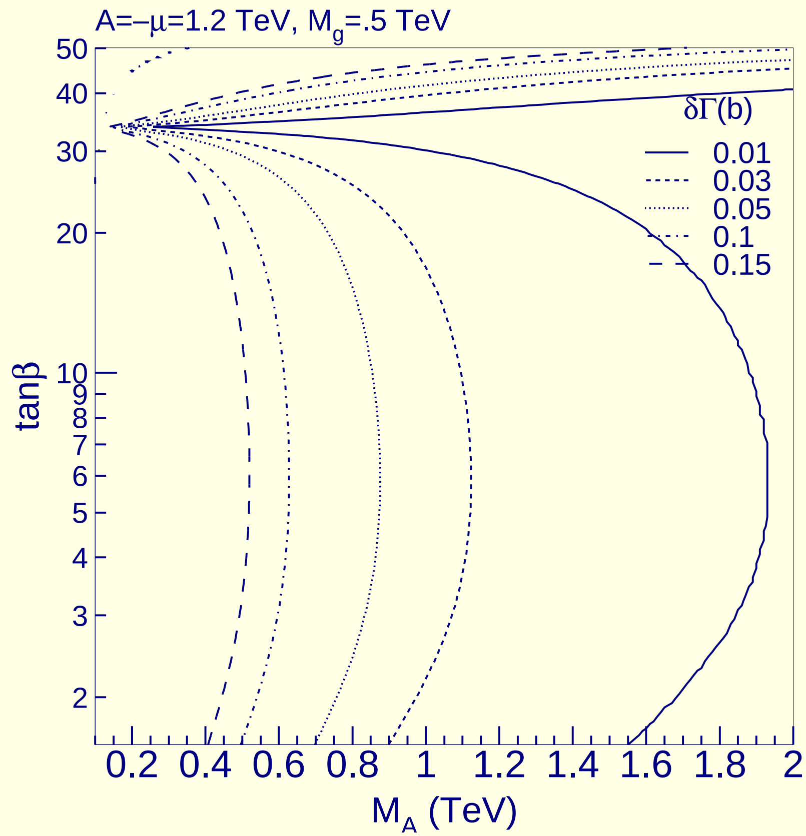


Figure 12: Deviations of $\Gamma(h^0 \to b\overline{b})$ relative to SM value for "normal" case and m_{A^0}-independent scenario.

If 5% deviations were measurable, we might see deviations for m_{A^0} as large as $1~\mathrm{TeV}$, but, we might also see no deviations even if m_{A^0} is small.

To interpret deviations, need knowledge of soft-SUSY-breaking parameters.

Branching Ratios and Widths of MSSM Higgs Bosons

- **We give just some sample plots.**

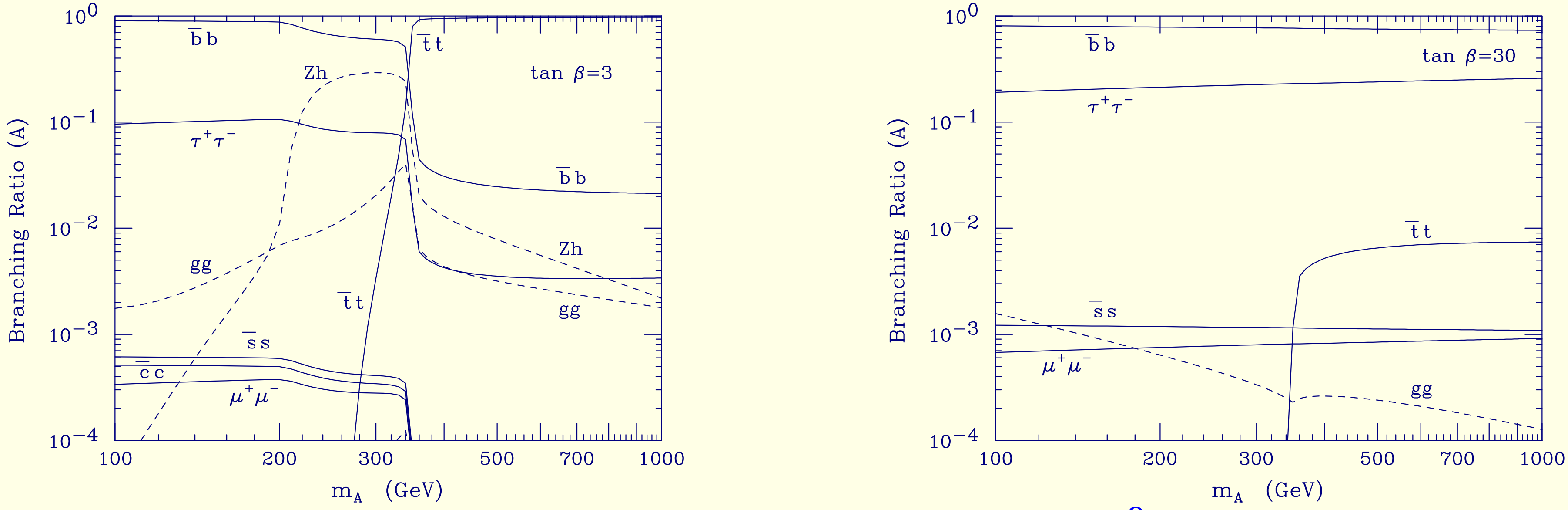


Figure 13: Branching ratios for A^0.

It's all $b\bar{b}$ for large $\tan\beta$.

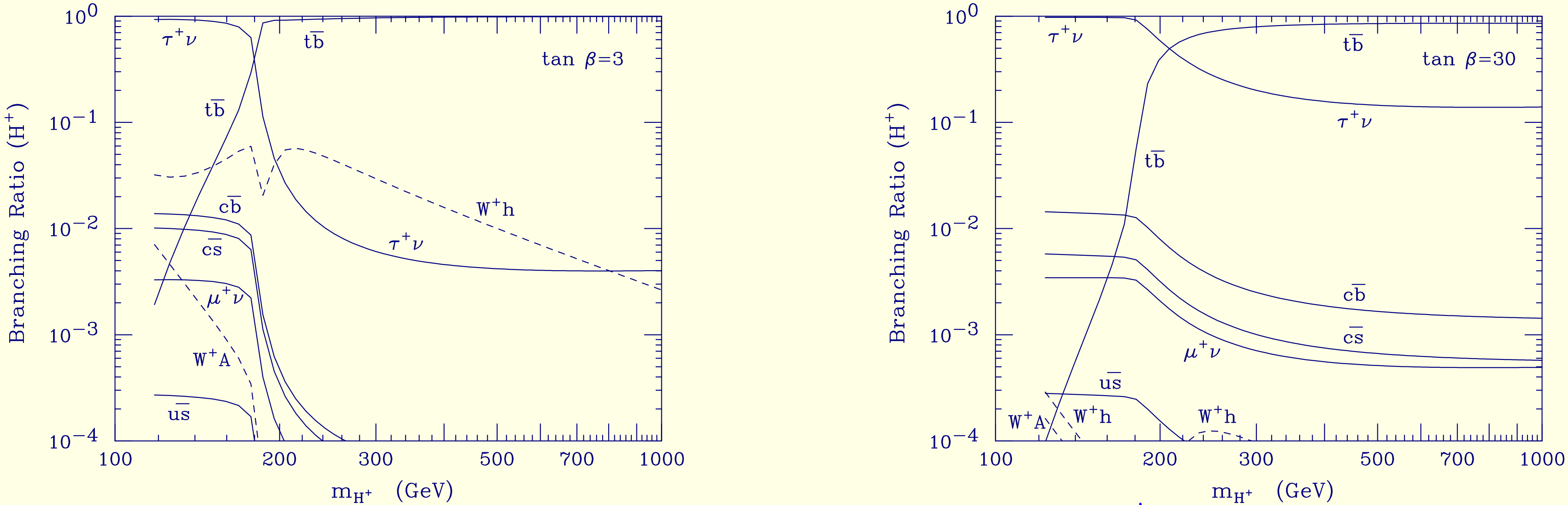


Figure 14: Branching ratios for $H^{\pm}$.

It's $\tau^{\pm}\nu$ until $m_{H^{\pm}} > m_t + m_b$, and then it is tb.

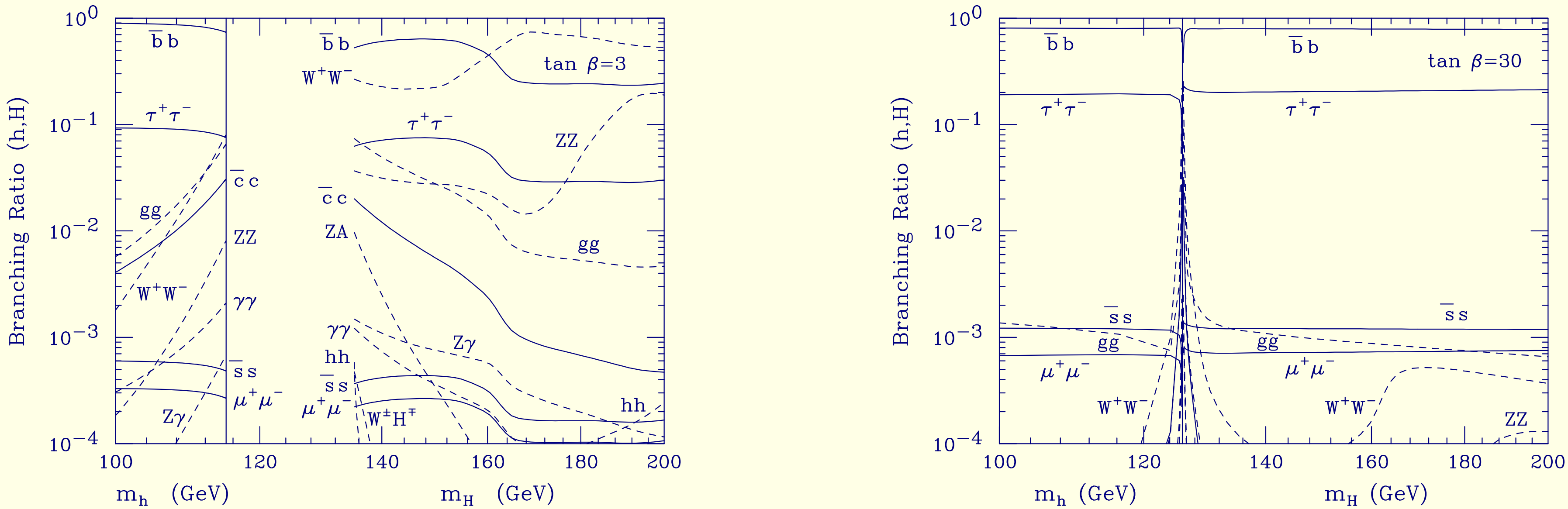


Figure 15: Branching ratios for h^0 and H^0 at lower mass.

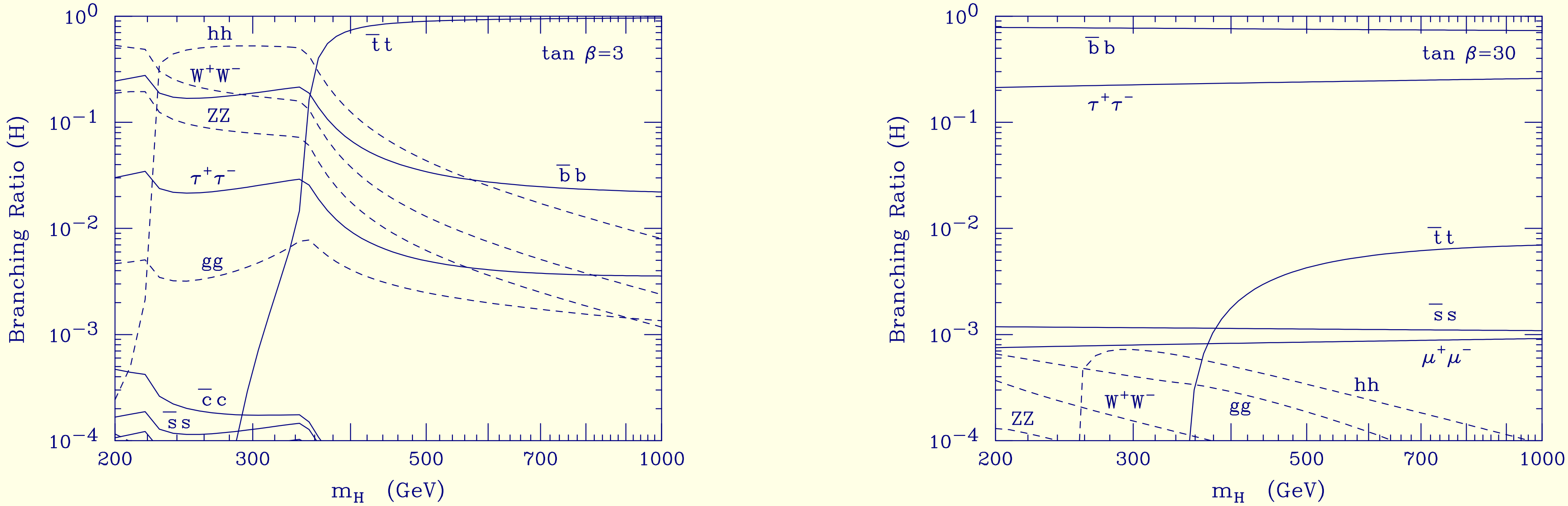


Figure 16: Branching ratios for H^0 to higher mass.

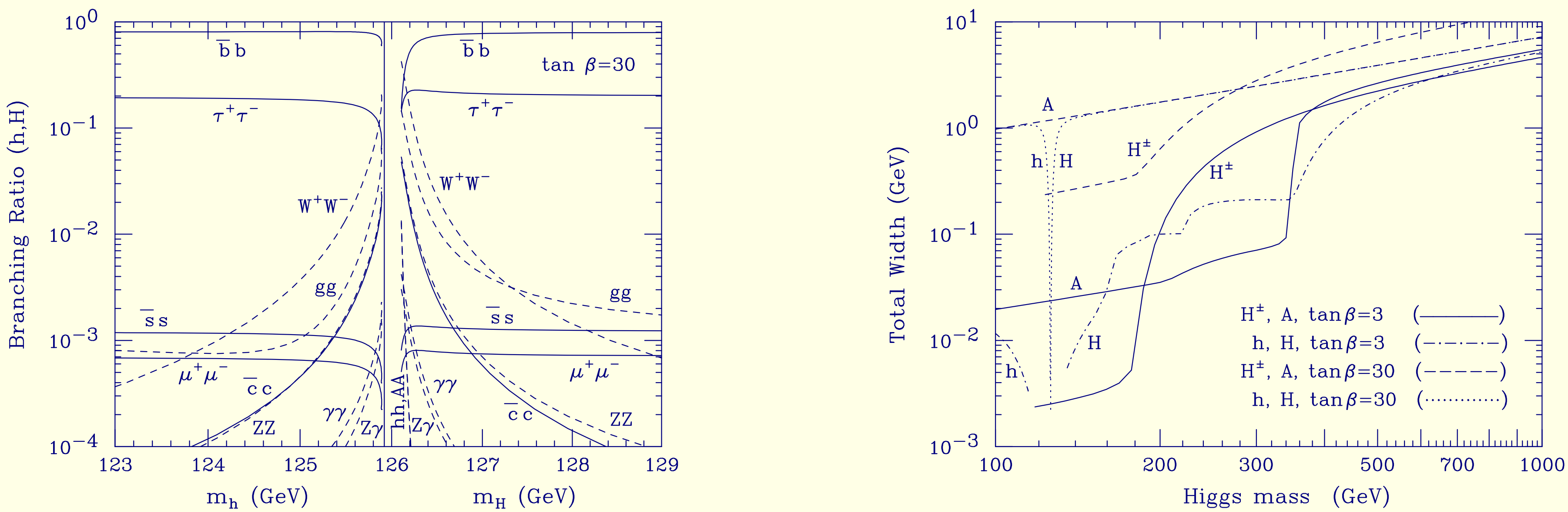


Figure 17: Left: expanded view of some h^0 and H^0 branching ratios. Right: the Higgs widths. All for $\tan\beta = 30$.

MSSM Higgs cross sections

At hadron colliders, the important cross sections are:

- $gg \to \phi$,
- $qq \to qqV^*V^* \to qqh^0, qqH^0$,
- $q\bar{q} \to V^* \to h^0V/H^0V$,
- $gg, q\bar{q} \to \phi b\bar{b}/\phi t\bar{t}$,

where $\phi = h^0$, H^0 or A^0.

At the LC, the most important cross sections are

- **Higgs-strahlung:** $e^+e^- \to Zh^0$, $e^+e^- \to ZH^0$,
- **Pair production:** $e^+e^- \to h^0A^0$, $e^+e^- \to H^0A^0$, $e^+e^- \to H^+H^-$,
- **Yukawa radiation:** $e^+e^- \to t\bar{t}\phi$, $e^+e^- \to b\bar{b}\phi$,

where $\phi = h^0, H^0, A^0$.

We show some results for the hadron collider cross sections.

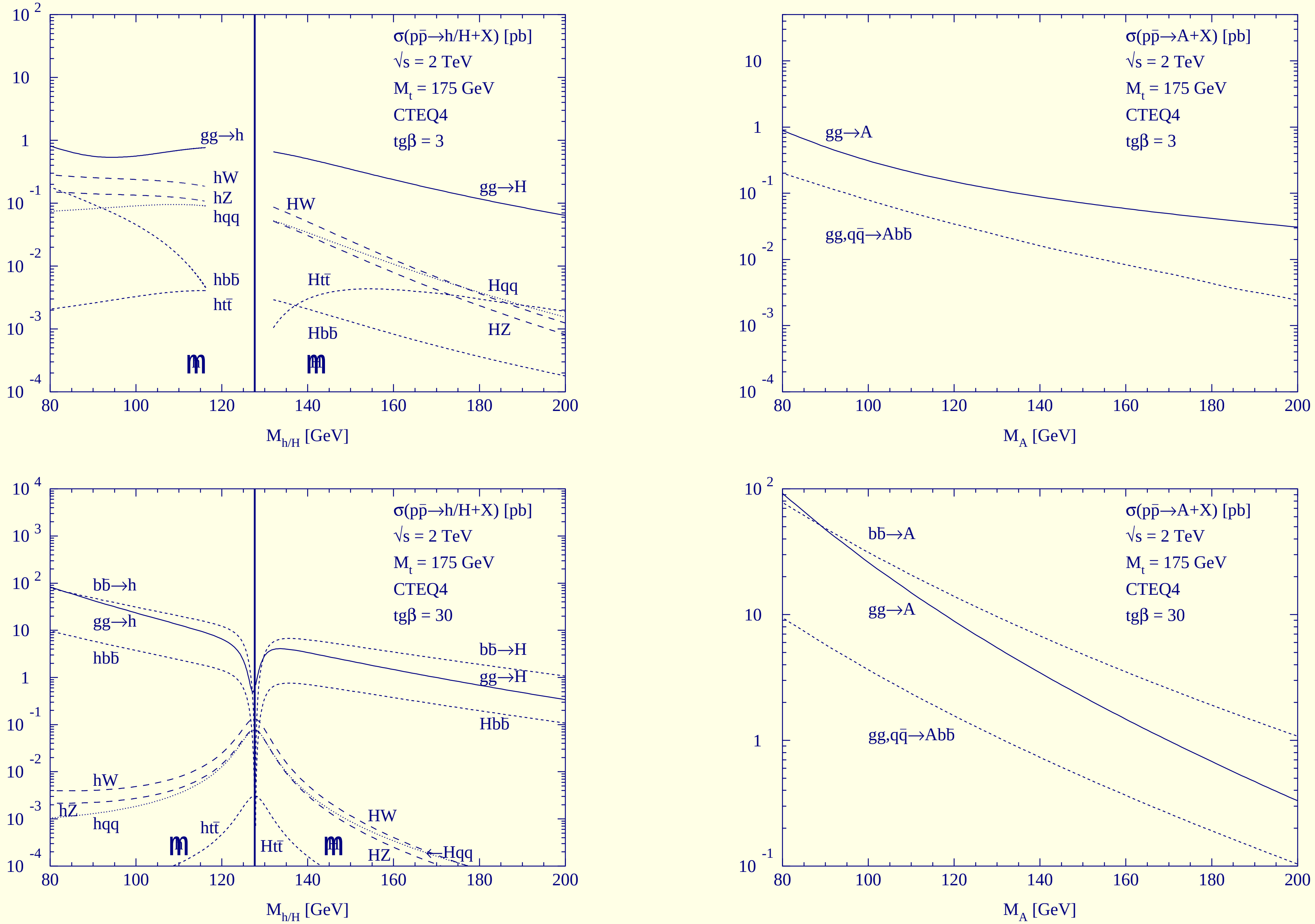


Figure 18: Tevatron cross sections.

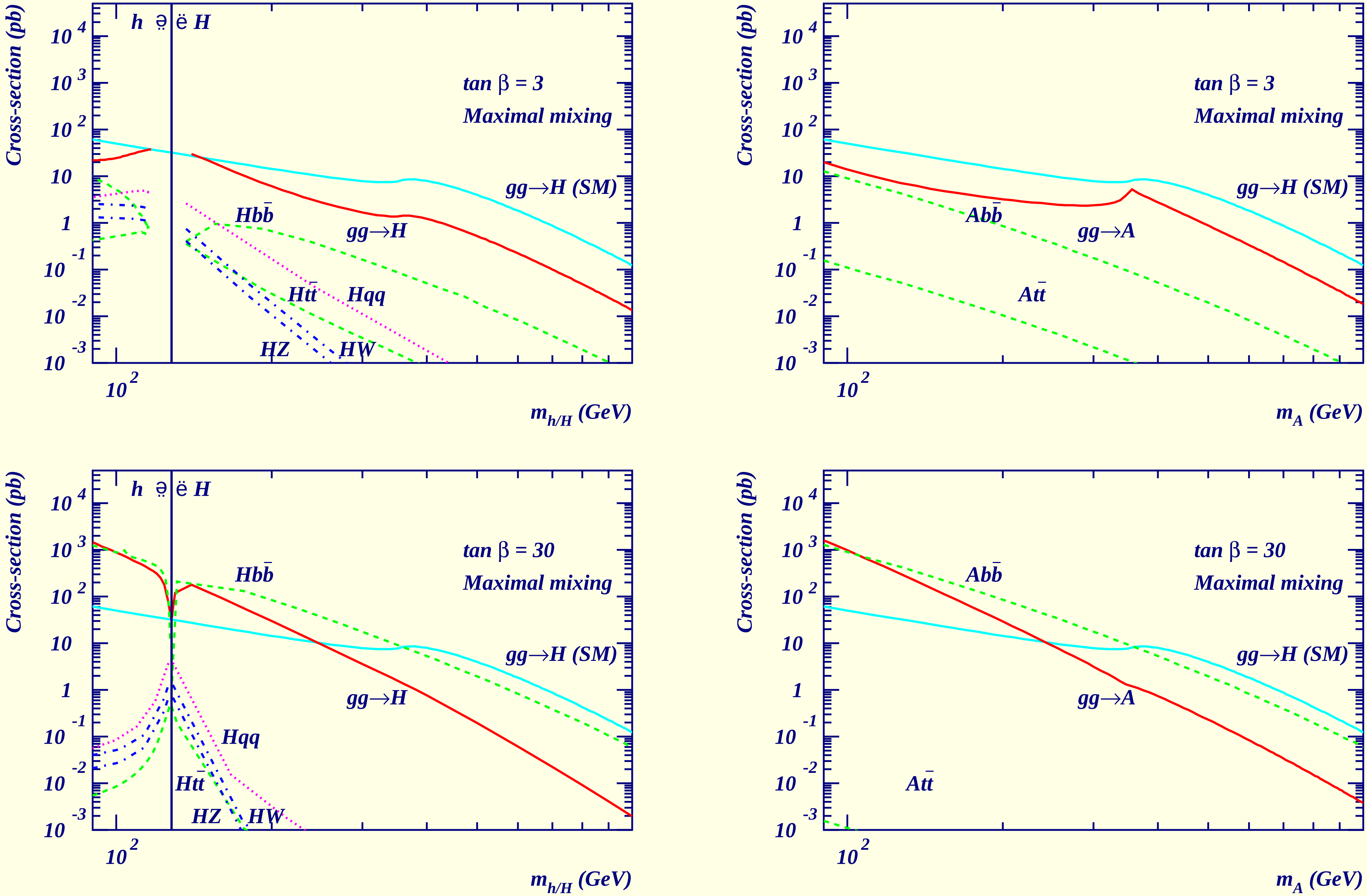


Figure 19: Neutral MSSM Higgs production cross-sections at the LHC. The cross section for gluon-gluon fusion to a SM Higgs boson is also shown.

- **Some remarks on Higgs discovery and measurements in the MSSM**

1. **LEP limits are really rather substantial, especially for the minimal-mixing scenario that is in many respects the "cleanest" model. There, we are being pushed to the decoupling limit.**

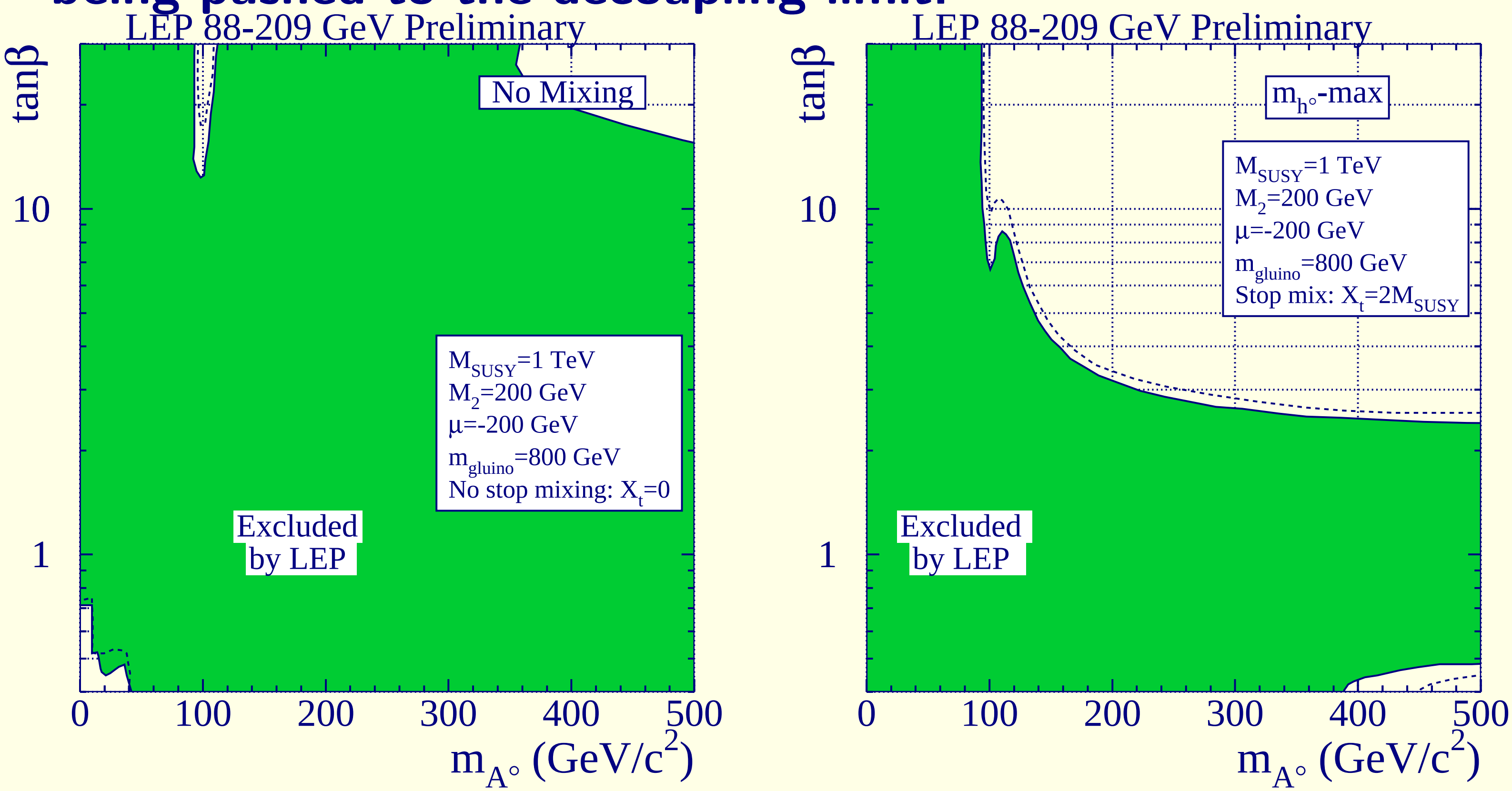


Figure 20: LEP2 limits for minimal and maximal mixing.

2. **If the Tevatron reaches full L, then it will be able to discover the h^0 in most cases. At very high $\tan\beta$ can see $b\bar{b}H^0/A^0$.**
3. **The LHC is guaranteed to find at least one MSSM Higgs boson.**

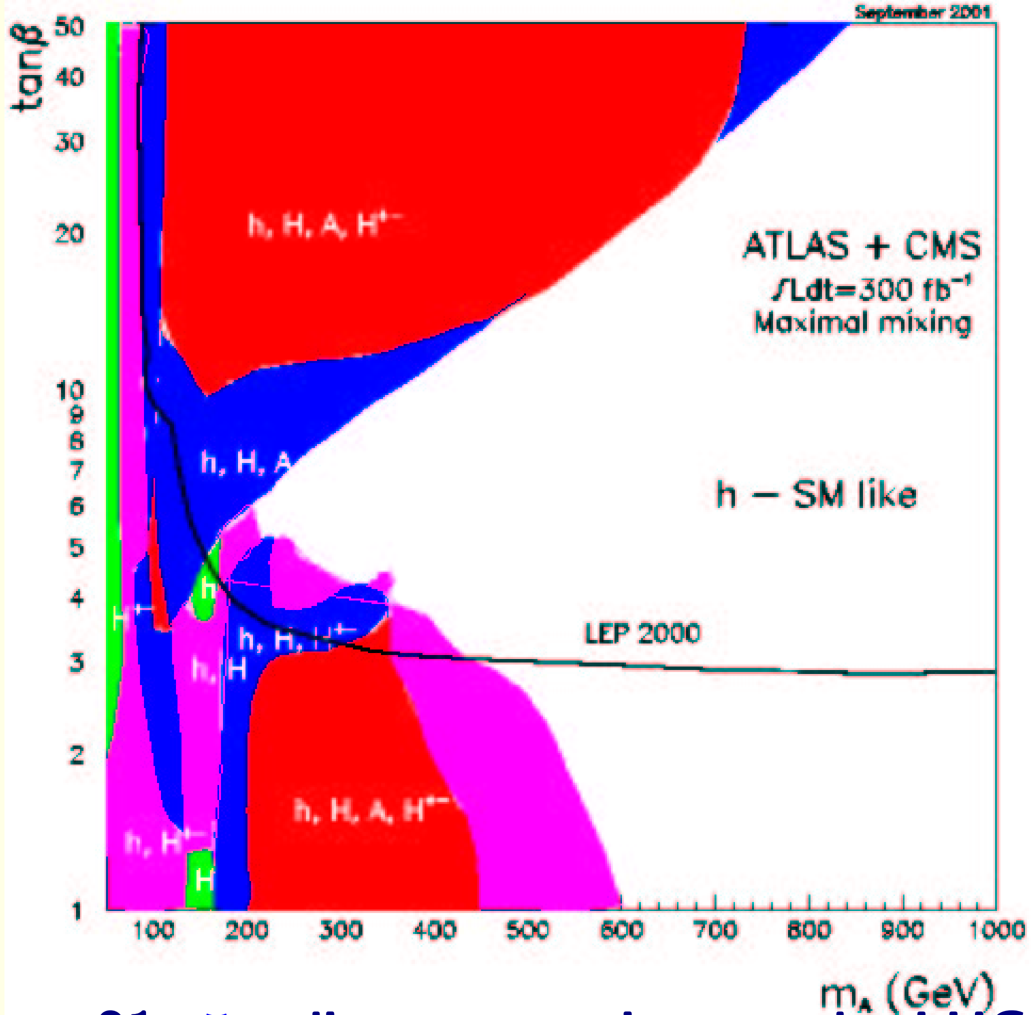


Figure 21: 5σ discovery regions at the LHC.

But, as one pushes further into the decoupling region, there is an increasingly large "wedge" of parameter space in which only the h^0 will be detectable.

4. A LC will certainly detect the h^0, and $e^+e^- \to H^0A^0$ will be observable if $m_{A^0} \lesssim \sqrt{s}/2$ (e.g. $\lesssim 300~\mathrm{GeV}$ for $\sqrt{s} = 600~\mathrm{GeV}$).

But, above this the LC wedge is even bigger than the LHC wedge.

If SUSY is observed at the LHC and/or LC and if the h^0 is seen, then one will know that there are (at least) the $H^0, A^0, H^\pm$ to be discovered.

If the MSSM parameters are in the "wedge" $\Rightarrow$ two options for direct discovery:

a) increase $\sqrt{s}$ past $2m_{A^0}$ if you know what m_{A^0} is (see below)?

b) operate the LC in the $\gamma\gamma$ collider mode;

$\Rightarrow$ H^0, A^0 discovery *precisely* in the "wedge" region up to $\sim 0.8\sqrt{s}$.

- Of course, even in the wedge region, decoupling is only approximate and one expects deviations from SM predictions. (Recall, for example, the $\Gamma(h^0 \to b\bar{b})$ deviations.)

  Can determine how much deviation in χ^2 for sensitive observables will arise for a given value of m_{A^0}.

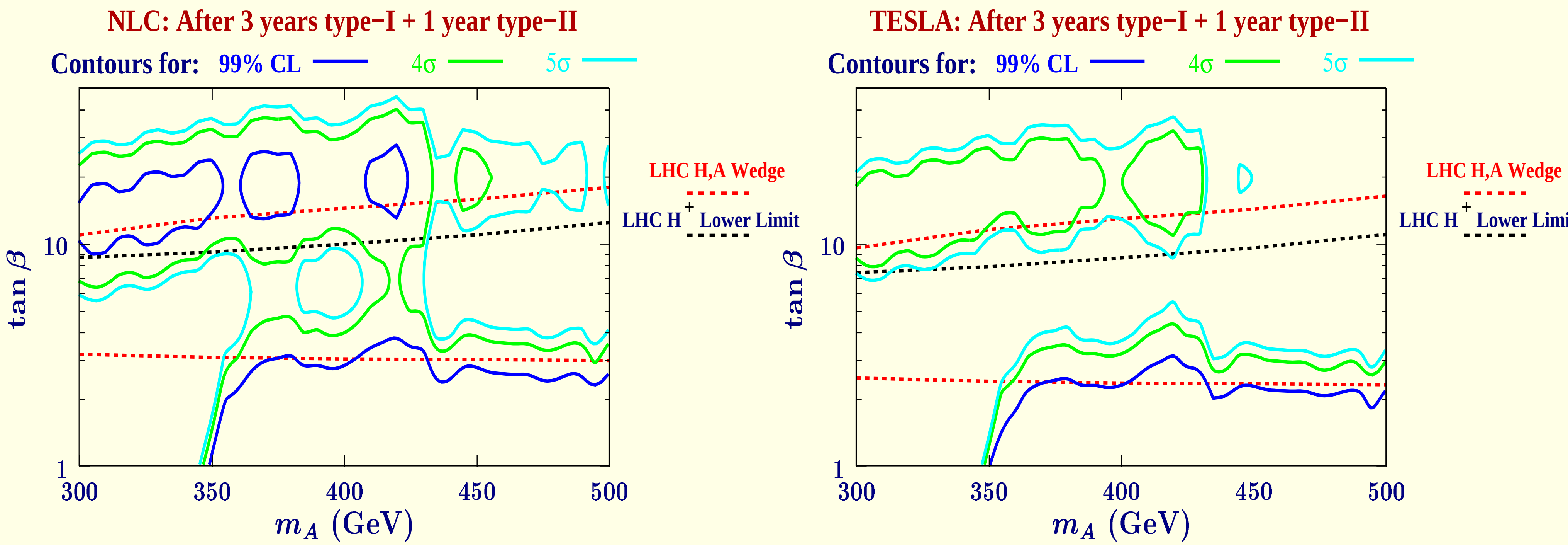


Figure 22: Contours for discovery and 99% CL exclusion after 4 years of NLC or TESLA $\gamma\gamma$ running. (Ignore the H^+ wedge line — it has moved up above the H^0, A^0 line.)

Obviously, the $\gamma\gamma$ option would be a priority at a certain point, and one could simultaneously have a very interesting overall $\gamma\gamma$ physics program.

A χ^2 deviation that could be reliably used to estimate a value of m_{A^0} would be a big help, allowing to center $E_{\gamma\gamma}$ peak on the approximate m_{A^0} value. $\Rightarrow$ much less running needed to detect the H^0, A^0.

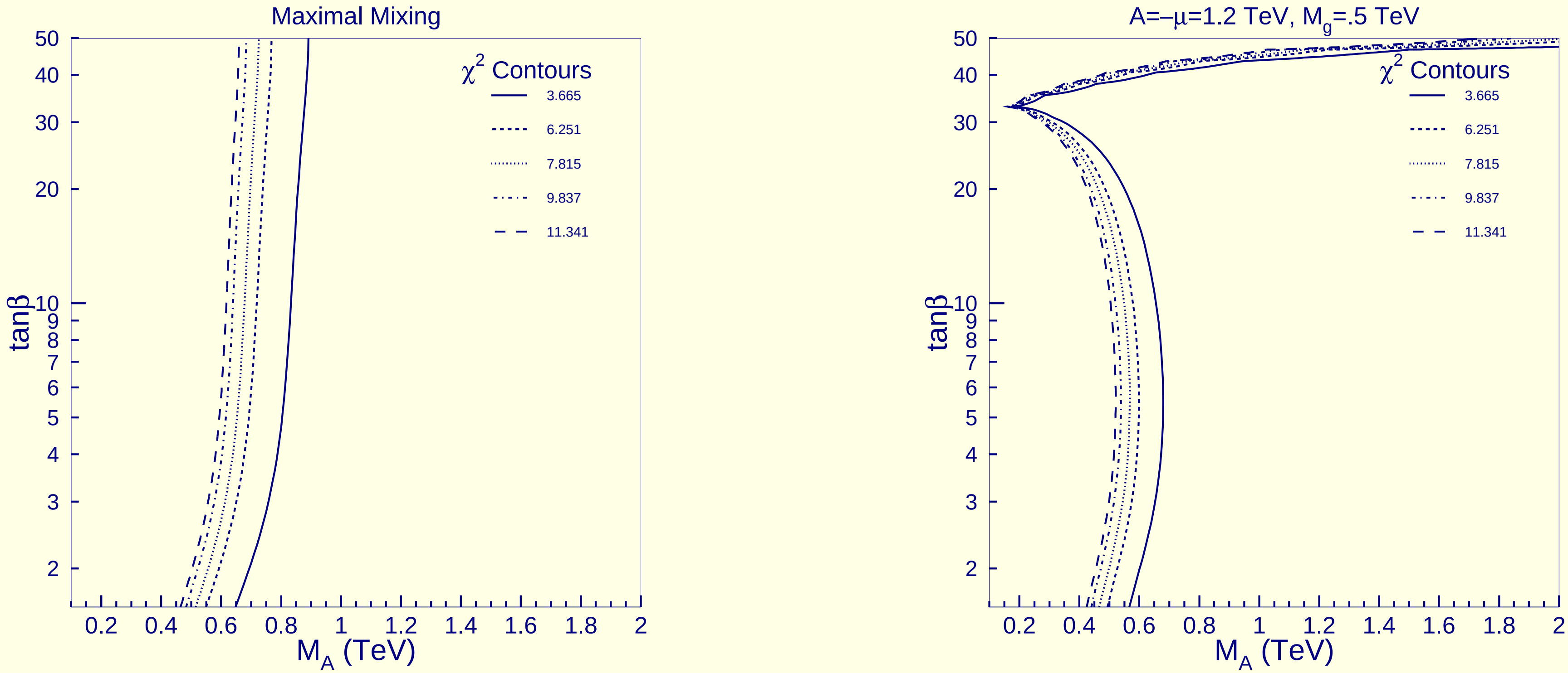


Figure 23: Contours of χ^2 for Higgs boson decay observables for (a) the maximal mixing scenario; and (b) a choice of MSSM parameters for which the loop-corrected $h^0 b\bar{b}$ coupling is suppressed large $\tan\beta$ and low m_{A^0}. Results are based on Higgs measurements anticipated at the LC with $\sqrt{s} = 500$ GeV and $L = 500\ fb^{-1}$. The contours correspond to 68, 90, 95, 98 and 99% confidence levels (right to left) for the observables g^2_{hbb}, $g^2_{h\tau\tau}$, and g^2_{hgg}.

$\Rightarrow$ may or may not get hint of value of m_{A^0}.

Also, may be difficult to interpret an observed deviation without knowledge of SUSY scenario.

CP Violation in the MSSM Higgs Sector induced at one-loop

- If the soft-SUSY-breaking parameters are complex, then δh_b, Δh_b, δh_t and Δh_t can all be complex.
- It is possible to find parameter choices consistent with EDM limits, and so forth, that give large CP-violation in the Higgs sector.
- Five crucial consequences
  1. The h^0, H^0 and A^0 all mix together and one has simply three neutral eigenstates $h_{1,2,3}$.
  2. The fermionic couplings of the $h_{1,2,3}$ will all have a mixture of $a + i\gamma_5 b$ couplings, where a is the CP-even part and b is the CP-odd part.
  3. The $h_{1,2,3}$ will share the VV coupling strength squared, generalizing the usual sum rule to $\sum_{i=1,2,3} g^2_{h_i VV} = g^2_{h_{\rm SM} VV}$.
  4. The $h_{1,2,3}$ could *at the same time* have somewhat similar masses, perhaps overlapping within the experimental resolution in certain channels.
  5. Or, in some regions of parameter space, one h_i has substantial VV coupling (which is the usual requirement for easy discovery), but instead

of decaying in the usual way, decays to a pair of lighter $h_j h_j$ or $h_j h_k$ or to Zh_k.

- There is even a region of parameter space such that there is a fairly light Higgs boson ($\lesssim 50~\mathrm{GeV}$) that would not have been seen at LEP.

- All these features are features as well of the general CP-violating two-Higgs-doublet (2HDM) model, and can potentially lead to some real problems for Higgs detection and analysis.

  1. The Tevatron could fail to see any one the h_i signals simply because all are weaker than predicted for the case where there is a single SM-like Higgs.
  2. The same could be true of the LHC. For example:
     The $\gamma\gamma$ decay modes are rapidly suppressed when the VV coupling is not full strength.
     The WW fusion cross section is also suppressed.
     A Higgs with good production cross section might not be detectable since it decays to two other Higgs bosons, each of which decays to $b\overline{b}$ (for example).
  3. A future LC would be guaranteed to find at least one of the Higgs bosons, provided there is no precision EW "conspiracy" such as discussed

earlier. This is because the PEW data (and RGE in the MSSM case) require significant $g^2_{ZZh_i}$ weight for $m_{h_i} \lesssim 200~\mathrm{GeV}$ *and* the Zh_i and $W^*W^* \to h_i$ cross sections cannot all be suppressed for the h_i in this mass region *and* the LC can probe to very small $g^2_{ZZh_i}$.

There is still a decoupling limit in the MSSM context. If $m_{H^\pm} \gg m_Z$, then the h^0 will become pure CP-even. The H^0 and A^0 will be heavy and can still mix strongly, but at least discovery of the h^0 would be guaranteed.

Determination of Higgs CP properties

- There are obviously quite a few situations in which we will need a way of precisely measuring the CP properties of one or more Higgs bosons.

  1. Separating the H^0 and A^0.
  2. Determining the CP admixture in the case of a CPV Higgs sector.
  3. Resolving overlapping Higgs resonances of different or mixed CP character.

  The $\gamma\gamma$ collider is clearly the best, in many cases only, way.

  One uses maximally polarized (either transverse or circular, depending upon whether the Higgs sector appears to be CPC or CPV, respectively) laser photon beams and looks at various rate asymmetries.

Why nature may prefer the decoupling limit?

- LEP limits tend to push in that direction in the MSSM context; they require large $\tan\beta$ and m_{A^0} in the minimal-mixing scenario, for example.

- Allowing the most general fermionic coupling structure in, for example, a general 2HDM leads to FCNC.

  However, in the decoupling limit, the FCNC couplings of the surviving light Higgs are suppressed by the small value of $\cos(\beta-\alpha)$.

- In similar fashion, it can be shown that all CP-violating couplings of the SM-like h^0 vanish as $\cos(\beta-\alpha)\to 0$ in the true decoupling limit.

- Of course, the H^0 and A^0 (in 2HDM for example) will generally have FCNC and CPV couplings, but their effects are suppressed by a factor of $m_{h^0}^2/m_{A^0}^2$ (propagator masses).

  As a result, all FCNC and CPV effects are at the same level for the h^0, H^0 and A^0 and are of order $\cos(\beta-\alpha)\sim \frac{m_{h^0}^2}{m_{A^0}^2}$.

Thus, we might in general anticipate that the Higgs sector will be in a decoupling limit, *unless the model contains other symmetries for suppressing the naturally present FCNC and CPV couplings*.

SUSY Left-Right Models can be constructed with the needed symmetries.

The NMSSM Higgs Sector

Motivation: Introducing an extra singlet superfield and the interaction $W \ni \lambda \hat{H}_1 \hat{H}_2 \hat{N}$ leads to natural explanation of μ term (as simply inserted in MSSM) when $\langle (\widehat{N})_{\text{scalar component}} \rangle = n$ with n at electroweak scale (as is natural in many cases).

Clearly, n can be traded for $\mu_{\rm eff}$ in describing parameter space.

We also include $\kappa \widehat{N}^3$ in W.

Assuming no CP violation, the NMSSM $\Rightarrow$ 3 CP-even Higgs bosons: $h_{1,2,3}$ and 2 CP-odd Higgs bosons: $a_{1,2}$.

Linear Collider

Many groups have shown that one can add a singlet, and indeed a continuum of singlets, and still find a signal.

LHC?

Old Snowmass96 Result (JFG+Haber+Moroi, hep-ph/9610337) $\Rightarrow$

Could find parameter choices for Higgs masses and mixings such that LHC would find no Higgs.

New Results (JFG+Ellwanger+Hugonie, hep-ph/0111179) $\Rightarrow$

An important new mode that allows discovery of many of the 'bad' points of SM96 is $t\bar{t}h \to t\bar{t}b\bar{b}$ (ref: ATLAS (Sapinski) + CMS (Drollinger) analysis for $h_{\rm SM}$).

But, we find new 'bad' points with just this one addition. $\Rightarrow$ include WW fusion modes to remove all bad points (subject to no Higgs pair ... decays).

Our procedure:

The modes employed in 1996 were:

1) $gg \to h \to \gamma\gamma$ at LHC;

2) $Wh, t\bar{t}h \to \ell + \gamma\gamma$ at LHC;

4) $gg \to h, a \to \tau^+\tau^-$ plus $b\bar{b}h, b\bar{b}a \to b\bar{b}\tau^+\tau^-$ at LHC;

5) $gg \to h \to ZZ^*$ or $ZZ \to 4\ell$ at LHC;

6) $gg \to h \to WW^*$ or $WW \to 2\ell 2\nu$ at LHC;

7) $Z^\star \to Zh$ and $Z^\star \to ha$ at LEP2;

To these we add:

3) $gg \to t\bar{t}h \to t\bar{t}b\bar{b}$; (JFG+ ..., Sapinski, ...)

8) $WW \to h \to \tau^+\tau^-$; (Zeppenfeld+...)

9) $WW \to h \to WW^{(*)}$. (Zeppenfeld+...)

We avoided regions of parameter space:

Where the highly model-dependent decays a) $h \to aa$; b) $h \to h'h'$; c) $h \to H^+H^-$; d) $h \to aZ$; e) $h \to H^+W^-$; f) $a \to ha'$; g) $a \to Zh$; h) $a \to H^+W^-$; are present, and where i) $a, h \to t\bar{t}$ j) $t \to H^\pm b$ decays are possible.

Parameter space:

λ, κ, μ, $\tan\beta$, A_λ, A_κ with RGE and perturbativity constraints.

Rates are made more marginal because:

- All WW, ZZ coupling shared among the h_i $\Rightarrow$ demotes decays and production using this coupling.

  In particular, it is easy to make $\gamma\gamma$ coupling and decays small — reduced W loop cancels strongly against t, b loops.

- $\tan\beta$ not very large $\Rightarrow$ well inside 'LHC wedge' for all Higgs bosons.

- **Need full $L = 300\mathrm{fb}^{-1}$ for ATLAS and CMS to guarantee discovery of at least one Higgs boson.**

- **Unfortunately, if we enter into parameter regions where the $h_i \to a_j a_j$, $a_j \to Z h_k$, $\ldots$ decays are allowed, these decays can be very strong and all the previous modes 1)-9) will not be useful.**

  $\Rightarrow$ much more work to do on how to detect Higgs bosons in Higgs pair or Z+Higgs decay modes at the LHC.

- **The $WW \to h_i \to a_j a_j, h_k h_k$ modes could also prove extremely valuable, but have not yet been simulated.**

- **Clearly, detection of a single isolated a_i or weakly-VV-coupled h_j would help put us on the right track.**

The Continuum Higgs Model

- **A still more difficult case for Higgs discovery is when there is a series of Higgs bosons separated by the mass resolution in the discovery channel(s), *e.g.* one every $\sim 10~\mathrm{GeV}$ (the detector resolution in the recoil mass spectrum for $Z+$Higgs.**

  For example, extra singlets are abundant in string models.

  Adding extra singlets to the two doublets of the MSSM does not affect the success of gauge unification!

- **In general, all these Higgs could mix with the normal SM Higgs (or the MSSM scalar Higgs bosons) in such a way that the physical Higgs bosons share the WW/ZZ coupling and decay to a variety of channels**

  May be forced to use $Z+X$ and look for broad excess in M_X.

- **Constraints? Use continuum notation. Important issue is value of m_C in**

$$\int_0^\infty dm K(m) m^2 = m_C^2\,, \quad \text{where} \quad \int_0^\infty K(m) = 1 \tag{43}$$

where $K(m)(gm_W)^2$ is the (density in Higgs mass of the) strength of the hWW coupling-squared.

- Precision electroweak suggests $m_C^2 \lesssim (200-250~\mathrm{GeV})^2$.
- For multiple Higgs reps. of any kind in the most general SUSY context, RGE + perturbativity up to $M_U \sim 2 \times 10^{16}~\mathrm{GeV}$ gives same result.
- Caution: Many types of new physics at low scale allow evasion of m_C^2 sizes above; e.g. large extra dimensions or appropriate extra Higgs structure.

Ignoring this caveat, assume sum rule and take $K(m)$=constant from $m_A = m_h^{\rm min}$ to $m_B = m_h^{\rm max}$: $K(m) = 1/(m_B - m_A)$.

$\Rightarrow m_B^2 + m_B m_A + m_A^2 = 3m_C^2$. For example, for $m_A = 0$, $m_B = \sqrt{3} m_C$.

To go beyond LEP constraints that do not allow much weight below $80~\mathrm{GeV}$, requires higher energy. $\sqrt{s} = 500~\mathrm{GeV}$ is more or less ideal.

- Use JFG, Han Sobey analysis (*Phys. Lett.* B429 (1998) 79) available for $Z \to e^+e^-, \mu^+\mu^-$, $\sqrt{s} = 500~\mathrm{GeV}$ and $m = 70 - 200~\mathrm{GeV}$ region.
- For $K(m)$ =constant, $m_C = 200~\mathrm{GeV}$ and $m_A = 70~\mathrm{GeV} \Rightarrow m_B = 300~\mathrm{GeV}$ and $m_B - m_A = 230~\mathrm{GeV}$.

  A fraction $f = 100~\mathrm{GeV}/230~\mathrm{GeV} \sim 0.43$ of the continuum Higgs signal lies in the $100-200~\mathrm{GeV}$ region (which region avoids Z peak region with largest background).

– Summing $Z \to e^+e^- + \mu^+\mu^-$, $\Rightarrow S \sim 540f$ with a background of $B = 1080$, for $100 - 200$ GeV window, assuming $L = 200\mathrm{fb}^{-1}$.

$$\frac{S}{\sqrt{B}} \sim 16f \left(\frac{L}{200\mathrm{fb}^{-1}}\right) \text{ for } m \in [100 - 200]\ \mathrm{GeV}\,. \qquad (44)$$

$\Rightarrow$ no problem!

- With $L = 500\mathrm{fb}^{-1}$, after a few years will be able to determine signal magnitude with reasonable error ($\sim 15\%$) in each 10 GeV interval.
- Hadron collider detection of continuum signal appears to be very challenging.

Left-Right Symmetric supersymmetric model

Motivations

- Using Higgs fields to break parity at some high scale m_R is an attractive idea.

- SO(10), which automatically includes ν_R fields for neutrino masses as well as usual SU(5) representation structures, contains

  $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$

  as a subgroup.

- SUSYLR context guarantees that R-parity is conserved.

- SUSYLR model guarantees no strong CP problem and no SUSY-CP problem (*i.e.* the generic problem of SUSY phases giving large EDM unless cancellations are carefully arranged) at m_R.

  It is then a matter of making sure that evolution from m_R down does not destroy these two properties.

Gauge unification?

I will not present details, but simply remark that gauge coupling unification is possible in certain variants of this approach.

Bottom line

There are certainly a lot of Higgs bosons in these theories, but all but the MSSM equivalent ones may be too heavy to detect.

CONCLUSIONS

Even in the context of the SM and Standard Supersymmetry Models there is a plethora of Higgs scenarios and possibilities.

- The Higgs sector may prove challenging to fully explore.
- The variety of models, complications due to unexpected decays (e.g. Higgs pair, Z+Higgs, SUSY), CP violation, overlapping signals **etc.** make attention to multi-channel analysis vital.
- There is enough freedom in the Higgs sector that we should not take Higgs discovery at the Tevatron or LHC for granted, *even in the case of the MSSM*.

  $\Rightarrow$ keep improving and working on every possible signature.

  $\Rightarrow$ LHC ability to show that WW sector is perturbative could be important.
- In the most general model, the precision electroweak data does not guarantee that a $\sqrt{s} = 600~\mathrm{GeV}$ LC will find some Higgs signal.

But, the scenarios of this type constructed so far always have a heavy SM-like Higgs that will be found by the LHC.

- The LC and the LHC will be vital to guarantee discovery of a Higgs boson in the most general case.

  The LHC, in case there is a heavy Higgs as in general 2HDM.

  The LC, in case of the NMSSM (probably), and certainly in the case of a continuum of strongly mixed Higgs bosons.

- Observation of the heavy H^0, A^0 may require $\gamma\gamma$ collisions to cover the "wedge" region.

  Once observed, the properties/rates for the H^0, A^0 will help enormously in determining important SUSY parameters, esp. $\tan\beta$.

- Exotic Higgs representations, e.g. triplet as motivated by seesaw approach to neutrino masses, will lead to exotic collider signals and possibilities.

- Direct CP determination will probably prove to be vital to disentangling any but the simplest SM Higgs sector.

The effort required to explore a complicated Higgs sector will be worth it, since understanding the Higgs sector will be crucial to a full understanding of the ultimate theory.