Determination of $\tan\beta$ at a Future e^+e^- Linear Collider

Jack Gunion
Davis Institute for High Energy Physics, U.C. Davis

Collaborators: Han, Jiang, Mrenna, Sopczak

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Some reminders

- **Determination of $\tan\beta$ at high $\tan\beta$ is very difficult using most supersymmetry observables.**

- **Most determinations of $\tan\beta$ at low $\tan\beta$ rely on neutralino/chargino couplings.**

  These do an acceptable job, but would be absent in a non-SUSY general two-Higgs-doublet model (2HDM).

- **It is the Higgs sector that defines $\tan\beta = v_2/v_1$ as the ratio of the vev that gives mass to up-type quarks to that responsible for down-type quark and lepton masses.**

  $\Rightarrow$ not surprising that the non-SM-like Higgs bosons provide the most direct access to $\tan\beta$.

We demonstrate that various Higgs sector measurables do indeed provide an excellent determination of $\tan\beta$ so long as appropriate processes are kinematically accessible.

What Collider? What Process?

- Because of background and other issues, an e^+e^- collider with sufficient energy to pair produce non SM-like Higgs bosons, $e^+e^- \to H^0A^0$, is the ideal.
  - At low $\tan\beta$, look at $e^+e^- \to H^0A^0 \to b\bar{b}b\bar{b}$ rate.
    Rate varies as $b\bar{b}b\bar{b}$ branching ratio goes from modest level to being dominant.
    Presence of modest SUSY decays helps in that $b\bar{b}b\bar{b}$ rate varies significantly out to much larger $\tan\beta$ than if no SUSY decays are present.
  - At high $\tan\beta$, look at $\langle\Gamma_{\rm tot}^{H^0}, \Gamma_{\rm tot}^{A^0}\rangle$.
    Recall that the decay widths become dominated at high $\tan\beta$ by $b\bar{b}$ and $\tau^+\tau^-$, growing as $\tan^2\beta$.

- For heavier masses and/or only one light non-SM-like Higgs, $e^+e^- \to b\bar{b}H^0 \to b\bar{b}b\bar{b}$ and/or $e^+e^- \to b\bar{b}A^0 \to b\bar{b}b\bar{b}$ rate(s) do the job at high $\tan\beta$.

  Here, the rates are very sensitive to the $b\bar{b}A^0, b\bar{b}H^0$ Yukawa couplings that are proportional to $\tan\beta$.

Since, $B(H^0, A^0 \to b\bar{b}b\bar{b}) \to 0.9$ at high $\tan\beta$ this $\Rightarrow b\bar{b}b\bar{b}$ rates that increase as $\tan^2\beta$.

- Especially important: high luminosity, $\mathcal{L} = 300 - 500\mathrm{fb}^{-1}$ per year, yielding after a number of years $\mathcal{L} = 2000\mathrm{fb}^{-1}$.

- Our study focused on $\sqrt{s} = 500~\mathrm{GeV}$ and not very heavy Higgs bosons: $m_{A^0} = 100 - 200~\mathrm{GeV}$.

  Work presented in this talk will be in MSSM context with $m_{A^0} = 200~\mathrm{GeV}$.

  Recall that in the MSSM $m_{H^0} \sim m_{A^0}$ once $\tan\beta \gtrsim 5$, and that the $e^+e^- \to H^0A^0$ cross section will be very nearly maximal.

- Earlier work:

  – Gunion and Kelly: Phys. Rev. D 56, 1730 (1997) [arXiv:hep-ph/9610495] and arXiv:hep-ph/9610421.
  – Barger, Han and Jiang, hep-ph/0006223.
  – Feng and Moroi, LBNL-40535, Proc. SUSY97.

The $b\overline{b}H^0 \to b\overline{b}b\overline{b}$, $b\overline{b}A^0 \to b\overline{b}b\overline{b}$ rates

A detailed discussion will be given by Andre Sopczak.

The considerations are:

- Rates become very large for high enough $\tan\beta$.

- There may be some small background from $H^0A^0 \to b\overline{b}b\overline{b}$ production diagrams that interfere with the $b\overline{b}H^0 + b\overline{b}A^0 \to b\overline{b}b\overline{b}$ signal.

  This interference will have to be either understood by changing cuts to allow varying amounts of this background to enter or computed in a specific model context.

- Assuming this systematic source of error is under control and using cuts that make the H^0A^0 background much smaller than the $b\overline{b}H^0 + b\overline{b}A^0$ signal, excellent accuracy on $\tan\beta$ at high $\tan\beta$ is possible.

  See later graph.

The $A^0H^0 \to b\bar{b}b\bar{b}$ event rates

As stated earlier, the key is the variation of $B(H^0 \to b\bar{b})B(A^0 \to b\bar{b})$ with $\tan\beta$ at low to moderate $\tan\beta$.

- Two different MSSM scenarios are considered:

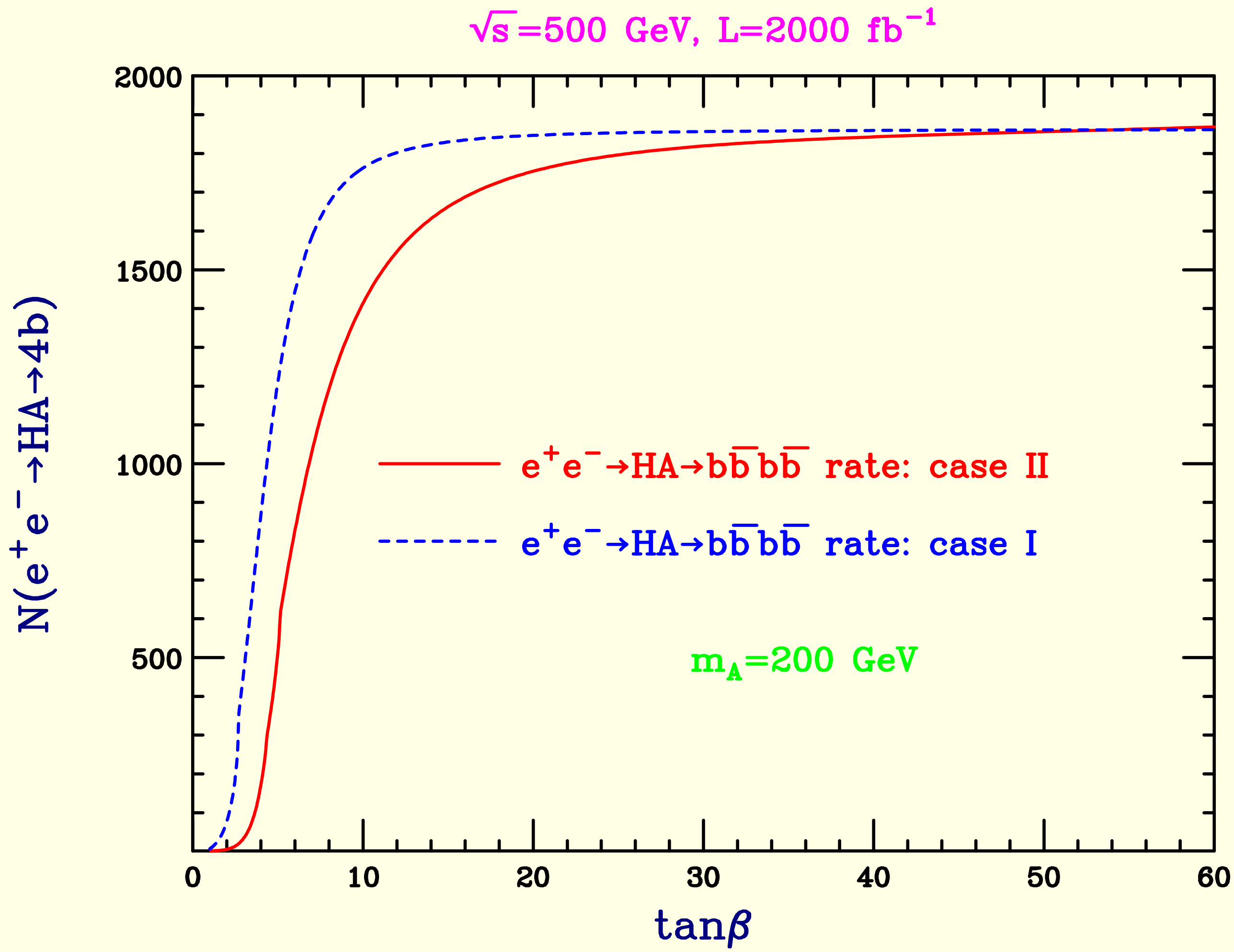
  (I) $m_{\tilde{g}} = 1$ TeV, $\mu = M_2 = 250$ GeV, $m_{\tilde{t}_L} = m_{\tilde{b}_L} = m_{\tilde{t}_R} = m_{\tilde{b}_R} \equiv m_{\tilde{t}} = 1$ TeV, $A_b = A_\tau = 0$, $A_t = \mu/\tan\beta + \sqrt{6} m_{\tilde{t}}$ (maximal mixing); SUSY decays of the H^0 and A^0 are kinematically forbidden.

  (II) $m_{\tilde{g}} = 350$ GeV, $\mu = 272$ GeV, $M_2 = 120$ GeV, $m_{\tilde{t}_L} = m_{\tilde{b}_L} = 356$ GeV, $m_{\tilde{t}_R} = 273$ GeV, $m_{\tilde{b}_R} = 400$ GeV, $A_\tau = 0$, $A_b = -672$ GeV, $A_t = -369$ GeV.
  SUSY decays of H^0, A^0 (mainly to $\widetilde{\chi}^0_1\widetilde{\chi}^0_1$) are allowed.

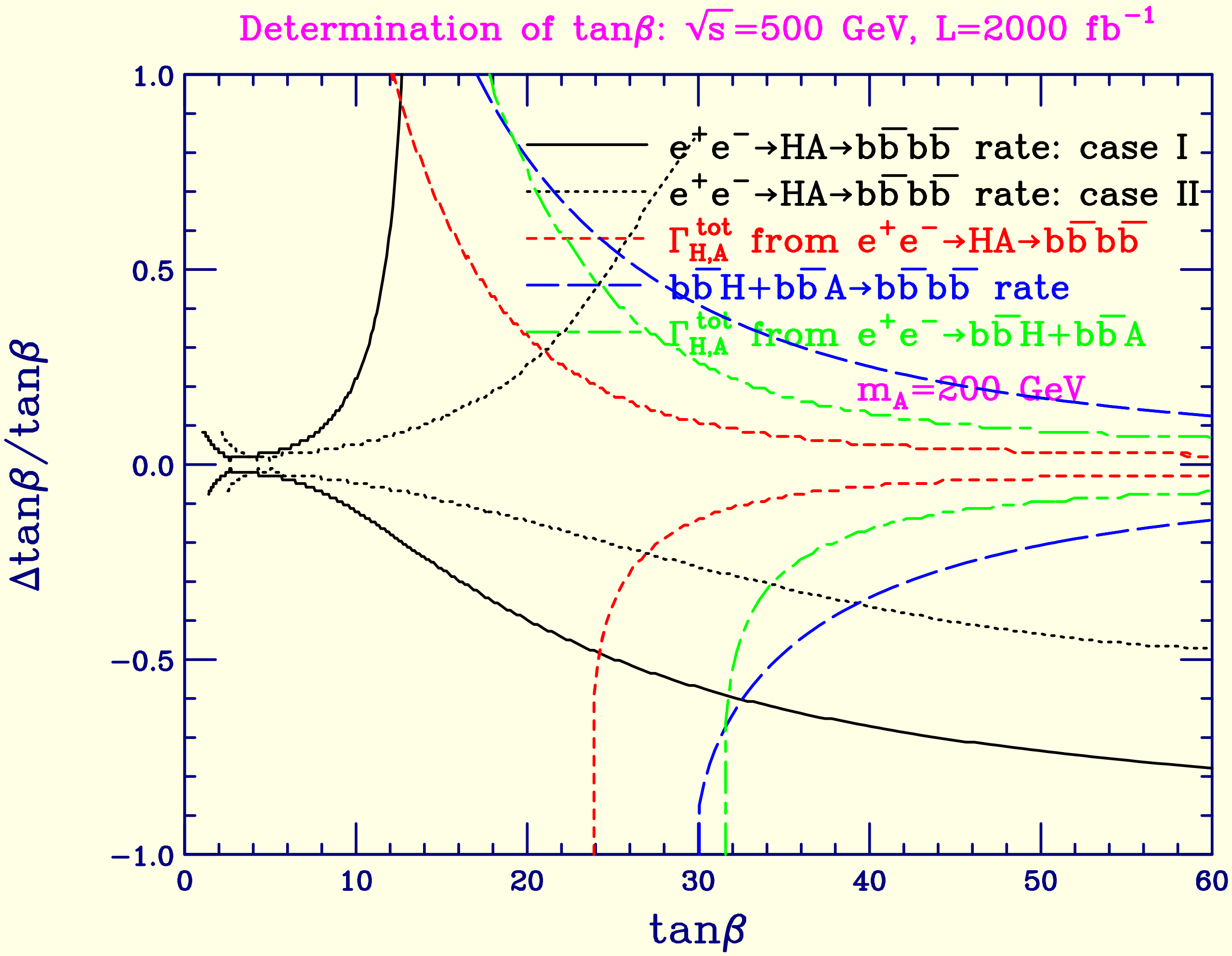
- In computing the statistical errors in $\tan\beta$, we assume that selection cuts having selection efficiency of 10% will achieve negligible background.

  Note: In the $b\bar{b}b\bar{b}$ final state, we can isolate 2 opposite b-tagged clusters of known (in the MSSM context, approximately equal) mass in order to eliminate backgrounds.

- To give an idea of the sensitivity of the $b\bar{b}b\bar{b}$ event rate to $\tan\beta$, we give a few numbers (assuming $\sqrt{s} = 500$ GeV and $\mathcal{L} = 2000\mathrm{fb}^{-1}$).
  - The $b\bar{b}b\bar{b}$ event rate, after 10% selection efficiency, is 1, 5, 34, 1415 1842 [8, 77, 464, 1762, 1859] at $\tan\beta = 1$, 2, 3, 10, 40, in scenarios (II) [(I)], respectively.
  - In case of scenario (I), $\Rightarrow$ good upside errors only for $\tan\beta < 10$, but downside errors remain acceptable to much higher $\tan\beta$ values.
  - In case of scenario (II), the substantial variation of $b\bar{b}b\bar{b}$ rate continues out to very large $\tan\beta$. $\Rightarrow$ some constraints at high $\tan\beta$ side.
    Don't really know if we should trust HDECAY out there; the H^0A^0 cross section keeps rising because $m_{H^0}(m_{A^0})$ starts to fall significantly for $\tan\beta > 40$ (e.g. as low as 194 GeV at $\tan\beta = 100$) so that $\sigma(e^+e^- \to H^0A^0)$ rises slowly from phase space increase. This does not happen in scenario (I).



The $e^+e^- \to H^0A^0 \to b\bar{b}b\bar{b}$ rates are plotted for scenarios (I) and (II). Note how the curve asymptotes at high $\tan\beta$ but varies rapidly at low $\tan\beta$ in case (I) but there is more variation in case (II) where SUSY decays are allowed ($\Rightarrow$ better high-$\tan\beta$ errors in case (II)).



We see significant sensitivity of the $\tan\beta$ errors from $H^0A^0 \to b\overline{b}b\overline{b}$ rates to the scenario choice, with the errors worse for scenario (I).

Errors for $\tan\beta$ from the $b\overline{b}H^0{+}b\overline{b}A^0 \to b\overline{b}b\overline{b}$ rate are essentially independent of the scenario choice. Running m_b has big impact on these errors.

All results employ couplings and widths ala HDECAY.

The average A^0, H^0 total width, $\langle\Gamma_{\rm tot}^{H^0}, \Gamma_{\rm tot}^{A^0}\rangle$

- Could be measured in either $b\overline{b}A^0 + b\overline{b}H^0$ or in H^0A^0 production.

  For the mass of $m_{A^0} = 200~{\rm GeV}$ being discussed and in the MSSM context, H^0A^0 production is best since it will be essentially background free after strong cluster cuts. (We again assume 10% signal efficiency to achieve this.)

  Still, a study of $b\overline{b}A^0 + b\overline{b}H^0$ is certainly warranted. So far, we have done only a somewhat naive estimate as described later.

The H^0A^0 analysis

- $\Gamma_{\rm tot}^{H^0}$ and $\Gamma_{\rm tot}^{A^0}$ start to become measurable despite the finite detector resolution (we adopt 5 GeV, see below) once $\tan\beta \gtrsim 10$.

- After 10% selection efficiency, there will be a large number of events (~ 1900) in the $H^0A^0 \to b\overline{b}b\overline{b}$ final state once $\tan\beta > 10$.

- **For experimental resolution, we look at previous studies (Drollinger, Sopczak) which found an LC detector resolution of $\Gamma_{\rm res} = 5~{\rm GeV}$ for the mass of interest.**

  Crucial: Small systematic error in knowledge of $\Gamma_{\rm res}$ will be crucial; we assume 10% systematic error, or $0.5~{\rm GeV}$.

  This systematic uncertainty considerably weakens our ability to determine $\tan\beta$ at the lower values of $\tan\beta$ for which $\Gamma_{\rm tot}^{H^0}$ and $\Gamma_{\rm tot}^{A^0}$ are smaller than $\Gamma_{\rm res}$.

- **Convolute detector resolution with a Breit-Wigner for the intrinsic width.**

- **An earlier study (Troncon etal) did this for H^0A^0 at $\tan\beta = 50$. They found:**
  - **An overall fit to the $b\overline{b}$ mass distribution gives a Higgs boson width about 2σ larger than expected from the convolution.**
  - **This is because the overall fit includes wings of the mass distribution that are present due to wrong pairings of the b-jets.**
  - **Restricting to the central peak retains 3/4 of the events and yields a width consistent with the convolution.**

$\Rightarrow$ **about 25% of the time wrong jet-pairings are made and contribute to the wings of the mass distribution.**

- **Therefore, our estimates of the error on the determination of the Higgs width will assume that only 3/4 of the events (*i.e.* those in the central peak) that are retained after our basic event selection cuts (with assumed selection efficiency of 10%) can be used in the statistics computation.**

- **The $m_{b\bar{b}}$ for each of the $b\bar{b}$ pairs identified with the H^0 or A^0 is binned in a single mass distribution.**

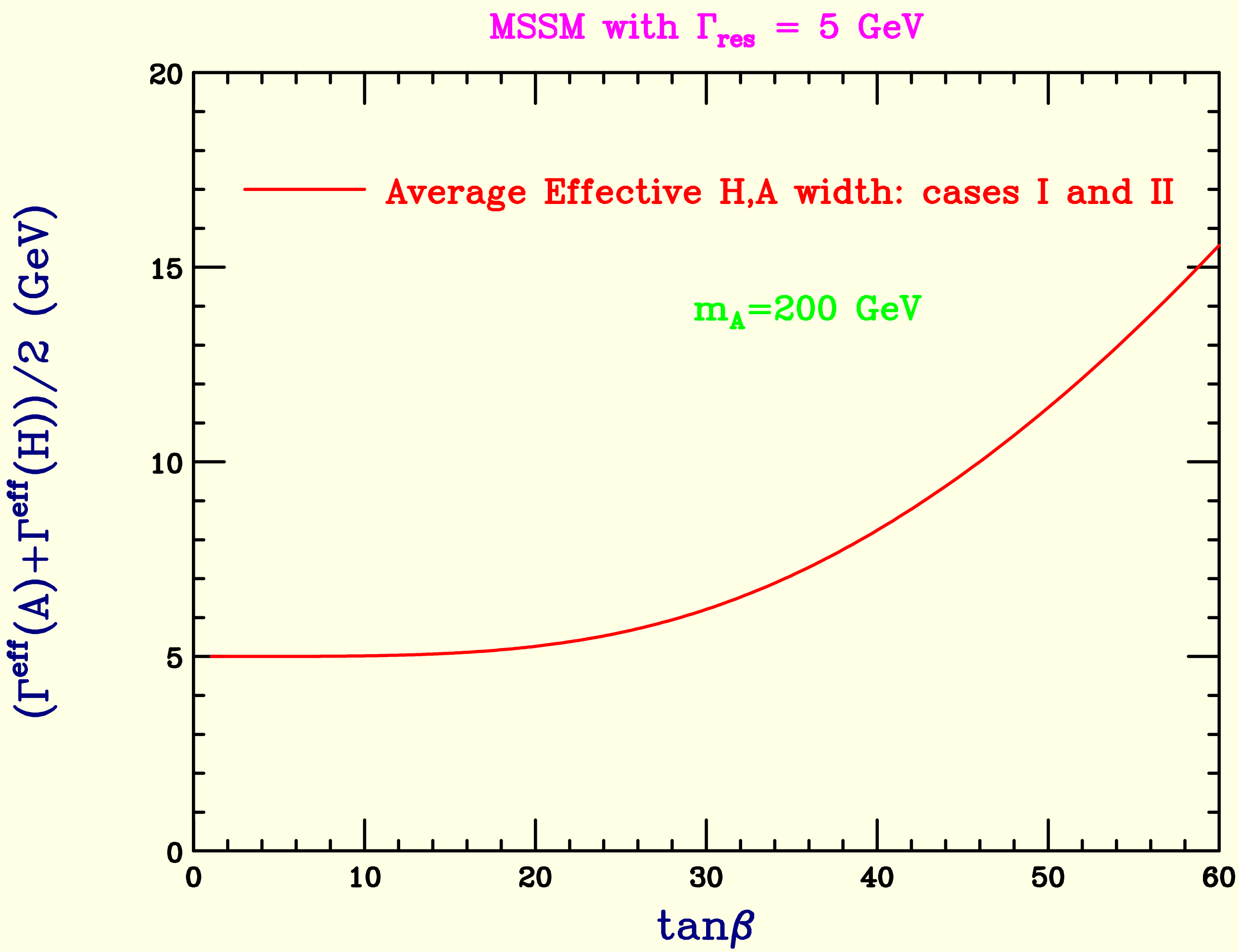
  This is appropriate since the H^0 and A^0 are highly degenerate for the large $\tan\beta$ values being considered.

- **Thus, our observable is the average of the widths $\Gamma_{\rm tot}^{H^0}$ and $\Gamma_{\rm tot}^{A^0}$.**

- **The resulting accuracy for $\tan\beta$ obtained from measuring the average H^0/A^0 width is shown in the earlier figure.**

  – **Good accuracy is already achieved for $\tan\beta$ as low as 20 with extraordinary accuracy predicted for large $\tan\beta$.**

- The sharp deterioration in the lower bound on $\tan\beta$ for $\tan\beta \lesssim 24$ occurs because the width falls below $\Gamma_{\rm res}$ as $\tan\beta$ is taken below the input value and sensitivity to $\tan\beta$ is lost.
  If there were no systematic error in $\Gamma_{\rm res}$, this sharp fall off would occur instead at $\tan\beta \lesssim 14$.
- We again give some numbers for scenario (II).
  At $\tan\beta = 55$ and 60, $\langle\Gamma^{H^0}_{\rm tot}, \Gamma^{A^0}_{\rm tot}\rangle \sim 12.5$ and 14.9 GeV, respectively.
  After including the detector resolution, the effective average widths become 13.4 and 15.7 GeV, respectively.
  In comparison, the total error in the measurement of the average width, including systematic error, is ~ 0.54 GeV.
  $\Rightarrow$ $\tan\beta$ can be determined to about ± 1, or to better than $\pm 2\%$.
- This high-$\tan\beta$ situation can be contrasted with $\tan\beta = 15$ and 20, for which $\langle\Gamma^{H^0}_{\rm tot}, \Gamma^{A^0}_{\rm tot}\rangle = 0.935$ and 1.64 GeV, respectively, which become 5.09 and 5.26 GeV after including detector resolution.
  Meanwhile, the total error, including the statistical error and the 0.5 GeV systematic uncertainty for $\Gamma_{\rm res}$, is about 0.54 GeV (i.e. systematics dominated).



The plot of $\langle \Gamma^{\rm eff}_{H^0}, \Gamma^{\rm eff}_{A^0} \rangle$ for scenarios (I) and (II) is shown (scenarios are indistinguishable).

Note how $\Gamma^{\rm res}$ dominates for lower $\tan\beta$, making $\tan\beta$ determination in the face of systematic $\Delta\Gamma^{\rm res} = 0.5$ GeV very difficult.

The $b\overline{b}H^0 + b\overline{b}A^0 \to b\overline{b}b\overline{b}$ width analysis

- Procedures are very similar to those for H^0A^0.
  - We assume same $\Gamma_{\rm res}$ and same systematic error.
    Whether this is reasonable in $b\overline{b}H^0, b\overline{b}A^0$ production, for which the A^0, H^0 have substantial boost in the e^+e^- c.o.m. frame, is not known. An experimental study is needed.
  - Statistics are based on the number of signal events retained after reducing backgrounds, as discussed in Andre's talk.
  - We assume that the signal peak in $m_{b\overline{b}}$ will stick up above a smooth background.
  - We have only one $b\overline{b}$ mass entry per event, since only the H^0 or the A^0 is present in a given event.
  - We reduce the number of events fitted in the peak by 50% to account for the possible presence of wings (this is what we have not studied in detail yet).

- Because of the weaker statistics, the plot shows that this determination takes higher $\tan\beta$ to be as effective as the $\tan\beta$ determinaton using widths as measured in H^0A^0 pair production.

- **However, if pair production is not kinematically allowed, it would be the only width-based technique!**

Further comments

- **The accuracies from the width measurement in H^0A^0 events are better than those achieved using the $b\overline{b}A^0 + b\overline{b}H^0 \to b\overline{b}b\overline{b}$ rate or width measurements.**

- **The width vs. rate high-$\tan\beta$ methods for determining $\tan\beta$ are beautifully complementary in that they rely on very different experimental observables.**

- **All 3 high-$\tan\beta$ determinations of $\tan\beta$ are nicely complementary in their $\tan\beta$ coverage to the $\tan\beta$ determination based on the $H^0A^0 \to b\overline{b}b\overline{b}$ rate, which comes in at lower $\tan\beta$.**

- **Still, there is a window $10 \lesssim \tan\beta \lesssim 20$ for which an accurate $\tan\beta$ determination using any of these techniques will not be possible.**
  - **This window expands rapidly as m_{A^0} increases (keeping $\sqrt{s}$ fixed).**
  - **Indeed, as m_{A^0} increases above $250~\mathrm{GeV}$, H^0A^0 pair production becomes kinematically forbidden at $\sqrt{s} = 500~\mathrm{GeV}$ and detection of**

the $b\overline{b}H^0 + b\overline{b}A^0$ processes at the LC (or the LHC) requires (Gunion, Kalinowski,Grzadkowski) increasingly large values of $\tan\beta$.

– This difficulty persists even at higher $\sqrt{s} \sim 1~\mathrm{TeV}$ and above; for $m_{A^0} > \sqrt{s}/2$, detection of the H^0 and A^0 must rely on $b\overline{b}H^0 + b\overline{b}A^0$ production, but the rates are too small for moderate $\tan\beta$ values.

- Our study is done in the context of the MSSM and assumes that the soft SUSY breaking parameters are known.

  Ambiguity can arise if the sign and magnitude of μ is not fixed since it significantly influences the one-loop corrections to the $b\overline{b}$ couplings of the H^0 and A^0 and the stop/sbottom mixing present in the one-loop corrections to the Higgs mass matrix.

  However, assuming that these parameters are known, the results for the error on $\tan\beta$ from the width measurement are quite insensitive to the precise scenario.

  Indeed, results for our two SUSY scenarios (I) and (II) are indistinguishable.

- In the above study, we have not made use of other decay channels of the H^0 and A^0, such as $H^0 \to WW, ZZ$, $H^0 \to h^0h^0$, $A^0 \to Zh^0$ and $H^0, A^0 \to$SUSY.

The Gunion+Kelly and Barger+Han+Jiang studies found that their inclusion will significantly aid in determining $\tan\beta$ at low to moderate $\tan\beta$ values.

- We have also not employed charged Higgs boson production processes.
  - In $e^+e^- \to H^+H^-$ production, the absolute event rates and ratios of branching ratios in various channels will increase the $\tan\beta$ accuracy at low $\tan\beta$ (Gunion+Kelly).
  - The total $H^\pm$ width measured in the tb decay channel will add further precision to the $\tan\beta$ measurement at high $\tan\beta$.
  - The rate for $e^+e^- \to t\bar{b}H^- + \bar{t}bH^+ \to t\bar{t}b\bar{b}$ is also very sensitive to $\tan\beta$ (Feng+Moroi).

    $\Rightarrow$ this channel yields errors for $\tan\beta$ that are only slightly larger than from the $b\bar{b}A^0 + b\bar{b}H^0 \to b\bar{b}b\bar{b}$ determination.

CONCLUSIONS

- A high-luminosity linear collider is unique in its ability to precisely measure the value of $\tan\beta$.

  This is because highly precise measurements of Higgs boson production processes will be essential and are only possible at the LC.

- In the context of the MSSM, we have shown that there are a variety of complementary methods that will allow an accurate determination of $\tan\beta$ over much of its allowed range, including, indeed especially for, large $\tan\beta$ values, provided appropriate processes are kinematically accessible.

  In particular, we have demonstrated the complementarity of employing: a) rates for $b\bar{b}A^0 + b\bar{b}H^0 \to b\bar{b}b\bar{b}$; b) measurements of the $H^0A^0 \to b\bar{b}b\bar{b}$ production rate; and c) measurement of the average H^0, A^0 total width in $H^0A^0 \to b\bar{b}b\bar{b}$ and/or $b\bar{b}H^0 + b\bar{b}A^0 \to b\bar{b}b\bar{b}$ production.

- The analogous charged Higgs observables — the $tbH^\pm \to tbtb$ rate, the $H^+H^- \to t\bar{b}\bar{t}b$ rate and the total $H^\pm$ width measured in H^+H^- and $tbH^\pm$ production — will further increase the sensitivity to $\tan\beta$.

- **In the general 2HDM, if there is only one Higgs boson with mass below $\sqrt{s}$ (say the A^0), the $b\bar{b}A^0 \to b\bar{b}b\bar{b}$ rate will be large and the production rate as well as the (independent) width measurement in this channel will (both) allow a good determination of $\tan\beta$ at high $\tan\beta$.**